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Chiral-symmetry breaking, the Dashen mass formula, and the decay $\eta \to 3\pi$

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The recently proposed extended partially conserved axial-vector current (EPCAC) hypothesis is used in order to obtain the chiral-SU(3) $\times$ SU(3)-symmetry-breaking corrections to (a) the Dashen mass formula, (b) the soft-meson theorem relevant to the $\eta \to 3\pi$ decay, and (c) the $\Delta I = 1$ baryon mass differences. It is found that the EPCAC hypothesis together with current algebra and an $\epsilon_3 u_3$ term in the chiral Hamiltonian provides a working explanation for the $\eta \to 3\pi$ puzzle in the framework of the $(3,3^*) + (3^*,3)$ model.

I. INTRODUCTION

Our present understanding of chiral-symmetry breaking[1] relies upon the model proposed by Gell-Mann, Oakes, and Renner (GMOR)[2] which may be summarized as follows: The strong-interaction Hamiltonian density H can be written as

$$H(x) = H_0(x) + H'(x) , \tag{1}$$

where H_0 is SU(3)$\times$SU(3) invariant, and the chiral-symmetry-breaking part, H', transforms according to the $(3,3^*)+(3^*,3)$ representation of SU(3) $\times$SU(3). On the other hand, vacuum symmetry is assumed to be spontaneously broken so that the symmetry of H_0 is realized in the Nambu-Goldstone manner. All this implies that SU(3) is broken at the same level as SU(3)$\times$ SU(3) but SU(2) $\times$SU(2) remains an exact symmetry in the limit $\mu_\pi{}^2 \to 0$[3]. These assumptions are well supported by present experimental evidence[1] and therefore they shall consititute the framework of our subsequent discussion.

In order to extract practical information from this set of assumptions one has to go further and propose specific techniques to calculate chiral-symmetry breaking. Among the different alternatives one has the current-algebra approach, pioneered by Fubini and his collaborators,[4] the Glashow-Weinberg method,[5] chiral perturbation theory, first suggested by Dashen[6] and subsequently developed by Pagels together with Li and Langacker,[1] and the quark-model approach.[7] The first two techniques rely upon the partially conserved axial-vector current (PCAC) hypothesis and current algebra. In this respect it is important to note that, as was stressed by Weinberg[8] and by Dashen and Weinstein,[6] the soft-meson theorems that follow from PCAC and current algebra are a consequence of symmetry so that there is a link between these results and the chiral-symmetry-limit theorems. The question is of course: How reliable is the PCAC hypothesis? It is often claimed that, at least at the pion level, PCAC and current-algebra predictions are in very good agreement with experiment. Although this agreement generally lies at the 10% level, a close look at the experimental errors shows that in many cases the predictions disagree with experiment by more than three standard deviations and in some instances by even more. On the other hand, kaon and η PCAC results are in more violent disagreement with experiment. The source of the problem could be traced back to the breakdown of the smoothness and/or saturation assumptions usually implied in this kind of approach.

It is interesting to observe that the numerical discrepancies just mentioned are approximately of the same size as the corrections to the Goldberger-Treiman relations (GTR)[9,10] in SU(2)$\times$SU(2) or SU(3)$\times$SU(3) depending upon whether one considers pion PCAC or kaon and η PCAC, respectively. It has been shown recently in a series of papers[11-14] that this apparent coincidence is not accidental but, rather, it is a natural consequence of an extended version of PCAC which incorporates heavy bosons in a model-independent fashion, i.e.,

$$\partial^\mu A_\mu = \sum_{n=0}^{N} \mu_{a_n}{}^2 f_{a_n} \phi_{a_n} \quad (a = \pi, K, \eta). \tag{2}$$

In order to derive practical consequences from Eq. (2) it is necessary to make an additional assumption about the behavior of the strong coupling constants of the heavy bosons to hadrons. Although in Ref. 11 it was assumed that this behavior was given by

$$g_{a_n AB} = \text{constant}, \tag{3}$$

where A and B denote hadrons, it is possible to replace Eq. (3) by the less restrictive hypothesis

$$g_{a_n AB}(q^2) = g_{a_0 AB}(\mu_{a_0}{}^2) h_n(q^2) , \tag{4}$$

where h_n is independent of A and B. This last assumption is supported e.g. in the context of the three-point-function Veneziano model[13] and it can be easily checked that all the results obtained in Refs. 11–14 remain unaltered if Eq. (4) is used instead of Eq. (3).

This extended PCAC (EPCAC) hypothesis has the virtue of unifying chiral-symmetry-breaking problems and of bringing soft-meson predictions into agreement with experiment at the 1-standard-deviation level. As it turns out, the corrections to the soft-meson theorems of PCAC and current algebra are given entirely in terms of the corrections to the GTR, Δ_π in SU(2)×SU(2) and Δ_K in SU(3)×SU(3).

In this paper we wish to discuss additional applications of EPCAC, namely, chiral-symmetry-breaking corrections to (a) the Dashen mass formula and Dashen's theorem,[6] (b) the soft-meson theorem relevant to the $\eta \to 3\pi$ decay, and (c) the $\Delta I = 1$ baryon mass differences. In the first case the EPCAC corrections cannot be evaluated in a model-independent fashion. This is due to the fact that the EPCAC corrections to a soft-meson limit for a two-point function are not directly related to Δ_π or Δ_K. Regarding $\eta \to 3\pi$ and the $\Delta I = 1$ baryon mass differences, we find that after including an $\epsilon_3 u_3$ term in the Hamiltonian and using the EPCAC version of the soft-meson theorem, the predicted mass differences are in close agreement with experiment.

II. DASHEN MASS FORMULA

As is well known, the Dashen mass formula for pseudoscalar mesons is obtained by means of a Ward identity and PCAC, and is valid to lowest order in SU(3)×SU(3) violations. In this section we shall incorporate into the Dashen mass formula the corrections due to SU(3)×SU(3) breaking which arise from the contribution of the heavy bosons.

To this end we start by considering the following Ward identity in the limit $q \to 0$:

$$q^\mu q^\nu \int d^4x\, e^{-iq\cdot x} \langle 0 | T(A_\mu^\alpha(x) A_\nu^\beta(0)) | 0 \rangle = \int d^4x\, e^{-iq\cdot x} \langle 0 | T(\partial^\mu A_\mu^\alpha(x) \partial^\nu A_\nu^\beta(0)) | 0 \rangle - i \langle 0 | [{}^5Q^\alpha, [{}^5Q^\beta, H(0)]] | 0 \rangle . \tag{5}$$

Using EPCAC, Eq. (2), the first term in the right-hand side of Eq. (5) becomes

$$\int d^4x\, e^{-iq\cdot x} \langle 0 | T(\partial^\mu A_\mu^\alpha(x) \partial^\nu A_\nu^\beta(0)) | 0 \rangle = \int d^4x\, e^{-iq\cdot x} \sum_{i=0}^{N} \sum_{j=0}^{N} (\mu_i^2 f_i)^{\alpha\lambda} \langle 0 | T(\phi_i^\lambda(x) \phi_j^\delta(0)) | 0 \rangle (\mu_j^2 f_j)^{\delta\beta} . \tag{6}$$

After diagonalizing Eq. (6) and extracting the meson poles, one has

$$\sum_{n=0}^{N} (\mu_n^2 f_n^2)^{\alpha\beta} = - \langle 0 | [{}^5Q^\alpha, [{}^5Q^\beta, H(0)]] | 0 \rangle . \tag{7}$$

Equation (7) is the EPCAC version of the Dashen mass formula where now the corrections due to SU(3)×SU(3) breaking are represented by the contribution of the heavy pseudoscalar mesons ($n \geq 1$).

The mass formula for the lowest-lying meson can be written as

$$(\mu_0^2 f_0^2)^{\alpha\beta} = - \langle 0 | [{}^5Q^\alpha, [{}^5Q^\beta, H(0)]] | 0 \rangle \frac{1}{1+\delta^{\alpha\beta}} , \tag{8}$$

where

$$\delta^{\alpha\beta} = \left(\frac{1}{\mu_0^2 f_0^2} \sum_{n=1}^{N} \mu_n^2 f_n^2 \right)^{\alpha\beta} . \tag{9}$$

Clearly the evaluation of the correction factor, $\delta^{\alpha\beta}$, is beyond the realm of EPCAC since a specific model is needed for the mass spectrum and the decay constants.

It is important to remark that in the chiral-symmetry limit $\delta^{\alpha\beta}$ vanishes because, by construction, the heavy pseudoscalar mesons do not become Goldstone bosons in that limit.[11]

In dispersion-theory language the contribution of the heavy bosons may be viewed as an approximation to the continuum integral in

$$(\mu_0^2 f_0^2)^{\alpha\beta} + \int_{m^2}^{\infty} \frac{ds}{s} \rho^{\alpha\beta}(s) = - \langle 0 | [{}^5Q^\alpha, [{}^5Q^\beta, H(0)]] | 0 \rangle , \tag{10}$$

where

$$\rho^{\alpha\beta}(q^2) = \sum_{n \neq \alpha} (2\pi)^3 \delta^4(p_n - q) \langle 0 | \partial^\mu A_\mu^\alpha | n \rangle \times \langle n | \partial^\nu A_\nu^\beta | 0 \rangle . \tag{11}$$

The EPCAC correction factor $\delta^{\alpha\beta}$, Eq. (9), might be infinite, corresponding to a divergent continuum integral in Eq. (10). If this were the case then a subtraction would be in order and hence the connection with chiral-symmetry would be lost.

It is important to note that the EPCAC corrections to this soft-meson theorem involving a two-point function are very different from those obtained previously for matrix elements between hadronic states.[11,12] In fact, in this last case the corrections due to heavy pseudoscalar mesons always sum up to give a factor of $(1-\Delta_\pi)$ or $(1-\Delta_K)$ for each meson that becomes soft. The appearance of the squares of masses and decay constants in Eq. (9) precludes this possibility since, we recall.

$$\Delta_a = -\frac{1}{f_{a_0}}\sum_{n=1}^{N} f_{a_n} \quad (a=\pi, K)\,. \tag{12}$$

We find it intriguing that in the case of two-point functions the chiral-symmetry-breaking corrections, as parametrized in terms of heavy pseudoscalar mesons, could easily become uncontrollable, especially since in all other applications they were shown to bring soft-meson predictions into excellent agreement with experiment.

A well known application of the Dashen mass formula is the relation between the electromagnetic mass differences of kaons and pions, i.e.,

$$(\mu_{K^+}{}^2 - \mu_{K^0}{}^2)_{\text{em}} = \frac{f\pi^2}{f_{K^2}}(\mu_{\pi^+}{}^2 - \mu_{\pi^0}{}^2)_{\text{em}}\,. \tag{13}$$

In view of the preceding discussion, Eq. (13) could be completely invalidated should the heavy bosons provide an infinite contribution to Eq. (9). However, Eq. (13) with $f_\pi = f_K$ can be easily obtained in the SU(3) limit without the need of current algebra.[6] In fact, writing

$$(m_{\pi^+}{}^2)_{\text{em}} = \langle \pi^+ | H_{\text{em}} | \pi^+ \rangle \tag{14}$$

and

$$(m_{K^+}{}^2)_{\text{em}} = \langle K^+ | H_{\text{em}} | K^+ \rangle\,, \tag{15}$$

and using the U-spin transformation $U|K^+\rangle = |\pi^+\rangle$, and $UH_{\text{em}}U^{-1} = H_{\text{em}}$, one finds

$$(\mu_{K^+}{}^2 - \mu_{K^0}{}^2)_{\text{em}} = (\mu_{\pi^+}{}^2 - \mu_{\pi^0}{}^2)_{\text{em}}\,. \tag{16}$$

Equation (16) is good up to SU(3)-breaking corrections.

III. THE DECAY $\eta \to 3\pi$ AND BARYON MASS DIFFERENCES

The decay $\eta \to 3\pi$ has been considered for a long time as a major theoretical puzzle since the Sutherland theorem[15] ensures that the electromagnetic amplitude vanishes in the SU(2)×SU(2)-chiral-symmetry limit, while the experimental decay widths are two orders of magnitude larger than what would be expscted from an electromagnetic effect. For this reason it has been suggested[16] that this process might be providing evidence for the presence of an $\epsilon_3 u_3$ term in the Hamiltonian, i.e., a nonelectromagnetic $\Delta I = 1$ interaction. However, the application of current algebra and PCAC (incorporating the $\epsilon_3 u_3$ term) provides a soft-pion prediction[17] which disagrees with experiment by a factor of 2–3. The same situation prevails in a chiral-perturbation-theory calculation of the decay widths[18] where extrapolation factors of the order of the η mass were ignored.

In this section we shall show that the EPCAC corrections to the standard result are enough to bring the predictions into agreement with experiment at the 1-to-2-standard-deviation level.

Before deriving the EPCAC soft-meson theorem let us introduce some definitions relevant to the $\eta \to 3\pi$ decay. The amplitude for this process may be written as

$$T_{ijk} = -\langle \pi_i \pi_j \pi_k | H | \eta \rangle\,, \tag{17}$$

where i, j, k, are isospin indices, and in particular

$$T_{000} = T_{+-0} + T_{-0+} + T_{0+-}\,. \tag{18}$$

According to present experimental evidence the T_{+-0} amplitude may be written as the linear expansion

$$T_{+-0} = A = BE_0\,, \tag{19}$$

where E_0 is the energy of the neutral pion and

$$B/A \approx -2.1/\mu_\eta\,. \tag{20}$$

Finally, the decay widths are given by

$$\Gamma_{000} = 827\,|3A + B\mu_\eta|^2 \text{ eV}\,, \tag{21}$$

$$\Gamma_{+-0} = 489\,|3A + B\mu_\eta|^2[1 + 0.02y(1+y)] \text{ eV}\,, \tag{22}$$

where

$$y = (\mu_\eta - 3\mu_\pi)/(\mu_\eta + 3A/B)\,. \tag{23}$$

The amplitude A can be written as

$$A = -i\,\frac{\epsilon_3}{2f_\pi^3}\,\frac{(q_\eta^2 - m_\eta^2)}{(1-\Delta_\pi)^3}\int d^4x\, e^{-iq_\eta x}\langle 0 | [{}^5Q^-, [{}^5Q^+, [{}^5Q^3, T(\phi_\eta(x) u_3(0))]]] | 0 \rangle\,, \tag{24}$$

where the factor $(1-\Delta_\pi)^3$ comes from the use of EPCAC, Eq. (2) and three pions have been taken soft. In this limit the η also becomes soft, but in order to avoid ambiguities ϕ_η should not be replaced by its corresponding axial divergence.[18]

Allowing for η-η' mixing and computing the commutators, Eq. (24) becomes

$$A = -\frac{4}{3\sqrt{3}}\,\frac{\tilde{\epsilon}_3}{f_\pi^3(1-\Delta_\pi)^3}\,, \tag{25}$$

where $\bar{\epsilon}_3 = \epsilon_3 Z_{8\eta}{}^{1/2}$ is the renormalized parameter in the chiral-symmetry-breaking Hamiltonian

$$H' = \epsilon_0 u_0 + \epsilon_3 u_3 + \epsilon_8 u_8 , \qquad (26)$$

and $Z_{8\eta}{}^{1/2}$ is the wave-function renormalization factor defined by

$$\langle 0 | v_8 | \eta \rangle = Z_{8\eta}{}^{1/2} . \qquad (27)$$

In the chiral-symmetry limit Δ_π vanishes and Eq. (25) reduces to the standard result.[19] It is interesting to observe that the correction term due to the heavy-boson contributions points in the opposite direction to the chiral-perturbation-theory result.[18] In this last case Langacker and Pagels[18] found a −34% correction to the symmetry limit while from Eq. (25) one reads a +20% correction if $\Delta_\pi = 0.06 \pm 0.02$ is used.[20] This +20% correction is clearly welcome since as we shall see in the following it will provide better results for the $\Delta I = 1$ baryon mass differences. Moreover, as has been mentioned before, the EPCAC corrections to the soft-meson theorems bring predictions into agreement with experiment at the 1-standard-deviation level both in SU(2)×SU(2) and in SU(3)×SU(3) problems; the $\eta \to 3\pi$ decay is just another successful application of EPCAC. In fact, from the experimental rates

$$\Gamma_{000} = 254 \pm 37 \text{ eV}, \qquad (28)$$

$$\Gamma_{+-0} = 200 \pm 29 \text{ eV}, \qquad (29)$$

and Eq. (25) one obtains the following values for $\bar{\epsilon}$:

$$\frac{\bar{\epsilon}_3}{f_\pi} = \begin{cases} -0.0056 \pm 0.0005 \text{ GeV}^2 \text{ (from } \Gamma_{000}), & \text{(30a)} \\ -0.0065 \pm 0.0006 \text{ GeV}^2 \text{ (from } \Gamma_{+-0}). & \text{(30b)} \end{cases}$$

Now these values may be used in the calculation of the baryon mass differences induced by the u_3 term, i.e.,

$$\begin{aligned} M_p - M_n &= E(-f/d + 1) , \\ M_{\Sigma^+} - M_{\Sigma^-} &= -2E(f/d) , \\ M_{\Xi^-} - M_{\Xi^0} &= E(f/d + 1) , \end{aligned} \qquad (31)$$

where

$$E = \frac{\sqrt{3}}{2} \frac{\epsilon_3}{\epsilon_8} (M_\Sigma - M_\Lambda) , \qquad (32)$$

and $f/d = 3.26$ from the medium-strong differences. The results are collected in Table I which shows that our predictions are in much better agreement with experiment than those obtained from chiral-perturbation theory. The value used for ϵ_8 in Eq. (32) is the standard one which follows from the GMOR model, i.e.,

$$\mu_\pi{}^2 f_\pi = Z_\pi{}^{1/2}[(\tfrac{2}{3})^{1/2}\epsilon_0 + (\tfrac{1}{3})^{1/2}\epsilon_8] , \qquad (33)$$

$$\mu_K{}^2 f_K = Z_K{}^{1/2}[(\tfrac{2}{3})^{1/2}\epsilon_0 - \tfrac{1}{2}(\tfrac{1}{3})^{1/2}\epsilon_8 + \tfrac{1}{2}\epsilon_3] , \qquad (34)$$

and then

$$\frac{\bar{\epsilon}_8}{f_\pi} = -0.324 \pm 0.006 \text{ GeV}^2 , \qquad (35)$$

where $Z_\pi{}^{1/2} = Z_K{}^{1/2} = Z_{8\eta}{}^{1/2}$ has been assumed. The kaon mass difference induced by the u_3 term in the Hamiltonian has been obtained from

$$(\mu_{K^+}{}^2 - \mu_{K^0}{}^2)_{u_3} = \frac{\bar{\epsilon}_3}{f_K} , \qquad (36)$$

which is independent of the Dashen sum rule. We have also shown in Table I the combination $M_p - M_n + M_{\Xi^0} - M_{\Xi^-}$ which appears in the Coleman-Glashow formula.[21]

As a final point we note that by means of the Dashen mass formula, Eq. (16), and experimental data it follows that

$$\begin{aligned} (\mu_{K^+}{}^2 - \mu_{K^0}{}^2)_{u_3} &= (\mu_{K^+}{}^2 - \mu_{K^0}{}^2)_{\text{exp}} - (\mu_{\pi^+}{}^2 - \mu_{\pi^0}{}^2)_{\text{exp}} \\ &= -0.0053 \text{ GeV}^2 , \end{aligned} \qquad (37)$$

in good agreement with the result of Eq. (36) (see Table I). We recall that Eq. (16) is good only up to SU(3)-breaking corrections. However, even if these corrections were large, the result of Eq. (37) would remain basically the same due to the smallness of the pion mass difference as compared to the kaon mass difference. Substituting

TABLE I. Predictions for the $\Delta I = 1$ mass differences induced by an $\epsilon_3 u_3$ term using (a) Eq. (30a), (b) Eq. (30b), and (c) experiment.

	(a)	(b)	(c)
$\mu_{K^+}{}^2 - \mu_{K^0}{}^2$ (GeV^2)	-0.0046 ± 0.0004	-0.0053 ± 0.0005	-0.004
$M_p - M_n$ (MeV)	-2.5 ± 0.2	-2.9 ± 0.2	-1.3
$M_{\Sigma^+} - M_{\Sigma^-}$ (MeV)	-7.2 ± 0.6	-8.5 ± 0.6	-7.98 ± 0.08
$M_{\Xi^-} - M_{\Xi^0}$ (MeV)	$+4.7 \pm 0.4$	5.5 ± 0.4	$+6.4 \pm 0.6$
$M_p - M_n + M_{\Xi^0} - M_{\Xi^-}$ (MeV)	-7.2 ± 0.4	-8.4 ± 0.4	-7.7 ± 0.6

Eq. (37) into Eq. (36) it follows that

$$\frac{\tilde{\epsilon}_3}{f_\pi} = -0.006\,5 \pm 0.000\,1\ \mathrm{GeV}^2\,, \qquad (38)$$

and using this value in Eq. (25) we find, with the aid of Eqs. (21) and (22),

$$\Gamma_{000} = 338 \pm 48\ \mathrm{eV}\,,$$
$$\Gamma_{+-0} = 200 \pm 28\ \mathrm{eV}\,,$$

in good agreement with experiment, Eqs. (28) and (29). Although these predictions for the rates look fine, they should be handled with some caution because they rely on the validity of the SU(3) limit of the Dashen mass formula. Even though Eq. (37) is not expected to change much in the presence of SU(3) breaking, a slight variation has a sizable effect in the rates. For instance, if we include vacuum breaking in Eq. (37) ($f_K \neq f_\pi$) then the rates become

$$\Gamma_{000} = 283 \pm 35\ \mathrm{eV}\,,$$
$$\Gamma_{+-0} = 166 \pm 20\ \mathrm{eV}\,.$$

IV. SUMMARY

In Sec. II we have derived the EPCAC version of the Dashen sum rule. As pointed out, the corrections due to the heavy-boson contributions cannot be evaluated in a model-independent fashion. This circumstance singles out soft-meson theorems for two-point functions as peculiar in the sense that the corrections are not determined entirely by Δ_π or Δ_K as is normally the case. The main issue of this paper, though, has been to show that if the EPCAC hypothesis is used together with current algebra and an $\epsilon_3 u_3$ term in the Hamiltonian, then there is no disagreement between the soft-meson prediction and experiment. Finally, the predicted $\Delta I = 1$ baryon mass differences as well as the kaon mass differences are reasonably close to their corresponding experimental values. All these conclusions are independent of the Dashen sum rule.

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