PHYSICAL REVIEW D VOLUME 18, NUMBER 3 1 AUGUST 1978

Pionic decays of $\phi(1020)$ and $f'(1514)$

J. Pasupathy*
Centre for Theoretical Studies, Indian Institute of Science, Bangalore-560012, India

C. A. Singh
Tata Institute of Fundamental Research, Homi Bhabha Road, Bombay-400 005, India
(Received 7 July 1977)

The decay amplitudes $\phi(1020)\to 3\pi$ and $f'(1514)\to\pi\pi$ are calculated using unitarity corrections to the Okubo-Zweig-Iizuka rule. We use dispersion relations for the amplitudes $\phi\to\rho\pi$ and $f'\to\pi\pi$ and evaluate the contributions arising from intermediate states such as $K\bar{K}$, $K^*\bar{K}$, etc. The $K\bar{K}$ intermediate state contributes to the absorptive part of the amplitude $\phi\to\rho\pi$ a value which corresponds to a partial width $\Gamma(\phi\to 3\pi)$ of 150 keV. The contribution of the $K\bar{K}$ and $K^*\bar{K}+\bar{K}K^*$ to the dispersive part of the amplitude $\phi\to\rho\pi$ within the context of approximations employed by us is somewhat larger and corresponds to a partial width of 900 keV for $\phi\to 3\pi$. For the $f'\to\pi\pi$ amplitude we have evaluated the contributions of the $K\bar{K}$, $\eta\eta$, and $K^*\bar{K}+\bar{K}^*K$ intermediate states using the dual model for the amplitudes $K\bar{K}\to\pi\pi$, $\eta\eta\to\pi\pi$, and $K^*\bar{K}\to\pi\pi$. The effective strength of the amplitude $f'\to\pi\pi$ arising from such unitarity corrections to the Okubo-Zweig-Iizuka rule corresponds to a partial width $\Gamma(f'\to\pi\pi)$ of 170 keV via the absorptive part and a value of 500 keV via the dispersive part.

I. INTRODUCTION

It is well known[1] that the pionic decays of the mesons $\phi(1020)$ and $f'(1514)$ are suppressed as compared to the pionic decays of their SU(3) partners $\omega(783)$ and $f(1270)$. A quantitative estimate of the suppression factors can be obtained as follows. It is known experimentally[2,3] from Dalitz-plot analyses of $\phi\to 3\pi$ that the process $\phi\to\pi^+\pi^-\pi^0$ is dominated by $\rho\pi$ production. In the BNL bubble-chamber experiment[2] the ratio of the number of $\phi\to 3\pi$ events with dipion mass above 714 MeV to the number below 714 MeV was found to be 0.55 ± 0.24 consistent with the value 0.45 expected from a $\rho\pi$ final state with a Breit-Wigner distribution for the ρ meson. In the storage-ring experiment by the Paris group[3] the observed Dalitz plot of events in $e^+e^-\to\phi\to\pi^+\pi^-\pi^0$ was compared with the prediction of the Gell-Mann–Sharp–Wagner[4] (GSW) model, that is, the process is assumed to proceed as $\phi\to\rho\pi\to 3\pi$. Their analysis shows that this mechanism accounts for at least 80% of all the events. We shall then compare the $\phi\rho\pi$ and $\omega\rho\pi$ couplings by using the GSW model for both $\phi\to 3\pi$ and $\omega\to 3\pi$. We write then the matrix element for $V\to\pi^+(p_1)+\pi^-(p_2)+\pi^0(p_3)$ as

$$T=2\epsilon_{\mu\nu\lambda\sigma}\epsilon^{\mu}p_1^{\nu}p_2^{\lambda}p_3^{\sigma}g_{V\rho\pi}g_{\rho\pi\pi}\left[\frac{1}{(p_1+p_2)^2-m_\rho^2+im_\rho\Gamma_\rho}+\frac{1}{(p_2+p_3)^2-m_\rho^2+im_\rho\Gamma_\rho}+\frac{1}{(p_3+p_1)^2-m_\rho^2+im_\rho\Gamma_\rho}\right], \quad (1.1)$$

where ϵ^{μ} is the polarization vector of the initial vector meson V (ϕ or ω). Using relativistic phase space and the values $m_\rho=770$ MeV, $\Gamma_\rho=150$ MeV we get from Eq. (1.1)

$$\frac{\Gamma(\phi\to\pi^+\pi^-\pi^0)}{\Gamma(\omega\to\pi^+\pi^-\pi^0)}=32.2(g_{\phi\rho\pi}^2/g_{\omega\rho\pi}^2). \quad (1.2)$$

Using further the values[1] $\Gamma(\omega\to 3\pi)\approx 9$ MeV and $\Gamma(\phi\to 3\pi)\approx 650$ keV we get for the ratio of the coupling strengths

$$g_{\phi\rho\pi}^2/g_{\omega\rho\pi}^2\approx 2.3\times 10^{-3} \quad (1.3)$$

and for their individual values

$$g_{\omega\rho\pi}^2/4\pi\approx 22\ \mathrm{GeV}^{-2}$$

and (1.4)

$$g_{\phi\rho\pi}^2/4\pi\approx 5.2\times 10^{-2}\ \mathrm{GeV}^{-2}.$$

It is worth noting that an independent determination[5] of $g_{\omega\rho\pi}$ using superconvergence relations for $\rho\pi$ scattering yields

$$g_{\omega\rho\pi}^2/4\pi\approx 20\ \mathrm{GeV}^{-2} \quad (1.5)$$

consistent with Eq. (1.4).

Comparison of $f'\to\pi\pi$ and $f\to\pi\pi$ is made from

$$\frac{\Gamma(f'\to\pi\pi)}{\Gamma(f\to\pi\pi)}=\frac{m_f^2(m_{f'}^2-4m_\pi^2)^{5/2}}{m_{f'}^2(m_f^2-4m_\pi^2)^{5/2}}\frac{g_{f'\pi\pi}^2}{g_{f\pi\pi}^2}. \quad (1.6)$$

A recent experiment[6] puts the value of the branching ratio for $f'\to\pi\pi$ at $(1.2\pm0.4)\times10^{-2}$ and $\Gamma(f'\to \text{all})$ at 66 ± 10 MeV. Using the median value $\Gamma(f'\to\pi\pi)\approx 790$ keV and $\Gamma(f\to\pi\pi)\approx150$ MeV (Ref. 1) we then get

$$g_{f'\pi\pi}{}^2/g_{f\pi\pi}{}^2\approx 2.8\times10^{-3}. \tag{1.7}$$

These suppressions Eq. (1.3) and Eq. (1.7) are to be contrasted with the fact that the partial widths $\Gamma(\phi\to K\bar{K}$, $\Gamma(f'\to K\bar{K})$ are characterized by normal coupling strengths and are in fact consistent with the expectations from SU(3) symmetry.[7]

The suppression of the pionic decays is attributed to the validity of the Okubo-Zweig-Iizuka (OZI) rule,[8] that is, since ϕ and f' are pure $s\bar{s}$ states, the transitions ϕ or $f'\to$ pions involve disconnected quark diagrams and are therefore forbidden. However, since the OZI rule violates unitarity of the S matrix, it cannot be an exact selection rule, and the experimentally observed small but finite amplitudes for ϕ or $f'\to$ pions can be attributed to unitarity corrections to the rule. In two brief earlier communications[9,10] we had computed the contributions to the absorptive parts of the amplitudes $\phi\to\rho\pi$ and $f'\to\pi\pi$ from intermediate states such as $K\bar{K}$ and found that the order of magnitude of these contributions is compatible with the experimentally observed magnitudes. In Refs. 9 and 10 we had used simple pole models for the amplitudes $K\bar{K}\to\rho\pi$ and $K\bar{K}\to\pi\pi$. In the present work we extend our earlier work by computing the dispersive parts of the amplitudes as well. We use the dual model for the scattering amplitudes involved in the calculations. We find that the dispersive parts of the couplings $g_{\phi\rho\pi}$ and $g_{f'\pi\pi}$ are comparable to the absorptive part and, in particular, within the context of the specific approximation schemes we use, are somewhat larger than the absorptive parts of the effective couplings.

The material of this paper is organized as follows. In Sec. II we outline the general procedure and evaluate the contribution of the $K\bar{K}$ intermediate state to the effective coupling $g_{\phi\rho\pi}$. In Sec. III we consider the contribution of the $K^*\bar{K}$ and $\bar{K}^*K$ intermediate states to $g_{\phi\rho\pi}$. In Secs. IV and V we extend our method to consider the decay $f'\to\pi\pi$. In Sec. IV the contribution to the absorptive part of $g_{f'\pi\pi}$ from intermediate states $K\bar{K}$, $\eta\eta$, and $K^*\bar{K}+\bar{K}^*K$ is evaluated while in Sec. V the contribution of these states to the dispersive part is evaluated. In the final section we present a discussion of our results. In the Appendix a variation of the calculations presented in Sec. IV is given.

II. $K\bar{K}$ INTERMEDIATE STATE CONTRIBUTION TO $\phi\to\rho\pi$

The invariant matrix element for the transition $\phi\to\rho\pi$ can be written as

$$M=\epsilon^\mu M_\mu=\epsilon_{\mu\nu\lambda\sigma}\epsilon^\mu P^\nu\epsilon_1^\lambda q_1^\sigma F(P^2,q_1^{\,2},q_2^{\,2}), \tag{2.1}$$

where the invariant amplitude F coincides with the coupling constant $g_{\phi\rho\pi}$ introduced in Eq. (1.1) when all the momenta are on their mass shells. ϵ^μ is the polarization vector of ϕ and P is its momentum, while ϵ_1 and q_1 denote those of ρ. We shall assume that F satisfies a standard dispersion relation in the variable $P^2=s$ with no subtractions, that is,

$$F(s)=\frac{1}{\pi}\int\frac{\mathrm{Im}F(s')}{s'-s-i\epsilon}\,ds'. \tag{2.2}$$

The absorptive part $\mathrm{Im}F(s)$ can be computed from unitarity as follows. Writing

$$M_\mu=D_\mu+iA_\mu \tag{2.3}$$

the absorptive part is given following standard procedure by

$$A_\mu=\tfrac{1}{2}(2q_1{}^0)^{1/2}\sum_n(2\pi)^4\delta^4(P_n-q_1-q_2)\times\langle q_1\epsilon_1|\,j_\pi|n\rangle\langle n|J_\mu|0\rangle, \tag{2.4}$$

where $j_\pi=(\Box+m_\pi{}^2)\Pi(x)$ is the source of the pion field $\Pi(x)$, and $J_\mu=(\Box+m_\phi{}^2)\Phi_\mu(x)$ is the source of the ϕ field $\Phi_\mu(x)$. In the sum over intermediate states $|n\rangle$ with $J^P=1^-$ and $I^G=0^-$, we shall restrict the sum to those which correspond to connected matrix elements for both terms in the product that appears on the right-hand side of Eq. (2.4). That is, we shall assume validity of the OZI rule in the right-hand side of Eq. (2.4) as a first approximation. The lowest-mass state of interest is then the $K\bar{K}$ state. Since the threshold for this state lies below the ϕ mass it contributes both to the real and imaginary part of the coupling constant

$$F(s=m_\phi{}^2)=g_{\phi\rho\pi}=g_R+ig_I. \tag{2.5}$$

The higher-mass intermediate states such as $K^*\bar{K}, K^*\bar{K}^*,\ldots,\Lambda\bar{\Lambda},\Sigma\bar{\Sigma}$, etc., however, contribute only to the dispersive part.

Consider the $K\bar{K}$ intermediate-state contribution. The $\phi K\bar{K}$ vertex appearing in Eq. (2.4) can be written as

$$\langle K(k_1)\bar{K}(k_2)|J_\mu|0\rangle=g_{\phi K\bar{K}}(4k_1^0k_2^0)^{-1/2}\times(k_1-k_2)_\mu\mathcal{K}((k_1+k_2)^2), \tag{2.6}$$

where we normalize the vertex function $\mathcal{K}(s)$ to unity at the ϕ mass. The other matrix element appearing in Eq. (2.4) is the scattering amplitude for the $K\bar{K}$ pair to go into the final ρ and π,

$$K(k_1)+\bar{K}(k_2)\to\rho(\epsilon_1,q_1)+\pi(q_2), \tag{2.7}$$

for which we can write from general invariance considerations

$$\langle q_1\epsilon_{1\mu}|\,j_\pi|k_1k_2\rangle=(8k_1^0k_2^0q_1^0)^{-1/2}\epsilon_{\mu\nu\lambda\sigma}\epsilon_1^\mu k_1^\nu k_2^\lambda q_1^\sigma R(s,t), \tag{2.8}$$

where $s=(k_1+k_2)^2$ and $t=(k_2-q_2)^2$. Using Eqs. (2.1), (2.4), (2.6), and (2.8) we find

$$\mathrm{Im}F(s)=\frac{-1}{32\pi}\frac{2k^3}{\sqrt{s}}\int_{-1}^{1}dz\,(1-z^2)\,\mathrm{Re}[g_{\phi K\bar K}\mathcal{K}(s)R^*(s,z)] \tag{2.9}$$

where $k=\frac{1}{2}(s-4m_K^{\,2})^{1/2}$ is the center-of-mass momentum and z is the cosine of the c.m. scattering angle.

To proceed further we shall assume that the amplitude $R(s,t)$ is given by a dual amplitude, analogous to the Veneziano formula[11] for $\pi\pi\to\omega\pi$ which has the same Lorentz structure as $K\bar K\to\rho\pi$. We can then write

$$R(s,t)=\beta\frac{\Gamma(1-\alpha_\omega(s))\Gamma(1-\alpha_{K^*}(t))}{\Gamma(2-\alpha_\omega(s)-\alpha_{K^*}(t))} \tag{2.10}$$

Using linear trajectories with $\alpha'_\omega=\alpha'_{K^*}\simeq 1\ \mathrm{GeV}^{-2}$ we can substitute Eq. (2.10) in Eq. (2.9) to get

$$g_I=\mathrm{Im}\,F(s=m_\phi^{\,2})$$

$$=\frac{-\beta g_{\phi K\bar K}}{32\pi}\frac{2k^3}{m_\phi(m_\omega^{\,2}-m_\phi^{\,2})}(-4.63) \tag{2.11}$$

$$=-\beta g_{\phi K\bar K}(4.4\times10^{-4})\,. \tag{2.12}$$

The numerical factor in Eq. (2.11) was arrived at by performing the angle integration in Eq. (2.9) numerically. The constant β appearing in Eq. (2.10) can be determined by computing the residue of $R(s,t)$ at the K^* pole occurring at $\alpha_{K^*}(t)=1$ and is found to be

$$\beta=2g_{K^*\bar K\pi}g_{\bar K^*K\rho}\,. \tag{2.13}$$

If, instead, we relate β to the residue of the s-channel ω pole at $\alpha_\omega(s)=1$ we obtain

$$\beta=2g_{\omega K\bar K}g_{\omega\rho\pi}\,. \tag{2.14}$$

If we use SU(3) symmetry to relate the coupling strengths these two values of β are identical and using the SU(3) relation $2g_{\omega K\bar K}=g_{\rho\pi\pi}$ we have

$$\beta=g_{\rho\pi\pi}g_{\omega\rho\pi}\,. \tag{2.15}$$

Substituting this in Eq. (2.12) we get

$$g_I=-g_{\rho\pi\pi}g_{\phi K\bar K}g_{\omega\rho\pi}(4.4\times10^{-4}) \tag{2.16}$$

$$\simeq-2.25\times10^{-2}g_{\omega\rho\pi}\,. \tag{2.17}$$

This effective coupling strength contributes a value of approximately 150 keV to the partial width $\Gamma(\phi\to3\pi)$.

To compute the dispersive part of the contribution of the $K\bar K$ state to the $\phi\rho\pi$ coupling we need to know both $R(s,t)$ and $\mathcal{K}(s)$ occurring in Eq. (2.6) and Eq. (2.8) for all s values. We shall assume that $R(s,t)$ is once again given by Eq. (2.10). However, since the β function has poles for real s, to avoid absurdities we shall approximate Eq. (2.10) by its asymptotic form,[12]

$$R(s,t)\approx\beta\Gamma(1-\alpha_{K^*}(t))e^{-i\pi\alpha_{K^*}(t)}(\alpha'_\omega s)^{\alpha_{K^*}(t)-1}\,. \tag{2.18}$$

The extra -1 in the exponent, of course, reflects the fact that in $K\bar K\to\rho\pi$ the ρ must be produced with helicity ±1 due to spin-parity selection rules. Very little is known about the vertex function $\mathcal{K}(s)$ except that it is also expected to decrease for large s. Since we are interested in only obtaining estimates of the dispersive part of the $\phi\rho\pi$ coupling, we shall use the approximation $\mathcal{K}(s)\approx\mathcal{K}(m_\phi^{\,2})=1$. There is no difficulty with convergence of the dispersion integral in Eq. (2.2) since $R(s,t)$ as given by Eq. (2.18) is a rapidly decreasing function of s. The latter fact also gives us some assurance that our approximation $\mathcal{K}(s)\approx\mathcal{K}(m_\phi^{\,2})$ will cause no violent error in our calculations. Given these two approximations for $R(s,t)$ and $\mathcal{K}(s)$ the rest of the calculation is straightforward.[13] Using Eqs. (2.2), (2.9), (2.15), and (2.18) we get

$$g_R(K\bar K)\approx-2.28\times10^{-2}g_{\omega\rho\pi}\,. \tag{2.19}$$

III. $K^*\bar K+\bar K^*K$ CONTRIBUTION

The next state of interest in the unitarity sum in Eq. (2.4) is the $K\bar K\pi$ state. An elementary angular momentum analysis shows that the relative angular momentum between the pion and either one of the kaons must be at least one. We shall therefore approximate the $K\bar K\pi$ state as the $K^*\bar K$ plus $\bar K^*K$ state. Returning to Eq. (2.4) the $\phi K^*\bar K$ vertex appearing in the right-hand side of that equation can be written as

$$(4k_1^0k_2^0)^{1/2}\langle K^*\bar K|J_\mu|0\rangle=G_1\epsilon_{\mu\nu\lambda\sigma}P^\nu\epsilon_2^\lambda k_2^\sigma H(s)\,, \tag{3.1}$$

where ϵ_2 is the polarization vector of K^* with momentum k_2, k_1 is the momentum of the kaon, and we normalize the vertex function $H(s)$ to unity at the ϕ mass, i.e., $H(m_\phi^{\,2})=1$. The other matrix element describing the transition $K^*\bar K\to\rho\pi$ has the structure

$$\langle q_1\epsilon_1|j_\pi|K^*\bar K\rangle=(8k_1^0k_2^0q_1^0)^{-1/2}T(\epsilon_2,k_2,k_1;\epsilon_1,q_1,q_2)\,. \tag{3.2}$$

The amplitude T has five invariant terms which can be chosen in the following way:

$$T(\epsilon_2,k_2,k_1;\epsilon_1,q_1,q_2)$$

$$=\epsilon_2\cdot\epsilon_1E(s,t)+\epsilon_2\cdot\Delta\epsilon_1\cdot\Delta D(s,t)+\epsilon_2\cdot Q\epsilon_1\cdot\Delta C(s,t)$$

$$+\epsilon_2\cdot\Delta\epsilon_1\cdot QB(s,t)+\epsilon_2\cdot Q\epsilon_1\cdot QA(s,t)\,, \tag{3.3}$$

with $\Delta=(k_1-q_2)$, $Q=(k_1+q_2)$, and $s=(k_1+k_2)^2$ and $t=\Delta^2$. The invariant amplitudes A, B, C, D, and E are free of kinematic singularities. It is clear from Eq. (2.4) that they enter linearly in the computation of Im$F(s)$. The amplitude $E(s,t)$ makes the dominant contribution since, unlike other amplitudes, it has no extra momentum factors and falls off less rapidly asymptotically. We describe below the computation of the contribution of $E(s,t)$ only. From Eq. (2.4) we find

$$\mathrm{Im}F^{(E)}(s)=\frac{-1}{32\pi}\frac{2k^2}{q\sqrt{s}}\int_{-1}^{1}dz\,z\,\mathrm{Re}[G_1H(s)E^*(s,t)]\,, \tag{3.4}$$

where k and q are the center-of-mass momenta of K^* and ρ, respectively and z is the cosine of the c.m. scattering angle. To proceed further we shall assume as in the previous section, that $E(s,t)$ is given by a dual amplitude

$$E(s,t)=\beta_E\frac{\Gamma(1-\alpha_\omega(s))\Gamma(1-\alpha_{K^*}(t))}{\Gamma(1-\alpha_\omega(s)-\alpha_{K^*}(t))}\,. \tag{3.5}$$

The constant β_E is determined by computing the residue of the K^* pole in the t channel,

$$\beta_E=g_{K^*\bar{K}^*\rho}g_{K^*\bar{K}\pi}=\tfrac{1}{2}g_{\rho\pi\pi}^{\ 2}\,, \tag{3.6}$$

where in the last step we have used universality of ρ couplings and SU(3) symmetry. To arrive at an estimate of the contribution of the amplitude $E(s,t)$ to $g_{\phi\rho\pi}$ we use the same approximations as in the preceding section, i.e., we replace Eq. (3.5) by its asymptotic form

$$E(s,t)\approx\beta_E\Gamma(1-\alpha_{K^*}(t))e^{-i\pi\alpha_{K^*}(t)}(\alpha'_\omega s)^{\alpha_{K^*}(t)} \tag{3.7}$$

and set $H(s)\simeq H(s=m_\phi^{\ 2})=1$. The $\phi K^*\bar{K}$ coupling constant G_1 appearing in Eq. (3.1) is related to $g_{\omega\rho\pi}$ by SU(3) symmetry

$$G_1=g_{\phi K^*\bar{K}}=\frac{1}{\sqrt{2}}g_{\omega\rho\pi}\,. \tag{3.8}$$

Using Eq. (2.4) and Eqs. (3.4) to (3.8) we get for the $K^*\bar{K}+\bar{K}^*K$ contribution[14]

$$G_R^{(E)}(K^*\bar{K}+\bar{K}^*K)\simeq-3.2\times10^{-2}g_{\omega\rho\pi}\,. \tag{3.9}$$

Adding the contribution of the $K\bar{K}$ intermediate state given by Eq. (2.19) we find

$$g_R\approx-5.5\times10^{-2}g_{\omega\rho\pi}\,. \tag{3.10}$$

The effective $\phi\rho\pi$ coupling given by Eq. (3.10) corresponds to a partial width $\Gamma(\phi\to3\pi)$ of approximately 900 keV.

IV. ABSORPTIVE PART OF THE $f'\pi\pi$ COUPLING STRENGTH

The procedure for computing the amplitude $f'(1514)\to\pi\pi$ is identical to that for $\phi\to3\pi$ except for the following slight differences. There are kinematical differences introduced by spin: f' has spin 2 and the decay is a d-wave decay. Further, there is more than one channel allowed by the OZI rule that is open at the f' mass, namely $K\bar{K}$, $K\bar{K}\pi$, and $\eta\eta$. These changes are easily taken into account. We can write the S-matrix element as

$$\begin{aligned}S&=\langle\pi(q_1)\pi(q_2)\,\mathrm{out}\,|f'(\epsilon_{\mu\nu},P)\rangle\\&=i(2\pi)^4\delta^{(4)}(P-q_1-q_2)(2P^0)^{-1/2}\\&\quad\times\langle\pi(q_1)\pi(q_2)|J_{\mu\nu}|0\rangle\epsilon^{\mu\nu}\,,\end{aligned} \tag{4.1}$$

where $\epsilon^{\mu\nu}$ is the polarization tensor of the f', P is its momentum, and q_1,q_2 are the four-momenta of the final pions. $J_{\mu\nu}$ is the source of the f' field. The transition matrix element is given by

$$\begin{aligned}M_{\mu\nu}&=i(2q_2^0)^{1/2}\int d^4x\,e^{iq_1\cdot x}\\&\quad\times\langle\pi(q_2)|[j_\pi(x),J_{\mu\nu}(0)]\theta(x_0)|0\rangle\,,\end{aligned} \tag{4.2}$$

for which we can write from general invariance grounds

$$M_{\mu\nu}=Q_\mu Q_\nu\mathcal{F}(P^2,q_1^{\ 2},q_2^{\ 2}) \tag{4.3}$$

with

$$Q_\mu=(q_1-q_2)_\mu\,.$$

When all the momenta are on the mass shell the invariant function $\mathcal{F}(P^2,q_1^{\ 2},q_2^{\ 2})$ is the coupling constant $g_{f'\pi\pi}$ introduced in Sec. I. We shall asume $\mathcal{F}(P^2,q_1^{\ 2},q_2^{\ 2})$ to satisfy an unsubtracted dispersion relation in the variable $P^2=s$,

$$\mathcal{F}(s)=\frac{1}{\pi}\int\frac{\mathrm{Im}\mathcal{F}(s')}{s'-s-i\epsilon}ds'\,, \tag{4.4}$$

where Im$\mathcal{F}(s)$ is to be computed from the absorptive part of $M_{\mu\nu}$. Writing $M_{\mu\nu}=D_{\mu\nu}+iA_{\mu\nu}$, the absorptive part $A_{\mu\nu}$ is given by

$$\begin{aligned}A_{\mu\nu}&=\tfrac{1}{2}(2q_2^0)^{1/2}\sum_n(2\pi)^4\delta^{(4)}(P_n-q_1-q_2)\\&\quad\times\langle\pi(q_2)|j_\pi(0)|n\rangle\langle n|J_{\mu\nu}|0\rangle\,.\end{aligned} \tag{4.5}$$

As in the case of our discussion in Sec. II of $\varphi\to\rho\pi$, we keep only those intermediate states which yield connected matrix elements in the right-hand side of Eq. (4.5). The lowest-mass state of interest is then the $K\bar{K}$ state for which we can write

$$\begin{aligned}\langle k_1k_2|J_{\mu\nu}|0\rangle&=(4k_1^0k_2^0)^{-1/2}g(k_1-k_2)_\mu\\&\quad\times(k_1-k_2)_\nu V((k_1+k_2)^2)\,,\end{aligned} \tag{4.6}$$

where g is the $f'K\bar{K}$ coupling constant and the vertex function $V(s)$ is normalized to unity at the f' mass. The $K\bar{K}\to\pi\pi$ transition matrix element can be written as

$$\langle\pi(q_2)|j_\pi|k_1k_2\rangle=(8k_1^0k_2^0q_2^0)^{-1/2}T(s,t)\,. \tag{4.7}$$

Using Eqs. (4.3), (4.5), (4.6), and (4.7) we get

$$\text{Im}\mathfrak{F}(s) = \frac{-1}{32\pi}\frac{k^3}{q^2\sqrt{s}}\int_{-1}^{1} dz\,(3z^2-1)\,\text{Re}[gV(s)T^*(s,t)]. \tag{4.8}$$

To proceed further we adopt a dual-model[15] amplitude for $K\bar{K}\to\pi\pi$,

$$T(s,t) = \beta\frac{\Gamma(1-\alpha_\rho(s))\Gamma(1-\alpha_{K^*}(t))}{\Gamma(1-\alpha_\rho(s)-\alpha_{K^*}(t))}. \tag{4.9}$$

The constant β is fixed by evaluating the residue of the K^* pole in the t channel to be

$$\beta = 2g_{K^*\bar{K}\pi}{}^2. \tag{4.10}$$

Using Eq. (4.3) and Eqs. (4.5) to (4.10) we get for the $K\bar{K}$ contribution to the absorptive part of the $f'\pi\pi$ coupling

$$\mathcal{G}_I(K\bar{K}) = -1.1\times10^{-3}g_{f\pi\pi}, \tag{4.11}$$

where $\mathcal{G}_I = \text{Im}\mathfrak{F}(P^2 = m_{f'}{}^2)$ and we have used SU(3) to relate the $f'K\bar{K}$ coupling constant occurring in Eq. (4.6) to the $f\pi\pi$ coupling constant

$$g = g_{f'K\bar{K}} = \frac{1}{\sqrt{2}}g_{f\pi\pi} \tag{4.12}$$

in arriving at Eq. (4.11).

The calculation of the $\eta\eta$ intermediate-state contribution proceeds along similar lines. For the $f'\eta\eta$ vertex we can write an expression analogous to Eq. (4.6) with the constant g now standing for the $f'\eta\eta$ coupling. Following Baacke, Jacob, and Pokorski,[16] we write for the $\eta\eta\to\pi\pi$ scattering amplitude $T_{\eta\eta}(s,t)$,

$$T_{\eta\eta}(s,t) = \beta'[V(s,t)+V(s,u)+V(u,t)], \tag{4.13}$$

where

$$V(s,t) = \frac{\Gamma(1-\alpha_f(s))\Gamma(1-\alpha_{A_2}(t))}{\Gamma(1-\alpha_f(s)-\alpha_{A_2}(t))} \tag{4.14}$$

and similarly for $V(s,u)$ and $V(u,t)$. Evaluating the residue at the f pole we find

$$\beta' = \frac{2}{\alpha'}g_{f\eta\eta}g_{f\pi\pi}. \tag{4.15}$$

The coupling constants $g_{f\eta\eta}$ and $g_{f'\eta\eta}$ can be related to $g_{f\pi\pi}$ using SU(3) symmetry and the OZI rule,

$$g_{f\eta\eta} = \cos^2\alpha\, g_{f\pi\pi}, \quad g_{f'\eta\eta} = \sqrt{2}\sin^2\alpha\, g_{f\pi\pi}, \tag{4.16}$$

where α in the mixing angle appearing in the η wave function

$$\eta = \cos\alpha\frac{1}{\sqrt{2}}(u\bar{u}+d\bar{d}) - \sin\alpha s\bar{s}. \tag{4.17}$$

Using the Gell-Mann–Okubo mass formula one finds $\cos^2\alpha\approx\sin^2\alpha\approx0.5$. Using Eqs. (4.13) to (4.17) we get for the $\eta\eta$ contribution to the absorptive part of the $f'\pi\pi$ coupling

$$\mathcal{G}_I(\eta\eta) = 1.04\times10^{-2}g_{f\pi\pi}. \tag{4.18}$$

The next more massive intermediate state of interest is the $K\bar{K}\pi$ state. Angular momentum analysis shows that the relative angular momentum between the pion and either of the kaons must be at least unity. Therefore we shall approximate this state as $K^*\bar{K}+\bar{K}^*K$. We can write for the $f'K^*\bar{K}$ vertex

$$(4k_1^0k_2^0)^{1/2}\langle K^*(k_2,\epsilon)K(k_1)|J_{\mu\nu}|0\rangle = g_1\epsilon_{\mu\rho\lambda\sigma}\epsilon^\rho(k_1-k_2)^\lambda(k_1+k_2)^\sigma(k_1-k_2)_\nu V_1((k_1+k_2)^2). \tag{4.19}$$

Here ϵ^ρ is the polarization vector of K^*, g_1 is the $f'K^*\bar{K}$ coupling strength, and $V_1\ (s=m_{f'}{}^2)=1$. The other matrix element describing the transition $K^*\bar{K}\to\pi\pi$ has the same Lorentz structure as the amplitude $K\bar{K}\to\rho\pi$ discussed in Sec. II. We can write

$$(8q_2^0k_1^0k_2^0)^{1/2}\langle\pi(q_2)|j_\pi|K^*\bar{K}\rangle = \epsilon_{\mu\nu\lambda\sigma}\epsilon^\mu k_1^\nu k_2^\lambda q_2^\sigma U(s,t). \tag{4.20}$$

The contribution of the $K^*\bar{K}$ intermediate state to the absorptive part is found from Eq. (4.19) and Eq. (4.20) to be

$$\text{Im}\mathfrak{F}(s) = \frac{-1}{32\pi}\frac{3\sqrt{s}\,k^4}{q}\int_{-1}^{1}dz\,z(1-z^2) \times \text{Re}[g_1V_1(s)U^*(s,t)], \tag{4.21}$$

where k and q are the initial and final center-of-mass momenta. As in the case of $K\bar{K}\to\rho\pi$, we adopt a dual model for the amplitude $U(s,t)$ appearing in Eq. (4.20), that is, we write

$$U(s,t) = \beta_1\frac{\Gamma(1-\alpha_\rho(s))\Gamma(1-\alpha_{K^*}(t))}{\Gamma(2-\alpha_\rho(s)-\alpha_{K^*}(t))} \tag{4.22}$$

with

$$\beta_1 = 8g_{fK^*\bar{K}}g_{f\pi\pi} \tag{4.23}$$

obtained by finding the residue of the f pole at $\alpha_\rho(s)=2$ in Eq. (4.22). Using Eqs. (4.21) to (4.23) we find for the $(K^*\bar{K}+\bar{K}^*K)$ contribution

$$\mathcal{G}_I(K^*\bar{K}+\bar{K}^*K) = 1.70\times10^{-2}g_{f\pi\pi}. \tag{4.24}$$

In arriving at this result we have used the following to determine $g_{f'K^*\bar{K}}$ and $g_{fK^*\bar{K}}$. By SU(3) symmetry

$$g_{f'K^*\bar{K}} = \sqrt{2}\,g_{fK^*\bar{K}} = g_{K^{**}\bar{K}^*\pi}. \tag{4.25}$$

The coupling strength $g_{K^{**}\bar{K}^*\pi}$ is determined from the partial width $\Gamma(K^{**}(1420)\to K^*(890)+\pi)\approx30$ MeV to be

$$\frac{1}{4\pi}(g_{K^{**}\bar{K}^*\pi}{}^2)\approx1.01\ \text{GeV}^{-4}. \tag{4.26}$$

Adding all three contributions to $\mathcal{G}_I$, Eq. (4.11), Eq. (4.18), and Eq. (4.24), we find

$$\mathcal{G}_I \simeq 2.63\times10^{-2}g_{f\pi\pi}, \tag{4.27}$$

which contributes a value of 170 keV to the partial width of $f' \to \pi\pi$.

V. DISPERSIVE PART OF THE $f'\pi\pi$ COUPLING STRENGTH

We proceed along the same line as in our calculation of the amplitude $\phi \to \rho\pi$. For calculating the $K\bar{K}$ intermediate-state contribution we smooth out the $K\bar{K} \to \pi\pi$ amplitude given by Eq. (4.9) by its asymptotic form

$$T(s,t) \approx \beta\Gamma(1-\alpha_{K^*}(t))e^{-i\pi\alpha_{K^*}(t)}(\alpha_\rho' s)^{\alpha_{K^*}(t)}, \qquad (5.1)$$

and approximate the vertex function $V(s)$ by its mass-shell value $V(s=m_{f'}^2)=1$. The rest of the calculation is then straightforward and we obtain

$$\mathrm{Re}\mathfrak{F}(s=m_{f'}^2) = \mathcal{G}_R(K\bar{K}) = -7.84\times 10^{-2} g_{f\pi\pi}. \qquad (5.2)$$

Similar calculations for the $\eta\eta$ and $(K^*\bar{K}+\bar{K}^*K)$ intermediate-state contributions yield

$$\mathcal{G}_R(\eta\eta) = -4.3\times 10^{-2} g_{f\pi\pi} \qquad (5.3)$$

and

$$\mathcal{G}_R(K^*\bar{K}+\bar{K}^*K) = +7.75\times 10^{-2} g_{f\pi\pi}. \qquad (5.4)$$

Adding all the contributions we find

$$\mathcal{G}_R = -4.4\times 10^{-2} g_{f\pi\pi}, \qquad (5.5)$$

which contributes to $\Gamma(f' \to \pi\pi)$ a value of about 500 keV.

The reason for the difference in sign between the $K\bar{K}$ intermediate-state contribution and the $(K^*\bar{K}+\bar{K}^*K)$ intermediate-state contribution to the dispersive part of the $f'\pi\pi$ coupling is the following. We can divide the integration in Eq. (4.4) into two regions, one from the threshold of the intermediate state up to $s=m_{f'}^2$ and the other from $s=m_{f'}^2$ to $s=\infty$. Since the transitions $f'K\bar{K}$ and $f' \to \bar{K}^*K$ are both d-wave transitions the dispersion integrands are strongly suppressed near the threshold, grow rapidly away from threshold, and eventually decrease asymptotically due to Regge behavior of the amplitudes $K\bar{K} \to \pi\pi$ and $K^*\bar{K} \to \pi\pi$. Because of the higher threshold of the $K^*\bar{K}$ intermediate state, the contribution from $s > m_{f'}^2$ dominates over the contribution from $(m_{K^*}+m_K)^2 < s < m_{f'}^2$, while in the case of the $K\bar{K}$ intermediate state it is the region $4m_K^2 < s < m_{f'}^2$ that is dominant. It is evident that our approximation of treating the vertex functions as a constant is especially poor for the d-wave transitions $f' \to K\bar{K}$, $\eta\eta$, and $K^*\bar{K}$ and is responsible for the large individual contributions of these states to $\mathcal{G}_R$. The almost exact cancellation of the $K\bar{K}$ and $K^*\bar{K}+\bar{K}^*K$ intermediate-state contributions is somewhat fortuitous. We should therefore regard our calculations to indicate only the order of magnitude of the partial width $\Gamma(f' \to \pi\pi)$.

VI. DISCUSSIONS

In the foregoing we have tried to estimate the amplitudes $\phi \to 3\pi$ and $f' \to \pi\pi$ using the following assumptions: (1) ϕ and f' are pure $s\bar{s}$ states. (2) The OZI rule can be used as a first approximation to the physical S-matrix element. In particular, in evaluating the corrections to the rule arising from unitarity as in Eq. (2.4) and Eq. (4.5) we can use the OZI rule in the right-hand side of the equations. We have used various models to estimate the absorptive parts of the amplitudes, and all of them yield results in agreement with the experimentally observed order of magnitude of the partial widths. Our calculations of the dispersive parts of the amplitudes $\phi \to \rho\pi$ and $f' \to \pi\pi$ are somewhat less reliable since in our calculations the various vertex functions involving ϕ and f' are assumed to be constant and, moreover, we have neglected the higher-mass intermediate states. Since within our model all the rescattering amplitudes proceed via the exchange of K^* trajectory (or strange-baryon trajectory in the case of baryon-antibaryon intermediate states), their contribution is expected to become smaller with increasing threshold for the higher-mass intermediate states.

It should be stressed that we have introduced no *ad hoc* mixing angles or arbitrary small parameters in our calculations. Although we are unable to evaluate the exact strengths of the transitions $\phi \to 3\pi$ and $f' \to \pi\pi$, it is indeed gratifying that the order of magnitude comes out correctly thus justifying our basic premises.

ACKNOWLEDGMENT

We thank Dr. Sunder and A. D. Gangal for their help in the numerical computations. One of us (C.A.S.) is grateful to Professor N. Mukunda for his hospitality at the Center for Theoretical Studies.

APPENDIX

In Table I we have listed the results of our calculation of $\mathrm{Im}\mathfrak{F}(s=m_{f'}^2)$ as computed in Sec. IV of this paper and we have also listed the results of our earlier calculation[10] where we have used a simple pole model for the amplitudes $K\bar{K} \to \pi\pi$, $\eta\eta \to \pi\pi$, and $K^*\bar{K} \to \pi\pi$. It is seen from the figures in the second and third columns of Table I that the $K\bar{K}$ contribution comes out to be very different in the dual model. The reason for this lies in the following. Our dual model expression for the $K\bar{K} \to \pi\pi$ amplitude [cf. Eq. (4.9)]

$$T(s,t) = \beta\frac{\Gamma(1-\alpha_\rho(s))\Gamma(1-\alpha_{K^*}(t))}{\Gamma(1-\alpha_\rho(s)-\alpha_{K^*}(t))} \qquad (A1)$$

TABLE I. Contribution of the intermediate states $K\bar{K}$, $\eta\eta$, and $K^*\bar{K}+\bar{K}^*K$ to the coupling $f'\to\pi\pi$ based on various models for the rescattering amplitudes $K\bar{K}$, $\eta\eta$, and $K^*\bar{K}\to\pi\pi$. An overall factor $g_{f\pi\pi}$ multiplying the numerical figures is understood.

Intermediate state	Absorptive part of the coupling $f'\to\pi\pi$: Pole model[a]	Dual model with real trajectories[b]	Dual model with complex trajectories[c]	Dispersive part[d] of the coupling $f'\to\pi\pi$
$K\bar{K}$	1.56×10^{-2}	-1.1×10^{-3}	1.05×10^{-2}	-7.84×10^{-2}
$\eta\eta$	0.52×10^{-2}	1.04×10^{-2}	0.75×10^{-2}	-4.3×10^{-2}
$K^*\bar{K}+\bar{K}^*K$	0.46×10^{-2}	1.70×10^{-2}	1.00×10^{-2}	$+7.75\times10^{-2}$

[a]Reference 10.
[b]Cf. Sec. IV.
[c]Cf. Appendix.
[d]Cf. Sec. V.

has a zero in the physical region of interest unlike the pole model used in Ref. 10. The argument of the denominator Γ function in Eq. (A1) can be written as

$$1-\alpha_\rho(m_{f'}^2)-\alpha_{K^*}(t)=-1.03-0.86z\,, \tag{A2}$$

where z is the cosine of the center-of-mass scattering angle. In arriving at Eq. (A2) we have used $\alpha'_\rho=\alpha'_{K^*}=1\ (\text{GeV})^{-2}$ and the values of the kaon and pion c.m. momenta corresponding to $s=m_{f'}^2$. Since in evaluating the absorptive part [cf. Eq. (4.8)] we integrate over z the presence of the zero, which is at $z\simeq-0.03$ according to Eq. (A2), produces a strong cancellation. Since this is somewhat artificial we have carried out the following modification. We have used the same expression as in Eq. (A1) but with the s-channel Regge trajectory now taken to be complex. That is, we assume $\alpha_\rho(s)$ to be given by[17]

$$\alpha_\rho(s)=\alpha_0+\alpha's+ia(s-s_0)\,, \tag{A3}$$

where the constants a and s_0 are fitted to the widths of ρ and f via the formula

$$\text{Im}\alpha(s=m_{\text{res}}^2)=\alpha' m_{\text{res}}\Gamma_{\text{res}} \tag{A4}$$

and are found to be

$$a=9.8\times10^{-2}\ \text{GeV}^{-2},\quad s_0=-0.58\ \text{GeV}^2\,. \tag{A5}$$

We can repeat our calculation of the imaginary part Eq. (4.8) with $T(s,t)$ now given by Eq. (A1) and Eq. (A3), and the result is listed in the fourth column of Table I. We have performed similar calculations for the $\eta\eta$ and $(K^*\bar{K}+\bar{K}^*K)$ states contributions also. It will be seen that the orders of magnitude obtained with all three models, (a) pole model, (b) dual model with real trajectories, and (c) dual model with complex trajectories, are the same.

*On leave of absence from the Tata Institute of Fundamental Research, Bombay-400 005, India.

[1]Particle Data Group, Rev. Mod. Phys. 48, S1 (1976).
[2]G. R. Kalbfleisch, R. C. Strand, and J. W. Chapman, Phys. Rev. D 13, 22 (1976).
[3]G. Parrour *et al*., Phys. Lett. 63B, 362 (1976).
[4]M. Gell-Mann, D. Sharp, and W. G. Wagner, Phys. Rev. Lett. 8, 261 (1962).
[5]V. De Alfaro, S. Fubini, G. Furlan, and C. Rossetti, *Currents in Hadron Physics* (North-Holland, Amsterdam, 1973). Dual models also yield the same value; cf. R. C. Brower and G. Chu, Phys. Rev. D 7, 56 (1973).
[6]A. J. Pawlicki *et al*., Phys. Rev. D 15, 3196 (1977). See also A. J. Pawlicki *et al*., Phys. Rev. Lett. 37, 971 (1976).
[7]N. P. Samios, M. Goldberg, and P. T. Meadow, Rev. Mod. Phys. 46, 49 (1974).
[8]S. Okubo, Phys. Lett. 5, 164 (1963); G. Zweig, CERN Reports Nos. TH 401 and TH 412, 1964 (unpublished); J. Iizuka, Prog. Theor. Phys. Suppl. 37–38, 21 (1966).
[9]J. Pasupathy, Phys. Rev. D 12, 2929 (1975).
[10]J. Pasupathy and C. A. Singh, Phys. Lett. 61B, 469 (1976).
[11]G. Veneziano, Nuovo Cimento 57A, 190 (1968).
[12]Equation (2.18) is obtained by taking the large-s limit of Eq. (2.10) slightly off the real axis.
[13]For the evaluation of the principal-value integral for the dispersive part we use the following formula for the Cauchy principal-value integral:

$$\text{P}\int_A^B dx\,\frac{f(x)}{x-x_0}=\int_{2x_0-A}^{B}dx\,\frac{f(x)}{x-x_0}+\int_0^{x_0-A}dx\,\frac{1}{x}\left[f(x_0+x)-f(x_0-x)\right],$$

where $f(x)$ is regular throughout the interval, and $A<x_0<(A+B)/2$.
[14]We have also computed the contribution of the invariant amplitude $D(s,t)$ which has the same asymptotic behavior as $E(s,t)$. Using the same procedure that we have used in the text for the amplitude $E(s,t)$ we find the contribution of the amplitude $D(s,t)$ to the dispersive part of the $\phi\to\rho\pi$ coupling to be $\approx-2\times10^{-3}g_{\omega\rho\pi}$, which is one order of magnitude smaller than Eq. (3.9). The amplitudes A,B,C are expected to make

negligible contributions since they decrease more rapidly than D and E.

[15]C. Lovelace, Phys. Lett. 28B, 264 (1968); J. Shapiro, Phys. Rev. 179, 1345 (1969).

[16]J. Baacke, M. Jacob, and S. Pokorski, Nuovo Cimento 62A, 332 (1969).

[17]Chan Hong-Mo, R. O. Raitio, G. H. Thomas, and N. A. Tornquist, Nucl. Phys. B19, 173 (1970).