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Dynamics and SU(3) violation in J/ψ decays

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The strong-interaction part of the amplitude for J/ψ decays into specific final hadronic states is computed by evaluating corrections arising from unitarity to the Okubo-Zweig-Iizuka rule. Dispersion relations for the relevant amplitudes are evaluated keeping only the contributions of charmed-hadron states to the absorptive part and assuming Regge asymptotic behavior for charmed hadrons→ordinary hadrons. To obtain an estimate of the absolute magnitude of an amplitude such as $\psi\to\rho\pi$ we use SU(4) symmetry for the connected vertices and find a suppression factor of the order of 10^{-4} for $g_{\psi\rho\pi}$ as compared to $g_{\omega\rho\pi}$. Assuming a mass splitting of 100 MeV for $m_F - m_D$ and $m_{F^*} - m_{D^*}$ leads to dynamical breaking of SU(3) symmetry even if the couplings characterizing connected vertices obey SU(3) symmetry. If the latter circumstance is true, we estimate the ratio of the amplitudes $\psi\to\rho^+\pi^-$ and $\psi\to K^{*+}K^-$ to be between 1.5 and 2. Such dynamical violation of SU(3) arising from mass splitting between charmed hadrons in the same SU(3) multiplet tends to cancel in $\psi\to K\bar{K}$. Related matters such as SU(3) symmetry in the decays of χ states and comparison of $\psi'\to\psi\eta$, $\psi'\to\rho\pi$, and $\psi\to\rho\pi$ are also discussed.

I. INTRODUCTION

The discovery of the narrow resonances[1,2,3] $J/\psi(3095)$ and $\psi(3684)$ has lead to the introduction of an additional quark (c) carrying a new quantum number called charm, with the states $\psi(3095)$ and $\psi(3684)$ themselves being interpreted as $c\bar{c}$ bound states. Introduction of a new quark requires the existence of a host of new hadrons[4] carrying the new quantum number, for example a set of a triplet of pseudoscalars $D^+ = c\bar{d}$, $D^0 = c\bar{u}$, $F^+ = c\bar{s}$, the corresponding vectors D^{*+}, D^{*0}, F^*, charmed baryons, etc. Among the experimental evidences for this new quantum number are (a) observation of charmed mesons in e^+e^- annihilation,[5] (b) observation of charmed baryons in a photoproduction experiment,[6] and (c) neutrino experiments involving dileptons in the final state[7] which require the existence of at least one more new quantum number. Further, experimental observation[9] of positive-C-parity states in the radiative decay of $\psi(3684)$ can be regarded as additional support for the $c\bar{c}$ bound-state picture of $\psi(3095)$ and $\psi(3684)$.

The narrow widths of ψ and $\psi' = \psi(3684)$ are attributed to the validity of the Okubo-Zweig-Iizuka (OZI) rule,[9] i.e., since ψ and ψ' are pure $c\bar{c}$ states while $p, \pi, K, \ldots$ are states made of u, d, s quarks, there is no connected quark diagram describing the transition $\psi\to$ ordinary hadrons (i.e., hadrons made of u, d, s quarks only). Even among the latter this suppression of transitions involving disconnected quark diagrams is manifest in the pionic decays of $\phi(1019)$ and $f'(1514)$. For example, the coupling $g_{\phi K\bar{K}}$ computed from the partial width $\Gamma(\phi\to K\bar{K})$ is in agreement with the SU(3) prediction[10]

$$g_{\phi K\bar{K}} = \frac{1}{\sqrt{2}}\, g_{\rho\pi\pi}\ ,$$

where $g_{\rho\pi\pi}$ is obtained from $\Gamma(\rho\to\pi\pi)$. On the other hand an estimate of the coupling constants $g_{\phi\rho\pi}$ and $g_{\omega\rho\pi}$ using the model of Gell-Mann, Sharp, and Wagner[11] for $\phi\to 3\pi$ and $\omega\to 3\pi$ and the experimental values of the partial widths[12] gives

$$g_{\phi\rho\pi}{}^2/g_{\omega\rho\pi}{}^2 \approx 2.3\times10^{-3}\ . \tag{1.1}$$

Similarly, while the couplings $g_{f\pi\pi}$, $g_{fK\bar{K}}$, and $g_{f'K\bar{K}}$ are comparable,[10] the recently measured value[13]

$$\Gamma(f'\to\pi\pi) \approx 550\ \text{keV} \tag{1.2}$$

leads to

$$g_{f'\pi\pi}{}^2/g_{f\pi\pi}{}^2 \approx 2.1\times10^{-3}\ . \tag{1.3}$$

In the case of decays of ψ and ψ' a large amount of experimental information already exists. Although numerous questions can be asked[14] about them, we would like to single out the following three features for our discussion.

(1) The narrow width itself, typified for example by the smallness of the partial width[15] $\Gamma(\psi\to\rho^+\pi^-) \approx 0.3$ keV. Writing the transition matrix element as

$$M(\psi(\epsilon_1, P)\to\rho^+(\epsilon_2, q_2) + \pi^-(q_1)) = g_{\psi\rho^-\pi^+}\epsilon_{\mu\nu\lambda\sigma}\epsilon_1^\mu\epsilon_2^\nu P^\lambda q_2^\sigma\ , \tag{1.4}$$

where ϵ_1 is the polarization vector of ψ, P is its momentum, and ϵ_2, q_2 similarly denote those of ρ, this leads to a value

$$g_{\psi\rho^-\pi^+}{}^2/4\pi \approx 3\times10^{-7}\ \mathrm{GeV}^{-2}\ , \tag{1.5}$$

to be compared with[11] $g_{\omega\rho\pi}{}^2/4\pi \approx 20\ \mathrm{GeV}^{-2}$.

(2) The partial width for the OZI-rule-forbidden process[16] $\psi' \to \psi\eta$ is relatively large compared to $\psi, \psi' \to \rho\pi$, which have identical Lorentz structure. Writing for the transition matrix element an expression similar to Eq. (1.4) with $g_{\psi'\psi\eta}$ replacing $g_{\psi\rho\pi}$, we have from the experimentally measured partial width[17] $\Gamma(\psi' \to \psi\eta) \approx 10$ keV a value

$$g_{\psi'\psi\eta}{}^2/4\pi \approx 3.6\times10^{-3}\ \mathrm{GeV}^{-2}\ . \tag{1.6}$$

(3) The apparent validity of SU(3) symmetry in certain decay channels such as $\psi \to K_L K_S$ is to be contrasted with substantial violations of SU(3) as indicated by comparison of $\psi \to \rho\pi$ and $\psi \to K^*\bar{K}$. Our discussion can be made quantitative if we adopt the following procedure due to Okubo.[18] Assume that the matrix element for the transition $\psi \to$ ordinary hadrons is described by an effective Hamiltonian

$$\mathcal{H}_\psi(x) = \left[\frac{\alpha}{2\sqrt{2}} j_\mu^{(0)}(x) + \beta j_\mu^{(8)}(x) + \frac{\sqrt{3}}{2}\gamma\left(j_\mu^{(3)}(x) + \frac{1}{\sqrt{3}} j_\mu^{(8)}(x)\right)\right]\psi^\mu(x)\ , \tag{1.7}$$

where $\psi_\mu(x)$ is the field of the ψ particle and $j_\mu^{(a)}(x)$ $(a = 0, 1, \ldots, 8)$ are the nonet of U(3) vector currents. α, β, and γ are assumed to be constants and, respectively, represent the strengths of the singlet part of the strong-interaction amplitude, the octet or the SU(3)-breaking part of the strong-interaction amplitude, and the photon-mediated amplitudes, i.e., $\psi \to$ photon $\to$ hadrons. The experimentally measured branching ratios for the $\rho\pi, K^*\bar{K}$ modes are[19,20,21]

$$\psi \to \rho^+\pi^- : K^{*+}K^- : K^{*0}\bar{K}^0 = 0.43 \pm 0.10 : 0.16 \pm 0.03 : 0.135 \pm 0.03\ . \tag{1.8}$$

If in Eq. (1.7) γ were absent we would expect the amplitudes $\psi \to K^{*+}K^-$ and $\psi \to K^{*0}\bar{K}^0$ to be equal, while if β were absent we would expect the amplitudes $\psi \to K^{*+}K^-$ and $\psi \to \rho^+\pi^-$ to be equal or, correcting for phase space as usual, $\Gamma(\psi \to K^{*+}K^-) = 0.85\Gamma(\psi \to \rho^+\pi^-)$. Comparing with the experimental data[15,19,21] [Eq. (1.8)] we reach the conclusion that the SU(3)-violating term β in Eq. (1.7) makes a larger contribution to the amplitude than the one-photon term γ in $\psi \to$ vector plus pseudoscalar.

On the other hand, the branching ratios R_B (Ref. 19) for $\psi \to$ two pseudoscalars,

$$R_B(\psi \to \pi^+\pi^-) = (1.6 \pm 1.6)\times10^{-4}\ , \tag{1.9a}$$

$$R_B(\psi \to K^+K^-) = (2.0 \pm 1.6)\times10^{-4}\ , \tag{1.9b}$$

$$R_B(\psi \to K_L K_S) < 0.89\times10^{-4}\ , \tag{1.9c}$$

are consistent with β being zero in Eq. (1.7). As explained in detail in Appendix A all the above three numbers can be understood to arise from $\psi \to$ photon $\to$ two pseudoscalars. The slight difference between the $\pi^+\pi^-$ and K^+K^- branching ratios can be attributed to the difference in the electromagnetic form factors of π^+ and K^+ at the ψ mass. This suggests that, when the strong-interaction part of the amplitude $\psi \to$ ordinary hadrons is forbidden by SU(3), it indeed vanishes. This behavior is to be contrasted with substantial SU(3) violation in the case of allowed decays.

In this paper we try to understand these features from the phenomenological S-matrix point of view. There are basically two questions to be answered. (1) Why is the OZI rule operative at all in the first instance? (2) The OZI rule clearly violates unitarity and therefore cannot be exact. What is the magnitude of the unitarity corrections to the rule? There exists a partial answer to the first question in literature.[22] It relates the approximate validity of the OZI rule to the absence of low-lying exotic states. For example using the absence of resonances in $\pi^+\pi^+$, π^+K^+, and K^+K^+ channels and duality, one can demonstrate[22] the decoupling of the f' trajectory from the $\pi\pi$ state. Such arguments can be easily extended to show that the ϕ trajectory decouples from the $\rho\pi$ state and the ψ trajectory decouples from pions and kaons. We do not pursue these points here. However, if we accept the validity of the OZI rule as a first approximation to the physical S-matrix element it becomes clear that the rule violates unitarity. For example in the case of $\phi \to 3\pi$, since both $\phi \to K\bar{K}$ and $K\bar{K} \to 3\pi$ are allowed by the OZI rule, unitarity leads to an effective coupling of $\phi \to 3\pi$. Since $\phi(1019)$ lies above the $K\bar{K}$ threshold, there is a contribution to the absorptive part of the transition amplitude $\phi \to 3\pi$ coming from the real intermediate state $K\bar{K}$. Using a simple pole model for the amplitude $K\bar{K} \to 3\pi$, one of us[23] estimated the contribution to the partial width $\Gamma(\phi \to 3\pi)$ coming from $\mathrm{Im}(\phi \to 3\pi) = (\phi \to K\bar{K} \to 3\pi)$ to be ~150 keV, to be compared with the experimental values[12] of 600 keV. Similar calculations for the $f' \to \pi\pi$ width gave the value[24] $\Gamma(f' \to \pi\pi) \approx 150$ keV to be compared with the experimental value[13] 550 keV.

There is no difficulty in extending the above procedure to a discussion of ψ decays. In a brief communication one of us[25] had estimated the effective coupling $g_{\psi\rho\pi}$ arising from unitarity corrections, i.e., from

$$\psi \to \text{charmed hadrons} \to \rho\pi\ .$$

Writing the transition amplitude as

$$M(\psi(\epsilon_1,P)\to\rho(\epsilon_2,q_2)+\pi(q_1))$$

$$=\epsilon_{\mu\nu\lambda\sigma}\epsilon_1^\mu\epsilon_2^\nu P^\lambda q_2^\sigma F(P^2,q_1^2,q_2^2) \qquad (1.10)$$

we see that the coupling constant $g_{\psi\rho\pi}$ introduced in Eq. (1.4) is the value of the function F when all the momenta are on their mass shell. Assuming the validity of dispersion relations in the variable $P^2=s$ we can write

$$g_{\psi\rho\pi}=F(s=m_\psi^2)=\frac{1}{\pi}\int\frac{\mathrm{Im}F(s',m_\pi^2,m_\rho^2)}{s'-m_\psi^2-i\epsilon}\,ds' \ . \qquad (1.11)$$

The absorptive part Im$F(s)$ occurring in (1.11) is to be computed from unitarity. In the sum over intermediate states we keep only the charmed-hadron states, that is, we consider only the connected amplitudes. In Ref. 25 it was stressed in particular that the key factor that keeps corrections to the OZI rule, in the case of $\psi\to\rho\pi$, so small is the smallness of the amplitude for charmed hadrons $\to\rho\pi$. Consider for example the contribution to ImF arising from $\psi\to D^*\bar{D}\to\rho\pi$. While the vertex $\psi D^*\bar{D}$ is expected to have a strength of the same order of magnitude as $\omega\rho\pi$, the amplitude $T(D^*\bar{D}\to\rho\pi)$ essentially behaves as

$$T\propto(\alpha'_\omega s)^{\alpha_{D^*}(t)} \ , \qquad (1.12)$$

where $\alpha'_\omega\approx1$ GeV^{-2} is the slope of the ω trajectory and s is the square of the center-of-mass energy of the $D^*\bar{D}$ state whose threshold begins at $s\approx16$ GeV^2. We expect the D^* trajectory to be below the ϕ trajectory, and since $\alpha_\phi(0)\approx0$ we expect $\alpha_{D^*}(0)<0$. In fact, reasonable estimates suggest an intercept $\alpha_{D^*}(0)\approx-1$, which means that T as given by Eq. (1.12) is small and rapidly decreases with increasing s. One of the purposes of the present paper is to discuss in detail the procedure outlined in Ref. 25.

The second feature of ψ,ψ' decays raised above, namely relative strengths of $\psi'\to\psi\eta$ and ψ or $\psi'\to\rho\pi$, has its origin in the fact that the amplitude for charmed hadrons $\to\psi\eta$ is substantially different from the amplitude for charmed hadrons $\to\rho\pi$. Consider for example $D^*\bar{D}\to\psi\eta$ and $D^*\bar{D}\to\rho\pi$. In the former the direct channel is exotic. Further there are three distinguishing features between the two amplitudes. One is the well-known $t_{\min}$ effect arising from the difference in the masses of the final states, namely, m_ψ and m_η in the former versus m_ρ and m_π in the latter. The second distinguishing feature is that the asymptotic region characterized by Regge behavior in $D^*\bar{D}\to\psi\eta$ sets in for much larger values of s than in $D^*\bar{D}\to\rho\pi$, and finally the asymptotic scales governing the two reactions can be different if the slopes of the D^* trajectory and ω trajectory are different. For example, unlike $D^*\bar{D}\to\rho\pi$, whose asymptotic form is given by Eq. (1.12), the amplitude for $D^*\bar{D}\to\psi\eta$ is given asymptotically as

$$T\propto(\alpha'_{D^*}s)^{\alpha_{D^*}(t)} \ , \qquad (1.13)$$

with $\alpha'_{D^*}<\alpha'_\omega$; the large-$s$ behavior of Eq. (1.12) and Eq. (1.13) are different. One of us[26] has discussed elsewhere in detail, along with model calculations, how these three differences can at least in part account for the large difference in the orders of magnitudes between $g_{\psi\rho\pi}$ and $g_{\psi'\psi\eta}$.

The SU(3) behavior of ψ can be understood as follows. To see how the difference between the effective couplings $g_{\psi\rho\pi}$ and $g_{\psi K^*\bar{K}}$ arises we can compare the contribution to the dispersion integral. Eq. (1.11) to the two couplings, from various multiplets of charmed hadrons, i.e., pseudoscalar pair, vector-pseudoscalar pair, charmed baryons, etc. Let us compare the contribution of the pseudoscalar pairs $D\bar{D}$ and $F\bar{F}$ in $\psi\to\rho\pi$ and $\psi\to K^*\bar{K}$. The $D\bar{D}$ contribution to $g_{\psi\rho\pi}$ in Eq. (1.11) is given essentially by (see Secs. III and IV)

$$g_{\psi\rho\pi}=\int_{4m_D^2}^\infty g_{\psi D\bar{D}}\beta_{\bar{D}^*D\rho}\beta_{D^*\bar{D}\pi}\frac{(\alpha'_\omega s)^{\alpha_{D^*}(t)-1}}{s-m_\psi^2}L(s)ds \ . \qquad (1.14)$$

Here $g_{\psi D\bar{D}}$ is the $\psi D\bar{D}$ coupling constant, the product $\beta_{\bar{D}^*D\rho}\beta_{D^*\bar{D}\pi}$ is the residue of the exchanged D^* Regge pole in $D\bar{D}\to\rho\pi$, and $L(s)$ is a kinematical factor which is derived later in Sec. III. The $F\bar{F}$ intermediate state does not contribute to $\psi\to\rho\pi$ in our approximation since $F\bar{F}\to\rho\pi$ involves a disconnected graph. On the other hand, in $\psi\to K^*\bar{K}$ the contribution from the $D\bar{D}$ intermediate state is analogous to that in $\psi\to\rho\pi$ except that the transition $D\bar{D}\to K^*\bar{K}$ involves an F^* exchange. The $F\bar{F}$ intermediate state also contributes to $\psi\to K^*\bar{K}$ with the transition $F\bar{F}\to K^*\bar{K}$ proceeding via D^* exchange. Analogous to Eq. (1.14) we have

$$g_{\psi K^*\bar{K}}^{D\bar{D}}=\int_{4m_D^2}^\infty g_{\psi D\bar{D}}\beta_{\bar{F}^*DK^*}\beta_{F^*\bar{D}\bar{K}}\frac{(\alpha'_\omega s)^{\alpha_{F^*}(t)-1}}{s-m_\psi^2}\times C_1(s)ds \ , \qquad (1.15)$$

$$g_{\psi K^*\bar{K}}^{F\bar{F}}=\int_{4m_F^2}^\infty g_{\psi F\bar{F}}\beta_{\bar{D}^*FK^*}\beta_{D^*\bar{F}\bar{K}}\frac{(\alpha'_\phi s)^{\alpha_{D^*}(t)-1}}{s-m_\psi^2}\times C_2(s)ds \ . \qquad (1.16)$$

The kinematical factors $C_1(s)$ and $C_2(s)$ are derived in Sec. III and are not significantly different from $L(s)$ in Eq. (1.14). If SU(3) were an exact symmetry we would expect

$$m_D=m_F\,,\quad \alpha_{D^*}=\alpha_{F^*}\,,\quad L(s)=C_1(s)=C_2(s)\,,$$

and with the use of appropriate Clebsch-Gordan coefficients for the g's and β's we have

$$g_{\psi K^* \bar{K}}^{D\bar{D}} = g_{\psi K^* \bar{K}}^{F\bar{F}} \qquad (1.17)$$

and

$$g_{\psi\rho^+\pi^-} = g_{\psi K^{*+}K^-} = g_{\psi K^{*+}K^-}^{D\bar{D}} + g_{\psi K^{*+}K^-}^{F\bar{F}} . \qquad (1.18)$$

However, with the breakdown of SU(3) symmetry we have $m_D < m_F$ and $\alpha_{F^*} < \alpha_{D^*}$. Taking this into account we find that the $D\bar{D}$ intermediate state contribution is smaller[27] in $\psi \to K^*\bar{K}$ than in $\psi \to \rho\pi$ since $\alpha_{F^*}(t) < \alpha_{D^*}(t)$, as can be seen by comparing Eq. (1.14) and (1.15) (whose values apart from the coupling constants g's and β's are tabulated in columns 2 and 4 of Table I). On the other hand, the dispersion integral occurring in Eq. (1.16) giving the $F\bar{F}$ intermediate-state contribution in $\psi \to K^*\bar{K}$ is smaller than that in Eq. (1.14) since the $F\bar{F}$ state has a higher threshold than the $D\bar{D}$ state, although both $F\bar{F} \to K^*\bar{K}$ and $D\bar{D} \to \rho\pi$ involve D^* exchange in the t channel. (See column 3 of Table I.) It will be seen from Table I that the change in the value of the dispersion integrals Eq. (1.15) and Eq. (1.16) are approximately the same, and since $g_{\psi K^*\bar{K}}$ is the sum of the two contributions there is substantial violation of SU(3) symmetry. In the case of $\psi \to K\bar{K}$, however, the $D\bar{D}$ and $F\bar{F}$ contributions to $g_{\psi K\bar{K}}$ come with opposite signs. Our calculations indicate (see Table IV) that the magnitude of the departures from the SU(3)-symmetric limit for these two are nearly the same so that the $D\bar{D}$ and $F\bar{F}$ state contributions almost cancel each other to yield an apparently exact SU(3)-symmetric result. We can repeat the above procedure for various multiplets of intermediate state $D^*\bar{D}$ and $F^*\bar{F}$, $D^*\bar{D}^*$ and $F^*\bar{F}^*$, etc. with similar results.

The material of this paper is organized as follows. In Sec. II we write down the unitarity equations and the contributions from the various intermediate states. In Sec. III we evaluate the dispersion integrals assuming Regge asymptotic behavior for the amplitudes charmed hadrons → ordinary hadrons. In Sec. IV we use the dual model to fix the Regge residues, and estimates of the couplings $\psi\rho\pi$ and $\psi K^*\bar{K}$ are obtained. In Sec. 5 we compute $\psi \to K\bar{K}$, and in the concluding section a discussion of related questions such as the comparison of $\psi' \to \rho\pi$ and $\psi \to \rho\pi$ and possible tests of SU(3) symmetry in the decays of the χ states is given.

II. $\psi \to$ VECTOR + PSEUDOSCALAR

We write the S-matrix element for the decay $\psi \to$ pseudoscalar + vector meson in the form

$$S = \langle q_1 q_2 \epsilon_2 \text{ out} | \psi(P, \epsilon_1)\rangle$$
$$= i(2\pi)^4 \delta^{(4)}(P - q_1 - q_2) \frac{1}{(2P^0)^{1/2}}$$
$$\times \langle q_1 q_2 \epsilon_2 \text{ out} | J_\mu | 0\rangle \epsilon_1^\mu , \qquad (2.1)$$

where P is the four-momentum and ϵ_1^μ is the polarization vector of the ψ particle, J_μ is the source of the ψ field, i.e., $(\Box + m_\psi^2)\psi_\mu = J_\mu$, q_2 and ϵ_2 refer to the momentum and polarization vector of the final vector meson (e.g., ρ meson in the case of $\rho\pi$ decay), and q_1 is the momentum of the pseudoscalar meson. The transition matrix element is therefore given by

$$M = \epsilon_1^\mu M_\mu , \qquad (2.2)$$

where

$$M_\mu = (4q_1^0 q_2^0)^{1/2} \langle q_1 q_2 \epsilon_2 \text{ out} | J_\mu | 0\rangle$$
$$= i(2q_2^0)^{1/2} \int d^4x\, e^{iq_1 \cdot x}$$
$$\times \langle q_2 \epsilon_2 | [j(x), J_\mu(0)] \theta(x) | 0\rangle , \qquad (2.3)$$

with

$$j(x) = (\Box + m^2)\phi(x) \qquad (2.4)$$

as the source of the pseudoscalar field $\phi(x)$ of mass m. There is only one invariant amplitude for the transition vector → vector + pseudoscalar so that we can write

$$M_\mu = \epsilon_{\mu\nu\lambda\sigma} P^\nu \epsilon_2^\lambda q_1^\sigma F(P^2, q_1^2, q_2^2) . \qquad (2.5)$$

We shall assume[28] that $F(P^2, q_1^2, q_2^2)$ satisfies a standard dispersion relation in the variable P^2 with no subtractions, i.e.,

$$F(s) = \frac{1}{\pi} \int \frac{\operatorname{Im} F(s', q_1^2, q_2^2)}{s' - s - i\epsilon} ds', \quad s \equiv P^2 . \qquad (2.6)$$

Writing

$$M_\mu = D_\mu + iA_\mu \qquad (2.7)$$

we obtain the absorptive part A_μ in the usual manner by inserting a complete set of intermediate states in Eq. (2.3):

$$A_\mu = (2q_2^0)^{1/2} \pi \sum_n \langle q_2 \epsilon_2 | j | n\rangle \langle n | J_\mu | 0\rangle$$
$$\times \delta^{(4)}(P_n - q_1 - q_2) , \qquad (2.8)$$

where P_n is the momentum of the intermediate state. The state $|n\rangle$ must have the same quantum number as ψ, i.e., $J^{PC} = 1^{--}$ and $I = 0$. In evaluating the absorptive part we shall keep only those intermediate states such that both the matrix elements occurring in Eq. (2.8) are of the type allowed by the OZI rule, i.e., involve only connected diagrams. This means that we keep only charmed-hardon states in the sum over intermediate states. Further, in the specific case of $\psi \to \rho\pi$ consider the contribution coming from the intermediate states consisting of charmed-psuedoscalar pairs D^+D^-, $D^0\bar{D}^0$, and F^+F^-. Since $F^+ = c\bar{s}$, the amplitude for $F^+F^- \to \rho\pi$ involves a disconnected diagram and will be neglected in a first approximation. We will be able to justify this *a posteriori*. This is because for the example under discussion, $F^+F^- \to \rho\pi$, we can again write dispersion relations and evaluate the amplitude by keeping in the sum over intermediate states contributing to the absorptive part only connected terms, and show that the resulting amplitude is smaller than $D^+D^- \to \rho\pi$. This point will become clear later.

The contribution of the charmed-pseudoscalar pairs to A_μ in Eq. (2.8) can be evaluated as follows. Let the momenta of the charmed mesons be k_1 and k_2. We can write

$$\langle k_1 k_2 | J_\mu | 0\rangle = \left(\frac{1}{4k_1^0 k_2^0}\right)^{1/2} (k_1 - k_2)_\mu \times GK((k_1+k_2)^2), \tag{2.9}$$

where G is the coupling constant and the vertex function is normalized to unity at the ψ mass, i.e., $K(m_\psi^2) = 1$. The other matrix element appearing in Eq. (2.8) is the scattering amplitude for charmed pseudoscalars going into the final pseudoscalar and vector meson, for example

$$D^+(k_1) + D^-(k_2) \to \pi^+(q_1) + \rho^-(\epsilon_2, q_2). \tag{2.10}$$

We can write for the matrix element

$$\langle q_2 \epsilon_{2\mu} | j | k_1 k_2 \rangle = (8k_1^0 k_2^0 q_2^0)^{-1/2} \epsilon_{\mu\nu\lambda\sigma} k_1^\nu k_2^\lambda q_2^\sigma R(s,t), \tag{2.11}$$

where $s = (k_1 + k_2)^2$ and $t = (k_2 - q_2)^2$. Using Eqs. (2.5)–(2.11) the contribution of the $D\bar{D}$ pair to the absorptive part is easily found to be

$$\text{Im} F(s) = \frac{-G}{32\pi} \frac{2k^3}{\sqrt{s}} \int_{-1}^{1} dz (1 - z^2) \,\text{Re}[K^*(s) R(s,z)], \tag{2.12}$$

where $k = (s - 4m_D^2)^{1/2}/2$, is the center-of-mass momentum of the D^+ and $z = \cos\theta$, where θ is the scattering angle. To compute the integral in Eq. (2.12) we must know the vertex function $K(s)$ and the scattering amplitude $R(s,z)$; this will be discussed in the following sections. For the present we turn our attention to the evaluation of the intermediate state consisting of a vector and pseudoscalar charmed-meson pair, for example $D\bar{D}^*$ and its charge conjugate $D^*\bar{D}$.

We can write for the $\psi D^*\bar{D}$ vertex

$$(4k_1^0 k_2^0)^{1/2} \langle D^*\bar{D} | J_\mu | 0\rangle = G_1 \epsilon_{\mu\nu\lambda\sigma} P^\nu \epsilon^\lambda k_2^\sigma H(s), \tag{2.13}$$

where ϵ^λ is the polarization vector of D^* with momentum k_2, G_1 is the $\psi D^*\bar{D}$ coupling constant, and $H(s)$ is normalized to unity at the ψ mass as before:

$$H(s = m_\psi^2) = 1. \tag{2.14}$$

The transition amplitude $D^*\bar{D} \to \rho\pi$ has five invariant terms which can be chosen as follows:

$$\begin{aligned} T(\bar{D}(k_1) + D^*(\epsilon, k_2) &\to \pi(q_1) + \rho(\epsilon_2, q_2)) \\ &= T(\epsilon, k_1, k_2; \epsilon_2, q_1, q_2) \\ &= \epsilon_2 \cdot \epsilon E(s,t) + \epsilon_2 \cdot \Delta\epsilon \cdot \Delta D(s,t) + \epsilon_2 \cdot \Delta\epsilon \cdot Q C(s,t) \\ &\quad + \epsilon_2 \cdot Q\epsilon \cdot \Delta B(s,t) + \epsilon_2 \cdot Q\epsilon \cdot Q A(s,t), \end{aligned} \tag{2.15}$$

with

$$\Delta = k_1 - q_1, \quad Q = k_1 + q_1 \tag{2.16}$$

and

$$s = (k_1 + k_2)^2 = (q_1 + q_2)^2, \quad t = (k_1 - q_1)^2 \tag{2.17}$$

as before. The amplitudes A, B, C, D, and E are free of kinematical singularities. It is clear from Eq. (2.8) that they enter linearly in the computation of $\text{Im} F(s)$. They are therefore easily calculated separately and are listed below:

contribution of $E(s,t)$

$$= -\frac{G_1}{32\pi} \frac{2k^2}{q\sqrt{s}} \int_{-1}^{1} dz\, z\, \text{Re}[H^*(s) E(s,t)], \tag{2.18}$$

contribution of $D(s,t)$

$$= -\frac{G_1}{32\pi} \frac{k^3}{\sqrt{s}} \int_{-1}^{1} dz (1 - z^2) \text{Re}[H^*(s) D(s,t)], \tag{2.19}$$

contribution of $C(s,t)$

$$= \frac{G_1}{32\pi} \frac{k^3}{\sqrt{s}} \int_{-1}^{1} dz (1 - z^2) \text{Re}[H^*(s) C(s,t)], \tag{2.20}$$

contribution of $B(s,t)$

$$= -\frac{G_1}{32\pi} \frac{k^3}{\sqrt{s}} \int_{-1}^{1} dz (1 - z^2) \text{Re}[H^*(s) B(s,t)], \tag{2.21}$$

contribution of $A(s,t)$

$$= \frac{G_1}{32\pi} \frac{k^3}{\sqrt{s}} \int_{-1}^{1} dz (1 - z^2) \text{Re}[H^*(s) A(s,t)]. \tag{2.22}$$

In the above k is the D^* momentum in the center-of-mass frame, q is the final ρ momentum in the same frame, and $z = \cos\theta$ with θ as the scattering angle. The above procedure can be repeated for any arbitrary intermediate state, $D^*\bar{D}^*$ or charmed-baryon–antibaryon-pair state, etc. Instead we now turn to the question of the magnitude of the expressions (2.12) and (2.18)–(2.22) and their contribution to $\psi\rho\pi$ coupling via the dispersion integral Eq. (2.6).

III. REGGE ASYMPTOTIC BEHAVIOR

From the foregoing it is not evident that the coupling $\psi\rho\pi$ would be orders of magnitude smaller than $\omega\rho\pi$, as is the case experimentally. The key element that makes it so is the smallness of the scattering amplitude for charmed hadrons $\to\rho\pi$. Having agreed to keep only the charmed-hadron states in the dispersion integral Eq. (2.6) the integral gets the first contribution from the $D\bar{D}$ state with $s \geq 4m_D^2 \approx 14$ GeV^2 corresponding to[5] $m_D \approx 1.87$ GeV. Since we are considering only connected amplitudes it is seen that the scattering $D\bar{D}\to\rho\pi$ proceeds through ω-like, i.e., $(u\bar{u}+d\bar{d})/\sqrt{2}$, composites in the s channel and $u\bar{c}$ states in the t channel. It follows from considerations of duality that the sum over s-channel resonances can be represented by Regge poles in the t channel and vice versa. We also know from discussion[22] of πp scattering that the energy scale that determines the asymptotic region, where the scattering amplitude is expected to have Regge behavior, is set by the slope of the Regge trajectory in the s channel, $\alpha'_N \approx \alpha'_\omega \approx 1$ GeV^{-2}. So even for values of center-of-mass energies of 2 to 3 GeV Regge representation is a good approximation to the scattering amplitude. For the problem at hand, namely $D\bar{D}\to\rho\pi$, the asymptotic scale is set by $\alpha'_\omega \approx 1$ GeV^{-2}, where ω is the s-channel Regge trajectory, and we can expect Regge representation to be a good approximation to the true amplitude since the charm threshold starts at $s\approx 14$ GeV^2. We shall therefore write for the amplitude $D\bar{D}\to\rho\pi$ occurring in Eq. (2.11)

$$M(D\bar{D}\to\rho\pi) = \epsilon_{\mu\nu\lambda\sigma}\epsilon_2^\mu k_1^\nu k_2^\lambda q_2^\sigma R(s,t)\,, \tag{3.1}$$

$$R(s,t)\approx\beta(\alpha'_\omega s)^{\alpha_{D^*}(t)-1}\,. \tag{3.2}$$

Here $\alpha_{D^*}(t)$ is the D^* Regge trajectory and the extra minus one that appears in the exponent in Eq. (3.2) reflects the fact that in the reaction $D\bar{D}\to\rho\pi$ parity conservation requires ρ to have helicity ± 1. Although at present $\alpha_{D^*}(t)$ is unknown experimentally, we definitely expect the D^* trajectory to lie blow the ϕ trajectory and since $\alpha_\phi(0) \approx 0$ we expect $\alpha_{D^*}(0)<0$. The plausible values for the D^* trajectory are discussed in Appendix B and a value of $\alpha_{D^*}(0)\approx -1$ does not seem unreasonable. This means $R(s,t)$ is a rapidly falling function of s. To evaluate the $D\bar{D}$ intermediate-state contribution to Im$F(s)$ we need to know the vertex function $K(s)$ also. Although in principle we can resort to dispersion relations to compute $K(s)$, such an effort at present is unwarranted since we expect $K(s)$ to be a slowly varying function of s as compared to $R(s,t)$, as given by Eq. (3.2), which is rapidly varying. Slight changes in $\alpha_{D^*}(t)$ significantly alter the value of the dispersion integral (2.6) (see Table I). Therefore variation in the value of the function $F(s)$ due to uncertainty in the determinations of α_{D^*} more than offsets slight changes in the vertex function $K(s)$. To evaluate the contribution to Eq. (2.6) from the $D\bar{D}$ state we have then set $K(s)=K(m_\psi^2)=1$ and performed the integration using Eq. (3.2) for $R(s,t)$ and different values of $\alpha_{D^*}(t)$ as described in Appendix B:

TABLE I. Values of the integrals I [see Eq. (3.4)] of the contributions of the charmed-pseudoscalar-pair intermediate states to $\psi\to\rho\pi$ and $\psi\to K^*\bar{K}$. The trajectory equations are $\alpha_{D^*}(t)=\alpha'(t-m_{D^*}^2)+1$ and $\alpha_{F^*}(t)=\alpha'(t-m_{F^*}^2)+1$.

Slope of exchanged trajectory α' (GeV^2)	$I(D\bar{D};\rho\pi)$ (GeV^2)	$I(F\bar{F};K^*\bar{K})$ D^* exchange (GeV^2)	$I(D\bar{D};K^*\bar{K})$ F^* exchange (GeV^2)
0.45	0.63×10^{-4}	0.40×10^{-4}	0.42×10^{-4}
0.5	0.25×10^{-4}	0.15×10^{-4}	0.16×10^{-4}
0.55	1.0×10^{-5}	0.59×10^{-5}	0.6×10^{-5}

$$F(D\bar{D};\rho\pi) = \frac{-\beta G}{32\pi^2}\int_{4m_D^2}^\infty ds\,\frac{8k^3}{\sqrt{s}}\,\frac{(\alpha'_\omega s)^{a-1}}{s-m_\psi^2}\,\frac{1}{R^2}\times\left(\cosh R-\frac{\sinh R}{R}\right) \tag{3.3}$$

$$= \frac{-\beta G}{32\pi^2}\,I(D\bar{D};\rho\pi) \tag{3.4}$$

The quantities a and R in Eq. (3.3) are defined by

$$a=\alpha_{D^*}(0)+\alpha'_{D^*}\tfrac{1}{2}(t_{\max}+t_{\min}) \tag{3.5}$$

and

$$R=2\alpha'_D\; kq\ln(\alpha'_\omega s)\,. \tag{3.6}$$

The value of $I(D\bar{D};\,\rho\pi)$ for various values of α_{D^*} is tabulated in Table I.

For the evaluation of the contribution of the $D^*\bar{D}$ intermediate state we proceed similarly. We shall assume Regge behavior for the five amplitudes defined in Eq. (2.15). Standard Regge-pole analysis[29] leads us to write

$$E(s,t)\approx\beta_E(\alpha'_\omega s)^{\alpha_{D^*}(t)}\,, \tag{3.7}$$

$$D(s,t)\approx\beta_D(\alpha'_\omega s)^{\alpha_{D^*}(t)}\,, \tag{3.8}$$

$$C(s,t)\approx\beta_C(\alpha'_\omega s)^{\alpha_{D^*}(t)-1}\,, \tag{3.9}$$

$$B(s,t)\approx\beta_B(\alpha'_\omega s)^{\alpha_{D^*}(t)-1}\,, \tag{3.10}$$

$$A(s,t)\approx\beta_A(\alpha'_\omega s)^{\alpha_{D^*}(t)-2}\,. \tag{3.11}$$

For reasons similar to the case of the $D\bar{D}$ intermediate state, we shall ignore the variation of the $\psi D^*\bar{D}$ vertex function $H(s)$, compared to the variation with s of the $D^*\bar{D}\to\rho\pi$ amplitude, and set $H(s)=H(m_\psi{}^2)=1$. Using Eqs. (3.7)–(3.11) we can carry out the evaluation of the dispersion integral Eq. (2.6) to get

$$F_E(D^*\bar{D};\rho\pi)=\frac{-\beta_E G_1}{32\pi^2}\int_{(m_D+m_{D^*})^2}^{\infty} ds\,\frac{4k^2}{q\sqrt{s}}\,\frac{(\alpha_\omega' s)^a}{s-m_\psi{}^2}\,\frac{1}{R}\times\left(\cosh R-\frac{\sinh R}{R}\right)$$

$$=\frac{-\beta_E G_1}{32\pi^2}J_E(D^*\bar{D};\rho\pi)\,, \qquad (3.12)$$

$$F_D(D^*\bar{D};\rho\pi)=\frac{-\beta_D G_1}{32\pi^2}\int_{(m_D+m_{D^*})^2}^{\infty} ds\,\frac{4k^3}{\sqrt{s}}\,\frac{(\alpha_\omega' s)^a}{s-m_\psi{}^2}\,\frac{1}{R^2}\times\left(\cosh R-\frac{\sinh R}{R}\right)$$

$$=\frac{-\beta_D G_1}{32\pi^2}J_D(D^*\bar{D};\rho\pi)\,, \qquad (3.13)$$

$$F_C(D^*\bar{D};\rho\pi)=\frac{\beta_C G_1}{32\pi^2}\int_{(m_D+m_{D^*})^2}^{\infty} ds\,\frac{4k^3}{\sqrt{s}}\,\frac{(\alpha_\omega' s)^{a-1}}{s-m_\psi{}^2}\,\frac{1}{R^2}\times\left(\cosh R-\frac{\sinh R}{R}\right)$$

$$=\frac{\beta_C G_1}{32\pi^2}J_C(D^*\bar{D};\rho\pi)\,, \qquad (3.14)$$

$$F_B(D^*\bar{D};\rho\pi)=\frac{-\beta_B G_1}{32\pi^2}\int_{(m_D+m_{D^*})^2}^{\infty} ds\,\frac{4k^3}{\sqrt{s}}\,\frac{(\alpha_\omega' s)^{a-1}}{s-m_\psi{}^2}\,\frac{1}{R^2}\times\left(\cosh R-\frac{\sinh R}{R}\right)$$

$$=\frac{-\beta_B G_1}{32\pi^2}J_B(D^*\bar{D};\rho\pi)\,, \qquad (3.15)$$

$$F_A(D^*\bar{D};\rho\pi)=\frac{\beta_A G_1}{32\pi^2}\int_{(m_D+m_{D^*})^2}^{\infty} ds\,\frac{4k^3}{\sqrt{s}}\,\frac{(\alpha_\omega' s)^{a-2}}{s-m_\psi{}^2}\,\frac{1}{R^2}\times\left(\cosh R-\frac{\sinh R}{R}\right) \qquad (3.16)$$

$$=\frac{\beta_A G_1}{32\pi^2}J_A(D^*\bar{D};\rho\pi)\,, \qquad (3.17)$$

where F_E, for example, is the contribution to F from the amplitude E in Eq. (2.15) and a and R are as defined before in Eqs. (3.5) and (3.6). We have tabulated J_E and J_D in Table II for various values of α_{D^*}'. The integral occurring in Eq. (3.15) is apart from a factor 2 identical in structure with $I(D\bar{D};\rho\pi)$ in Eq. (3.4). Numerically it is smaller than $I(D\bar{D};\rho\pi)$ because of the higher threshold for the $D^*\bar{D}$ state. The contribution of the A term to the $\psi\rho\pi$ coupling F in Eq. (2.6) is even smaller because of the extra minus two in the exponent in Eq. (3.11) and will be ignored in the following.

TABLE II. Values of the integrals J_E [see Eq. (3.12)] of the contribution of the charmed vector-pseudoscalar intermediate state to $\psi\to\rho\pi$ and $\psi\to K^*\bar{K}$. The subscript E denotes that these quantities are contributions of the invariant amplitude $E(s,t)$ [see Eq. (3.12)].

Slope of the exchanged trajectory α' (GeV^2)	$J_E(D^*\bar{D};\rho\pi)$	$J_E(F^*\bar{F};K^*K)$ D^* exchange	$J_E(D^*\bar{D};K^*\bar{K})$ F^* exchange
0.45	0.15×10^{-2}	0.11×10^{-2}	0.92×10^{-3}
0.50	0.62×10^{-3}	0.45×10^{-3}	0.36×10^{-3}
0.55	0.26×10^{-3}	0.18×10^{-3}	0.14×10^{-3}

The calculation of the transition $\psi\to K^*\bar{K}$ proceeds along identical lines. Both $D\bar{D}$ and $F\bar{F}$ intermediate states contribute in this case, the former proceeding via F^* exchange while the latter with D^* exchange. Although there is little doubt that $m_F>m_D$ and $m_{F^*}>m_{D^*}$, their precise values will be known only after the experimental identification of these states. We have assumed uniformly $m_F=1.97$ GeV and $m_{F^*}=2.12$ GeV in our calculation of the dispersion integrals. The hundred-MeV mass difference between the F and D states, which is not an unreasonable value, that we use in our calculations illustrates how the breakdown of SU(3) symmetry occurs dynamically in ψ decays. This can be seen from Table I, where we have tabulated the contribution of the integrals analogous to $I(D\bar{D};\rho\pi)$ in Eq. (3.4) for the case of $\psi\to K^*\bar{K}$. The contribution $I(D\bar{D};K^*\bar{K})$ is smaller because the exchanged F^* trajectory lies lower than D^* while $I(F\bar{F};K^*\bar{K})$ is smaller because the threshold of the dispersion integral now starts at $4m_F{}^2$ instead of $4m_D{}^2$ [although the exchanged object is the same as in $I(D\bar{D};\rho\pi)$].

IV. DUAL MODEL AND ESTIMATES OF THE SUPPRESSION FACTORS IN ψ DECAYS

To be able to obtain numerical estimates of the effective coupling $\psi\to\rho\pi$ we must know the coupling strengths $\psi D\bar{D}$, $\psi D^*\bar{D}$, etc., as well as the Regge residues occurring in Eqs. (3.2), (3.7), and (3.8). The former can be obtained using SU(4) symmetry.[30] The residue of the D^* Regge pole coupling to the D and π states occurring in Eq. (3.2) can also be obtained by using SU(4) symmetry and the knowledge of the ρ-trajectory residue in πN charge-exchange scattering. We shall instead use a slightly different procedure which we believe brings out the suppression factors involved in the effective $\psi\rho\pi$ coupling more clearly. To this end we write for the amplitude $D\bar{D}\to\rho\pi$, which is similar in Lorentz structure to the amplitude $\pi\pi\to\omega\pi$ originally considered by Veneziano,[31] the analogous expression

$$T(D^+(k_1)+D^-(k_2)\to\pi^+(q_1)+\rho^-(\epsilon_2,q_2)$$

$$=\gamma_1\epsilon_{\mu\nu\lambda\sigma}\epsilon_2^\mu k_1^\nu k_2^\lambda q_2^\sigma\,\frac{\Gamma(1-\alpha_{D^*}(t))\Gamma(1-\alpha_\omega(s))}{\Gamma(2-\alpha_{D^*}(t)-\alpha_\omega(s))}\,. \tag{4.1}$$

There is only one term in Eq. (4.1) since the u channel is exotic. We can fix the constant γ_1 by evaluating the residue at the D^* pole in the t channel and relating it to the on-shell coupling constant. We get

$$\gamma_1=-2\alpha'_{D^*}g_{\bar{D}^*\pi D}\,g_{D^*\rho\bar{D}}\,. \tag{4.2}$$

We can rewrite the Γ functions in Eq. (4.1) in the form

$$\Gamma(1-\alpha_{D^*}(t))\,\frac{-\sin\pi[\alpha_{D^*}(t)+\alpha_\omega(s)]}{\sin\pi\alpha_\omega(s)}\times\frac{\Gamma(\alpha_\omega(s)+\alpha_{D^*}(t)-1)}{\Gamma(\alpha_\omega(s))}\,. \tag{4.3}$$

Using the linear trajectory form $\alpha_\omega(s)=\alpha_\omega(0)+\alpha'_\omega s$ we see that even at the $D\bar{D}$ threshold $\alpha'_\omega s\approx 14$ is large compared to unity. We can then use the asymptotic form for the Γ functions and smooth out the poles of $\sin\pi\alpha_\omega(s)$ by taking the large-s limit slightly off the real axis with the result

$$T(D\bar{D}\to\rho\pi)=2\alpha'_{D^*}g_{\bar{D}^*\pi D}\,g_{D^*\rho\bar{D}}\Gamma(1-\alpha_{D^*}(t))e^{i\pi\alpha_{D^*}(t)}\times\epsilon_{\mu\nu\lambda\sigma}\epsilon_2^\mu k_1^\nu k_2^\lambda q_2^\sigma(\alpha'_\omega s)^{\alpha_{D^*}(t)-1}\,. \tag{4.4}$$

This expression for the $D\bar{D}\to\rho\pi$ is identical to Eq. (3.1) and (3.2) except for the signature factor $e^{-i\pi\alpha_{D^*}(t)}\Gamma(1-\alpha_{D^*}(t))$. The dispersion integrals Eq. (3.3) whose values [for different $\alpha_{D^*}(t)$] are tabulated in Table I were evaluated without the signature factor. We have also carried out the dispersion integral with this factor included, that is, with Eq. (4.4) instead of Eq. (3.2), and find that the value of the integral decreases by a factor of two or three. As already pointed out in Sec. III, the uncertainty in the value of the dispersion integrals due to uncertainty in $\alpha_{D^*}(t)$ is larger than that due to variation in the $\psi D\bar{D}$ vertex function $K(s)$ or the signature factor mentioned above. Therefore, to obtain an estimate of the order of magnitude of the $\psi\rho\pi$ coupling we set β occurring in Eq. (3.2) equal to γ_1 as given by Eq. (4.2). We can then write for the contribution of the $D\bar{D}$ state to the coupling (adding the contributions of D^+D^- and $D^0\bar{D}^0$)

$$F_{\psi\rho\pi}^{D\bar{D}}=4\alpha'_{D^*}\,\frac{g_{\bar{D}^*D\pi}\,g_{\psi D\bar{D}}}{4\pi}\,g_{D^*\bar{D}\rho}\,\frac{I(D\bar{D};\rho\pi)}{8\pi}\,, \tag{4.5}$$

where I is tabulated in Table I. Now if SU(4) symmetry is used in the sense of Okubo's ansatz,[9] then

$$\frac{g_{\bar{D}^*D\pi}\,g_{\psi D\bar{D}}}{4\pi}=\frac{1}{2}\,\frac{g_{\rho\pi\pi}{}^2}{4\pi}\approx 1.4\,, \tag{4.6}$$

$$g_{D^*\bar{D}\rho}=\frac{1}{\sqrt{2}}\,g_{\omega\rho\pi}\,. \tag{4.7}$$

Since $2\alpha'_{D^*}\approx 1$ GeV^{-2} it follows from Eqs. (4.5) and (4.6) that the factor $I/8\pi$ (in units of GeV^2) is a measure of the magnitude of the unitarity corrections to the OZI rule.

To estimate the contribution of $D^*\bar{D}$ intermediate state we can proceed similarly and write for $E(s,t)$

$$E(s,t)=\gamma_2\,\frac{\Gamma(1-\alpha_{D^*}(t))\Gamma(1-\alpha_\omega(s))}{\Gamma(1-\alpha_{D^*}(t)-\alpha_\omega(s))}\,, \tag{4.8}$$

with

$$\gamma_2=\frac{\alpha'_{D^*}}{\alpha'_\omega}\,g_{D^*\bar{D}^*\rho}\,g_{\bar{D}^*D\pi}\,. \tag{4.9}$$

Identifying β_E occurring in Eq. (3.7) with γ_2 as given by Eq. (4.9) we have

$$F_{\psi\rho\pi}^{D^*\bar{D}}=\frac{\alpha'_{D^*}}{\alpha'_\omega}\,\frac{g_{D^*\bar{D}^*\rho}\,g_{\bar{D}^*D\pi}}{4\pi}\,g_{\psi\bar{D}^*D}\,\frac{J_E(D^*\bar{D}\to\rho\pi)}{8\pi}\,. \tag{4.10}$$

Usual universality arguments would suggest $g_{D^*\bar{D}^*\rho}=g_{\bar{D}^*D\pi}$ so that we expect

$$\frac{g_{D^*\bar{D}^*\rho}\,g_{\bar{D}^*D\pi}}{4\pi}\approx\frac{1}{2}\,\frac{g_{\rho\pi\pi}{}^2}{4\pi}\,.$$

From Eq. (4.10) then $J_E/8\pi$ emerges as the measure of the unitarity correction factor. The large difference between the contribution of the $D^*\bar{D}$ state as compared to the $D\bar{D}$ state is of course to be traced to the difference between Eq. (3.2) and Eq. (3.7). To obtain the value of $g_{\psi\rho\pi}$ we must add the contribution of all the intermediate states. It is evident from Tables I, II, and III that the $D\bar{D}$ and the $D^*\bar{D}+\bar{D}^*D$ states alone contribute a unitarity

TABLE III. Values of the integrals J_D [see Eq. (3.13)] of the contributions of the charmed-vector-pseudoscalar intermediate state to $\psi\to\rho\pi$ and $\psi\to K^*\bar{K}$. The subscript D indicates that these quantities are the contribution of the invariant amplitude $D(s,t)$ [see Eq. (3.13)].

Slope of the exchange trajectory α' (GeV^2)	$J_D(D^*\bar{D};\rho\pi)$ (GeV^2)	$J_D(F^*\bar{F};K^*\bar{K})$ D^* exchange (GeV^2)	$J_D(D^*\bar{D};K^*\bar{K})$ F^* exchange (GeV^2)
0.45	0.51×10^{-3}	0.36×10^{-3}	0.31×10^{-3}
0.50	0.19×10^{-3}	0.15×10^{-3}	0.11×10^{-3}
0.55	0.73×10^{-4}	0.49×10^{-4}	0.41×10^{-4}

correction factor or a suppression factor as compared to OZI-rule-allowed coupling (like $\psi D^*\bar{D}$) between 10^{-4} and 10^{-5} in the amplitude. This is to be compared with the experimentally observed suppression factor $\approx 10^{-4}$ in the amplitude [see Eq. (1.5)]. We can easily extend our procedure to calculate the contribution of the higher-mass charmed-hadron states like $D^*\bar{D}^*$, etc. However, we have *a priori* some assurance that these higher-mass states would make a smaller contribution compared to the $D^*\bar{D}$ state because of their higher thresholds, so that the order-of-magnitude estimates of the corrections to the OZI rule by the lower-mass charmed-hadron states remain meaningful.

To get an estimate of $\psi \to K^*\bar{K}$ coupling we can repeat the above procedure for $\psi \to \rho\pi$ taking into account this time both $D\bar{D}$ and $F\bar{F}$, $D^*\bar{D}$ and $F^*\bar{F}$, etc. for the intermediate states. If we assume that coupling strengths $\psi D\bar{D}$, $\psi F\bar{F}$ and the Regge residues β's (which all involve connected vertices) retain their SU(3)-symmetric values, there is still an SU(3) violation in the effective coupling $\psi \to K^*\bar{K}$ induced by the mass splittings among the charmed hadrons. If, for example, charmed pseudoscalars were the sole intermediate state contributing to ψ decay, then clearly the ratio of $I(D\bar{D};\rho\pi)$ to $\frac{1}{2}[I(D\bar{D};K^*\bar{K})+I(F\bar{F};K^*\bar{K})]$ is the ratio of the amplitudes $\psi\to\rho\pi$ and $\psi \to K^*\bar{K}$. Table I then suggests that this ratio lies between 1.5 and 2. The relative contribution of the $D^*\bar{D}$ and $F^*\bar{F}$ states listed in Tables II and III also point to a similar value for the ratio. We can clearly expect the higher-mass multiplets such as $D^*\bar{D}^*$ and $F^*\bar{F}^*$ also to maintain this behavior as the basic mechanism remains the same. Summarizing then, if the hadron masses within a charmed SU(3) multiplet are split by a hundred MeV [$m(c\bar{s})-m(c\bar{u})\approx 100$ MeV] then our calculations indicate that the strong-interaction part of the amplitudes $\psi\to\rho^+\pi^-$ and $\psi\to K^{*+}K^-$ can have a ratio between 1.5 and 2.

V. $\psi \to K\bar{K}$

Among the strong-interaction transitions $\psi\to$ two pseudoscalars the only one that is allowed by G-parity selection rule is the decay mode $\psi\to K\bar{K}$. If SU(3) symmetry were exact this decay would also be forbidden, and the interesting feature of the experimental data as pointed out in the Introduction and in Appendix A is that the branching ratios for $\psi\to K^+K^-$ and $\psi\to K_L K_S$ are consistent with the one-photon term only, i.e., no SU(3) violation in the strong-interaction part of amplitude [$\beta=0$ in Eq. (1.7)] in contradistinction to $\psi\to K^*K$.

We can compute the strong-interaction part of the transition $\psi\to K\bar{K}$ by following the same procedure adopted for $\psi\to K^*\bar{K}$. The transition matrix element is given by

$$\tilde{M}=\epsilon_1^{\mu}\tilde{M}_{\mu}\,, \tag{5.1}$$

with

$$\tilde{M}_{\mu}=(4q_1^0q_2^0)^{1/2}\langle q_1q_2 \text{ out}|J_{\mu}|0\rangle \tag{5.2}$$

$$=(2q_2^0)^{1/2}\int d^4x\, e^{iq_1\cdot x} \times\langle q_2|[j(x),J_{\mu}(0)]\theta(x)|0\rangle, \tag{5.3}$$

where J_{μ} is the source of the ψ field with polarization ϵ_1^{μ} as in Eq. (2.1) and $j(x)$ is the source of the kaon field and q_1 and q_2 are the momenta of the final kaons. There is only one invariant matrix element for the transition for which we can write, analogous to Eqs. (2.9) and (2.10),

$$\tilde{M}_{\mu}=(q_1-q_2)_{\mu}\tilde{F}(P^2,q_1^{\,2},q_2^{\,2}), \tag{5.4}$$

$$\tilde{F}(P^2=s)=\frac{1}{\pi}\int ds'\,\frac{\mathrm{Im}\tilde{F}(s')}{s'-s-i\epsilon}. \tag{5.5}$$

The function $\mathrm{Im}\tilde{F}(s')$ is computed from the absorptive part $\tilde{A}_{\mu}$ of $\tilde{M}_{\mu}$, where

$$\tilde{M}_{\mu}=\tilde{D}_{\mu}+i\tilde{A}_{\mu}\,,$$

with

$$\tilde{A}_{\mu}=(2q_2^0)^{1/2}\pi\sum_n\langle q_2|j|n\rangle\langle n|J_{\mu}|0\rangle \times\delta^{(4)}(P_n-q_1-q_2). \tag{5.6}$$

As before we shall keep only the charmed-hadron states in the sum over the intermediate states $|n\rangle$ in Eq. (5.6) and the lowest-mass state of interest is then the $D\bar{D}$ state. The $\psi D\bar{D}$ vertex function has been defined previously in Eq. (2.9) and the other factor that occurs in Eq. (5.6) is the scattering amplitude

$$D(k_1)+\bar{D}(k_2)\to K(q_1)+\bar{K}(q_2).$$

Writing

$$\langle\bar{K}(q_2)|j|D(k_1)\bar{D}(k_2)\rangle=\left(\frac{1}{8k_1^0k_2^0q_2^0}\right)^{1/2}P_1(s,t)\,, \tag{5.7}$$

we can insert Eq. (5.7) and Eq. (2.9) in Eq. (5.6) to get the $D\bar{D}$ intermediate-state contribution to the absorptive part as

$$\mathrm{Im}\tilde{F}(s)=-\frac{G}{32\pi}\,\frac{2k^2}{q\sqrt{s}}\int dz\,\mathrm{Re}[K^*(s)P_1(s,t)]z\,, \tag{5.8}$$

where q is the center-of-mass momentum of the kaons and z is the cosine of the c.m. scattering angle.

The contribution of the $D^*\bar{D}$ intermediate state can be evaluated by writing the scattering amplitude

$$D^*(\epsilon, k_2) + \bar{D}(k_1) \to K(q_1) + \bar{K}(q_2) \tag{5.9}$$

as

$$T(\epsilon, k_1, k_2; q_1, q_2) = \epsilon_{\mu\nu\lambda\sigma}\epsilon^{\mu} k_1^{\nu} q_1^{\lambda} q_2^{\sigma} V(s, t) \tag{5.10}$$

and using for the $\psi D^*\bar{D}$ vertex function the expression given by Eq. (2.13). We get for the absorptive part

$$\mathrm{Im}\tilde{F}(s) = -\frac{G_1}{32\pi}\frac{k^3}{\sqrt{s}}\int dz(1-z^2)\,\mathrm{Re}[H^*(s)V(s,t)]. \tag{5.11}$$

To proceed further we can assume, following the arguments given in Sec. III, that the amplitudes $P_1(s,t)$ and $V(s,t)$ can be approximated by their Regge asymptotic forms, i.e.,

$$P_1(s,t) \approx \beta_P(\alpha'_\omega s)^{\alpha_{F^*}(t)}, \tag{5.12}$$

$$V(s,t) \approx \beta_V(\alpha'_\omega s)^{\alpha_{F^*}(t)-1}, \tag{5.13}$$

and approximate the vertex functions $K(s)$ and $H(s)$ by their mass-shell value unity. The contribution to $\tilde{F}(s)$ of the $D\bar{D}$ intermediate state is then computed from Eqs. (5.6), (5.7), (5.8), and (5.12) with the result

$$\tilde{F}_{D\bar{D}}(s = m_\psi^2) = \frac{-G\beta_P}{32\pi^2}\int_{4m_D^2}^{\infty} ds \frac{k^2}{q\sqrt{s}}\frac{(\alpha'_\omega s)^{a'}}{s - m_\psi^2}\frac{4}{R'} \times\left(\cosh R' - \frac{\sinh R'}{R'}\right) \tag{5.14}$$

$$= \frac{-G\beta_P}{32\pi^2} J_P(D\bar{D}; K\bar{K}), \tag{5.15}$$

where the quantities a' and R' differ from a and R defined by Eqs. (3.5) and (3.6) in that α_{D^*} is to be replaced by α_{F^*}. The value of the integral J_P is listed in the fourth column of Table IV for different values of the F^* trajectory.

In addition to the $D\bar{D}$ intermediate state we must take into account the contribution of the $F\bar{F}$ state. The calculation is identical to the $D\bar{D}$ case and we obtain in place of Eqs. (5.14) and (5.15)

$$\tilde{F}_{F\bar{F}}(s = m_\psi^2) = \frac{\beta'_P G'}{32\pi^2}\int_{4m_F^2}^{\infty} ds\left[\frac{k^2}{q\sqrt{s}}\frac{(\alpha'_\phi s)^a}{s - m_\psi^2}\frac{4}{R} \times\left(\cosh R - \frac{\sinh R}{R}\right)\right] \tag{5.14'}$$

$$= \frac{\beta'_P G'}{32\pi^2} J_P(F\bar{F}; K\bar{K}) \tag{5.15'}$$

These are obtained from Eqs. (5.14) and (5.15) with the replacement $m_D \to m_F$, $\alpha_{F^*} \to \alpha_{D^*}$ with the coupling strength G' referring to the $\psi F\bar{F}$ vertex and β'_P to the square of the Regge residue of the exchanged D^* pole in the $F\bar{F} \to K\bar{K}$ amplitude. Notice that there is an overall difference in sign between Eq. (5.15) and Eq. (5.15′).

Consider now the SU(3)-symmetric limit. We then have $G = G'$, $\beta_P = \beta'_P$, $m_D = m_F$, $\alpha_{D^*} = \alpha_{F^*}$, etc. so that the dispersion integrals (5.15) and (5.15′) will be equal. In the $\psi \to K\bar{K}$ amplitude then the $D\bar{D}$ and $F\bar{F}$ contributions cancel each other exactly. But we have seen in the case of $\psi \to K^*\bar{K}$ the effect of symmetry breakdown manifests itself dynamically in the dispersion integrals if we take into account the mass splittings among the charmed-meson multiplets even if the vertex functions retain their SU(3)-symmetric value. To illustrate this point we have computed the dispersion integral (5.15) for the hypothetical SU(3)-symmetric limit with $m_D = m_F = 1.87$ GeV, $\alpha_{D^*} = \alpha_{F^*}$, $m_{D^*} = m_{F^*} = 2.02$ GeV and listed it in the second column of Table IV. It is seen from the table that the values listed in columns 3 and 4 (which are calculated with $m_F - m_D = m_{F^*} - m_{D^*} = 100$ MeV) are substantially different from those in column 2. However, the difference between the dispersion integrals $J_P(D\bar{D}; K\bar{K})$ and $J_P(F\bar{F}; K\bar{K})$ is much smaller than the difference between either of them and column 2. Therefore, if we assume as we did for $\psi \to K^*\bar{K}$ that the vertex functions retain their SU(3)-symmetry values, the $D\bar{D}$ and $F\bar{F}$ contributions (5.15) and (5.15′) continue to cancel

TABLE IV. Values of the integral J_P [cf. Eqs. (5.15) and (5.15′)] for the contribution of the charmed-pseudoscalar-pair intermediate states to $\psi \to K\bar{K}$. Column 2 gives the value of J_P in the SU(3)-symmetry limit $m_D = m_F = 1.87$ GeV, $m_{D^*} = m_{F^*} = 2.02$ GeV. Columns 3 and 4 gives the values of J_P with a 100-MeV splitting (see text) between the masses, namely $m_D = 1.87$ GeV, $m_F = 1.97$ GeV and $m_{D^*} = 2.02$ GeV, $m_{F^*} = 2.12$ GeV.

Slope of the exchanged trajectory α' (GeV^2)	J_P $m_D = m_F$, $m_{D^*} = m_{F^*}$	$J_P(F\bar{F}; K\bar{K})$ D^* exchange	$J_P(D\bar{D}; K\bar{K})$ F^* exchange
0.45	0.34×10^{-3}	0.21×10^{-3}	0.19×10^{-3}
0.50	1.7×10^{-4}	1.0×10^{-4}	0.89×10^{-4}
0.55	0.87×10^{-4}	0.49×10^{-4}	0.42×10^{-4}

TABLE V. Values of the integral J_V [cf. Eqs. (5.17) and (5.17′)] for the contribution of the charmed vector-pseudoscalar intermediate states to $\psi \to K\bar{K}$. Column 2 gives the value of J_V in the SU(3) limit, $m_D = m_F = 1.87$ GeV, $m_{D^*} = m_{F^*} = 2.02$ GeV. Columns 3 and 4 give the value of J_V with $m_D = 1.87$ GeV, $m_F = 1.97$ GeV and $m_{D^*} = 2.02$ GeV, $m_{F^*} = 2.12$ GeV.

Slope of the exchanged trajectory α' (GeV^2)	J_V $m_D = m_F$, $m_{D^*} = m_{F^*}$ (GeV^2)	$J_V(F^*\bar{F}; K\bar{K})$ D^* exchange (GeV^2)	$J_V(D^*\bar{D}; K\bar{K})$ F^* exchange (GeV^2)
0.45	0.53×10^{-3}	0.34×10^{-3}	0.29×10^{-3}
0.50	0.20×10^{-3}	0.12×10^{-3}	0.10×10^{-3}
0.55	0.76×10^{-4}	0.45×10^{-4}	0.36×10^{-4}

each other nearly exactly.

To see whether this type of cancellation is operative for the $D^*\bar{D}$ and $F^*\bar{F}$ state contributions we have first computed the $D^*\bar{D}$ contributions using Eqs. (5.10) and (5.13). We find

$$\tilde{F}_{D^*\bar{D}}(s = m_\psi{}^2) = \frac{-G_1\beta_V}{32\pi^2}\int_{(m_D + m_{D^*})^2}^{\infty} ds \times \left[\frac{k^3}{\sqrt{s}}\,\frac{(\alpha'_\omega s)^{\alpha'_*-1}}{s - m_\psi{}^2}\,\frac{4}{R'^2} \times \left(\cosh R' - \frac{\sinh R'}{R'}\right)\right] \tag{5.16}$$

$$= \frac{-G_1\beta_V}{32\pi^2} J_V(D^*\bar{D}; K\bar{K}) . \tag{5.17}$$

The values of the integral $J_V(D^*\bar{D}; K\bar{K})$ are listed in the fourth column of Table V (for different values of the F^* trajectory). The $F^*\bar{F}$ state contribution analogous to Eqs. (5.16) and (5.17) is

$$\tilde{F}_{F^*\bar{F}}(s = m_\psi{}^2) = \frac{G'_1\beta'_V}{32\pi^2}\int_{(m_F + m_{F^*})^2}^{\infty} ds \times \left[\frac{k^3}{\sqrt{s}}\,\frac{(\alpha'_\phi s)^{\alpha-1}}{s - m_\psi{}^2}\,\frac{4}{R^2} \times \left(\cosh R - \frac{\sinh R}{R}\right)\right] \tag{5.16′}$$

$$= \frac{G'_1\beta'_V}{32\pi^2} J_V(F^*\bar{F}; K\bar{K} , \tag{5.17′}$$

with G'_1 referring to the strength of the $\psi F^*\bar{F}$ vertex and β'_V the product of the Regge residues of the D^* pole in $F^*\bar{F} \to K\bar{K}$. We have listed the values of $J_V(F^*\bar{F} - K\bar{K})$ in the third column of Table V. We see again that the difference between the values listed in columns 3 and 4 is much smaller than the difference between either one of them and the hypothetical SU(3)-symmetric value listed in column 2. We can expect this behavior to be repeated with higher-mass multiplets $D^*\bar{D}^*$ and $F^*\bar{F}^*$, charmed-baryon–antibaryon pairs, etc., so that pairwise cancellations take place between contributions coming from members of each charmed-hadron multiplet, thus explaining the apparent absence of SU(3) violation in the strong-interaction part of the amplitude $\psi \to K\bar{K}$.

VI. DISCUSSION

Following the arguments given in the previous section we can expect the SU(3)-violating terms in the strong-interaction part of the amplitudes to be negligible in the decays for $\psi \to K^*(892)\bar{K}^*(892)$ and $\psi \to K^*(1420)\bar{K}$. These two decays then proceed mostly through the one-photon term. Current experimental upper limits[19,20] on the branching ratios for these as well as on $\psi \to \pi A_2$ are consistent with our expectation.

For the χ states, charge-conjugation invariance plus SU(3) forbids the decays[32]

$$\chi \to \bar{K}K^*(892) , \tag{6.1}$$

$$\chi \to \bar{K}^*(892)K^*(1420) . \tag{6.2}$$

In analogy with our discussion of $\psi \to K\bar{K}$ we can once again expect that SU(3) symmetry is better respected in (6.1) and (6.2) than in allowed decays such as

$$\chi \to \rho\rho, K^*(892)\bar{K}^*(892) , \tag{6.3}$$

$$\chi(^3P_{0,2}) \to \pi\pi, K\bar{K} . \tag{6.4}$$

Since there is no interference from the one-photon terms as in the case of ψ decays, comparison of the decay rates for the reactions (6.1)–(6.4) should provide tests of dynamics of $c\bar{c}$ decays as discussed in this paper. In the case of baryon-antibaryon decays our considerations imply that the decay amplitudes satisfy the relation

$$T(\psi \to p\bar{p}) > T(\psi \to \Lambda\bar{\Lambda}) > T(\psi \to \Xi\bar{\Xi}) . \tag{6.5}$$

This inequality should hold also for the decay of χ states into $B\bar{B}$ pairs.

The calculations for the amplitude $\psi' \to \rho\pi$ pro-

TABLE VI. Values of the integrals of the contribution of the $D\bar{D}$ and $D^*\bar{D}$ intermediate states to $\psi' \to \rho\pi$. $I'(D\bar{D} \to \rho\pi)$ is defined by Eq. (3.4) by replacing ψ by ψ'. Similarly $J'_E(D^*\bar{D} - \rho\pi)$ is defined by Eq. (3.12) by replacing ψ by ψ'.

Slope of the exchanged trajectory α' (GeV^2)	$I'(D\bar{D};\rho\pi)$ (GeV^2)	$J'_E(D^*\bar{D};\rho\pi)$
0.45	1.09×10^{-4}	0.20×10^{-2}
0.50	0.43×10^{-4}	0.84×10^{-3}
0.55	1.7×10^{-5}	0.36×10^{-3}

ceed in an identical manner to that of $\psi\to\rho\pi$. In fact, in the expression for the absorptive part, $\psi'\to$ charmed hadrons $\to\rho\pi$, the term involving the amplitude charmed hadrons $\to\rho\pi$ is identical to that in $\psi\to\rho\pi$. It may be thought at first sight that since the dispersion denominator in Eq. (2.6) is smaller in the case of ψ' decays, because of the proximity of charm threshold, it may provide an enhancement factor. Firstly, since the vertex function $\psi,\psi'\to$ charmed hadrons as well as the scattering amplitudes charmed hadrons $\to\rho\pi$ in our calculations incorporate the correct angular momentum factors, the effect of the denominator is not as large as may be imagined. In Table VI we have given the analogs of the integrals $I(D\bar{D};\rho\pi)$ [Eq. (3.4)] and $J_E(D\bar{D};\rho\pi)$ for the ψ' case. It is seen that the enhancement factor is less than two, in all cases. This slight enhancement is more than offset by the difference in the $\psi' D\bar{D}(\psi' D^*\bar{D})$ vertex as compared to the $\psi D\bar{D}(\psi D^*\bar{D})$ vertex. We know from experiment[33] that, in the case of the higher resonances, the coupling of $\rho(1250)$ and $\rho(1600)$ to $\pi\pi$ are much weaker than $\rho(770)$ to $\pi\pi$. Using SU(4) symmetry (in the sense of generalized Okubo ansatz[9]) we can therefore analogously expect the $\psi' D\bar{D}$ coupling to be much smaller than that of $\psi D\bar{D}$. In other words, progressive decrease of the couplings of the excited ψ states to the lowest charmed-meson multiplets, such as $D\bar{D}, D^*\bar{D}, D^*\bar{D}^*$, should result in a progressive decrease of the effective couplings of these ψ states to ordinary hadrons.

It is interesting to compare our phenomenological S-matrix approach to understand the decays of ψ with the approach[34] based on quantum chromodynamics (QCD), which is known to be an asymptotically free theory.[35] In this latter method the decays of ψ and ψ' are viewed as proceeding via the annihilation of $c\bar{c}$ quarks into 3 gluons which subsequently materialize into ordinary hadrons. The narrow width of ψ and ψ' is attributed to the smallness of the quark-gluon coupling constant which asymptotically tends to zero, while we have attributed it to the smallness of the amplitude for charmed hadrons $\to$ ordinary hadrons. If QCD is to be regarded as the fundamental theory of hadrons, one should, besides deriving the spectrum of hadrons, be able to derive Regge asymptotic behavior of the various physical scattering amplitudes. Thus, although in principle one should derive the basic ingredient of our calculation, namely asymptotic decrease of the amplitude for charmed hadrons $\to$ ordinary hadrons, from the asymptotic-freedom property of the underlying theory, no practical calculation exists at present. Our phenomenological approach is somewhat closer to experiment; besides, the asymptotic scale in our calculations is also clearly set, namely by the slope of the ω trajectory which governs the high-energy behavior of the reactions $D\bar{D}, D^*\bar{D}, D^*\bar{D}^*, \ldots \to \rho\pi$. We may also add that in the context of calculations[34] based on QCD, the partial width for an exclusive channel such as $\psi\to\rho\pi$ is quite difficult.

APPENDIX A: SU(3) IN $J/\psi(3095)$ DECAYS

We shall assume that the transitions $J/\psi\to$ ordinary hadrons is described by an effective Hamiltonian as suggested by Okubo[18]

$$\mathcal{H}_\psi(x) = \left[\frac{\alpha}{2\sqrt{2}}\, j_\mu^{(0)}(x) + \beta j_\mu^{(8)}(x) + \frac{\sqrt{3}}{2}\,\gamma\left(j_\mu^{(3)}(x) + \frac{1}{\sqrt{3}}\, j_\mu^{(8)}(x)\right)\right]\psi^\mu_{(x)}, \tag{A1}$$

where $\psi_\mu(x)$ is the field of the ψ particle and $j_\mu^{(a)}(x)$ ($a = 0, 1, 2, \ldots, 8$) are the nonet of U(3) vector currents. In terms of quark fields the currents are

$$j_\mu^{(a)}(x) = \sum_{i,j} \bar{q}_i(x)\lambda_{ij}^{(a)}\gamma_\mu q_j(x), \tag{A2}$$

where

$$a = 0, 1, 2, \ldots, 8; \quad i, j = u, d, s,$$

although for our purpose we shall not require this specific form. In Eq. (A1) α, β, and γ are assumed to be constants (the extra factors $1/2\sqrt{2}$ and $\sqrt{3}/2$ in front of α and γ are introduced for convenience). The term proportional to γ arises from $\psi\to$ photon $\to$ hadrons. The first two terms represent the effect of pure strong interactions. If SU(3) were an exact symmetry and ψ is an SU(3) singlet then β would be zero. A nonvanishing β can be taken to represent any of the following. (a) There are dynamical violations of SU(3) in ψ decay of the type discussed in the present work but which need not be specified for a pure group-theoretical analysis. (b) ψ is not a pure SU(3) singlet but has admixtures of an octet wave function such as $(u\bar{u}+d\bar{d}-2s\bar{s})/\sqrt{6}$. (c) The charmed quark is not an SU(3) singlet but is a member of an SU(3) triplet of heavy quarks of the kind as considered for example by Harari.[36]

We shall consider two-body or quasi-two-body decays into states belonging to pseudoscalar (P), vector (V), and tensor (T) nonets. Because of charge-conjugation invariance there is no contribution of the singlet current in (A1) to decays of the type $\psi\to PP, VV, PT$, while it is the dominant term in $\psi\to PV, VT$. What the currently available experimental data indicate is the following. When a decay mode such as $\psi\to K^+K^-$ is forbidden by SU(3), its experimental magnitude can be solely understood in terms of the one-photon contribution. On the other hand, in decay modes such as $\psi\to\rho^+\pi^-$, $K^{*+}K^-$, there is need to invoke a substantial presence of the β term (octet part).

The various amplitude ratios in $\psi\to PP$, PV, etc. are straightforward to calculate from Eq. (A1). We take the vector and tensor nonets to be ideally mixed, while for the pseudoscalar nonet we make the identifications

$$\begin{aligned}\eta&=\cos\theta\,\eta_8+\sin\theta\,\eta_1,\\ \eta'&=-\sin\theta\,\eta_8+\cos\theta\,\eta_1,\end{aligned}\tag{A3}$$

with $\theta=+10.4^\circ$ as given by the Gell-Mann–Okubo mass formula.[37] The amplitude ratios are the following:

$$\psi\to\pi^+\pi^-:K^+K^-:K_L^0K_S^0=\gamma:(\gamma-\beta):-\beta\tag{A4}$$

For the decay into a vector and pseudoscalar the amplitudes are apart from an overall normalization factor

$$\begin{aligned}\psi&\to\rho^+\pi^-=\rho^0\pi^0=\rho^-\pi^+=\alpha+2\beta+\gamma,\\ \psi&\to K^{*+}K^-=K^{*-}K^+=\alpha-\beta+\gamma,\\ \psi&\to K^{*0}\bar{K}^0=\bar{K}^{*0}K^0=\alpha-\beta-2\gamma,\\ \psi&\to\phi\eta=\left(\frac{2}{3}\right)^{1/2}\left(\frac{\sin\theta}{\sqrt{2}}-\cos\theta\right)(\alpha-4\beta-2\gamma),\\ \psi&\to\phi\eta'=\frac{1}{\sqrt{3}}(\sqrt{2}\,\sin\theta+\cos\theta)(\alpha-4\beta-2\gamma),\\ \psi&\to\omega\eta=\frac{1}{\sqrt{3}}(\cos\theta+\sqrt{2}\,\sin\theta)(\alpha+2\beta+\gamma),\\ \psi&\to\omega\eta'=\frac{1}{\sqrt{3}}(\sqrt{2}\,\cos\theta-\sin\theta)(\alpha+2\beta+\gamma),\\ \psi&\to\omega\pi^0=3\gamma.\end{aligned}\tag{A5}$$

Of these, the first three require only SU(3) symmetry while the rest require nonet symmetry, with ideal mixing for the vector nonet and with the pseudoscalars mixing as in Eq. (A3). Equations (A5) lead to the following relations for the branching ratios:

$$\begin{aligned}&R_B(\psi\to\rho^+\pi^-):R_B(\psi\to K^{*+}K^-):R_B(\psi\to K^{*0}\bar{K}^0)\\ &\quad=|\alpha+2\beta+\gamma|^2:0.85\,|\alpha-\beta+\gamma|^2:0.85\,|\alpha-\beta-2\gamma|^2\end{aligned}\tag{A6}$$

$$=0.43\pm0.10:0.16\pm0.03:0.13\pm0.035,\tag{A7}$$

where the numerical values are proportional to experimental branching ratios (see Refs. 19–21). This has led the authors of Ref. 19 to suggest an SU(3) violation of 10 to 20%. Comparison of Eqs. (A6) and (A7) suggests that the SU(3)-violating term β is larger than the one-photon term γ.

A somewhat different picture emerges from the two pseudoscalar modes. Experimentally we have[19]

$$\begin{aligned}R_B(\psi\to\pi^+\pi^-)&=(1.6\pm1.6)\times10^{-4},\\ R_B(\psi\to K^+K^-)&=(2.0\pm1.6)\times10^{-4},\\ R_B(\psi\to K_LK_S)&<0.89\times10^{-4}.\end{aligned}\tag{A8}$$

If SU(3) were exact, i.e., $\beta=0$, we expect, correcting for phase space,

$$\Gamma(\psi\to K^+K^-):\Gamma(\psi\to\pi^+\pi^-)=1:1.15.\tag{A9}$$

If we take the median values in Eq. (A8) as the true values of the branching ratio then the slight discrepancy between Eqs. (A8) and (A9) can be explained as follows. Suppose there was no strong-interaction contribution to $\psi\to K^+K^-$ as in the case of $\psi\to\pi^+\pi^-$, then

$$\frac{\Gamma(\psi\to K^+K^-)}{\Gamma(\psi\to\pi^+\pi^-)}=\left|\frac{F_K(s=m_\psi^2)}{F_\pi(s=m_\psi^2)}\right|^2\frac{1}{1.15}.\tag{A10}$$

We know that, because of breakdown of SU(3) symmetry, $F_\pi\neq F_K$. Experimentally it is known[38] that in e^+e^- annihilation the following phenomenological expression:

$$\begin{aligned}\frac{\sigma(e^+e^-\to K^+K^-)}{\sigma(e^+e^-\to\pi^+\pi^-)}&=\frac{\beta_K^{\,3}}{\beta_\pi^{\,3}}\,\frac{|F_{K^+}|^2}{|F_{\pi^+}|^2}\\ &=\frac{\beta_K^{\,3}}{\beta_\pi^{\,3}}\,\frac{(\frac{1}{2}P_\rho+\frac{1}{6}P_\omega+\frac{1}{3}P_\phi)^2}{P_\rho^{\,2}},\end{aligned}\tag{A11}$$

with $P_V=m_V^{\,2}/(s-m_V^{\,2})$, fits the data for $\sqrt{s}$ between 1.2 and 1.7 GeV. Using the right-hand side (rhs) of Eq. (A11) at $s=m_\psi^{\,2}$ to evaluate the rhs of Eq. (A10) we get a value of 1.4, which is close to the experimental ratio of the median values. Using the same approximation we have

$$\frac{F_{K^0}(s)}{F_{K^+}(s)}=\frac{-\frac{1}{2}P_\rho+\frac{1}{6}P_\omega+\frac{1}{3}P_\phi}{\frac{1}{2}P_\rho+\frac{1}{6}P_\omega+\frac{1}{3}P_\phi}\tag{A12}$$

$$\approx0.25\text{ at }s=m_\psi^{\,2},\tag{A13}$$

so that $\Gamma(\psi\to\gamma\to K_LK_S)/\Gamma(\psi\to\gamma\to K^+K^-)\approx6.2\times10^{-2}$ is well below the experimental upper limit [Eq. (A8)]. To summarize, then there is no evidence for the presence of the β term in $\psi\to K\bar{K}$.

It must be emphasized that we cannot explain the difference between $\psi \to \rho\pi$ and $\psi \to K^*\bar{K}$ as due to the difference between the form factors of the currents $j_{\text{e.m.}}$ or $j^{(0)}$. Such a procedure would clearly make $\psi \to K^*\bar{K}$ larger than $\psi \to \rho\pi$.

APPENDIX B: COMMENTS ON THE CHOICE OF $\alpha_{D^*}(t)$ AND $\alpha_{F^*}(t)$

It is known that the experimental masses[12] of ρ, f, g, h as well as $\omega, A_2, \omega'(1670)$ lie approximately on a straight line with slope $(\alpha')^{-1} = 1.11$ GeV^2. Similarly the $1^-, 2^+, 3^-, K^*$ states[12] with masses 892, 1420, and 1776 MeV lie approximately on a straight line with slope $(\alpha')^{-1} \approx 1.2$ GeV^2 a slightly large value than $(\alpha_\rho')^{-1}$ reflecting perhaps the effect of the larger s-quark mass. Encouraged by this we try to approximate the D^* trajectory by a straight line connecting the 1^- and 2^+ states. There is good evidence that the vector D^* state has a mass ≈ 2.02 GeV. For the tensor D^* state various theoretical calculations[39] indicate a value anywhere between 350 to 500 MeV above the vector D^* state. We have taken therefore three typical values $\alpha'_{D^*} = 0.45$ GeV^2, 0.50 GeV^{-2}, and 0.55 GeV^{-2} which put the tensor state mass, respectively, at 2.51 GeV, 2.47 GeV, and 2.43 GeV.

The pseudoscalar F meson has not yet been experimentally identified. Again theoretical estimates[39] of the F-D splitting give $m_F - m_D \approx 100$ MeV. To determine the F^* trajectory we have taken $m_{F^*} = m_{D^*} + 100$ MeV ≈ 2.12 MeV and used the same slope as for D^*.

For the ψ trajectory if we use a linear fit using the 1^- and 2^+ with masses[40] 3.095 and 3.552 then we have

$$\alpha'_\psi \approx 0.33 \text{ GeV}^{-2}. \tag{B1}$$

One of us has suggested elsewhere[41] that the Regge slope α'_ρ, α'_{D^*}, and α'_ψ factorize, that is

$$(\alpha'_{D^*})^2 = \alpha'_\rho \alpha'_\psi. \tag{B2}$$

Equations (B1) and (B2) then yield $\alpha'_{D^*} \approx 0.54$ GeV^{-2}.

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