PHYSICAL REVIEW D VOLUME 18, NUMBER 3 1 AUGUST 1978

Problem of the η-meson mass

Richard H. Capps
Physics Department, Purdue University, West Lafayette, Indiana 47907
(Received 8 August 1977; revised manuscript received 11 May 1978)

In quark-gluon theory a conventional assumption is that the surprisingly large mass of the η meson is caused by virtual annihilation and re-creation of a $q\bar{q}$ pair, the intermediate states being gluon states. The simplest component of the gluon states consists of two gluons. The virtual-annihilation mechanism is usally defined in terms of transitions through states containing gluons but no quarks. It is shown that if this definition is accepted, the virtual-annihilation mechanism is of the wrong sign to explain the large η mass, unless the theory is more pathological than is commonly supposed. An alternate, diagrammatic definition of virtual annihilation is discussed briefly.

I. THE HAMILTONIAN PICTURE OF VIRTUAL ANNIHILATION

A. The sign argument

In the quark-gluon model the basic structure of the lightest P and V (pseudoscalar and vector) mesons is $q\bar{q}$. It has been shown that a simple quark-model Hamiltonian predicts fairly accurately the masses of the light V-meson SU(3) nonet and all members of the P nonet except the η(549 MeV) and the η'(958).[1] The predictions of these two masses are too small. Consequently, a convenient representation of the Hamiltonian is

$$H = H_q + H' , \qquad (1)$$

where H_q can be approximated in a simple quark model and H' is small or zero for all the P- and V-nonet particles except the η and η'.

It is assumed conventionally that H' is the result of virtual annihilation and re-creation of the $q\bar{q}$ pair, as shown in Fig. 1(a).[1-6] The intermediate states are "gluon states". These may be complicated structures; the simplest components of the gluon states in question are GG states, where the gluon G transforms as an octet under color SU(3). The lowest-order contribution is shown in Fig. 1(b). This idea is attractive because the mechanism can contribute only to self-conjugate meson states, since a gluon state has no additive, flavor quantum numbers. Furthermore, of the mesons considered, only the two-dimensional subspace of the η and η' can have a contribution from GG states, because of charge-conjugation and isotopic-spin invariance.

The purpose of this note is to point out that if the annihilation Hamiltonian is defined in the usual way, the contribution of this mechanism to the η mass is of the wrong sign. The argument is simple; the theorem involved is well known. However, I have found that the implication of the argument for the annihilation contribution to the η mass is not generally known. An alternative, diagrammatic definition of virtual annihilation is discussed in Sec. II.

It is necessary to define the quantities in Eq. (1) more precisely, in a way consistent with the general procedure of Refs. 1–6. The symbol H denotes the actual, correct Hamiltonian. I take H_q to be equal to H for all non-self-conjugate meson states, so that H' is zero for these states. We next consider the self-conjugate states. The problem arises because the measured masses of the V mesons and non-self-conjugate P mesons satisfy regularities that suggest masses and wave functions for the η and η'. The suggested masses differ greatly from the experimental masses. I define H_q so that these suggested masses and wave functions are the relevant eigenvalues and eigenstates of H_q. The smallest eigenvalue of H_q for isoscalar P states is denoted by $E_q^{\min}$; it is shown in Refs. 1–6 that

$$E_q^{\min} = m_\pi . \qquad (2)$$

It is not necessary to define H_q in more detail; only the smallest eigenvalues are relevant here.

The operator H' is then defined by Eq. (1), so the anomalous η and η' masses are associated with H' by definition. One may use experimental numbers to estimate the matrix elements of H' in the η-η' subspace. I will make the usual assumption that H' corresponds to virtual annihilation into gluon states, and show that this leads to a contradiction for the η mass. It is assumed that electromagnetic interactions may be neglected, and that

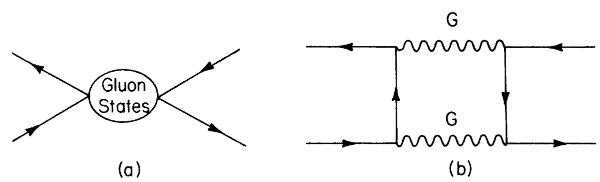

FIG. 1. Virtual annihilation of a $q\bar{q}$ pair.

the η is the lightest, color-singlet, isoscalar, P state composed of one of more hadrons. Physical states composed of two or more gluons may exist, but presumably, they are heavier than the η.[7]

We consider a complete set of states that is the direct sum of states of two classes, "gluon states" that contain no $q\bar{q}$ pairs, and "quark states" that contain one or more $q\bar{q}$ pairs. The number of gluons in any of these states need not be a good quantum number. One of these quark states, denoted by χ, is taken to be the eigenstate of H_q with the eigenvalue m_π of Eq. (2). In the simple models this state is obtained by changing the relative sign of the $u\bar{u}$ and $d\bar{d}$ terms of the pion wave function, where u and d are the up and down quarks.

The usual definition of the annihilation Hamiltonian H' in this basis is the term in H that connects quark states to gluon states. This term has no elements that do not involve gluon states. I consider the effect of H' on the η mass. The contradiction follows immediately. As discussed above,

$$\langle \chi | H_q | \chi \rangle = m_\pi .$$

However, since $\langle \chi | H' | \chi \rangle = 0$, this implies

$$\langle \chi | H | \chi \rangle = m_\pi . \tag{3}$$

It is assumed that the eigenfunctions of H are a complete set, so that χ may be expanded in these eigenfunctions. The result of Eq. (3) contradicts the assertion that the η is the lightest isoscalar P state, thus completing the argument. The crucial assumption that the eigenfunctions of H are a complete set is discussed in Sec. I B.

In order to make the point clear I will relate the argument to the concepts of Refs. 1–6. The mass matrix is the Hamiltonian in the frame of zero three-momentum. All the standard references consider mass matrices or mass-squared matrices for the η mesons, in three or four dimensions.[1-6] I will consider the mass matrix, and will refer to the paper of Fuchs[4]; the argument would be the same if one of the other of Refs. 1–6 were taken. Fuchs considers a four-by-four mass-squared matrix referring to three $q\bar{q}$ states and one gluon state. However, the basic interaction Hamiltonian connects these states to other states including other gluon states. These interactions affect the masses of the η mesons. One way to account for some of these effects is through perturbation theory; one partially diagonalizes the Hamiltonian so the basic four states pick up pieces of higher states, and the interactions under consideration no longer connect these four states to other states. The modified four-by-four Hamiltonian then contains terms that are actually of higher order in the basic interaction Hamiltonian. Some of these extra terms are diagonal in the modified four-by-four Hamiltonian.

In Ref. 4 the contribution of the virtual-annihilation mechanism is not calculated, but is treated phenomologically. The parameter λ is used to re-represent the effect in the four-by-four matrix; the λ term is included both in diagonal and nondiagonal terms. I have no criticism of this general procedure; it is quite reasonable. In Ref. 4 no theoretical argument is given concerning the sign of λ; fitting to experimental data requires that λ is positive. My point is that if the Hamiltonian is written as an infinite matrix in the complete gluon-state–quark-state basis, the most natural definition of the annihilation Hamiltonian has no diagonal elements. This would require that the λ of Ref. 4 be negative.

B. The completeness assumption

I next discuss the possibility that the assumption that the eigenfunctions of H are a complete set is violated. This is conceivable, because a quark-gluon theory must contain some sort of confinement mechanism. However, it is usually assumed that confinement is a property only of non-color-singlet states, while the states involved in the completeness assumption are color singlets. If the confinement is absolute, all $q\bar{q}$ and GG states are bound. However, the simplest picture we can make of absolute confinement involves using an unbounded potential for which the bound states are a complete set.

On the other hand, the completeness assumption is not valid if physical color-singlet states do not exist. In this case the confinement mechanism is more extreme than is commonly supposed.

II. THE DIAGRAM PICTURE

A different definition of the virtual-annihilation process can be made in terms of Feynman diagrams. It is assumed that the physical quantities may be expressed as sums of contributions of sets of diagrams. Virtual annihilation is defined as the sum of diagrams for which the initial quark line becomes the initial antiquark line, rather than the final quark line.

If this definition is made, the sign of the effect cannot be determined by a general argument such as that of Sec. I. In order to understand the difference between the two definitions, we can imagine that the vertices of a diagram are written in terms of gluon and quark annihilation and creation operators. A diagram includes different time orderings of the vertices, and so includes other terms besides those involving intermediate gluon states. For example, the final $q\bar{q}$ pair may be created before the initial pair is annihilated. An example of such a term is obtained by regarding Fig. 1(b) as a $q\bar{q}$ exchange rather than as a virtual annihilation into a GG state.

Consequently, it is possible theoretically that if the diagram definition of virtual annihilation is made, the process might be responsible for the anomalous η mass. However, if this should turn out to be the case, it would be quite interesting that the numerical consequences of the two definitions are so different. Furthermore, it would be misleading to describe the process as annihilation into intermediate gluon states.

ACKNOWLEDGMENTS

I would like to thank T. K. Kuo, L. A. P. Balazs, and B. F. L. Ward for helpful conversations. This work was supported in part by the U.S. Energy Research and Development Administration.

[1]A. De Rújula, H. Georgi, and S. L. Glashow, Phys. Rev. D 12, 147 (1975).
[2]H. Fritzsch and P. Minkowski, Nuovo Cimento 30A, 393 (1975).
[3]N. Isgur, Phys. Rev. D 13, 122 (1976).
[4]N. Fuchs, Phys. Rev. D 14, 1912 (1976).
[5]H. Fritzsch and J. D. Jackson, Phys. Lett. 66B, 365 (1977).
[6]Ch. de la Vaissiere, CERN Report No. CERN/EP/PHYS 77-7 (unpublished).
[7]A discussion of the possible spectrum of gluon states is given in Ref. 2.

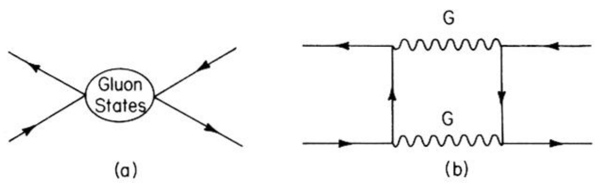

FIG. 1. Virtual annihilation of a $q\bar{q}$ pair.