Leptonic and radiative meson decays: A phenomenologically broken symmetry approach

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An extension, that incorporates the ω-ϕ mixing, of a model proposed by Schwinger in order to include U(3)-symmetry-breaking effects in vector-meson interactions is presented. The model is applied to vector-meson leptonic decays and to the one- and two-photon radiative meson decays. With our three parameters chosen as $\beta = 0.75$, $\theta_V = 4.5°$, and $\theta_P = 54.7°$ the agreement with the current data is good with the exception of the decay $\rho^- \to \pi^- \gamma$.

Since the measurements of the decays $\eta \to 2\gamma$ (Ref. 1) and $\rho^- \to \pi^-\gamma$ (Ref. 2) in 1974, the necessity of reviewing and improving our understanding of the "old" radiative meson decays, both from the experimental and the theoretical points of view, has become apparent. The discrepancy between the experimental results and the standard theory, nonrelativistic quark model and vector-meson dominance based on SU(3), is at the level of 2 standard deviations on the average.[3] This fact has motivated a great deal of work within this area in an attempt to bring closer together the theoretical and the experimental results. The main lines of research that one encounters in the literature can be roughly classified into three different categories that sometimes overlap each other. In the first place, we find various kinds of modifications to the usual quark-model approach that, for example, use harmonic-oscillator wave functions to calculate some overlap integrals,[4] leave them as free parameters to fit the data[5] or consider the corrections due to recoil.[6] Secondly, there are extended vector-meson-dominance models which include higher-mass vector mesons in the description of the radiative meson decays.[7] Finally, we are left with those models that try to take symmetry-breaking effects seriously into account.[8]

We subscribe to this latter point of view and present here an extension, that includes the ω-φ mixing, of a phenomenological model proposed by Schwinger in order to incorporate symmetry-breaking effects in the non-Abelian vector interactions.[9]

Our paper is organized as follows: Section I contains a somewhat detailed review of the above-mentioned model. The necessary modifications needed to incorporate the ω-ϕ mixing effects are made in Sec. II. In Sec. II we also present the results of the application of the extended model to the vector-meson leptonic decays together with one- and two-photon radiative meson decays.

I. NON-ABELIAN VECTOR INTERACTION WITHOUT ω-ϕ MIXING

In a first approach to the non-Abelian vector interactions, this model[9] ignores the ω-ϕ mixing, assumes that ρ and ω are mass degenerate, and does not pay any attention to the electromagnetic mass differences within the ρ and K^* multiplets. The observable particle fields are conveniently unified in the following 3×3 array:

$$v = \begin{bmatrix} \frac{\rho^0+\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & \frac{-\rho^0+\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \overline{K}^{*0} & \phi \end{bmatrix}, \tag{1}$$

where the vector index has been suppressed for simplicity.

The starting point for the dynamics of these particles is the U(3) non-Abelian gauge-invariant Lagrangian

$$\mathcal{L}_0 = \mathrm{Tr}\left[-\tfrac{1}{2}\overline{v}_{\mu\nu}\left(\partial^\mu \overline{v}^\nu - \partial^\nu \overline{v}^\mu - \frac{ig}{\sqrt{2}}[\overline{v}^\mu, \overline{v}^\nu]\right) + \tfrac{1}{4}\overline{v}_{\mu\nu}\overline{v}^{\mu\nu} - \tfrac{1}{4}m_\rho^{\,2}\overline{v}_\mu \overline{v}^\mu\right] \tag{2}$$

which would correspond to a nonet of degenerate vector mesons if $\overline{v}_\mu$ were the field that describes the observed particles. This symmetry is broken in nature and one takes the attitude that the Lagrange function Eq. (2) represents, in terms of the fields $\overline{v}_\mu$ and $\overline{v}_{\mu\nu}$, the U(3)-invariant piece of the full vector-meson Lagrangian. The U(3)-symmetry breaking is introduced not only by allowing each vector meson to have its real mass, but more fundamentally, by distinguishing the observable particle fields v_μ and $v_{\mu\nu}$ from the carriers of the partial symmetry $\overline{v}_\mu$ and $\overline{v}_{\mu\nu}$ by means of the linear transformations

$$\bar{v}_\mu = \left(\frac{m}{m_\rho}\right)^\beta v_\mu , \tag{3a}$$

$$\bar{v}_{\mu\nu} = \left(\frac{m_\rho}{m}\right)^\beta v_{\mu\nu} , \tag{3b}$$

where m signifies the physical mass of a particle, and β is a parameter to be selected empirically. In other words, the right-hand side of Eq. (3a), for example, is intended to be just a shorthand notation for

$$\left(\frac{m}{m_\rho}\right)^\beta v = \begin{bmatrix} \frac{\rho^0+\omega}{\sqrt{2}} & \rho^+ & \left(\frac{m_{K^*}}{m_\rho}\right)^\beta K^{*+} \\ \rho^- & \frac{-\rho^0+\omega}{\sqrt{2}} & \left(\frac{m_{K^*}}{m_\rho}\right)^\beta K^{*0} \\ \left(\frac{m_{K^*}}{m_\rho}\right)^\beta K^{*-} & \left(\frac{m_{K^*}}{m_\rho}\right)^\beta \bar{K}^{*0} & \left(\frac{m_\phi}{m_\rho}\right)^\beta \phi \end{bmatrix} , \tag{4}$$

which specifies the action of the operator $(m/m_\rho)^\beta$ upon a matrix v written in terms of the particle fields.

The above-mentioned linear transformations (or operators) provide us with a way of assigning arbitrary symmetry-breaking factors to any one of the particle fields that are contained in the array v_μ. It is worth noticing that this goal cannot be accomplished in the simpler framework of matrix multiplication.

The problem of extending the definition of the linear transformations introduced in Eqs. (3) to include any 3×3 matrix $A=[a_{ij}]$ is readily solved here by noticing that each element a_{ij} of the matrix A can be naturally associated with a definite symmetry-breaking factor as suggested by Eq. (4). One is thus led to the generalization

$$\left(\frac{m}{m_\rho}\right)^\beta A = \begin{bmatrix} a_{11} & a_{12} & \left(\frac{m_{K^*}}{m_\rho}\right)^\beta a_{13} \\ a_{21} & a_{22} & \left(\frac{m_{K^*}}{m_\rho}\right)^\beta a_{23} \\ \left(\frac{m_{K^*}}{m_\rho}\right)^\beta a_{31} & \left(\frac{m_{K^*}}{m_\rho}\right)^\beta a_{32} & \left(\frac{m_\phi}{m_\rho}\right)^\beta a_{33} \end{bmatrix} , \tag{5}$$

which completely defines the operator $(m/m_\rho)^\beta$ in the matrix vector space.

The partially invariant Lagrange function that describes the vector interaction is taken to be

$$\mathcal{L} = \mathrm{Tr}\left[-\tfrac{1}{2}\bar{v}_{\mu\nu}\left(\partial^\mu \bar{v}^\nu - \partial^\nu \bar{v}^\mu - \frac{ig}{\sqrt{2}}[\bar{v}^\mu, \bar{v}^\nu]\right) + \tfrac{1}{4}\bar{v}_{\mu\nu}\bar{v}^{\mu\nu} - \tfrac{1}{2}m_\rho{}^2 \bar{v}_\mu \bar{v}^\mu\right] + \mathrm{Tr}\left\{\tfrac{1}{4}\bar{v}_{\mu\nu}\left[\left(\frac{m}{m_\rho}\right)^{2\beta} - 1\right]\bar{v}^{\mu\nu} - \tfrac{1}{2}m_\rho{}^2\bar{v}_\mu\left[\left(\frac{m}{m_\rho}\right)^{2-2\beta} - 1\right]\bar{v}^\mu\right\} . \tag{6}$$

The first part of it reproduces Eq. (2) and constitutes the invariant part of the Lagrangian. This partial symmetry is realized in terms of the $\bar{v}_\mu$ and $\bar{v}_{\mu\nu}$ transformation properties provided the second part of the Lagrange function is absent. The latter part breaks this invariance and is designed in such a way that the nonderivative terms of the full Lagrangian expressed in terms of the physical fields are

$$\mathrm{Tr}[\tfrac{1}{4}v_{\mu\nu}v^{\mu\nu} - \tfrac{1}{2}v_\mu m^2 v^\mu] , \tag{7}$$

where m is the vector mass operator. It is worth noticing that the definitions Eqs. (3a) and (3b) provide the right derivative terms for the physical fields in the Lagrange function Eq. (6). The cubic terms in the interaction do indeed acquire explicit mass factors which simply means that we are introducing symmetry-breaking effects in the effective coupling constants.

Before considering the electromagnetic interaction, we write down the transformation properties of the field $\bar{v}_\mu$ under a non-Abelian gauge transformation generated by $\delta\lambda$,

$$\delta\bar{v}_\mu = i[\delta\lambda, \bar{v}_\mu] + \frac{\sqrt{2}}{g}\partial_\mu \delta\lambda , \tag{8}$$

and the expression for $\bar{v}_{\mu\nu}$ that is produced by applying the action principle to the Lagrangian, Eq. (6),

$$\bar{v}_{\mu\nu}=\left(\frac{m_\rho}{m}\right)^{2\beta}\left[\partial_\mu \bar{v}_\nu-\partial_\nu \bar{v}_\mu-\frac{ig}{\sqrt{2}}[\bar{v}_\mu,\bar{v}_\nu]\right]. \qquad (9)$$

From the last two equations it is apparent that the general transformation property for $\bar{v}_{\mu\nu}$ is given by

$$\delta\left[\left(\frac{m}{m_\rho}\right)^{2\beta}\bar{v}_{\mu\nu}\right]=i\left[\delta\lambda,\left(\left(\frac{m}{m_\rho}\right)^{2\beta}\bar{v}_{\mu\nu}\right)\right]. \qquad (10)$$

In other words, $\bar{v}_{\mu\nu}$ transforms covariantly (or unitarily) only if one stays within a particle multiplet, that is to say, under any combination of isospin and hypercharge gauge transformations.

In order to introduce the electromagnetic interaction one makes the substitution

$$\bar{v}_\mu \to \bar{v}_\mu-\sqrt{2}\,\frac{e}{g}q A_\mu\,, \quad q=\begin{bmatrix}\frac{2}{3} & 0 & 0\\ 0 & -\frac{1}{3} & 0\\ 0 & 0 & -\frac{1}{3}\end{bmatrix} \qquad (11)$$

in the mass term of the Lagrangian Eq. (6). The matrix q is chosen in such a way that it reproduces the correct U(3) normalization for the electric charge stated in the expression

$$\begin{aligned} Q&=T_{11}-\tfrac{1}{3}(T_{11}+T_{22}+T_{33})\\ &=\tfrac{2}{3}T_{11}-\tfrac{1}{3}T_{22}-\tfrac{1}{3}T_{33}\,. \end{aligned} \qquad (12)$$

Now it is crucial to check that the full Lagrangian Eq. (6) is invariant under the Abelian electromagnetic gauge transformation

$$e\delta A_\mu(x)=\partial_\mu\delta\gamma(x)\,. \qquad (13)$$

Taking the compensating non-Abelian gauge transformation to be the one generated by

$$\delta\lambda(x)=q\,\delta\gamma(x)\,, \qquad (14)$$

one finds that

$$\delta\left(\bar{v}_\mu-\sqrt{2}\,\frac{e}{g}q A_\mu\right)=i\delta\gamma(x)\left[q\,,\bar{v}_\mu-\sqrt{2}\,\frac{e}{g}q A_\mu\right] \qquad (15a)$$

and

$$\delta\bar{v}_{\mu\nu}=i\delta\gamma(x)[q\,,\bar{v}_{\mu\nu}]\,, \qquad (15b)$$

which is obtained from Eq. (10) by noticing that the matrix q in Eq. (11) can be written as a linear combination of the identity matrix, the hypercharge generator, and the third component isospin generator. That is to say, the matrix q can be moved across any operator of the type $(m/m_\rho)^{\pm\beta}$ that appears in the Lagrangian Eq. (6). In this way the invariance of this Lagrange function under the combined electromagnetic and induced non-Abelian gauge transformations is made apparent because the problem reduces to show that the trace of the product of any two matrices is invariant under the unitary transformation generated by q according to Eqs. (15a) and (15b).

This concludes our review of Schwinger's model which constitutes the basic framework within which we will discuss ω-φ mixing effects in the next section.

II. NON-ABELIAN VECTOR INTERACTION WITH ω-ϕ MIXING

In order to include ω-φ mixing effects we start by redefining the 3×3 array of observable particle fields v_μ in the following way:

$$v=\begin{bmatrix}\dfrac{\rho^0+\omega\cos\theta_V+\varphi\sin\theta_V}{\sqrt{2}} & \rho^+ & K^{*+}\\ \rho^- & \dfrac{-\rho^0+\omega\cos\theta_V+\varphi\sin\theta_V}{\sqrt{2}} & K^{*0}\\ K^{*-} & \bar{K}^{*0} & \phi\cos\theta_V-\omega\sin\theta_V\end{bmatrix}, \qquad (16)$$

which introduces the ϕ-ω mixing angle θ_V. We still consider each of the ρ and K^* multiplets to be separately degenerate in mass.

The distinction between the observable particle fields and the carriers of the partial symmetry is stated again by means of Eqs. (3a) and (3b). The explicit expression for the action of the operator $(m/m_\rho)^\beta$, for example, upon the new array v_μ poses no problem because this matrix is written in terms of particle fields, whose masses identify uniquely the corresponding symmetry-breading factors.

Unfortunately the situation is not so simple when we want to consider the action of such linear transformations upon a general matrix A. The prescription given in Eq. (5) is clearly not satisfactory here because the diagonal terms of the matrix should be associated with a mixture of symmetry-breaking factors as suggested by $(m/m_\rho)^\beta v_\mu$. A natural way of getting around this problem arises by noticing that there exists a basis in the matrix vector space that allows v_μ to be written as a superposition of terms, each of which refers to a definite particle and consequently has associated with it a definite symmetry-breaking factor.[10] This basis, which we will call the V (vector) basis, is formed by the following set of nine 3×3 matrices:

$$V_{\rho^0}=\begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & -\frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 \end{bmatrix},\quad V_{\rho^+}=\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$V_{\rho^-}=\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},\quad V_{K^{*0}}=\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix},$$

$$V_{K^{*+}}=\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$V_{K^{*-}}=\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix},\quad V_{\bar{K}^{*0}}=\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \tag{17}$$

$$V_{\omega}=\begin{bmatrix} \frac{\cos\theta_V}{\sqrt{2}} & 0 & 0 \\ 0 & \frac{\cos\theta_V}{\sqrt{2}} & 0 \\ 0 & & -\sin\theta_V \end{bmatrix},$$

$$V_{\phi}=\begin{bmatrix} \frac{\sin\theta_V}{\sqrt{2}} & 0 & 0 \\ 0 & \frac{\sin\theta_V}{\sqrt{2}} & 0 \\ 0 & 0 & \cos\theta_V \end{bmatrix}.$$

We notice in passing that this is an orthonormal basis when the scalar product in the matrix vector space is defined as

$$(A,B)=\mathrm{Tr}(A^TB), \tag{18}$$

where the superscript T means transposed. The components of v_μ in the V basis are the respective particle fields as can be directly recognized from Eqs. (16) and (17).

The V basis constitutes an adequate coordinate system where we can characterize in general the linear transformations introduced in Eqs. (3a) and (3b). For example, the action of the operator $(m/m_\rho)^\beta$ is defined as

$$(m/m_\rho)^\beta A=\sum_{\{i\}}\left(\frac{m_i}{m_\rho}\right)^\beta A_i V_i, \tag{19}$$

where $\{i\}$ is an appropriate set of indices that refers to individual particles, and $A_i=\mathrm{Tr}(V_i^TA)$ is the i component of the matrix A in the V basis. In other words, the action of the operator upon the off-diagonal matrix elements reduces to the multiplication by the respective symmetry-breaking factor as before, while the diagonal matrix elements are rearranged in a way that incorporates the mixing angle θ_V according to the definite prescription given in Eq. (19).

The nontrivial aspect of the definition Eq. (19), which is related only to the diagonal sector of the matrices A and $\tilde{A}\equiv(m/m_\rho)^\beta A$ as we have just discussed, can be presented in the following equivalent way:

$$\begin{bmatrix} \tilde{A}_{11} \\ \tilde{A}_{22} \\ \tilde{A}_{33} \end{bmatrix} = R^T\begin{bmatrix} 1 & 0 & 0 \\ 0 & (m_\omega/m_\rho)^\beta & 0 \\ 0 & 0 & (m_\phi/m_\rho)^\beta \end{bmatrix} R \begin{bmatrix} A_{11} \\ A_{22} \\ A_{33} \end{bmatrix}, \tag{20}$$

where R is the orthogonal matrix given by

$$R=\begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{\cos\theta_V}{\sqrt{2}} & \frac{\cos\theta_V}{\sqrt{2}} & -\sin\theta_V \\ \frac{\sin\theta_V}{\sqrt{2}} & \frac{\sin\theta_V}{\sqrt{2}} & \cos\theta_V \end{bmatrix}. \tag{21}$$

Equation (20) is the statement that the diagonal elements A_{ii}, $i=1,2,3$ are first unmixed by the transformation R in such a way that a definite symmetry-breaking factor can be assigned to each one of the resulting terms as described by the diagonal matrix that operates next. Once this is done, the terms are mixed again according to the inverse transformation R^T to produce the final result.

The Lagrange function that corresponds to this extended vector interaction is the same Eq. (6) provided we use Eq. (16) for the observed particle field v_μ. The definitions of the linear transformations involved here are such that this Lagrangian remains hypercharge- and isospin-invariant. Electromagnetism is introduced, as before, by the substitution Eq. (11) in the mass term of the Lagrangian and the electromagnetic gauge invariance of the interaction follows due to the same arguments already discussed in Sec. I.

Now that we have settled the basic ideas of this extended model we turn to the calculation of the related decay widths. To begin with we present the electromagnetic coupling between the photon and the zero-charged vector mesons produced by the substitution Eq. (11)

$$\mathcal{L}_{\rm em}=\frac{e}{g}m_\rho{}^2A^\mu\left[\rho_\mu{}^0+\left(\frac{m_\omega}{m_\rho}\right)^{2-\beta}\tfrac{1}{3}(\cos\theta_V+\sqrt{2}\,\sin\theta_V)\omega_\mu-\left(\frac{m_\phi}{m_\rho}\right)^{2-\beta}\frac{\sqrt{2}}{3}\left(\cos\theta_V-\frac{\sin\theta_V}{\sqrt{2}}\right)\phi_\mu\right]. \tag{22}$$

Letting $\beta=0$ we recover the standard vector-meson-dominance result.[11] From the interaction Lagrangian Eq. (22) and neglecting finite-width effects, the ω-ρ interference, and the possible modifications to the photon propagator, we obtain the following for the leptonic decays of the vector mesons:

$$\begin{aligned}\Gamma(\omega\to e^+e^-)&=\frac{(\cos\theta_V+\sqrt{2}\,\sin\theta_V)^2}{9}\\&\quad\times\left(\frac{m_\rho}{m_\omega}\right)^{2\beta-1}\Gamma(\rho\to e^+e^-)\,,\\\Gamma(\phi\to e^+e^-)&=\tfrac{2}{9}\left(\cos\theta_V-\frac{\sin\theta_V}{\sqrt{2}}\right)^2\\&\quad\times\left(\frac{m_\rho}{m_\phi}\right)^{2\beta-1}\Gamma(\rho\to e^+e^-)\,,\end{aligned} \tag{23}$$

where the mass factors $(m_\rho/m_v)^{2\beta-1}$ should be noticed.

Now we consider the meson radiative decays. The starting point is the vector-pseudoscalar interaction and our attitude is to write a Lagrange function in terms of the fields $\overline{v}_{\mu\nu}$ that respect the partial symmetry in the Lagrangian Eq. (6). That is to say, we begin with the interaction

$$\mathcal{L}_{pvv}=-\frac{g}{\sqrt{2}}\frac{1}{m_\rho}\mathrm{Tr}({}^*\overline{v}_{\mu\nu}\overline{v}^{\mu\nu}p)\,, \tag{24}$$

where

$${}^*\overline{v}_{\mu\nu}=\tfrac{1}{2}\epsilon_{\mu\nu\kappa\lambda}\overline{v}^{\kappa\lambda} \tag{25}$$

is the dual tensor of $\overline{v}_{\mu\nu}$ and the pseudoscalar matrix p corresponds to the 3×3 array

$$p=\begin{bmatrix}\dfrac{\pi^0+\eta\cos\theta_P+\eta'\sin\theta_P}{\sqrt{2}} & \pi^+ & K^+\\ \pi^- & \dfrac{-\pi^0+\eta\cos\theta_P+\eta'\sin\theta_P}{\sqrt{2}} & K^0\\ K^- & \overline{K}^0 & \eta'\cos\theta_P-\eta\sin\theta_P\end{bmatrix}, \tag{26}$$

which defines the η-η' mixing angle θ_P. At this level of approximation in the dynamics we do not make any distinction between the observable fields and the partial symmetry carriers for the pseudoscalar particles.

The Lagrange functions for one- and two-photon radiative meson decays are obtained from Eq. (24) by making the substitution

$$\overline{v}_{\mu\nu}\to\overline{v}_{\mu\nu}-\left(\frac{m_\rho}{m}\right)^{2\beta}\sqrt{2}\,\frac{e}{g}qF_{\mu\nu}\,, \tag{27}$$

which is a direct consequence of Eqs. (9) and (11), and by rewriting the vector fields with bars in terms of the physical ones.

In performing this calculation it is convenient to introduce the following parameters:

$$R_1=R+\frac{r}{2}\sqrt{3}\,\cos(2\theta_V-\theta_0)\,, \tag{28}$$

$$R_2=R+\frac{r}{2}(\tfrac{3}{2})^{1/2}\sin(2\theta_V-\theta_0)\,, \tag{29}$$

$$R=\tfrac{1}{2}\left[\left(\frac{m_\rho}{m_\omega}\right)^{2\beta}+\left(\frac{m_\rho}{m_\phi}\right)^{2\beta}\right], \tag{30}$$

$$r=\left(\frac{m_\rho}{m_\omega}\right)^{2\beta}-\left(\frac{m_\rho}{m_\phi}\right)^{2\beta}, \tag{31}$$

where $\theta_0=54.7°$ is defined by $\cos\theta_0=1/\sqrt{3}$ and $\sin\theta_0=(\tfrac{2}{3})^{1/2}$. The quantities R_1 and R_2 reduce to $(m_\rho/m_\omega)^{2\beta}$ and $(m_\rho/m_\phi)^{2\beta}$, respectively, when the vector mixing angle θ_V is zero, and to 1 when we have complete mass degeneracy. Finally we are led to the following Lagrangians for each of the interactions:

$$\mathcal{L}_{P\gamma\gamma} = -\frac{e^2}{g}\frac{1}{m_\rho}\,{}^*F_{\mu\nu}F^{\mu\nu}\left[\frac{\pi^0}{3}R_1+\eta\left(\frac{9+R_1^2}{18}\cos\theta_P-\frac{\sqrt{2}}{9}R_2^2\sin\theta_P\right)+\eta'\left(\frac{9+R_1^2}{18}\sin\theta_P+\frac{\sqrt{2}}{9}R_2^2\cos\theta_P\right.\right. \quad (32)$$

and

$$\begin{aligned}\mathcal{L}_{PV\gamma} = \frac{e}{m_\rho}\,{}^*F^{\mu\nu}\Bigg\{&\rho^0_{\mu\nu}\left(\frac{R_1}{3}\pi^0+\eta\cos\theta_P+\eta'\sin\theta_P\right)+\omega_{\mu\nu}\left(\frac{m_\rho}{m_\omega}\right)^2\Bigg[\pi^0\cos\theta_V+\eta'\left(\frac{R_1}{3}\cos\theta_V\cos\theta_P-\tfrac{2}{3}R_2\sin\theta_V\sin\theta_P\right)\\&+\eta'\left(\frac{R_1}{3}\cos\theta_V\sin\theta_P+\tfrac{2}{3}R_2\sin\theta_V\cos\theta_P\right)\Bigg]\\&+\varphi_{\mu\nu}\left(\frac{m_\rho}{m_\phi}\right)^\beta\Bigg[\pi^0\sin\theta_V+\eta\left(\frac{R_1}{3}\sin\theta_V\cos\theta_P+\tfrac{2}{3}R_2\cos\theta_V\sin\theta_P\right)\\&+\eta'\left(\frac{R_1}{3}\sin\theta_V\;\sin\theta_P-\tfrac{2}{3}R_2\cos\theta_V\cos\theta_P\right)\Bigg]\\&-\tfrac{1}{2}\left(1-\frac{R_1}{3}+\tfrac{2}{3}R_2\right)\left(\frac{m_\rho}{m_{K^*}}\right)^\beta\left(\bar{K}^{*0}_{\mu\nu}K^0+K^{*0}_{\mu\nu}\bar{K}^0\right)+\tfrac{1}{2}\left(1+\frac{R_1}{3}-\tfrac{2}{3}R_2\right)\left(\frac{m_\rho}{m_{K^*}}\right)^\beta\left(K^{*+}_{\mu\nu}K^-+K^{*-}_{\mu\nu}K^+\right)\\&+\tfrac{1}{3}R_1\left(\rho^+_{\mu\nu}\pi^-+\rho^-_{\mu\nu}\pi^+\right)\Bigg\}\,. \end{aligned} \quad (33)$$

If we specialize our results in Eqs. (22), (32), and (33) to the case where $\theta_V=0$ and $m_\rho=m_\omega$, we recover the corresponding Lagrange functions previously found by Schwinger.[9]

Now we have all the ingredients to start the calculation of the widths corresponding to the different decay modes. By comparing these predictions with the experimental results we will be able to fix the parameters β, θ_V, and θ_P of our model.

In the first place, we consider the decay widths that are functions only of β and θ_V. Within this class of processes, those that depend more dramatically on these parameters are $\varphi\to\pi\rho$, $\phi\to\pi\gamma$, $\phi\to e^+e^-$, and $K^*\to K\gamma$ either because of their rapid variation as in the first two cases, or because the predicted number is just in the border of the experimental error bars as in the φ leptonic decay or in the K^* radiative decay. After all, we must remind ourselves that the decays $\varphi\to\pi\rho$ and $\varphi\to\pi\gamma$ are forbidden when $\theta_V=0$ and that we are probing symmetry-breaking factors of the form $(m_v/m_\rho)^\beta$ which are more sensitive to the ϕ and K^* mesons than to the ω meson.

The decay $\phi\to\pi\rho$ ($\phi\to\pi^0\rho^0+\pi^+\rho^-+\pi^-\rho^+$) is described by the Lagrange function Eq. (24) and the relevant piece of it is

$$\mathcal{L}_{\phi\pi\rho} = -\frac{g}{m_\rho}\sin\theta_V\left(\frac{m_\rho}{m_\phi}\right)^\beta {}^*\vec{\rho}_{\mu\nu}\,\phi^{\mu\nu}\cdot\vec{\pi}\,, \quad (34)$$

from which we obtain the width

$$\Gamma_M(\phi\to\pi\rho)=\frac{\sin^2\theta_V}{2}\left(\frac{g^2}{4\pi}\right)\left(\frac{m_\rho}{m_\phi}\right)^{2\beta-2}\times\left\{\left[1-\left(\frac{M+m_\pi}{m_\phi}\right)^2\right]\times\left[1-\left(\frac{M-m_\pi}{m_\phi}\right)^2\right]\right\}^{3/2} m_\phi\,. \quad (35)$$

In the above equation M is the mass of the ρ meson. We would like to estimate the finite-width corrections in this process due to the ρ-meson instability. To this end, we neglect the interference between the different ρ channels and simply average the mass-dependent width Eq. (35) using a Breit-Wigner type of weight factor in such a way that

$$\Gamma(\phi\to\pi\rho)=\int_{2m_\pi}^{m_\phi-m_\pi} dM\,\frac{2}{\pi}\,\frac{M^2\Gamma_M(\rho)}{(M^2-m_\rho^2)^2+M^2\Gamma_M(\rho)^2}\times\Gamma_M(\varphi\to\pi\rho)\,. \quad (36)$$

The mass dependence of the ρ width $\Gamma_M(\rho)$ (accounted for entirely by the process $\rho\to\pi\pi$) is calculated from the familiar coupling

$$\mathcal{L}_{\rho\pi\pi}=g\vec{\rho}_\mu\cdot\partial^\mu\vec{\pi}\times\vec{\pi}\,, \quad (37)$$

and is presented as

$$\Gamma_M(\rho)=0.152\left(\frac{M}{m_\rho}\right)\left[\frac{1-4m_\pi^2/M^2}{1-4m_\pi^2/m_\rho^2}\right]^{3/2}\text{GeV}\,. \quad (38)$$

This expression is normalized to the nominal ρ width of 0.152 GeV for $M=m_\rho=0.773$ GeV. The result of carrying on this estimation is conveniently written in the form

$$\Gamma(\phi\to\pi\rho)=1.06\Gamma_{m_\rho}(\phi\to\pi\rho)\,, \tag{39}$$

which shows that ρ finite-width effects are of the order of 6% in our approximation.

The calculation of the $\phi\to\pi\rho$ width includes the coupling constant g and, for the sake of definiteness, we adopt the value

$$\frac{g^2}{4\pi}=3.1\pm 5\%\,, \tag{40}$$

which is consistent with the strong-interaction relation

$$\tfrac{1}{2}\left(\frac{g}{m_\rho}\right)^2=\left(\frac{f_0}{m_\pi}\right)^2 \tag{41}$$

that gives $g^2/4\pi=3.2$ when one uses the low-energy s-wave pion-nucleon coupling constant $f_0=0.8$.

In dealing with the leptonic decays of the vector mesons, we face a problem that is common to most of the models that use some version of vector-meson dominance to introduce the electromagnetic interaction. If we were to calculate the width $\Gamma(\rho\to e^+e^-)$ according to the Lagrange function Eq. (22) using the chosen value $g^2/4\pi=3.1$, we would obtain a number that is approximately 70% of the experimental result. One needs the smaller value $g^2/4\pi=2.1$ to reproduce the observed ρ leptonic width. As a reflection of our ignorance in understanding the q^2 dependence of the coupling constant g in the region $0<q^2\lesssim 1$ GeV, we will take the width $\Gamma(\rho\to e^+e^-)=6.54\pm 0.76$ keV (Ref. 12) as the input to predict the other leptonic decays of the vector mesons according to Eq. (23).

The widths $\Gamma(v\to p\gamma)$ for the one-photon radiative vector-meson decays are calculated according to the expression

$$\frac{\Gamma(v\to p\gamma)}{\Gamma(\omega\to\pi^0\gamma)}=\left[\frac{C(vp\gamma)}{(m_\rho/m_\omega)^\beta\cos\theta_V}\right]^2\left(\frac{m_v}{m_\omega}\right)^3\left[\frac{1-m_p{}^2/m_v{}^2}{1-m_\pi{}^2/m_\omega{}^2}\right]^3, \tag{42}$$

where the coupling constant $C(vp\gamma)$ can be readily read from the Lagrangian Eq. (33) as the factor of $(e/m_\rho)\,{}^*F^{\mu\nu}v_{\mu\nu}p$. We use the experimental value $\Gamma(\omega\to\pi^0\gamma)=880\pm 60$ keV as the input in the calculation of the widths $\Gamma(v\to p\gamma)$.

The remaining decay widths that are independent of the pseudoscalar mixing angle θ_P are presented in the following formulas:

$$\Gamma(\pi^0\to 2\gamma)=\tfrac{1}{9}\frac{\alpha^2}{(g^2/4\pi)}R_1{}^2\left(\frac{m_\pi}{m_\rho}\right)^2 m_\pi\,, \tag{43}$$

$$\Gamma(\omega\to\pi^0\gamma)=\frac{\alpha}{6}\cos^2\theta_V\left(\frac{m_\rho}{m_\omega}\right)^{2\beta-2}\left(1-\frac{m_\pi{}^2}{m_\omega{}^2}\right)^3 m_\omega\,, \tag{44}$$

where α is the fine-structure constant.

Some comments concerning the general characteristics of the above-mentioned widths and how they favor one or another choice of the parameters are now in order.

In the first place, we remark that the widths for the decays $\phi\to\pi\rho$, $\phi\to\pi^0\gamma$, $\omega\to\pi^0\gamma$, and $K^*\to K\gamma$ depend only on the absolute value of θ_V,[13] while those corresponding to $\phi\to e^+e^-$, $\omega\to e^+e^-$, $\pi^0\to\gamma\gamma$, and $\rho\to\pi\gamma$ are sensitive to the sign of the vector mixing angle θ_V.

The widths $\Gamma(\phi\to\pi\rho)$ and $\Gamma(\phi\to\pi^0\gamma)$ can be fitted simultaneously only by taking into account the errors involved through $g^2/4\pi$ and $\Gamma(\omega\to\pi^0\gamma)$, respectively. Then, it is possible to find a continuous pair of solutions for β and $|\theta_V|$ that ranges from $\beta=0.6$, $|\theta_V|=4.35°$ to $\beta=1.5$, $|\theta_V|=5.5°$ in the region we have been exploring. The consistency with the measured values is obtained from the lower [upper] experimental limit for $\Gamma(\varphi\to\pi\rho)$ [$\Gamma(\phi\to\pi^0\gamma)$].

The decay $K^{0*}\to K^0\gamma$ is essentially insensitive to the vector mixing angle for $|\theta_V|\leqslant 10°$, but its width depends very strongly on the value of β thus imposing the constraint $\beta\geqslant 0.75$.

The leptonic mode $\varphi\to e^+e^-$ is sensitive to both β and θ_V favoring $\beta\leqslant 0.8$ for $\theta_V>0$ while the sector $\theta_V<0$ admits solutions in the range $0.6\leqslant\beta\leqslant 1.2$ with $-4.35°\geqslant\theta_V\geqslant -5.1°$, respectively. On the contrary, the decay width for $\omega\to e^+e^-$ is almost insensitive to either β or θ_V in the range $4.35\leqslant|\theta_V|\leqslant 5.1$ and depends only on the sign of the vector mixing angle.

The two-photon decay $\pi^0\to 2\gamma$ favors positive vector mixing angles and in this region the dependence upon β is very small.

If we restrict ourselves to positive vector mixing angles (we will see later that the negative sector appears to be ruled out by the current data) the best overall fit to the experimental results corresponds to the choice $\beta=0.75$ and $\theta_V=4.5°$. The comparison between our predictions and the experimental values is presented in Table I where we have also included the results for the choice $\beta=1.0$ and $\theta_V=-4.85°$. The latter value for β is appealing from the aesthetical point of view, but unfortunately for the moment, it requires a negative vector mixing angle.

Now we turn our attention to those decay widths that depend on the pseudoscalar mixing angle θ_P. The most stringent constraint on this parameter is imposed by the ratio

$$\frac{\Gamma(\eta\to 2\gamma)/m_\eta{}^3}{\Gamma(\pi^0\to 2\gamma)/m_\pi{}^3}=\left[\frac{[(9+R_1{}^2)/6]\cos\theta_P-(\sqrt{2}/3)R_2{}^2\sin\theta_P}{R_1}\right]^2. \tag{45}$$

TABLE I. Meson decay widths that are independent of the pseudoscalar mixing angle θ_P, for two choices of the parameters β and θ_V.

Decay mode	Widths (keV) Theory $\beta=0.75$, $\theta_V=4.5°$	$\beta=1.0$, $\theta_V=-4.85°$	Experiment
$\phi\to\pi\rho$	0.52 ± 0.03	0.52 ± 0.03	0.57 ± 0.03 (Ref. 14)
$\phi\to\pi^0\gamma$	8.41 ± 0.59	8.57 ± 0.60	$5.7\ \pm2.1$ (Ref. 15)
$\phi\to e^+e^-$	1.12 ± 0.13	1.23 ± 0.15	1.31 ± 0.10 (Ref. 12)
$\omega\to e^+e^-$	0.89 ± 0.11	0.55 ± 0.07	0.76 ± 0.17 (Ref. 12)
$\pi^0\to\gamma\gamma$	$(8.09\pm0.40)\times10^{-3}$	$(6.72\pm0.34)\times10^{-3}$	$(7.92\pm0.42)\times10^{-3}$ (Ref. 1)
$\omega\to\pi^0\gamma$	870	864	$880\ \pm60$ (Ref. 12)
$K^{*0}\to K^0\gamma$	$118.4\ \pm8.3$	$100\ \pm7.0$	$75\ \pm35$ (Ref. 16)
$K^{*+}\to K^+\gamma$	$76.9\ \pm5.2$	$81.2\ \pm5.7$	<80 (Ref. 17)
$\rho^-\to\pi^-\gamma$	$98.4\ \pm6.7$	$82.3\ \pm5.8$	$35\ \pm10$ (Ref. 2)

Its experimental value is 0.61 ± 0.09. The implied restrictions upon θ_P are

$$52.3°\le\theta_P\le56.8° \text{ for } \beta=0.75,\quad \theta_V=4.5° \tag{46a}$$

and

$$57.1°\le\theta_P\le61.1° \text{ for } \beta=1.0,\quad \theta_V=-4.85°. \tag{46b}$$

The other decay modes for which the experimental widths are known, such as $\rho^0\to\eta\gamma$, $\omega\to\eta\gamma$, and $\phi\to\eta\gamma$ are consistent with the bound given in Eq. (46a) and the general tendency is to favor the lower angles in this range. In particular, the width $\Gamma(\rho^0\to\eta\gamma)$ which is independent of β and θ_V produces the constraint $\theta_P\le56.5°$. When compared with Eq. (46b), this constraint is the first indication that negative values for θ_V might be inconsistent with the data. A complementary conclusion about this matter can be drawn from the ratio

$$\frac{\Gamma(\eta'\to\rho^0\gamma)}{\Gamma(\eta'\to\omega\gamma)}=\left(\frac{m_\omega}{m_\rho}\right)^{2\beta}\left(\frac{1-m_\rho^2/m_{\eta'}^2}{1-m_\omega^2/m_{\eta'}^2}\right)^3\left[\frac{\sin\theta_P}{(R_1/3)\cos\theta_V\sin\theta_P+(2/3)R_2\sin\theta_V\cos\theta_P}\right]^2, \tag{47}$$

which depends strongly upon the sign of θ_V and has the experimental value 9.9 ± 2.0.[18] In fact, this ratio varies from 8.9 to 9.5 (increasing) for the choice $\beta=0.75$ and $\theta_V=4.5°$ in the range $51°\le\theta_P\le61°$. When $\beta=0.75$ and $\theta_V=-4.5°$ this variation is from 14.5 to 13.5 (decreasing) while for the choice $\beta=1.0$ and $\theta_V=-4.85°$, the ratio is between 15 and 14 (decreasing), all this for the same range

TABLE II. Meson decay widths that include the η-η' system, using the same parameters β and θ_V chosen in Table I, together with adequate values for θ_P. The experimental upper limits for the η' decays are obtained assuming $\Gamma(\eta'\to\text{all})<1$ MeV.

Decay mode	Widths (keV) Theory $\beta=0.75$, $\theta_V=4.5°$, $\theta_P=54.7°$	$\beta=1.0$, $\theta_V=-4.85°$, $\theta_P=59.1°$	Experiment
$\eta\to2\gamma$	0.322 ± 0.022	0.323 ± 0.016	$0.324\pm\ 0.046$ (Ref. 1)
$\rho^0\to\eta\gamma$	$39\ \pm3$	$30.8\ \pm2.2$	$50\ \pm13$ (Ref. 19)
$\omega\to\eta\gamma$	$3.6\ \pm0.2$	$4.3\ \pm0.3$	$3.0\ ^{+2.5}_{-1.8}$ (Ref. 19)
$\phi\to\eta\gamma$	$78\ \pm5$	$41.3\ \pm2.9$	$62\ \pm17$ (Ref. 15) $55\ \pm12$ (Ref. 19)
$\eta'\to2\gamma$	$6.2\ \pm0.4$	$7.3\ \pm0.4$	<20 (Ref. 12)
$\phi\to\eta'\gamma$	$0.14\ \pm0.01$	$0.12\ \pm0.01$	...
$\eta'\to\rho^0\gamma$	152	168	<304 (Ref. 12)
$\eta'\to\omega\gamma$	17	12	<50 (Ref. 20)

of θ_P mentioned above. As we can see, the present experimental upper limit 12.9 for the ratio given in Eq. (47) does not favor negative values for the vector mixing angle and we will rule them out for the time being.

The decay width $\Gamma(\eta' \to 2\gamma)$ is calculated from Eq. (45) after changing m_η into $m_{\eta'}$ and θ_P into $-(\pi/2 - \theta_P)$ as can be seen immediately from the Lagrange function Eq. (32). For the widths $\Gamma(v \to p\gamma)$, we use Eq. (42) and the decay $\eta' \to \rho^0\gamma$ is calculated according to

$$\Gamma(\eta' \to \rho^0\gamma) = \frac{\alpha}{2}\sin^2\theta_P\left(\frac{m_{\eta'}}{m_\rho}\right)^2\left(1 - \frac{m_\rho{}^2}{m_{\eta'}{}^2}\right)^3 m_{\eta'} \,. \tag{48}$$

In order to present our results for the preferred solution $\beta = 0.75$ and $\theta_V = 4.5°$, we choose $\theta_P = \theta_0 = 54.7°$ that corresponds to the "ideal" mixing angle in which case the η' meson belongs entirely to the singlet representation of U(3). Table II contains our predictions together with the experimental data when known. For completeness, we have also included in this table the predictions for the choice $\beta = 1.0$ and $\theta_V = -4.85°$. In this case, we have taken $\theta_P = 59.1°$ which corresponds to the value 0.61 for the ratio given in Eq. (45).

We have presented here a rather successful model that incorporates in a consistent picture the leptonic decays of the vector mesons together with the one- and two-photon meson radiative decays. Moreover, we have been able to see how our particular way of introducing the U(3)-symmetry breaking has played a crucial role in obtaining good agreement with the data, notably in some ϕ and K^* decays. The only exception in which our model fails is in the decay $\rho^- \to \pi^-\gamma$ which, to our knowledge, still constitutes a challenge for all present models of radiative meson decays.

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