Higher-order mass formulas in SU(4)

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We derive mass formulas among hadrons ($\frac{1}{2}^+$, $\frac{3}{2}^+$, 0^-, 1^-) in SU(4) framework including second-order mass contributions from $\underline{84}$ and $\underline{20}''$ representations. Some hybrid mass relations are obtained by relating second-order parameters. Masses of some hadrons are estimated.

I. INTRODUCTION

Several mass sum rules relating the mass splittings of hadrons have been obtained in charm-quark models[1] and in SU(4) symmetry considerations.[2] In SU(4) symmetry, the strong mass-breaking operator has been assumed[3] to transform like the $T_3^3 + yT_4^4$ component of the adjoint representation $\underline{15}$. A large mass difference between ψ and $(\rho, \omega, \phi, K^*)$ indicates, however, that SU(4) is so badly broken that the mass sum rules derived in first-order breaking are likely to be subjected to large corrections due to higher-order SU(4) breaking.[3]

In the present paper we examine the second-order SU(4)-breaking effects on the masses of hadrons ($\frac{1}{2}^+$, $\frac{3}{2}^+$, 0^-, and 1^-). Higher-order effects have been considered earlier in the SU(3) framework.[4] Some hybrid mass formulas have also been obtained by relating higher-order parameters.[4] In our considerations electromagnetic (em) mass breaking is also included. The first-order mass-breaking operator is taken to transform like the

$$aT_1^1 + bT_3^3 + cT_4^4 \tag{1.1}$$

component of $\underline{15}$. The second-order mass-breaking Hamiltonian then would have an SU(4) transformation property dictated by the direct product:

$$\underline{15} \otimes \underline{15} = \underline{1} \oplus 2(\underline{15}) \oplus \underline{20}'' \oplus \underline{45} \oplus \underline{45}^* \oplus \underline{84}\,. \tag{1.2}$$

The representation $\underline{15}$ has already been taken in the first-order breaking (1.1). We wish to consider the effects of $\underline{20}''$ and $\underline{84}$ as second-order mass contributions. Thus the second-order mass-breaking operator transforms as

$$T_{11}^{11}\,, T_{33}^{33}\,, T_{44}^{44}\,, T_{13}^{13}\,, T_{14}^{14}\,, T_{34}^{34} \tag{1.3}$$

components of $\underline{20}''$ and $\underline{84}$. The general mass operator in SU(4) then can be written as

$$\begin{aligned} m_i &+ a_i^{D/F}T_1^1 + b_i^{D/F}T_3^3 + c_i^{D/F}T_4^4 + d_i\,T_{(1,1)}^{(1,1)} \\ &+ e_i T_{(3,3)}^{(3,3)} + f_i\,T_{(4,4)}^{(4,4)} + g_i\,T_{(1,3)}^{(1,3)} + h_i\,T_{(1,4)}^{(1,4)} \\ &+ k_i\,T_{(3,4)}^{(3,4)} + g_i'\,T_{[1,3]}^{[1,3]} + h_i'\,T_{[1,4]}^{[1,4]} + k_i'\,T_{[3,4]}^{[3,4]}\,, \end{aligned} \tag{1.4}$$

where i stands for the spin of the multiplet.

In Sec. II various mass relations are obtained assuming $\underline{84}$ dominance [as an extension of $\underline{27}$ in SU(3)][4] in second-order effects. $\underline{20}''$ is also included later in order to have more general mass breaking. We express the higher-order parameters in terms of masses thereby relating the discrepancies present in the first-order mass formulas. We observe that the more general mass operator (1.4) gives the Coleman-Glashow sum rule and its charmed analog (2.2b). It also gives many relations which have been obtained in quark models.[1] Mixing masses are also derived in Sec. II.

In Sec. III, we give some hybrid mass relations assuming universality of certain ratios of the higher-order parameters. The hybrid mass formulas (3.1) among the uncharmed hadrons have been obtained earlier in SU(3) considerations.[4] Because general mass-breaking formulas have a large number of parameters, we are unable to predict mass values of hadrons as such. In order to have some idea about the values of masses we assume that the terms like $T_{13}^{13}, T_{14}^{14}, T_{34}^{34}$ do not contribute appreciably to the mass breaking. This may be understandable if symmetry breaking is considered in a tadpole scheme.[5]

II. SECOND-ORDER EFFECTS

In this section we describe various mass relations among baryons and mesons assuming $\underline{84}$ dominance of second-order mass breaking. The $\underline{20}''$ representation is included later. In writing mass relations we have used the particle symbol to denote its mass.

A. $\frac{1}{2}^+$ baryons

Second-order mass contributions are obtained from the contraction:

$$\bar{B}^i_{[k,m]} B_l^{[j,m]} H_{ij}^{kl}\,, \tag{2.1}$$

where H_{ij}^{kl} represents a second-order mass-breaking spurion. Both upper and lower indices are symmetric and antisymmetric for $\underline{84}$ and $\underline{20}''$ representations, respectively.

(a) 84 dominance. This gives the following:

$$\Xi^0 - \Xi^- - \Sigma^+ + \Sigma^- = n - p$$

$$(1.6 \pm 0.7 \text{ MeV} = 1.29 \pm 0.01 \text{ MeV}), \quad (2.2a)$$

$$\Xi_2^{++} - \Xi_2^+ - \Sigma_1^{++} + \Sigma_1^0 = n - p, \quad (2.2b)$$

$$3(\Xi_1'^+ - \Xi_1'^0) - 4(\Sigma_1^{++} - \Sigma_1^0) - (\Xi_1^+ - \Xi_1^0) = 2(\Xi^0 - \Xi^-) - 4(\Sigma^+ - \Sigma^-) \sim 20 \text{ MeV}, \quad (2.2c)$$

$$-d_{1/2} = \Sigma_1^{++} + \Sigma_1^0 - 2\Sigma_1^+ = \Sigma^+ + \Sigma^- - 2\Sigma^0 \quad (1.8 \pm 0.2 \text{ MeV}), \quad (2.2d)$$

$$2g_{1/2} = (\Sigma_1^+ - \Sigma_1^0) - (\Xi_1^+ - \Xi_1^0), \quad (2.2e)$$

$$2h_{1/2} = 4(\Sigma^0 - \Sigma^-) - 3(\Sigma_1^+ - \Sigma_1^0) - (\Xi_1^+ - \Xi_1^0) \quad (2.2f)$$

In the absence of em breaking we get

$$(\Omega_2 - \Xi_2) - (\Omega_1 - \Sigma_1) = (\Sigma - \Xi) \quad (-125 \text{ MeV}), \quad (2.3a)$$

$$3(\Xi_1 - \Sigma_1) - (\Xi_1' - \Lambda_1') = 2(\Lambda - N) \quad (353 \text{ MeV}), \quad (2.3b)$$

$$-2e_{1/2} = 2N + 2\Xi - 3\Lambda - \Sigma = 2(\Omega_1 + \Sigma_1 - 2\Xi_1) \quad (-26.83 \text{ MeV}), \quad (2.3c)$$

$$-2f_{1/2} = 2N + 2\Xi_2 - 3\Lambda_1' - \Sigma_1, \quad (2.3d)$$

$$2k_{1/2} = (3\Lambda + \Sigma - 2\Xi - 2N) - 2[2(\Xi_1 - \Sigma_1) - (\Xi - N)] \quad \text{(see Ref. 6)} \quad (2.3e)$$

In this multiplet some of the states are mixed because of SU(2) and SU(3) breakings. We calculate the mixing masses to be

$$\sqrt{3}\, m_{\Lambda\Sigma^0} = (\Sigma^0 - \Sigma^- - \Xi^0 + \Xi^-) + (\Xi_1^+ - \Xi_1^0) - (\Sigma_1^+ - \Sigma_1^0), \quad (2.4a)$$

$$\sqrt{3}\, m_{\Lambda_1^+\Sigma_1^+} = (\Sigma^0 - \Sigma^- - \Xi^0 + \Xi^-), + (\Xi_1^+ - \Xi_1^0) - (\Sigma_1^+ - \Sigma_1^0) + 3[(\Sigma_1^+ - \Sigma_1^0) - (\Sigma^0 - \Sigma^-)], \quad (2.4b)$$

$$2/\sqrt{3}\, m_{\Xi_1'\Xi_1} = 2(\Xi_1 - \Sigma_1) - (\Lambda - N) - (\Sigma - N). \quad (2.4c)$$

(b) 20″ effects. If we include the 20″ representation, relations (2.2a), (2.2b); (2.2d), (2.2e), (2.3a); (2.3c), (2.3d); and (2.4a) remain unaffected. In addition, the following relations are obtained:

$$4h_{1/2} = 2(\Xi^0 - \Xi^-) - 3(\Xi_1'^+ - \Xi_1'^0) - (\Xi_1^+ - \Xi_1^0) + 2(\Sigma_1^+ - \Sigma_1^0), \quad (2.5a)$$

$$4(g_{1/2}' - h_{1/2}') = 4(\Sigma^+ - \Sigma^-) - 4(\Sigma_1^{++} - \Sigma_1^0) + 3(\Xi_1'^+ - \Xi_1'^0) - (\Xi_1^+ - \Xi_1^0) - 2(\Xi^0 - \Xi^-), \quad (2.5b)$$

$$4k_{1/2} = (\Xi_1 - \Sigma_1) + 3(\Lambda_1' - \Xi_1') + 2(\Sigma - N), \quad (2.5c)$$

$$2k_{1/2}' = (\Lambda_1' - \Xi_1') - 2(\Lambda - N) + 3(\Xi_1 - \Sigma_1), \quad (2.5d)$$

$$2\sqrt{3}\, m_{\Lambda_1\Sigma_1^+} = 4(\Sigma^0 - \Sigma^- + \Xi^- - \Xi^0) + 3[(\Xi_1'^+ - \Xi_1'^0) - (\Sigma_1^+ - \Sigma_1^0)] + (\Xi_1^+ - \Xi_1^0) - (\Sigma_1^+ - \Sigma_1^0), \quad (2.5e)$$

$$2/\sqrt{3}\, m_{\Xi_1'\Xi_1} = (\Lambda - \Sigma) + (\Xi_1' - \Xi_1) - (\Lambda_1' - \Sigma_1). \quad (2.5f)$$

The Coleman-Glashow relation[5] (2.2a) and its charmed analog (2.2b), already obtained in quark models,[1] remain valid in our general mass breaking (1.4). Relations (2.2d) and (2.3a) have been obtained in quark models by Franklin[1] and by Hendry and Lichtenberg,[1] respectively. In other relations, discrepancies present in first-order mass formulas are related.[6] It may be worthwhile noting that the most general mass operator (1.4) gives the $\Lambda - \Sigma^0$ mixing mass to be the same as predicted in SU(3) breaking up to first order,[7] if the $T^{\{1,3\}}_{\{1,3\}}$ component of 84 can be ignored. For the mass breaking, the tadpole considerations[5] do not favor this contribution ($T^{\{1,3\}}_{\{1,3\}}$).

B. $\frac{3}{2}^+$ baryon resonances

For $\frac{3}{2}^+$ isobars, the 20″ representation gives a null contribution, as it is not present in the direct product

$$\underline{20}^* \otimes \underline{20} = \underline{1} \oplus \underline{15} \oplus \underline{84} \oplus \underline{300}. \quad (2.6)$$

Second-order mass effects from the 84 are obtained from the contraction:

$$\bar{B}^{ijm} B_{klm} H^{\{k,l\}}_{\{i,j\}}. \quad (2.7)$$

This relates the discrepancies present in the equal-spacing rule in the following manner:

$$\Delta^{++} - \Delta^- = 3(\Delta^+ - \Delta^0), \quad (2.8a)$$

$$(\Xi^{*0} - \Xi^{*-}) + (\Delta^0 - \Delta^-) = 2(\Sigma^{*0} - \Sigma^{*-}), \quad (2.8b)$$

$$(\Xi_2^{*++} - \Xi_2^{*+}) + (\Delta^0 - \Delta^-) = 2(\Sigma_1^{*+} - \Sigma_1^{*0}), \quad (2.8c)$$

$$\tfrac{8}{3} d_{3/2} = \Sigma_1^{*++} + \Sigma_1^{*0} - 2\Sigma_1^{*+} = \Sigma^{*+} + \Sigma^{*-} - 2\Sigma^{*0} = \Delta^+ + \Delta^- - 2\Delta^0, \quad (2.8d)$$

$$\tfrac{16}{3} g_{3/2} = (\Xi_1^{*+} - \Xi_1^{*0}) - (\Sigma_1^{*+} - \Sigma_1^{*0}) = (\Sigma^{*0} - \Sigma^{*-}) - (\Delta^0 - \Delta^-), \quad (2.8e)$$

$$\tfrac{16}{3} h_{3/2} = (\Sigma_1^{*+} - \Sigma_1^{*0}) - (\Delta^0 - \Delta^-), \quad (2.8f)$$

In the absence of the electromagnetic interaction we get

$$\Omega - \Delta = 3(\Xi^* - \Sigma^*)$$

$$(440 \text{ MeV} = 447 \text{ MeV}), \quad (2.9a)$$

$$\Omega_3^* - \Omega = 3(\Omega_2^* - \Omega_1^*)\,, \qquad (2.9b)$$

$$\Omega_3^* - \Delta = 3(\Xi_2^* - \Sigma_1^*)\,, \qquad (2.9c)$$

$$\tfrac{8}{3} e_{3/2} = \Omega_1^* + \Sigma_1^* - 2\Xi_1^* = \Omega + \Sigma^* - 2\Xi^* = 3\ \text{MeV}\,, \qquad (2.9d)$$

$$\tfrac{8}{3} f_{3/2} = \Omega_2^* + \Sigma^* - 2\Xi_1^* = \Xi_2^* + \Delta - 2\Sigma_1^*\,, \qquad (2.9e)$$

$$\tfrac{16}{3} k_{3/2} = (\Xi_1^* - \Sigma_1^*) - (\Sigma^* - \Delta)\,. \qquad (2.9f)$$

Relations (2.8a) and (2.8d) have been obtained in the quark model by Franklin[1] and (2.9a), (2.9b), (2.9c), (2.9d) by Hendry and Lichtenberg.[1]

C. Pseudoscalar and vector mesons

We consider sixteen 0^- mesons to belong to the $\underline{15} \oplus \underline{1}$ irreducible representations of SU(4). Second-order effects with $\underline{84}$ dominance predict

$$3P_0^{\,2} = 2(F^2 - D^2 - K^2 + \pi^2) + \pi^2 + P_8^{\,2} + P_{15}^{\,2}\,. \qquad (2.10a)$$

We express the higher-order parameters in terms of meson masses as follows:

$$d_0 = 2[(\pi^0)^2 - (\pi^+)^2] = -0.0026\ \text{GeV}^2\,, \qquad (2.10b)$$

$$2g_0 = (K^+)^2 - (K^0)^2 + (\pi^0)^2 - (\pi^+)^2 - \sqrt{3}\, m^2(\pi^0\eta)\,, \qquad (2.10c)$$

$$2h_0 = 2[(D^0)^2 - (D^+)^2] + (\pi^0)^2 - (\pi^+)^2 + (K^0)^2 - (K^+)^2 - \sqrt{3}\, m^2(\pi^0\eta)\,. \qquad (2.10d)$$

In the absence of em breaking we get

$$2e_0 = 3\eta^2 - 4K^2 + \pi^2 = -0.062\ \text{GeV}^2\,, \qquad (2.11a)$$

$$2f_0 = (8P_{15}^{\,2} + 5\pi^2 - 12D^2 - \eta^2) + 4(F^2 + \pi^2 - K^2 - D^2)\,, \qquad (2.11b)$$

$$k_0 = F^2 + \pi^2 - K^2 - D^2\,. \qquad (2.11c)$$

The pseudoscalar mesons $\pi^0, \eta, \eta', \eta_c$ are mixed. The $\pi^0\eta$ mixing is expected to be small; hence $P_3 \to \pi^0$ and $P_8 \to \eta$ limits have been used in the relations given above. However, for completeness we write the mixing masses as follows:

$$(2\sqrt{2}/\sqrt{3})m^2(P_3P_{15}) = (2/\sqrt{3})m^2(P_3P_8) - (D^0)^2 + (D^+)^2 - (K^0)^2 + (K^+)^2\,, \qquad (2.12a)$$

$$2\sqrt{2}\, m^2(P_0P_3) = (K^+)^2 - (K^0)^2 + (D^0)^2 - (D^+)^2 + 2[(P_3)^2 - (\pi^+)^2]\,, \qquad (2.12b)$$

$$2\sqrt{2}\, m^2(P_8P_{15}) = (\pi^2 - P_8^{\,2}) + 2(F^2 + \pi^2 - K^2 - D^2)\,, \qquad (2.12c)$$

$$2\sqrt{6}\, m^2(P_0P_8) = 3(\pi^2 - P_8^{\,2}) - 2(F^2 + \pi^2 - K^2 - D^2)\,, \qquad (2.12d)$$

$$2\sqrt{3}\, m^2(P_0P_{15}) = (\pi^2 + P_8^{\,2} - 2P_{15}^{\,2}) - 2(F^2 + \pi^2 - K^2 - D^2)\,. \qquad (2.12e)$$

When the $\underline{20}''$ representation is also included, relations (2.10b), (2.10c) and (2.11a) are maintained. Other relations are modified to

$$2h_0 = (P_3)^2 - (\pi^+)^2 + (D^0)^2 - (D^+)^2 - (\tfrac{8}{3})^{1/2} m^2(P_3P_{15}) - (1/\sqrt{3})m^2(P_3P_8)\,, \qquad (2.13a)$$

$$2g_0' = (K^0)^2 - (K^+)^2 - (P_3)^2 + (\pi^+)^2 + \sqrt{2}\, m^2(P_0P_3) - (1/\sqrt{3})m^2(P_3P_8) + (2/\sqrt{6})m^2(P_3P_{15})\,, \qquad (2.13b)$$

$$2h_0' = (D^+)^2 - (D^0)^2 - (P_3)^2 + (\pi^+)^2 - (2/\sqrt{6})m^2(P_3P_{15}) + \sqrt{2}\, m^2(P_0P_3) + (1/\sqrt{3})\, m^2(P_3P_8)\,, \qquad (2.13c)$$

$$2f_0 = 2P_{15}^{\,2} + 2P_0^{\,2} - 4D^2 - P_8^{\,2} + \pi^2\,, \qquad (2.13d)$$

$$4k_0 = 2(F^2 + \pi^2 - K^2 - D^2) + 3P_0^{\,2} - P_8^{\,2} - P_{15}^{\,2} - \pi^2\,, \qquad (2.13e)$$

$$4k_0' = 3P_0^{\,2} - P_8^{\,2} - P_{15}^{\,2} - \pi^2 - 2(F^2 + \pi^2 - K^2 - D^2)\,, \qquad (2.13f)$$

$$2\sqrt{2}\, m^2(P_8P_{15}) = 3P_0^{\,2} - P_{15}^{\,2} - 2P_8^{\,2}\,, \qquad (2.13g)$$

$$4\sqrt{6}\, m^2(P_0P_8) = 6(\pi^2 - P_8^{\,2}) - 2(F^2 + \pi^2 - K^2 - D^2) - (3P_0^{\,2} - P_8^{\,2} - P_{15}^{\,2} - \pi^2)\,, \qquad (2.13h)$$

$$8\sqrt{3}\, m^2(P_0P_{15}) = 4(\pi^2 + P_8^{\,2} - 2P_{15}^{\,2}) - 2(F^2 + \pi^2 - K^2 - D^2) - 3(3P_0^{\,2} - P_8^{\,2} - P_{15} - \pi^2)\,. \qquad (2.13i)$$

Vector mesons can also be treated in a similar manner.

III. HYBRID MASS FORMULAS

Using SU(3) considerations Coleman and Glashow[5] derived hybrid mass formulas among baryon and meson masses in a dynamical model of symmetry breaking in the first-order limit, neglecting the nontadpole-type contributions. Eliezer and Singer also got many hybrid mass formulas by relating the higher-order SU(3) mass breaking.[4] In this paper we obtain some hybrid mass relations among the hadron multiplets by assuming the universality of the ratios of higher-order parameters. Our hybrid mass formula among the uncharmed hadrons has already been obtained by Eliezer and Singer.[4] The assumption of the universality of $d/e, e/f, g/h, g/k, d/g$ for baryons and mesons yields:

$$\frac{\Sigma^+ + \Sigma^- - 2\Sigma^0}{2N + 2\Xi - 3\Lambda - \Sigma} = \frac{2[(\pi^+)^2 - (\pi^0)^2]}{4K^2 - 3\eta^2 - \pi^2} = \frac{\Sigma^{*+} + \Sigma^{*-} - 2\Sigma^{*0}}{\Omega^* + \Sigma^* - 2\Xi^*} = \frac{2[(\rho^+)^2 - (\rho^0)^2]}{4K^{*2} - \rho^2 - 3V_8^2}\ , \tag{3.1}$$

$$\begin{aligned}\frac{2N + 2\Xi - 3\Lambda + \Sigma}{2N + 2\Xi_2 - 3\Lambda_1' + \Sigma_1} &= \frac{3\eta^2 - 4K^2 + \pi^2}{(8P_{15}^2 + 5\pi^2 - 12D^2 - \eta^2) + 4(F^2 + \pi^2 - K^2 - D^2)} \\ &= \frac{\Omega + \Sigma^* - 2\Xi^*}{\Omega_2^* + \Sigma^* - 2\Xi_1^*} \\ &= \frac{3V_8^2 - 4K^{*2} + \rho^2}{(8V_{15}^2 + 5\rho^2 - 12D^{*2} - V_8^2) + 4(F^{*2} + \rho^2 - K^{*2} - D^{*2})}\ ,\end{aligned} \tag{3.2}$$

$$\begin{aligned}\frac{\Sigma_1^+ - \Sigma_1^0 - \Xi_1^+ + \Xi_1^0}{4(\Sigma^0 - \Sigma^- - \Sigma_1^+ + \Sigma_1^0)} &= \frac{(K^+)^2 - (K^0)^2 + (\pi^0)^2 - (\pi^+)^2 - \sqrt{3}\, m^2(\pi^0\eta)}{2[(D^0)^2 - (D^+)^2 - (K^+)^2 + (K^0)^2]} \\ &= \frac{\Sigma^{*0} - \Sigma^{*-} - \Delta^0 + \Delta^-}{\Sigma_1^{*+} - \Sigma_1^{*0} - \Sigma^{*0} + \Sigma^{*-}} \\ &= \frac{(K^{*+})^2 - (K^{*0})^2 + (\rho^0)^2 - (\rho^+)^2 - \sqrt{3}\, m^2(\rho^0 V_8)}{2[(D^{*0})^2 - (D^{*+})^2 - (K^*)^2 + (K^{*0})^2]}\ ,\end{aligned} \tag{3.3}$$

$$\begin{aligned}\frac{\Sigma_1^+ - \Sigma_1^0 - \Xi_1^+ + \Xi_1^0}{3\Lambda + \Sigma - 2N - 2\Xi - 2[2(\Xi_1 - \Sigma_1) - (\Xi - N)]} &= \frac{(K^+)^2 - (K^0)^2 + (\pi^0)^2 - (\pi^+)^2 - \sqrt{3}\, m^2(\pi^0\eta)}{F^2 + \pi^2 - K^2 - D^2} \\ &= \frac{\Sigma^{*0} - \Sigma^{*-} - \Delta^0 + \Delta^-}{\Xi_1^* - \Sigma_1^* - \Sigma^* + \Delta} \\ &= \frac{(K^{*+})^2 - (K^{*0})^2 + (\rho^0)^2 - (\rho^+)^2 - \sqrt{3}\, m^2(\rho^0 V_8)}{F^{*2} + \rho^2 - K^{*2} - D^{*2}}\ ,\end{aligned} \tag{3.4}$$

$$\begin{aligned}\frac{\Sigma^+ + \Sigma^- - 2\Sigma^0}{\Xi_1^+ - \Xi_1^0 - \Sigma_1^+ + \Sigma_1^0} &= \frac{2[(\pi^0)^2 - (\pi^+)^2]}{(K^+)^2 - (K^0)^2 + (\pi^0)^2 - (\pi^+)^2 - \sqrt{3}\, m^2(\pi^0\eta)} \\ &= \frac{\Sigma^{*+} + \Sigma^{*-} - 2\Sigma^{*0}}{\Sigma^{*0} - \Sigma^{*-} - \Delta^0 + \Delta^-} \\ &= \frac{2[(\rho^0)^2 - (\rho^+)^2]}{(K^{*+})^2 - (K^{*0})^2 + (\rho^0)^2 - (\rho^+)^2 - \sqrt{3}\, m^2(\rho^0 V_8)}\ .\end{aligned} \tag{3.5}$$

Most of these formulas are not amenable to testing experimentally because of scanty experimental information available about the masses of charmed hadrons and about em mass differences of $\frac{3}{2}^+$ isobars.

IV. ESTIMATION OF MASSES OF HADRONS

As the Hamiltonian under consideration (1.4) involves several parameters, we are unable to predict the numerical mass values of most of the particles. In order to reduce the number of parameters, we drop terms like $T^{13}_{13}, T^{14}_{14}, T^{34}_{34}$ in the Hamiltonian, which is plausible in the tadpole mechanism of Coleman and Glashow.

Neglecting the T^{13}_{13}, T^{14}_{14}, and T^{34}_{34} components in the mass-breaking operator (1.4) we get the following mass relations

A. $\frac{1}{2}^+$ baryons

Relations (2.2a), (2.2b), (2.3c), and (2.3d) are maintained. Other relations are reduced to

$$\begin{aligned}\Xi_1^+ - \Xi_1^0 = \Sigma_1^+ - \Sigma_1^0 &\\ = \Sigma^0 - \Sigma^- &\\ = (-4.87 \pm 0.06\ \text{MeV})\ ,&\end{aligned} \tag{4.1a}$$

$$\begin{aligned}\Sigma_1^{++} - \Sigma_1^+ = \Sigma^+ - \Sigma^0 &\\ = (-3.12 \pm 0.08\ \text{MeV})\ ,&\end{aligned} \tag{4.1b}$$

$$\begin{aligned}3(\Xi_1'^+ - \Xi_1'^0) = 2(\Xi^0 - \Xi^-) + \Sigma^0 - \Sigma^- &\\ = (-17.7 \pm 0.6\ \text{MeV})\ ,&\end{aligned} \tag{4.1c}$$

$$\begin{aligned}\Omega_1 - \Sigma_1 = \Xi - N &\\ = (378\ \text{MeV})\ ,&\end{aligned} \tag{4.1d}$$

$$\begin{aligned}4(\Xi_1 - \Sigma_1) = 3\Lambda + \Sigma - 4N &\\ = (784\ \text{MeV})\ ,&\end{aligned} \tag{4.1e}$$

$$\begin{aligned}\Omega_2 - \Xi_2 = \Sigma - N &\\ = (254\ \text{MeV})\ ,&\end{aligned} \tag{4.1f}$$

$$\begin{aligned}4(\Xi_1' - \Lambda_1') = \Lambda + 3\Sigma - 4N &\\ = (939\ \text{MeV})\ ,&\end{aligned} \tag{4.1g}$$

$$\sqrt{3}\, m_{\Lambda\Sigma^0} = \sqrt{3}\, m_{\Lambda_1'^+\Sigma_1^+}$$
$$= \Sigma^0 - \Sigma^- - \Xi^0 + \Xi^-$$
$$= (2.66 \text{ MeV}), \quad (4.1\text{h})$$
$$4/\sqrt{3}\, m_{\Xi_1'\Xi_1} = \Lambda - \Sigma^0$$
$$= (-77 \text{ MeV}). \quad (4.1\text{i})$$

Recently a candidate for $\overline{\Lambda}_1'^+$ has been observed as a narrow state at 2.26 GeV in $\overline{\Lambda}\pi^-\pi^-\pi^+$ decay mode.[8] Another state is also observed at 2.5 GeV which decays to $\overline{\Lambda}_1'^+$ and positive pion. This state can be either $\overline{\Sigma}_1^0(\frac{1}{2}^+)$ or an isobar $\overline{\Sigma}_1^{*0}$. Putting $\overline{\Lambda}_1'^+$(2.26 GeV) and Σ_1^0 (2.5 GeV) as input we are able to give numerical values of sextet and antitriplet ($C=1$) baryons in Table I. Mass values of triplet ($C=2$) have a parameter f which gives the extent of second-order SU(4) breaking. Mixing masses are estimated and mixing angles are found to be

$$\theta_{\Lambda\Sigma^0} = 1°4'; \quad \theta_{\Lambda_1'^+\Sigma_1^+} = 0°22'; \quad \theta_{\Xi_1'\Xi_1} = 9°16'.$$

B. $\frac{3}{2}^+$ baryons

For $\frac{3}{2}^+$ isobars we get

$$\Xi_2^{*++} - \Xi_2^{*+} = \Xi_1^{*+} - \Xi_1^{*0}$$
$$= \Sigma_1^{*+} - \Sigma_1^{*0}$$
$$= \Xi^{*0} - \Xi^{*-}$$
$$= \Sigma^{*0} - \Sigma^{*-}$$
$$= \Delta^0 - \Delta^-, \quad (4.2\text{a})$$
$$\Sigma_1^{*++} - \Sigma_1^{*+} = \Sigma^{*+} - \Sigma^{*0}$$
$$= \Delta^+ - \Delta^0, \quad (4.2\text{b})$$
$$\Omega_2^* - \Xi_2^* = \Xi_1^* - \Sigma_1^*$$
$$= \Sigma^* - \Delta$$
$$= 152 \text{ MeV}, \quad (4.2\text{c})$$
$$\Omega_1^* - \Xi_1^* = \Xi^* - \Sigma^*$$
$$= 149 \text{ MeV}, \quad (4.2\text{d})$$

Relations (2.8a), (2.8d), (2.9a), (2.9b), (2.9c), (2.9d), and (2.9e) are maintained.

Using Σ_1^{*0} (2.5 GeV) as input we give various masses in Table I. Masses of triplet ($C=2$) and singlet ($C=3$) have a parameter f which gives second-order SU(4) breaking. Discovery of a charmed isobar $C \geq 2$ will determine this parameter.

B. $\frac{3}{2}^+$ baryons

In case of 0^- mesons, (2.10b) and (2.11a) remain unaffected. In addition we get

$$(D^0)^2 - (D^+)^2 = (\pi^0)^2 - (\pi^+)^2$$
$$= -0.0013 \text{ GeV}^2, \quad (4.3\text{a})$$
$$F^2 = K^2 - \pi^2 + D^2$$
$$= (1.93 \text{ GeV})^2, \quad (4.3\text{b})$$
$$3P_0^2 - P_{15}^2 - P_8^2 = \pi^2$$
$$= 0.019 \text{ GeV}^2, \quad (4.3\text{c})$$
$$2f_0 = 8P_{15}^2 - P_8^2 + 5\pi^2 - 12D^2, \quad (4.3\text{d})$$
$$\sqrt{3}\, m^2(P_3P_8) = (K^+)^2 - (K^0)^2 - (\pi^0)^2 + (\pi^+)^2$$
$$= 0.0053 \text{ GeV}^2, \quad (4.3\text{e})$$
$$2\sqrt{6}\, m^2(P_3P_{15}) = 5[(K^+)^2 - (K^0)^2] - (\pi^0)^2 - (\pi^+)^2$$
$$= 0.0213 \text{ GeV}^2, \quad (4.3\text{f})$$
$$2\sqrt{3}\, m^2(P_0P_3) = 3[(\pi^0)^2 - (\pi^+)^2] + (K^+)^2 - (K^0)^2$$
$$= 0.0001 \text{ GeV}^2, \quad (4.3\text{g})$$
$$2\sqrt{3}\, m^2(P_8P_{15}) = (2\sqrt{2}/\sqrt{3}) m^2(P_0P_8)$$
$$= \pi^2 - P_8^2, \quad (4.3\text{h})$$
$$2\sqrt{3}\, m^2(P_0P_{15}) = \pi^2 + P_8^2 - 2P_{15}^2. \quad (4.3\text{i})$$

TABLE I. Estimated masses of baryons (see text).

$\frac{1}{2}^+$ isomultiplets		Mass (GeV)	$\frac{3}{2}^+$ isomultiplets		Mass (GeV)
$B(6)$	Σ_1	2.50 (input)	$D(6)$	Σ_1^*	2.50 (input)
	Ξ_1	2.69		Ξ_1^*	2.65
	Ω_1	2.88		Ω_1^*	2.80
$B(3)$	Ξ_2	$3.75 - 2f_{1/2}$	$D(3)$	Ξ_2^*	$3.77 + 8/3f_{3/2}$
	Ω_2	$4.00 - 2f_{1/2}$		Ω_2^*	$3.92 + 8/3f_{3/2}$
$B(3^*)$	Λ_1'	2.26 (input)	$D(1)$	Ω_3^*	$5.04 + 8f_{3/2}$
	Ξ_1'	2.49			

From the above relations (4.3c) to (4.3i) we can obtain the mass relations among physical states making the following transformation

$$\eta_c = P_0 \sin\phi - P_{15}\cos\phi ,$$
$$\eta' = -P_8 \sin\theta + \cos\theta(P_0 \cos\phi + P_{15}\sin\phi) ,$$
$$\eta = P_8 \cos\theta + \sin\theta(P_0\cos\phi + P_{15}\sin\phi) ,$$
$$\pi^0 = P_3\cos\psi + \sin\psi[P_8\cos\theta - \sin\theta(P_0\cos\phi + P_{15}\sin\phi)] .$$

Here we have three mixing angles θ, ϕ, ψ. π^0 can be assumed to be pure P_3 state, i.e., $\psi = 0$. Similarly, relations among vector mesons can be obtained. In the calculations of the F-meson mass we take the D meson to be the 1.87-GeV boson, recently observed as a narrow state decaying to $(K\pi)^0$ and $(K\pi\pi\pi)^0$ decay modes.[9]

V. SUMMARY AND DISCUSSIONS

In this work we have considered second-order effects on the masses in addition to first-order breaking by including contributions from $\underline{20}''$ and $\underline{84}$ representations. We summarize our results as follows:

(i) The Coleman-Glashow sum rule and its charmed analog are reproduced by the general mass-breaking operator (1.4). Some of the relations obtained in quark models[1] are also regained.

(ii) Contributions of $\underline{84}$ and $\underline{20}''$ representations for the baryons and mesons are expressed in terms of particle masses; thereby, discrepancies in the relations obtained in first-order limit are estimated.

(iii) Assuming the universality of the ratios of the higher-order parameters for the baryons and mesons, we obtain a few hybrid mass formulas. Similar hybrid mass formulas have been obtained earlier in SU(3). Hybrid mass formulas obtained by Coleman and Glashow[5] neglect the nontadpole contribution corresponding to higher-order breaking (in the pure tadpole model $\pi^0 = \pi^\pm$, a result which is not present in our case). We have derived hybrid mass formulas among charmed states also.

(iv) Neglecting the T^{13}_{13}, T^{14}_{14}, T^{34}_{34} components of the mass operator (1.4), we obtain additional mass relations given in Sec. IV. Mass values for charmed hadrons are given in Table I.

(v) It is noted that the most general mass-breaking operator (1.4), in the absence of the T^{13}_{13} component of $\underline{84}$, predicts the SU(3) value of the $\Lambda - \Sigma^0$ mixing mass.[7] Mixing angles of other states in $\underline{20}'$ are also calculated.

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