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Thresholds and the rising pion inclusive cross section

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In the context of the hypothesis of the Pomeron-f identity, it is shown that the rising pion inclusive cross section can be explained over a wide range of energies as a series of threshold effects. Low-mass thresholds are seen to be important. In order to understand the contributions of high-mass thresholds (flavoring), a simple two-channel multiperipheral model is examined. The analysis sheds light on the relation between thresholds and Mueller-Regge couplings. In particular, it is seen that inclusive- and total-cross-section threshold mechanisms may differ. A quantitative model based on this idea and utilizing previous total-cross-section fits is seen to agree well with experiment.

I. INTRODUCTION

Chiu and Tow[1] have recently shown that the rise in the inclusive pion cross section in the central region is consistent with the increase expected from baryon-antibaryon ($B\bar{B}$) production at CERN ISR energies. Using simple arguments and parametrizations of the data, they conclude that the increase is primarily due to those pions produced in events which also contain a $\bar{B}$. The energy dependence is thus a threshold effect, the $B\bar{B}$ threshold being delayed until high energies because of the high $B\bar{B}$ mass and because of multiperipheral kinematic effects. This explanation is consistent with the Pomeron-f identity scheme proposed by Chew and Rosenzweig (CR),[2] and can be contrasted to the standard Regge scheme[3] which predicts an $s^{-1/4}$ approach to an asymptotic limit. The primary goal of this paper is to study these effects in the context of a specific model.

Inclusive distributions present a significant challenge to the CR scheme. If we consider the average of the π^+ and π^- cross sections, then only the vacuum trajectories contribute, and in the CR scheme there is only one vacuum trajectory. The conventional Regge model assumes separate Pomeron and f trajectories, and arrives at a rising inclusive cross section if the interference term is negative.[3] In the CR approach, when high-mass thresholds are taken into account, the Pomeron (f) singularity consists of a leading real pole α and a series of complex poles. The threshold effects have been termed "flavoring", and a detailed study of total cross sections[4] has shown that strange-particle and baryon production (as $K\bar{K}$ and $B\bar{B}$ pairs) can explain the NN, πN, and KN amplitudes at $t=0$ over a wide range of energies. At moderate energies, below flavoring thresholds, the Pomeron is well represented by a "bare" pole at $\hat{\alpha}(0)=0.85$. If we naively use a Mueller-Regge model to describe inclusive cross sections, then we will find, at $y=0$, that $d\sigma/dy \sim s^{-0.15}$ below flavoring thresholds in the *subenergy* $s_1 \approx \sqrt{s}$. In other words, if the threshold for $B\bar{B}$ production is $s_{\rm th}$, we do not expect a contribution to the *inclusive* cross section (at $y=0$) until $s_1 \geq s_{\rm th}$, which means $s \geq s_{\rm th}^2$.[5] This means we might expect flavoring to contribute to inclusive cross sections at a much higher energy than it contributes to total cross sections. Experimentally, the inclusive plateau rises monotonically through ISR energies, while total cross sections do not start rising until $s \sim 400$ GeV^2. Chiu and Tow found flavoring to be important at ISR energies. Thus we are faced with an apparent inconsistency. We need flavoring effects to explain rising total *and* inclusive cross sections at about the same energies, yet the Mueller-Regge theory applied to a simple flavoring model tells us that flavoring should not contribute to inclusives until perhaps the highest ISR energy.

We can demonstrate the above problem in two ways. First, the Mueller-Regge theory tells us that the inclusive cross section is roughly

$$s\frac{d\sigma}{dy} \cong A\left(\frac{Y}{2}+y\right) A\left(\frac{Y}{2}-y\right) , \tag{1}$$

where A is the Pomeron amplitude and $Y=\ln s$. At $y=0$, we must evaluate A at $\frac{1}{2}Y$, and if A includes more than one Regge pole, then $A^2(\frac{1}{2}Y) \neq A(Y)$. In the CR scheme, A is represented by a series of poles (one real, the rest complex), which is well approximated at moderate energies by a single real pole with $\hat{\alpha}(0)=0.85$. Thus, if Y is "large" and $\frac{1}{2}Y$ is "moderate", we will have $A^2(\frac{1}{2}Y) \approx s^{\hat{\alpha}} = s^{0.85}$, while $A(Y) \approx s^{\alpha} \cong s^{1.08}$. Thus, $d\sigma/dy$ will decrease as energy increases while $\sigma_{\rm tot}$ is increasing.

A second way of looking at this may shed additional light on the problem we wish to address. High thresholds for $B\bar{B}$ pairs mean essentially that the $B\bar{B}$ pair consumes a lot of rapidity. Near $B\bar{B}$ threshold, *most* of the rapidity interval is consumed by the $B\bar{B}$ pair. If pions are produced as well, they can be produced only at the ends of the interval ($y \approx \pm\frac{1}{2}Y$). *Central* pion production occurs only when the rapidity interval is large enough to allow a $B\bar{B}$ pair whose rapidity gap does not overlap $y=0$. This assumes, of course, that the $B\bar{B}$ pair is treated as a unit.

The authors of Ref. 1 overcame this difficulty in part by convoluting the pion distribution with that of the $B\bar{B}$'s. Furthermore, their thresholds for $N\bar{N}$ production in σ_{tot} and $d\sigma/dy$, which they take from data, are not consistent with one another. They assume a rapidity gap of $\ln s_{N\bar{N}} = 1.6$ (s in GeV^2), when analyzing the inclusive distribution. This will lead to a threshold of $s \sim 25\ \text{GeV}^2$ for $pp \to ppN\bar{N}$, if one uses $s \approx s_p s_{N\bar{N}} s_p$. The experimental threshold for $N\bar{N}$ production (which they use since they take $d\sigma^{\bar{p}}/dy_N$ from data) is around 150 GeV^2. This explains why they do not detect the problem we have described. This is not to be considered a serious criticism of their work, however, as their results depend only weakly on the assumption that the N and $\bar{N}$ appear as an $N\bar{N}$ unit. For instance, if we take their $\ln s_{N\bar{N}}$ and call it $\ln s_N$, the rapidity gap of a single nucleon or antinucleon, then their philosophy will be identical to that to be described here, and their phenomenology will proceed essentially intact.

Chiu and Tow did not attempt to formulate an explicit model for generating *both* σ_{tot} and $d\sigma/dy$, concentrating instead on the systematics of the data interpreted in multiperipheral terms. We have found in fact, as shown above, that a consistent description of thresholds in σ_{tot} and $d\sigma/dy$ cannot be achieved using the explicit model of Ref. 4, and that more general models that assume the Pomeron and f to be identical might have difficulty confronting inclusive data. Examining our arguments, we see that the second applies to simple one-dimensional (or "strong-ordering") models which assume $N\bar{N}$ production as a unit. The first is more general, but can be shown to apply primarily to one-channel models. We have found that if the problem is properly treated as a multichannel problem incorporating N and K exchanges,[6] then the above difficulties disappear. In particular, N and $\bar{N}$ no longer need to be produced as a unit. Consistency with the Mueller-Regge theory derives from the fact that Eq. (1) becomes a matrix equation with nonvanishing off-diagonal contributions. The model retains the basic philosophies of Refs. 1 and 4, and does not seriously affect the phenomenology in these papers, if the appropriate reinterpretation is made. We believe the arguments given here to be quite general, and apply to any multiperipheral model which identifies the Pomeron with the f and utilizes flavoring thresholds to renormalize the Pomeron intercept. Since such models must be consistent with the Mueller-Regge theory, it is hard to see how one can assume the N and $\bar{N}$ are produced as a unit and still correctly describe inclusive data.

We have been able to achieve consistency by showing that thresholds in *subenergies* need not be the same as thresholds in total energies. Specifically, since new flavors (or baryon number) must be produced in *pairs* of particles ($K\bar{K}$, $B\bar{B}$,$D\bar{D}$), the threshold for such production is roughly proportional to twice the mass of the particle ($2m_K, \ldots$). (Kinematic effects in peripheral models cause the effective thresholds to be higher than the literal masses.) However, when we consider inclusive pion production, the threshold in the *subenergy* $\sqrt{s_1}$ is only the mass of a *single* particle ($m_K, \ldots$), since we can have strange-particle exchange with the pion emitted from this exchange. Thus, the threshold in the subenergy is *less* than the corresponding threshold in the total energy. We can see that flavoring will then contribute to inclusive cross sections at very nearly the same total energy as it contributes to total cross sections.

At first glance the above description may seem still to conflict with the Mueller-Regge picture. This is not the case, however. We will show that the Regge poles are precisely the same in both inclusive and elastic amplitudes, but that the Mueller-Regge couplings are not simply related to the Regge–external-particle couplings. This difference reflects the different thresholds and allows consistency of the flavoring scheme.

In the following section, we describe a two-channel model which incorporates the effects discussed here, and illuminates the relation of flavoring thresholds to the Mueller-Regge model. In Sec. III, we attempt a quantitative description of the inclusive cross section, adapting for this purpose the model of Ref. 4. We must make some additional assumptions, based on the ideas presented in Sec. II, but are still able to find rough agreement with inclusive data without significantly affecting the total-cross-section fits of Ref. 4. We close with some remarks on additional effects not considered here.

II. THRESHOLDS AND THE MUELLER-REGGE MODEL

We consider now a simple multiperipheral model which will incorporate the effects described qualitatively in the Introduction. We wish in particular to demonstrate that subenergy thresholds need not be the same as total-energy thresholds, and that this is realized in the Mueller-Regge model as new contributions to the Mueller-Regge coupling. As discussed in the Introduction, one can achieve these results by incorporating strange-particle exchanges in the model.

Our model therefore contains two input trajectories, one carrying a heavy-quark flavor (say, strangeness), called generically K in this discussion. The nonstrange trajectory will be called the ρ. We could as easily consider baryon exchange[6] instead of (or in addition to) strangeness, but we limit ourselves to the one flavor for simplicity, as we are only concerned with qualitative effects at this point. We will allow ρ production from either a ρ or K exchange. It is here that the model differs from that of Ref. 4. We will ignore considerations of isospin and other quantum numbers, except, of course, strangeness, which is crucial to our discussion. Our amplitude can then be written in matrix form (in the J-plane) as (we suppress J dependence where it is obvious)

$$A(J)=\overline{V}(J)P(J)V(J)+\overline{V}PGPV+\overline{V}PGPGPV+\cdots$$
$$=\overline{V}(1-PG)^{-1}PV. \qquad (2)$$

Here,

$$V(J)=\begin{pmatrix}V_\rho(J)\\ 0\end{pmatrix},$$

$$P(J)=\begin{pmatrix}\dfrac{1}{J-\beta_\rho} & 0\\ 0 & \dfrac{1}{J-\beta_K}\end{pmatrix}, \qquad (3)$$

and

$$G(J)=\begin{pmatrix}g_\rho & g_K e^{-b_K J}\\ g_K e^{-b_K J} & g_K\end{pmatrix}. \qquad (4)$$

V is the coupling to external particles, and we assume that only the ρ exchange couples, for simplicity. The derivation to follow becomes more complicated if we allow a coupling V_K (corresponding to associated production), but the conclusions are not changed qualitatively. $P(J)$ represents the propagators, with $\beta_i \equiv 2\alpha_i - 1$. The couplings in $G(J)$ are $g_\rho \equiv g_{\rho\rho\rho}$ and $g_K \equiv g_{K\overline{K}\rho}$, assumed to be the same for Reggeons and particles. We include a threshold term $e^{-b_K J}$ for production of K's. On transforming back to $Y=\ln s$, we will get a term proportional to $\theta(Y-b_K)$ from this exponential. Note that a term in the expansion (2) which goes as $g_K{}^N$ does not imply production of N K's. Instead, the term proportional to $\exp(-Nb_K J)$ represents production of N strange particles ($N/2$ K's and $N/2$ $\overline{K}$'s). Clearly, N must be even. It is easy

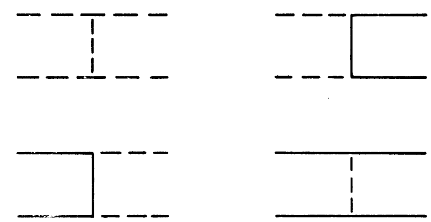

FIG. 1. The four terms in the kernel GP, diagrammatically. Dashed lines are ρ's, solid lines are K's.

to verify that this will be the case, and that in fact the expansion (2) does incorporate strangeness conservation. Figure 1 shows the four terms (diagrammatically) in the kernel $K=PG$. Finally, note that the overall threshold for strange-particle production is $\ln s = 2b_K$.

The amplitude (2) will be determined in the s plane by its poles in the J plane. These poles are the zeros of the determinant of $(1-PG)$, and will have residues $\beta_i \equiv \beta(\alpha_i)$. β_i will be a matrix, of course, in our model. If we write

$$(1-PG)^{-1}=\sum_i \frac{\beta_i}{J-\alpha_i}+\text{nonpole terms}, \qquad (5)$$

then we find

$$A(Y)\equiv\int_c \frac{dJ}{2\pi i}A(J)e^{JY}$$
$$=\sum_i\,[\overline{V}(\alpha_i)\beta_i P(\alpha_i)V(\alpha_i)]e^{\alpha_i Y}. \qquad (6)$$

Since we allow only the ρ to couple at the end, the full coupling of the ith pole is

$$[\beta_i P(\alpha_i)]_{11}V_\rho^2(\alpha_i)\equiv\overline{\beta}_i, \qquad (7)$$

where $(\beta P)_{11}$ is the $\rho\rho$ component of the matrix βP.

Let us next consider the inclusive cross section. We are interested in ρ production in the vicinity of flavoring thresholds. The first term of interest will be that containing a single produced ρ, and a $K\overline{K}$ pair. If we are interested in *central* ρ production, the pertinent diagram is that in which the ρ is produced *between* the K and $\overline{K}$, as shown in Fig. 2. We see that this term will have a threshold at

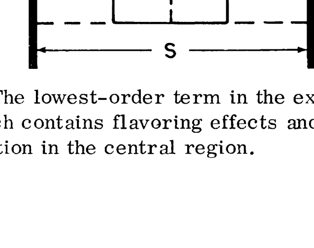

FIG. 2. The lowest-order term in the expansion of Eq. (1) which contains flavoring effects and contributes to ρ production in the central region.

$\ln s = 2b_K$, and $\ln s_1 = b_K$. (There is no threshold for ρ production in this model.) Thus, the subenergy threshold is less than the corresponding threshold in the total energy. In the models considered in Ref. 1 and 4, the K and $\bar{K}$ (or B and $\bar{B}$) had to *both* be produced on one side of the π, so the subenergy threshold would be the *same* as the flavoring threshold in the total cross section. If the inclusive cross section is written (at $y = 0$ in the center of mass) as

$$\frac{d\sigma}{dy} = \frac{1}{s}\bar{A}(Y/2)\bar{A}(Y/2)\,, \tag{8}$$

then the threshold for K production in $\bar{A}(y)$ is at $y = b_K$, not $y = 2b_K$.

The inclusive cross section is determined by an amplitude (in Mueller's generalized optical theorem) which depends on two angular-momentum variables in the J plane. If we let

$$F(Y-y,y) \equiv s\frac{d\sigma}{dy}(Y)\,, \tag{9}$$

then

$$F(j_1,j_2) \equiv \int e^{-j_1(Y-y)}e^{-j_2 y}F(Y-y,y)d(Y-y)dy$$
$$= \bar{V}(j_1)[1-PG(j_i)]^{-1}K^\rho(j_1)$$
$$\times[1-PG(j_2)]^{-1}P(j_2)V(j_2)\,, \tag{10}$$

where

$$K^\rho_{ij} \equiv (PG)_{ij}\delta_{ij} \tag{11}$$

represents the ρ-production components of the kernel PG. Note that $F(j_1,j_2)$ has the *same poles* as $A(J)$, since the kernels are identical. Equivalently, $\bar{A}(J)$, whose transform appears in (8), has the same poles as $A(J)$.

We have already seen explicitly that the pertinent thresholds in $\bar{A}(y)$ and $A(Y)$ are different. We now wish to show how this is reflected in the pole decomposition of these amplitudes. In this model there will be in general two real poles (the f and f'), and associated complex poles arising from the threshold exponentials. The detailed pole structure is unimportant for our present considerations. Since the poles in $\bar{A}$ and A are the same, differences in thresholds must arise from residue effects. Referring to Eq. (5), we can write the inclusive amplitude F as

$$F(Y-y,y) = \sum_{i,j}\bar{V}(\alpha_i)\beta_i K^\rho(\alpha_i)\beta_j P(\alpha_j)e^{\alpha_i(Y-y)}e^{\alpha_j y}$$
$$= \sum_{i,j}\{(\beta_i K^\rho_i)_{11}[\beta_j P(\alpha_j)]_{11} + (\beta_i K^\rho_i)_{12}[\beta_j P(\alpha_j)]_{21}\}V_\rho(\alpha_i)V_\rho(\alpha_j)e^{\alpha_i(Y-y)}e^{\alpha_j y}\,. \tag{12}$$

The important aspect of Eq. (12) is the presence of the off-diagonal terms in the residues (the second term in the brackets). The first term is just $g_\rho A(Y-y)A(y)$, since $(\beta K^\rho)_{11} = g_\rho(\beta P)_{11}$. The second term, however, represents the contribution from events where the ρ is emitted from a K exchange. This term is the manifestation in the Regge-pole expansion of the different threshold structures for the total and inclusive amplitudes. The significant feature is that the Mueller-Regge residues are not simply related to the ordinary Regge residues. If there were just a single pole, of course, the difference in residues would not change the energy dependence. In the presence of thresholds, the energy dependence of the amplitude is determined by both real and complex poles, and it is easy to see how the different terms can combine to give us the threshold effects we have described. In particular, since the threshold for ρ production with concurrent $K\bar{K}$ production is $s \geq 2b_K$, we see that flavoring will affect the inclusive cross section at about the same total energies as the total cross section.

Before considering a quantitative model for the inclusive plateau, let us consider the rapidity distribution one might expect from the present model. Without going into detail, we can see that the inclusive ρ distribution will be roughly flat in y, typical of multiperipheral models. This will be true both below and above flavoring thresholds, since the ρ can be emitted from any part of the multiperipheral chain. (Quantitatively, this statement depends on the relative sizes of g_ρ and g_K.) In contrast, if the ρ must be produced *outside* the $K\bar{K}$ or $B\bar{B}$ *pair* (as in Refs. 1 and 4), then ρ production will be excluded from the central region in those events containing a $K\bar{K}$ ($B\bar{B}$), near the corresponding flavoring threshold. This exclusion could lead to a bowl-shaped rapidity distribution, and is directly connected to the threshold effects we have been discussing. Such a distribution conflicts with experiment, and gives us added incentive to take seriously the model described in this section. Investigation of the π rapidity distribution in events with simultaneous $\bar{p}$ production would provide a test of our model. We turn now to a quantitative look at inclusive cross sections.

III. A QUANTITATIVE MODEL

Our model is adapted from that of Ref. 4, with two important changes. First, since the model in that paper assumed that all pions had been summed, we need to reinstate pion production. Secondly, as described in Sec. II, we will need to reinterpret $K\bar{K}$ and $B\bar{B}$ production mechanisms to allow pion production between the K and $\bar{K}$ (B and $\bar{B}$).

Using notation analogous to that of Sec. II (except that now we have a one-channel model), we write the imaginary part of the NN elastic amplitude in the J plane as

$$A(J)=V^2(J)P(J)/[1-K(J)]. \tag{13}$$

In the present form of the model, we let

$$K(J)=G(J)P(J)\,,\qquad G(J)=G_\rho(J)+G_K(J)+G_B(J)\,, \tag{14}$$

representing ρ, $K\bar{K}$, and $B\bar{B}$ production, respectively. We will assume that all pions come from ρ decay, though we will find later that this will not be sufficient phenomenologically. In fact, we will want to use the term ρ to represent all low-mass pion clusters, whatever their nature. The propagator $P(J)$ is

$$P(J)=(J-\beta_0)^{-1}\,, \tag{15}$$

where $\beta_0=2\alpha_0-1\approx 0$. Summing over pions ($\rho$'s) (i.e., summing the power series in G_ρ) yields the *bare* Pomeron, $\hat{\alpha}(0)\approx 0.85$. The renormalized Pomeron, the singularity in the full amplitude $A(J)$, is a real pole around $\alpha(0)\approx 1.08$, accompanied by a series of complex poles which result from the $K\bar{K}$ and $B\bar{B}$ thresholds. When all non-diffractive thresholds have been taken into account, the fully flavored Pomeron is considered to be the Gribov bare Pomeron input to Reggeon field theory.

Now we wish to identify our amplitude with that of Ref. 4, to take advantage of the extensive phenomenology there. This will give us a quantitative prediction of flavoring effects in inclusive cross sections. The identification can be made if we let

$$V^2(J)\equiv N(J)\,,\qquad [1-K(J)]/P(J)\equiv D(J)\,, \tag{16}$$

in the notation of Ref. 4. Recall that $N(J)$ and $D(J)$ are written as

$$N(J)=\beta e^{-b_0 J}[1+g_\Lambda e^{-b_\Lambda J}/(J-J_\Lambda)^2-g_D e^{-b_D J}/(J-J_D)^2] \tag{16a}$$

$$D(J)=J-\hat{\alpha}-g_K e^{-b_K J}/(J-J_K)-g_B e^{-b_B J}/(J-J_B)\,. \tag{16b}$$

The notation is that of Ref. 4. The second and third terms in $N(J)$ refer to associated production and inelastic diffraction, respectively. The diffractive (cut) term is negative in the full amplitude, due to absorption.[7] It is shown in Ref. 7 that this cut does not contribute to inclusive cross sections in the central region, so we omit it in our calculation. The last two terms in $D(J)$ correspond to $K\bar{K}$ and $B\bar{B}$ production. The reader is referred to Ref. 4 for further details. With the identifications (16), we then have

$$A(J)=N(J)/D(J)\,. \tag{17}$$

The total cross section is given by the first line of Eq. (6). Note that $N(J)$ in (16a) is not strictly a complete square, so we will have to approximate it as one when we extract $V(J)$.

Now, if $G_\rho(J)\equiv g_\rho$, the inclusive ρ distribution in rapidity y_ρ is given by

$$\frac{d\sigma}{dy_\rho}=\frac{g_\rho}{s}\bar{A}\left(\frac{Y}{2}-y_\rho\right)\bar{A}\left(\frac{Y}{2}+y_\rho\right)\,, \tag{18}$$

where

$$\bar{A}(J)\equiv V(J)/D(J)\,. \tag{19}$$

(Note that we are using center-of-mass rapidities.) If there is a threshold associated with ρ production, we will have (for example)

$$G_\rho(J)=g_\rho e^{-b_\rho J} \tag{20}$$

and

$$\frac{d\sigma}{dy_\rho}=\frac{g_\rho}{s}\bar{A}\left(\frac{Y-b_\rho}{2}-y_\rho\right)\bar{A}\left(\frac{Y-b_\rho}{2}+y_\rho\right)\,. \tag{21}$$

In order to explain the rising pion plateau at moderate energies we will have to assume a ρ-threshold effect. Equation (21) will then form the basis for our calculation. We will need an estimate of b_ρ and g_ρ, but since we will be comparing to data at fixed transverse momentum we will end up normalizing the model to data at a particular energy, and g_ρ will be unimportant. Furthermore, we use the specific form (20) of $G_\rho(J)$ only to illustrate the dependence on b_ρ. The actual calculations use Eq. (21) and incorporate the exact $D(J)$ from Eq. (16b).

We could estimate g_ρ and b_ρ by attempting to simultaneously fit $\langle n_\pi\rangle$ and σ_{tot}. However, it is well known that it is difficult to generate a Pomeron and get the proper value for $\langle n\rangle/\ln s$ with just ρ production. We are not concerned with these difficulties here, since our results would be unchanged qualitatively if the ρ were a three- or four-pion cluster. Of course, parametrizing the decay distribution of such a cluster is considerably more difficult than that of the ρ. We will use in our calculation of $d\sigma/dy$ the value $b_\rho=0.6$, but the results are insensitive to the precise value of b_ρ.

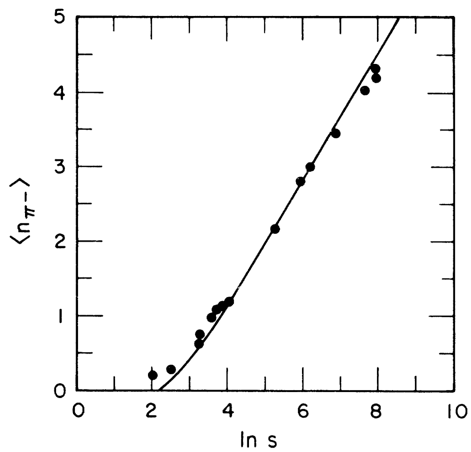

FIG. 3. Negative-pion multiplicity distribution. Data are from Ref. 6. s is in GeV^2.

We are, however, interested in the effects of flavoring on the energy dependence of $\langle n_\pi\rangle$, since it has been predicted[5] that the slope of $\langle n\rangle/\ln s$ ought to oscillate. Setting $b_\rho = 0$ for the moment, we can calculate $\langle n_\pi\rangle$ from the model of Ref. 4, using

$$\langle n_{\pi^-}\rangle = \tfrac{2}{3} g_\rho \frac{\partial}{\partial g_\rho}(\ln\sigma)\,. \tag{22}$$

One such fit to $\langle n_{\pi^-}\rangle$, using $g_\rho = 1.6$, is shown in Fig. 3. Note that $\langle n_{\pi^-}\rangle$ is virtually linear in $\ln s$ through the available energies, so that threshold effects have not had much influence on it. It is easy to see that these effects ought to make $\langle n_{\pi^-}\rangle$ turn concave downward (or, in fact, develop an oscillating slope, beginning with a decrease in slope[5]), since $K\bar{K}$ and $B\bar{B}$ events will initially be at low-pion multiplicities. These high-mass clusters will contribute to σ_{tot}, but will be insignificant in the sum $\sum_n n\sigma_n$, so $\langle n_\pi\rangle$ will increase less rapidly. This statement can be made more precise mathematically in the context of our model, but it happens that the effect is not noticeable until $\ln s \approx 10$.

Having fixed our parameters, we now wish to extract the pion distribution from Eq. (21). We need first to incorporate the $\rho \to 2\pi$ decay distribution into our model. Then we will also need to adapt the model to the ideas of Sec. II. As it stands, the model excludes ρ emission from a K or B exchange, and will predict a rapidity distribution (above the $K\bar{K}$ threshold) that is concave upwards. From our discussion in Sec. II, we know that this is unrealistic; the distribution above flavoring thresholds ought to be similar to those without flavoring, if we allow ρ emission from the flavored (K or B) exchange. Since the model of Ref. 4. *does* predict the right total cross sections, and (using clusters) the proper pion multiplicity, we conclude that its normalization is correct, but that it merely predicts the wrong shape for the rapidity distribution above $K\bar{K}$ threshold. Our procedure will therefore be to assume that the *shape* of the rapidity distribution at any energy is just what we would get in the absence of flavoring. The *magnitude* of the distribution is determined by the model. This amounts to reinterpreting the model of Ref. 4 in the manner of Sec. II, and should in no way change the results of Ref. 4.

The ρ rapidity distribution predicted by Eq. (21) is actually a plateau of width $(Y - b_\rho)$, whose height falls like $s^{-0.15}$. This may be approximately valid for the ρ, but is certainly not true for pions. When we incorporate the ρ-decay distribution, the pion distribution is seen to be reasonable. Pions resulting from a ρ at rapidity y_ρ can have a range of rapidities $y_\rho - \eta \le y_\pi \le y_\rho + \eta$. Convoluting this decay distribution with $d\sigma/dy_\rho$ provides an enhancement of the central region in y_π, which rises in nice agreement with experiment.

We will assume that the ρ decay distribution is isotropic in the overall center of mass. Although this is certainly not true for individual decays, it is reasonable to assume that it is approximately valid as an average of many ρ decays of varying transverse momenta. With this assumption, we find that the pion rapidity distribution from a single ρ decay is (normalized to unity)

$$F(y_\rho - y_\pi) \equiv \frac{dN}{dy_\pi} \approx \frac{2}{\pi}\left[\tanh^2\eta - \tanh^2(y_\rho - y_\pi)\right]^{1/2}. \tag{23}$$

For $\rho \to 2\pi$, $\eta = 1.6$. We then have

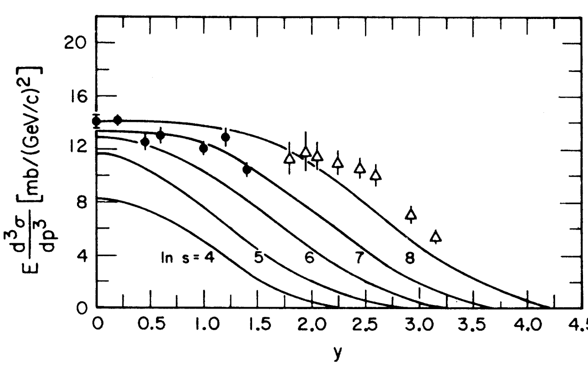

FIG. 4. The single-particle inclusive π^- distributions at various values of $\ln s$ (s in GeV^2), as described in text. The data are from Alper *et al.*, (solid) and Capiluppi *et al.*, (open), Ref. 9, for $\sqrt{s} = 53$ and $p_\perp = 0.4$ GeV/c. Note the apparent disagreement in normalization of the data.

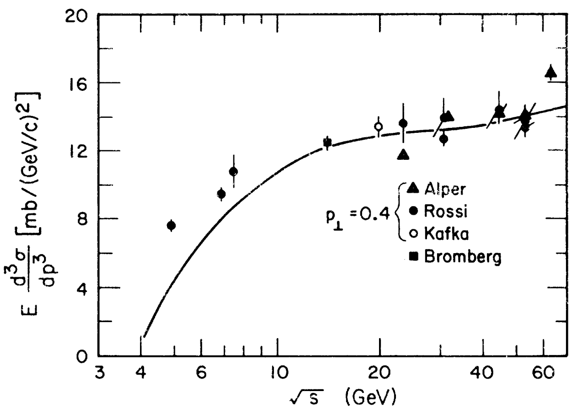

FIG. 5. The single-particle inclusive π^- distribution at $y=0$, plotted vs $\sqrt{s}$. Data are at $p_\perp = 0.4$ GeV/c, except for Bromberg's. The latter were normalized assuming the same $p_\perp$ distribution at $\sqrt{s} = 13.9$ GeV and $\sqrt{s} = 19.7$ GeV.

$$\frac{d\sigma}{dy_\pi} = \int_{-\infty}^{\infty} \frac{d\sigma}{dy_\rho} F(y_\rho - y_\pi) dy_\rho. \tag{24}$$

With this formulation, the inclusive rapidity distribution (π^+ and π^- averaged) is as shown in Fig. 4. We include data[9] at $p_\perp = 0.4$ GeV/c at one energy, for comparison.[10] Remember that the *shape* of $d\sigma/dy$ has been determined by the unflavored distribution—the *magnitude* comes from the full model. We show in Fig. 5 the energy dependence of the central plateau, also compared to data at $p_\perp = 0.4$ GeV/c. In each case we see that the model is in approximate agreement with the data. Since the model has no $p_\perp$ dependence, we have normalized to Alper's data at $\sqrt{s} = 53$ GeV and $y = 0$. Although the inclusive cross section does have an energy-dependent $p_\perp$ distribution, this dependence is primarily at large $p_\perp$, where the cross section is small. We would expect only small corrections if we were to plot $d\sigma/dy$ integrated over $p_\perp$.

Owing to the displacement by $\frac{1}{2}b_\rho$ in the argument of Eq. (21), thresholds in $d\sigma/dy$ are delayed somewhat, relative to the corresponding thresholds in σ_{tot}. The reason, of course, is that we are not interested in the $K\bar{K}$ threshold but rather the $K\bar{K}\rho$ threshold. A similar displacement effect has been noted by Chew and Koplik,[11] using a realistic multi-Regge model. The flattening of the energy dependence of the central plateau is due to $K\bar{K}$ contributions, and the subsequent rise results from $B\bar{B}$ events. If flavoring is omitted, the plateau falls like $s^{-0.15}$ above $\sqrt{s} \approx 15$ GeV, and does not rise as high in absolute magnitude. The $K\bar{K}$ contribution to the inclusive cross section is about 20% at $\sqrt{s} = 15$ GeV.

The model is only qualitatively correct, as one can see by examining Figure 5. For example, we are low at low energies, indicating we have missed important contributions from single-pion emission. In addition, there is evidence that three- and four-pion clusters play an important role in nondiffractive events.[12] However, in general our parametrization in terms of ρ production can be generalized to clusters, without changing our basic results.

Of greater concern is the high-energy region, where the model predicts only a 10% rise in the inclusive cross section. Alper's data[8] show a 20% rise of the central plateau between $\sqrt{s} = 23$ GeV and $\sqrt{s} = 53$ GeV, for all values of $p_\perp \leq 0.5$ GeV/c. Assuming these data are the most reliable, the present model falls short. We believe we have realistically calculated the contributions expected from thresholds within the context of this model, and that the increase expected from pions produced in conjunction with K's and baryons cannot be much greater than calculated here. This means that additional mechanisms must exist to explain the remaining increase observed at ISR. One mechanism not included in this calculation is high-mass resonance production (K^*, N^*). Our model has assumed only multiperipheral pion production. If a significant percentage of K's and N's are actually resonances, we will get additional pions produced. This would in no way alter the cross-section fits of Ref. 4, nor the calculations of this section, since the nature of the K's and N's has never been specified. A crude calculation based on the results of Ref. 4 shows that if one π^- is emitted, on the average, from each $K\bar{K}$ and $B\bar{B}$ pair, then we will get an additional 7% contribution to the inclusive cross section at $\sqrt{s} = 63$ GeV. This is close to what is needed to bring agreement with Alper's data.

IV. CONCLUSIONS

The principal result of this investigation is the demonstration of a specific and reasonable mechanism whereby threshold effects can influence both total and inclusive cross sections at about the same energy. This mechanism is consistent with the Mueller-Regge picture, and indicates that strange-particle and baryon exchanges are likely to play an important role at high energies. Our numerical example has shown how a detailed model incorporates threshold effects. We have shown that ρ, $K\bar{K}$, and $B\bar{B}$ thresholds all must be taken into account in order to understand the overall energy behavior of the central region. Due to ρ-threshold effects, the high-mass thresholds ($K\bar{K}$

and $B\bar{B}$) are somewhat higher in the inclusive than in the total cross section, but still much lower than would be expected naively. This effect, a result of our use of ρ production, explains in part the difference between our results and those of Chiu and Tow.[1] Our results indicate that resonance production (K^*, N^*) is also important for producing pions in the central region. Finally, we observe no effects of flavoring on the pion multiplicity, in agreement with the prediction of Chew and Koplik.[5]

ACKNOWLEDGMENTS

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[10] At ISR energies, the rapidity distribution at $p_\perp = 0.4$ GeV/c is not significantly different from $d\sigma/dy$ integrated over $p_\perp$.

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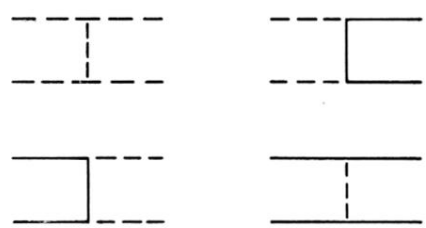


FIG. 1. The four terms in the kernel GP, diagrammatically. Dashed lines are ρ's, solid lines are K's.

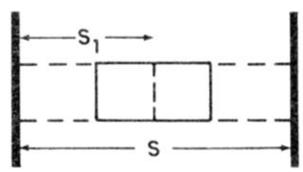


FIG. 2. The lowest-order term in the expansion of Eq. (1) which contains flavoring effects and contributes to ρ production in the central region.

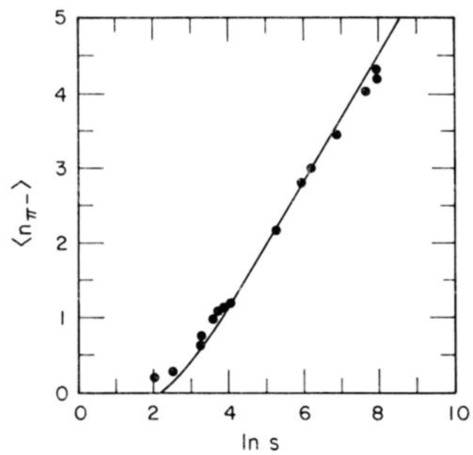
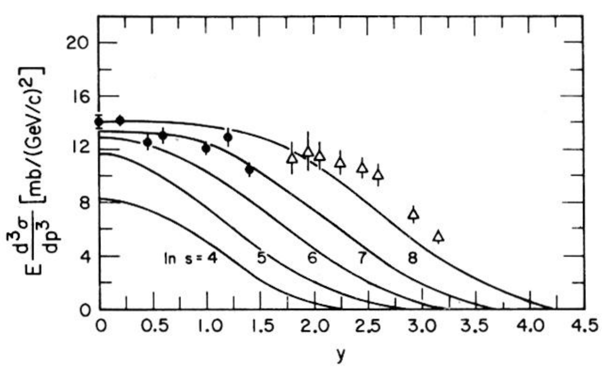


FIG. 3. Negative-pion multiplicity distribution. Data are from Ref. 6. s is in GeV^2.



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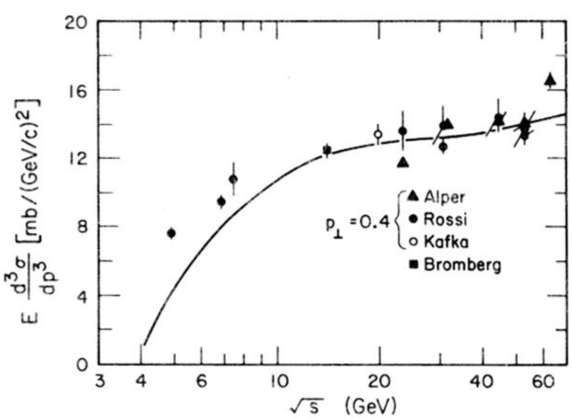

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