PHYSICAL REVIEW D VOLUME 18, NUMBER 3 1 AUGUST 1978

Iteration models and elastic reactions beyond the diffraction peaks

Alex. Martin* and R. Peschanski
Cen-Saclay, Boite Postale No 2, 91190 Gif-sur-Yvette, France
(Received 29 September 1977)

We investigate the consequences of the assumption that elastic reactions are described in a large domain of energy and transfer momentum by an iteration model. We use a method which leads to a simple and general approximation of the iteration-model amplitudes at intermediate momentum transfer ($0.5 \lesssim |t| \lesssim 10$ GeV^2). Using this approximation, we obtain a good description of the available data on hadron-hadron elastic scattering at different energies. A study of the generating "Born term" of the iteration series can be performed directly from the intermediate-momentum-transfer analysis. At low energy, it is compatible with Regge-pole models with exchange-degenerate Regge trajectories. At very high energy, however, one has to add to the Pomeron singularity a new real contribution. We study the implications of this real contribution which will serve as a test for the iteration approach.

I. INTRODUCTION-MOTIVATION

As is well known, elastic scattering of two hadrons gives rise to a diffraction peak in the differential cross section. This diffraction peak is related to the "shadow" contribution of the multiparticle states through unitarity and is commonly described by the exchange of a Regge singularity, the Pomeron. Beyond the diffraction peak, a different behavior appears, with a flattening of the momentum-transfer dependence and the existence of structures in the differential cross sections.

It has long been thought that the diffraction peak and the larger-momentum-transfer structures might be related by an iteration procedure.[1] In this appraoch, one starts with an amplitude ϕ describing the diffraction peak as a primary interaction, or Born term. Then one adds the contributions corresponding to the multiple interactions, which are commonly given by the repeated convolution—or iteration—of the single-interaction amplitude

$$A=\sum_n c_n \phi^{*n}.$$

The amplitude is specified by an input term ϕ and a set of c_n coefficients which is expected to be simple or derived from field-theoretical considerations.[1]

However, this plan suffers from two main difficulties. First, phenomenologically, the attempts made to date have not been completely convincing. At very high energy (CERN ISR range), the dip at $t \sim 1.4$ GeV in pp scattering has been predicted correctly, but other expected structures at larger momentum transfer are lacking.[2] At moderate energies [CERN Proton Synchrotron (PS) range], the usual models do not extend to a large domain in momentum transfer (for $|t|$ larger than 1–2 GeV^2 there are problems).

Second, a more fundamental question has been raised about the multiple-interaction contribution. In a space-time picture of the interaction[3] process they do not correspond to successive interaction but to simultaneous ones. In terms of Reggeon graphs, the difference is depicted in Fig. 1. The incoming particles dissociate themselves into their constituents, which then interact and finally recombine. The overall result is to give a more complicated contribution than the simple convolution.[3] Many iteration models are possible, though none can claim any strong theoretical justification.

Our aim is to discuss phenomenologically the validity of this approach. We first look for some general features, common to the various iteration models. The general features are then compared with the experimental data in their range of validity. For instance, for a large class of input Born terms ϕ and different sets of coefficients c_n, we find a general approximation, valid in the large-momentum-transfer region. This is shown in Sec. II, where we review the general features of this approximation. The phenomenological study

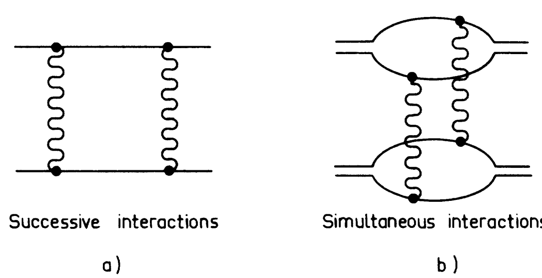

FIG. 1. (a) Two successive interactions—no contribution to the two-Reggeon cut. (b) Two simultaneous interactions.

of the data, performed in Sec. III, allows a thorough discussion of the applicability of iteration models. Pursuing this approach, we discuss the properties of the Born term, as revealed by the large-momentum-transfer behavior and compare them to the properties already known from the usual low-momentum-transfer models. In Sec. IV, we draw the general conclusions of our work, under the form of a status of the iteration models as used in elastic scattering. This discussion is completed by a comparison of these models with the geometrical and constituent approaches.

II. A LARGE-MOMENTUM-TRANSFER APPROXIMATION TO ITERATION-MODEL AMPLITUDES

Let us first consider a simple model of iteration, defined by the following formula:

$$A = \sum (-1)^n e^{-na} \phi^{*n}(s,t). \tag{1}$$

In this formula one may recognize the usual alternating sign and the strength parameter (e^{-a}) of the Regge-cut models. The convolution is defined as follows:

$$\begin{aligned} \phi(s,t) &= \int_0^\infty b\,db\, \tilde{\phi}(s,b) J_0(b\sqrt{-t}), \\ \tilde{\phi}(s,b) &= \int_0^\infty \sqrt{-t}\, d\sqrt{-t}\, \phi(s,t) J_0(b\sqrt{-t}). \end{aligned} \tag{2}$$

Then, $\tilde{\phi}$ is the profile function of ϕ, and $\tilde{\phi}^n$ is the profile function of ϕ^{*n}. We write

$$\phi^{*n}(s,t) = \int_0^\infty b\,db [\tilde{\phi}(s,b)]^n J_0(b\sqrt{-t}). \tag{3}$$

Inserting formula (3) in the amplitude (1) and inverting summation and integration, we get

$$\begin{aligned} A(s,t) &= \sum_{n=1}^{\infty} (-1)^n e^{-na} \int_0^\infty b\,db [\tilde{\phi}(s,b)]^n J_0(b\sqrt{-t}) \\ &= \int_0^\infty b\,db\, \tilde{A}(s,b^2) J_0(b\sqrt{-t}), \end{aligned} \tag{4}$$

with

$$\tilde{A}(s,b) = \frac{\tilde{\phi}(s,b)e^{-a}}{1+\tilde{\phi}(s,b)e^{-a}}.$$

We may replace the integration over the real axis by a contour in the complex b^2 plane. We note that the Hankel function $H_0(z)$ is analytic with a cut on the real axis, the discontinuity across this cut being given by $J_0(z)$. Thus we may rewrite the amplitude as an integral over the contour C (see Fig. 2):

$$A(s,t) = \frac{1}{4i} \int_C db^2 \frac{\tilde{\phi} e^{-a}}{1+\tilde{\phi} e^{-a}} iH_0(b\sqrt{-t}). \tag{5}$$

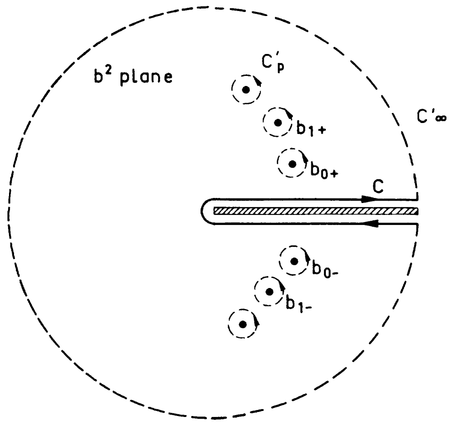

FIG. 2. The complex-impact-parameter plane and the deformation of the integration contour. Continuous line: the original contour C. Dashed line: the pieces of the deformed contour C'. C'_p corresponds to the contour at infinity.

The amplitude can thus be represented as a sum of residues of the poles arising from the zeros of the denominator. This continuation into the complex b^2 plane, with resulting exploitation of the poles, has been used by Dean,[4] though from a different point of view. This author was principally interested in impact-parameter poles in the crossed channel ($t>0$). We will be concerned strictly with the s-channel impact-parameter poles, which as we shall see below is the proper tool for investigating two-body scattering outside the very forward region.

Closing the contour at infinity[5] and assuming that $\tilde{\phi}$ is meromorphic in b^2, we get

$$A(s,t) = \sum_k iH_0(b_k\sqrt{-t}) \times (\text{residue at pole } k), \tag{6}$$

with the b_k given by the solutions of

$$1+\tilde{\phi}(s,b_k^2)e^{-a} = 0. \tag{7}$$

The physical content of formula (6) may be illustrated by considering a simple but realistic example, that of an input Born term consisting of an exponential. The relevant convolution formulas (2) and (3) give in this case

$$\phi(s,t) = 2R^2 e^{R^2 t}, \quad \tilde{\phi}(s,b) = e^{-b^2/4R^2},$$

$$\tilde{\phi}^n(s,t) = e^{-nb^2/4R^2}, \quad \phi^{*n}(s,t) = \frac{2R^2}{n} e^{R^2 t/n}.$$

The iteration series then has the form

$$A = 2R^2 \sum \frac{(-1)^n}{n} e^{-na} e^{R^2 t/n}. \tag{8}$$

At small momentum transfer, one can check that the iteration is a good perturbative approach to the amplitude. The first term ($n=1$) describes the diffraction peak and the other terms give only small contributions, provided the parameter a is large enough (a is of order 1 in all types of fits). *At large momentum transfer*, the situation is quite different. A large number of terms of the iteration contribute, since the smallness of the term e^{-na} is compensated by the smaller slope of the exponential $e^{R^2t/n}$ in formula (8).

Now in this case, let us apply our method of approximating the amplitude. The poles $b_k^{\,2}$, solutions of Eq. (7), are easily obtained

$$e^{-b^2/4R^2-a}=-1=e^{\pm i\pi+2ki\pi},$$

or (9)

$$b_{k\pm}^{\,2}=-4R^2(a\pm i\pi+2ki\pi).$$

We get a series of poles $b_{k\pm}^{\,2}$, with increasing imaginary part. Noting that at large momentum transfer

$$H_0(b_kz)\simeq\frac{1}{(\frac{1}{2}\pi i b_k z)^{1/2}}\,e^{ib_kz},$$

one obtains

$$\begin{aligned}A&\simeq\frac{\pi}{2}\sum_k\frac{ie^{ib_kz}}{(\frac{1}{2}\pi i b_k z)^{1/2}}\\&\quad\times\left(\text{Residue at } b^2=b_k^{\,2} \text{ of } \frac{e^{-a-b^2/4R^2}}{1+e^{-a-b^2/R^2}}\right)\\&=\frac{\pi}{2}\sum_k\frac{ie^{ib_kz}}{(\frac{1}{2}\pi i b_k z)^{1/2}}\,4R^2.\end{aligned}\qquad(10)$$

This characteristic form of a sum of exponentials in the variable $\sqrt{-t}$ will be the basis of our subsequent phenomenological investivation in Sec. III.

The validity of our approximation method is confirmed by the direct estimation of the iteration sum given by Anselm and Dyatlov in a series of articles.[6] Their method leads to the following qualitative picture: At finite t there is a balance between the damping in n provided by the strength factor e^{-na} and the growth of the $e^{R^2t/n}$ factor. There exitst thus a value of n (in fact two complex values $n_\pm$ because of the alternating character of the series) which dominates the sum for a given, nonzero, value of t. These values are given by

$$n_\pm=\frac{R\sqrt{-t}}{(a\pm i\pi)^{1/2}},$$

leading to the approximation

$$A\simeq\lambda_+e^{2R^2t/n_+}-\lambda_-e^{2R^2t/n_-},$$

with $\lambda_\pm$ being slowly varying functions of t. One readily recognizes that these are but the first two terms ($k=0$) of Eq. (10). A direct check of the exponential behavior yields

$$\frac{2R^2t}{n_\pm}=-2R\sqrt{-t}\,(a\pm i\pi)^{1/2}=ib_{0\pm}\sqrt{-t}\,.$$

Furthermore, one can verify that the Anselm-Dyatlov method leads to the same coefficients $\lambda_\pm$ as found in Eq. (10). There is thus a perfect identity in this case between the impact-parameter-pole approximation and the asymptotic estimate given by the Anselm-Dyatlov method.

The complex-impact-parameter method offers the advantage that it can be extended to more general interactions than can be treated by the method of Anselm and Dyatlov. In particular we can consider a sum of two exponentials in t, representing a Pomeron plus a secondary Regge contribution, as we know must be the case for low to moderately high energies. This justifies the application of our method to a large domain in energy in the phenomenological analysis of the next section.

Another desirable generalization consists in introducing a set of coefficients c_n into the iterative series. In fact, our method of approximation can indeed be generalized to other iteration series than (1), though the nature of the singularities in the complex b^2 plane may change. Let us consider, for instance, the important example of the eikonal series. Instead of formulas (1) and (5) we have

$$\begin{aligned}A(s,t)&=\sum_{n=1}^{\infty}(-1)^n\frac{e^{-na}}{n!}\,\phi^{*n}(s,t)\\&=\frac{1}{4i}\int_c b\,db\,[\exp(-\phi e^{-a})-1]\,iH_0(b\sqrt{-t}\,).\end{aligned}\qquad(11)$$

Instead of poles in the complex plane, the integral is dominated by saddle points. (Note that the constant -1 in the integrand does not contribute outside the forward direction.) In the large-momentum-transfer region, the amplitude is thus reduced to a simple expression, considering the approximation of the Hankel function in the same region ($H_0(bz)\sim[1/(ibz)^{1/2}]e^{ibz}$).

$$A(s,t)\simeq\tfrac{1}{4}\int_c\frac{b\,db}{(\frac{1}{2}\pi i b z)^{1/2}}\exp[-\tilde{\phi}(b^2,s)e^{-a}+ib\sqrt{-t}\,].$$

This integral is dominated by saddle points $b_k^{\,2}$, which are easily derived once the function $\tilde{\phi}(b,s)$ is given. This leads to results very similar to the previous ones: Up to logarithms, the functions which appear are again exponentials in $\sqrt{-t}$.[7]

A last extension of our approximation method concerns spin. For helicity amplitudes with a net helicity flip ν, one has to use in all formulas Hankel functions with index ν. Since all these functions possess the same asymptotic approximation up to phases, one gets the same expression. Moreover, the location of the singularities in the complex b^2 plane is the same. Indeed in most iteration models, the flip amplitudes are defined by the convolution of a flip Born term times the nonflip kernel.

$$A_{\text{flip}} = \phi_{\text{flip}} * (1 + 2iA_{\text{nonflip}}) .$$

The singularities contained in the nonflip amplitudes dominate also the flip amplitudes. Only the residues differ in formula (7). This means that the exponential behavior will also be the same. Note that the new contributions one has to consider in the nonflip term (such as flip-flip contributions) can be absorbed in the nonflip Born term and do not change our conclusions.

III. EXPERIMENTAL CHECKS AND DISCUSSION

A. Parametrization and fits

The results of the preceding section suggest that the moderate-momentum-transfer region of elastic reactions can be described by a small number (2 at most) of exponentials in the variable $\sqrt{-t}$. Let us look at the data.

The cross section is given by the sum over the different helicity amplitudes. As shown previously, it is legitimate to consider the same exponentials for all amplitudes. We then write

$$\frac{d\sigma}{dt} = \sum_{\nu = \text{helicity indices}} |M_\nu|^2$$

$$= \sum_\nu |\lambda_\nu^+ e^{\mu^+ z} - \lambda_\nu^- e^{\mu^- z}|^2 , \tag{12}$$

where μ^+, μ^- are independent of ν. We shall from now on assume that the λ are constant at nonzero transfer, focusing our attention upon the exponential behavior.

Collecting the terms with different exponents, we get

$$d\sigma/dt = \sum_\nu |\lambda_\nu^+|^2 e^{(\mu^+ + \bar{\mu}^+)z} + \sum_\nu |\lambda_\nu^-|^2 e^{(\mu^- + \bar{\mu}^-)z} - 2\,\text{Re}\left[\left(\sum_\nu \lambda_\nu^+ \bar{\lambda}_\nu^-\right) e^{(\mu^+ + \bar{\mu}^-)z}\right] , \tag{13}$$

where the bar stands for complex conjugation.

One can rewrite formula (13), taking into account the real and imaginary parts of the complex numbers $\mu^\pm$, $\mu^\pm = R^\pm + iI^\pm$,

$$\frac{d\sigma}{dt} = \left(\sum_\nu |\lambda_\nu^+|^2\right)^{1/2} \left(\sum_\nu |\lambda_\nu^-|^2\right)^{1/2} e^{(R^+ + R^-)} \left[+ \frac{(\sum_\nu |\lambda_\nu^+|^2)^{1/2}}{(\sum_\nu |\lambda_\nu^-|^2)^{1/2}} e^{(R^- - R^+)z} - 2\,\text{Re}\, \frac{\sum_\nu \lambda_\nu^+ \bar{\lambda}_\nu^-}{(\sum_\nu |\lambda_\nu^-|^2 \sum_\nu |\lambda_\nu^+|^2)^{1/2}} e^{i(I^+ - I^-)z} \right] .$$

Then, we get a seven-parameter formula

$$\frac{d\sigma}{dt} = 2Ge^{2\Sigma z}[\cosh(2\Delta z + 2\psi) - \rho \cos(2Iz + 2\varphi)] ,$$

with

$$G = \left(\sum_\nu |\lambda_\nu^+|^2\right)^{1/2} \left(\sum_\nu |\lambda_\nu^-|^2\right)^{1/2} , \quad \Sigma = \frac{R_+ + R_-}{2} ,$$

$$\Delta = \frac{R_+ - R_-}{2} , \quad I = \frac{I_+ - I_-}{2} ,$$

$$\psi = \tfrac{1}{4} \ln\left(\frac{\sum_\nu |\lambda_\nu^+|^2}{\sum |\lambda_\nu^-|^2}\right), \quad \rho e^{2i\varphi} = \frac{\sum_\nu \lambda_\nu \bar{\lambda}_\nu^-}{(\sum_\nu |\lambda_\nu^-|^2 \sum |\lambda_\nu^+|^2)^{1/2}} .$$

Note that $\rho \leq 1$, owing to the Schwarz inequality. The equality is obtained only if $\lambda^- / \lambda_\nu^+ =$ constant. We shall make this assumption in order to consider the simplest parametrization, and thus we get a six-parameter formula

$$\frac{d\sigma}{dt} = 4Ge^{2\Sigma z}[\sinh^2(Rz + \psi) + \sin^2(Iz + \varphi)] . \tag{14}$$

For each value of the incident energy, one has to use formula (14) with different parameters. This means that the experimental checks will be meaningless unless an extended-momentum-transfer region can be considered. This leads us to consider the following data (see Table I):

(a) *pp scattering at PS*[8] *and ISR*[2] *energies*. One has results up to 10 GeV^2.

(b) *Low-energy* $\pi^\pm p$, $K^\pm p$, *and* $\bar{p}p$ *scattering*. We will concentrate on the data at 5 GeV/c,[9] which cover all these two-body elastic reactions for all angles.

The curves of Fig. 3 show the resulting fits for pp scattering. Those of Fig. 4 show the reactions at 5 GeV/c. All fits are satisfactory in a momentum-transfer interval from 0.5 GeV^2 to 8–10 GeV^2. At this point, the data available are compatible with the hypothesis of an iteration mechanism. Concerning the possibility of a more complete study, we point out that in both cases, the results from Fermilab or SPS are needed to cover satisfactorily the entire range of reactions and energy available.

Note that pp and K^+p cross sections—the exotic channels—differ qualitatively from the others.

TABLE I. Parameters of the fits to elastic scattering at large momentum-transfer values.

Reactions \ Parameters		4G (mb GeV^{-2})	Σ (GeV^{-1})	R (GeV^{-1})	I (GeV^{-1})	ψ (rad)	φ (rad)
		(a) pp scattering					
pp	10 GeV	1470	5.1	2.9	3.4	−2.5	2.1
pp	19 and 24 GeV	1142	4.97	1.86	2.95	−1.62	2.81
pp	1.500 GeV	1960	5.39	1.65	1.19	−1.96	1.63
		(b) 5-GeV reactions					
π^-p		170	3.46	0.61	4	−1.3	−0.51
π^-p		150	3.45	1.65	3.34	−2.5	~0.4
K^-p		166	3.73	0.27	4.27	0.54	−1
K^+p		170.7	2.98	1.01	~0	−1.64	−0.15
$\bar{p}p$		116	3.18	0.18	5.25	0.36	−0.88

This well-known difference[10] can be understood in our approach, as we shall discuss now.

B. The physical interpretation: the properties of the Born term

In the discussion of the difference between exotic and nonexotic channels in the moderate-momentum-transfer region, it is natural to look for a physical interpretation using the usual models for the Born term, namely the Reggeon exchanges: Pomeron and secondary Regge trajectories. The properties at moderate or large momentum transfer would then be related to those at low momentum transfer. In this respect, one is led to separate the discussion of the low-energy cross sections which mainly depend on the secondary Regge exchanges from that of the high-energy ones, where one can study the Pomeron singularity alone.

1. Low-energy cross sections

If we include Regge poles in the input Born term, we know that there exists a difference at the level of the secondary Regge poles (ρ, A_2, P', ω) between exotic and nonexotic channels. These poles give a mainly real contribution in the former case and one with a rotating phase in the latter one. The question is whether the iteration of such a Born term will give the observed effects. For its study, we consider a standard realization of an iteration model, based on the eikonal series with a Born term consisting of a dipole Pomeron[1,11] and a pair of strongly degenerate Regge poles

$$A^{\pm} = i\sum \frac{(-1)^n}{n!} e^{-na}(\phi^{\pm})^{*n}, \tag{15}$$

$$\phi^{\pm} = \frac{\lambda}{(t-t_a)(t-t_b)} + \frac{g}{i} e^{bt}(\pm s)^{\alpha(t)-1} \equiv \mathcal{P} + \mathcal{R}^{\pm}(s,t),$$

with $a=0$ (pure eikonal model[1]) and $\alpha(t) \equiv \alpha(0) + \alpha' t \simeq 0.5 + 0.9t$. The ± factor gives the rules for Reggeon phases (+ for exotic channels, − for the s-u crossed nonexotic channels such as, respectively, pp and $\bar{p}p$ elastic scattering). We fix the parameters λ, t_a, t_b, g, b through consistency with the data at 5 GeV/c (see Table II).

The results are shown in Fig. 5 for a set of typical energies (5 GeV/c, 10, 24, 100, 200, and 2000). As expected, one obtains oscillations for the nonexotic reaction and a break for the exotic one. This is in agreement with the gross features of the data. Changing its position with energy, the break in the exotic cross section is transferred to larger momentum transfer and the second dip of the nonexotic one disappears. Note the profound first dip for which it could be interesting to look

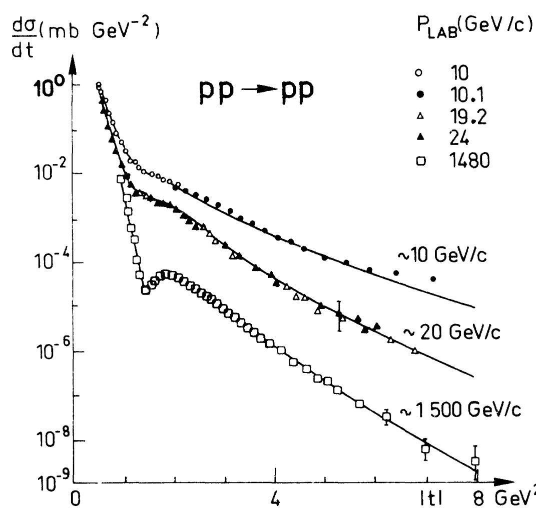

FIG. 3. Results of the fits with the approximation formula: pp scattering. The data are from Refs. 9 (PS range) and 2 (ISR range).

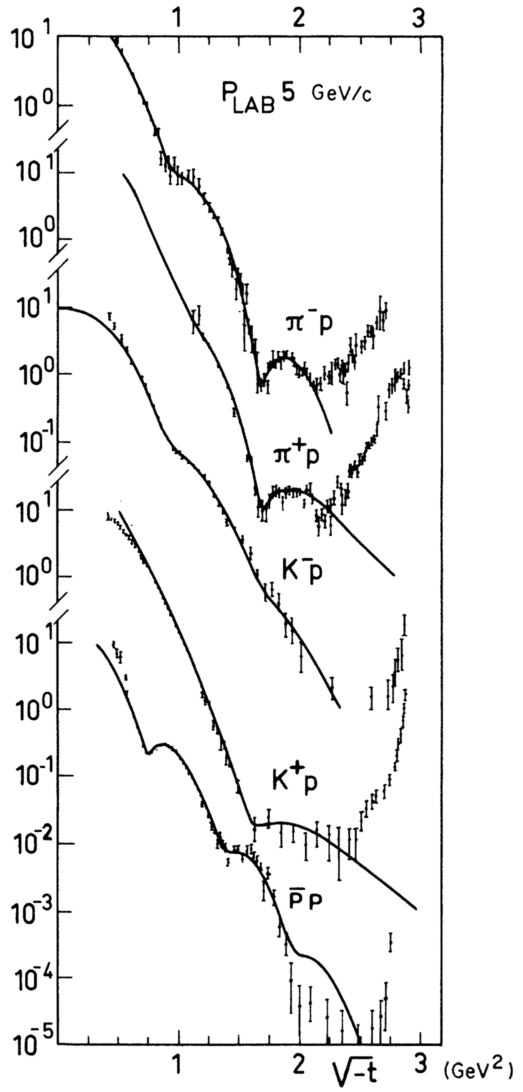

FIG. 4. Results of the fits with the approximation formula: elastic reactions at 5 GeV/c. The data are from Ref. 11.

in $\bar{p}p$ experiments between 15 and 50 GeV/c.

Confirming this phenomenological analysis, the features of exotic and nonexotic reactions can be related to the location of the singularities b_k^2 in the complex plane (see Sec. II). Rewriting the formula (7), one gets the equation for the singularities (assuming poles for simplicity):

$$1+[\tilde{\mathcal{P}}(b_k^2,s)+\tilde{\mathcal{R}}^{\pm}(b_k^2,s)]e^{-a}=0. \qquad (16)$$

Since the intercept of the Reggeons is $\alpha_0=0.5$, the Reggeon amplitude for nonexotic reactions $\mathcal{R}^{-}(s,t=0)$ is real, like the Pomeron (with our conventions), in the forward direction. Then their profile functions are also essentially real, and the solution of (18) consists of pairs of complex-conjugate poles $b_{k\pm}^2$, in order to be consistent with a purely real equation. One then obtains oscillations in the cross sections since the poles give two complex-conjugate contributions [$R=\psi=0$ in formula (14); only the $\sin^2$ term remains].

In exotic channels, Pomeron and Reggeons are no longer in phase, the Pomeron being real and the Reggeon contribution imaginary. Assuming for simplicity the Pomeron contribution to be real and constant in the region where we look for b^2 singularities, we write

$$1+[\text{const}+\tilde{\mathcal{R}}^{+}(b^2,s)]e^{-a}=0,$$

and in the simple example of formula (15), we get

$$\tilde{\mathcal{R}}^{+}(b^2,s)\propto\frac{g}{i}\,\frac{s^{\alpha(0)-1}}{4\alpha'\ln s}\exp\left(-\frac{b^2}{4\alpha'\ln s}\right).$$

The phase shift between the Reggeon contribution and the Pomeron prevents Eq. (7) from being real, and the solutions are no longer complex conjugate leading to two different contributions [$R,\psi\neq 0$ in formula (14), with a damping or a suppression of the oscillations due to the $\sinh^2$ term].

2. *High-energy cross section*

In low-energy checks, the model for the Pomeron is not properly tested. In order to discuss the Pomeron singularity in isolation, one has to consider the ISR data for pp scattering. At these energies, the usual iteration models of the Pomeron—including the dipole Pomeron, see Fig. 6—are inadequate. They predict a second dip not seen in the data.[2] This general result[12] can easily be understood from our approximation formulas.

TABLE II. Parameters of the eikonal models of pp and $\bar{p}p$ scattering with a dipole Pomeron.

Pomeron parameters			Reggeons or other secondary contributions	
(a) Eikonal model + Reggeons				
λ	t_a	t_b	g	b
14.1	0.71	0.71	13.85	2.28
(b) Eikonal model for ISR data "classical"				
λ	t_a	t_b		
11	0.466	1.46		
(c) Eikonal model for ISR data "modified"				
λ	t_a	t_b	h	a
13.6	0.36	2.074	0.71	1.37

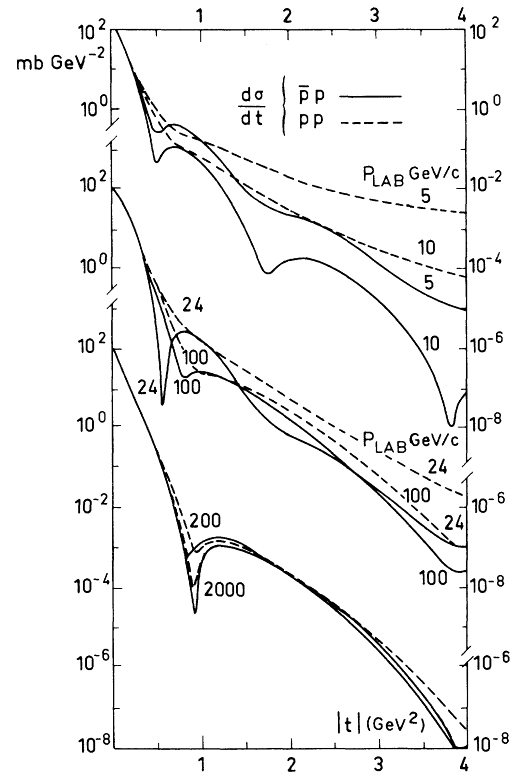

FIG. 5. Qualitative description and predictions for $\bar{p}p$ and pp scattering at various energies in a model with exchange-degenerate Regge poles.

Using an approximately real Pomeron, one gets large oscillations, corresponding to two complex-conjugate poles.

These oscillations cannot be arbitrary since the lower-momentum-transfer data ($|t|<3$ GeV^2) fix their period and imply a second dip in a specific momentum-transfer region ($t \simeq 4-6$ GeV^2).

One can verify that secondary Regge poles cannot provide us with a solution to this problem. Their rapid decrease with the energy leads to very small contributions at ISR energies, and the slope of the input Regge terms ($\alpha' = 0.9$ GeV^{-2}) gives too much peaking to be of relevance at large momentum transfer. Thus the only possibility is to modify the Pomeron singularity itself.

The physical idea is to introduce in the Born term a new contribution, out of phase with the Pomeron. We already know that this way we avoid the oscillations (low-energy exotic reaction), the problem then being to describe the dip at 1.4 GeV^2 (instead of a break). We propose the following parametrization [the values of the parameters are in Table II(c)]:

$$\phi(t) = \frac{\lambda}{(t-t_a)(t-t_b)} + \frac{h}{i} e^{at} .$$

The nature of this new contribution—a "core" term as we call it—has to be elucidated, but its introduction seems unavoidable in the framework of the iteration models.

It is important to look for other possible effects of the core term. In our model, it is predicted that there is no additional dip beyond the first one at $|t| \simeq 1.4$ (GeV/c)^2 and that at sufficiently large momentum transfer the cross section will behave as an exponential in $\sqrt{-t}$. This flattening, which is apparent in our calculated curve (Fig. 6), appears to be seen in Fermilab[13] and ISR recent data.[14] More work is needed to clarify completely this important question.

IV. CONCLUSIONS AND OUTLOOK

We have shown that it is possible to describe approximately a large family of iteration models by two exponentials in the variable $\sqrt{-t}$, or equivalently by the contribution of two singularities in the complex plane of the impact parameter.

Using the property, we are led to a first conclusion: The large-momentum-transfer data of elastic scattering are compatible with the iteration-model approach.

In order to apply a realistic model, one has to investigate the properties of the input Born term as given by Regge-exchange models. We are led to the following conclusions:

(i) At low energy, one clearly identifies the effects of the exchange-degenerate Regge poles (ρ-A_2 or P'-ω).

(ii) At very high energy, one has to add a new "core" contribution to the usual models of the Pomeron. The core is responsible for the absence of additional dips in pp scattering and the flattening of the cross section after $t=8$ GeV. The introduction of a core contribution appears to be essential if one wants to use iteration models for the description of the data we have considered.

The approximation we have proposed is abstracted from the iteration-model approach, and its success as a description of large-t elastic scattering in all its variety should be considered as a hint for this kind of approach. We see that the structure of the differential cross section depends rather weakly on the details of the primary interaction assumed and on the specific form of the iteration. This structure is characteristic of geometrical models, e.g., that of Schrempp and Schrempp[15] in which the exponentials in $\sqrt{-t}$ have their origin in surface waves creeping along the interaction interface.

The energy dependence should come from the input Born term. In the absence of a detailed mod-

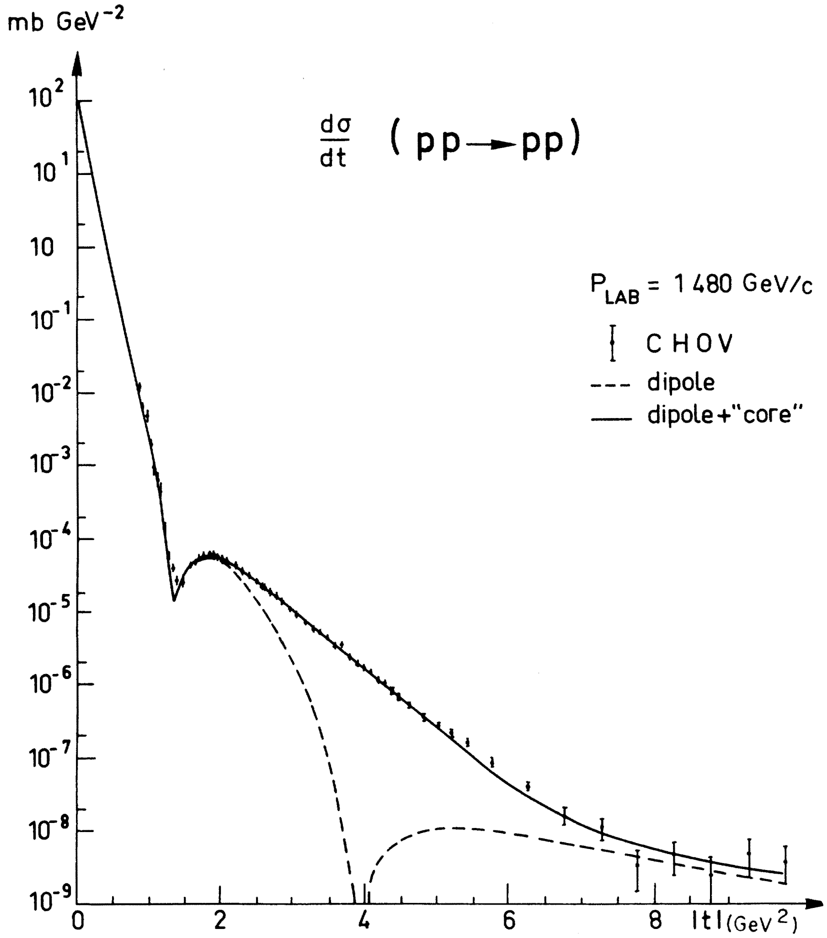

FIG. 6. Comparison of an eikonal model with (continuous line) and without (dashed line) an additive real contribution. The data are from Ref. 12.

el for the Pomeron and secondary Reggeons which would constitute this input, it is hard to give more than a qualitative discussion of the s dependence. In particular, we would like to compare our results with those of Dean and of the parton model, both of which look at the fixed-angle energy behavior.

First let us look at 90° pp scattering which has been studied by Dean, using a technique formally similar to ours, but working in the t channel. Freely parametrizing the motion of the t channel impact-parameter pole, Dean was able to fit the energy dependence below and above $s \sim 17$ GeV^2. Now it has been pointed out since that these data (including the "break" at $s \sim 17$) lend themselves quite naturally to a description by Eq. (14), as we might expect if the b^2-pole parameters vary slowly with energy.

Parton models[16] predict a power law in energy at fixed angle

$$\frac{d\sigma}{dt} = s^{-N} f(\cos\theta), \tag{18}$$

with $N \sim 8$ for incident mesons and $N \sim 10$ for baryons incident. These predictions are generally in accord with existing data. Iteration models can simulate a power law over a finite energy range, though asymptotically, as the influence of the secondary Regge poles vanishes, one will arrive at a form of the type of Eq. (14) independently of energy. Thus we expect eventually an exponential behavior in the center-of-mass energy. Experimental data at higher energies should allow us to decide on this point.

There is a possibility that both processes, Regge-pole iteration and hard-parton collisions, contribute to elastic scattering. In that case, the parton processes will eventually dominate fixed-angle scattering. However, we can define a transition t_0 such that for $|t| < |t_0|$ the iterative process dominates. Equating the two contributions.

$$s^{-N}\sim e^{-2\Sigma\sqrt{-t_0}}\,, \qquad (19)$$

we obtain

$$-t_0\cong\left(\frac{N}{2\Sigma}\ln s\right)^2\sim \ln^2 s\,, \qquad (20)$$

since for baryons we have $N\sim 8$, $\Sigma\sim 4$, and for mesons $N\sim 10$, $\Sigma\sim 5$. Taking $|t_0|\simeq 4$ GeV^2 at $p_L = 5$ GeV/c leads to $t_0\sim 36$ GeV^2 at $p_L = 500$ GeV/c. The experimentally accessible domain should be considerably enlarged before we can decide on these issues.

ACKNOWLEDGMENTS

We acknowledge many fruitful discussions with G. Cohen-Tannoudji, F. Hayot, H. Navelet, and R. Schaeffer. One of us (A.M.) would like to thank C. De Dominicis and the members of the Service de Physique Théorique for their kind hospitality.

---

*Present address: Centre de Physique Théorique, Marseille, France. CEN-Saclay, Boite Postale No. 2, 91190 Gif-sur-Yvette, France,

[1]The best known models of this type are the following: the inelastic overlap model—L. Van Hove, Nuovo Cimento 28, 798 (1963); the eikonal models—T. T. Chou and C. N. Yang, Phys. Rev. 170, 1591 (1963); H. Cheng and T. T. Wu, Phys. Rev. Lett. 24, 1456 (1970).

[2]A. Böhm *et al.*, Phys. Lett. 49B, 491 (1974); H. DeKerret *et al.*, *ibid.* 62B, 363 (1976).

[3]A. Krzywicki, private communication; and in *Proceedings of the Gif-sur-Yvette Summer School*, 1973, edited by M. S. Detoeuf (Institut National de Physique Nucleaire et de Physique des Particules. Paris. 1973).

[4]N. W. Dean, Nuovo Cimento 52A, 1129 (1967); 62A, 604 (1969).

[5]The contour at infinity does not contribute if $\tilde{\varphi}(s,b)$ does not grow exponentially, as is explicitly the case here. For all Bessel function properties, we refer to *Handbook of Mathematical Functions*, edited by M. Abramowitz and I. A. Stegun, National Bureau of Standards Applied Mathematics Series No. 55 (Dover, New York, 1965).

[6]A. A. Anselm and I. I. Dyatlov, Phys. Lett. 24B. 479 (1967); Yad. Fiz. 6, 591 (1967); 6, 603 (1967) [Sov. J. Nucl. Phys. 6, 430 (1968); 6, 439 (1968)]. Anselm and Dyatlov use a Sommerfeld-Watson representation in the complex n plane. The contour is deformed to pass through the saddle point at $n_\pm$ and is evaluated by the method of steepest descent.

[7]A rough argument is the following: Let us consider an exponential input term. The saddle points are given by the equation

$$\frac{\partial}{\partial b^2}\left[-e^{-b^2/4R^2-a}+ib\sqrt{-t}-\tfrac{1}{2}\ln\left(\frac{i\pi}{2}\,b\sqrt{-t}\right)\right] \equiv \frac{1}{4R^2}\,e^{-b^2/4R^2-a}+\frac{i\sqrt{-t}}{2b}-\frac{1}{4b^2}=0\,,$$

or

$$e^{-b^2/4R^2-a}+\frac{2iR^2\sqrt{-t}}{b}-\frac{R^2}{b^2}=0\,.$$

This equation is very similar to Eq. (9), except that there are slowly t-dependent contributions.

[8]$\bar{p}p$ data at 10, 12, 14, 24 GeV/c: J. V. Allaby *et al.*, Nucl. Phys. B52, 316 (1973); pp data at 19 GeV/c: J. V. Allaby *et al.*, Phys. Lett. 28B, 67 (1968).

[9]Cross-section data at 5 GeV/c: A. Eide *et al.*, Nucl. Phys. B60, 173 (1973).

[10]See, for instance, H. Harari, lectures at the Brookhaven Summer School, Brookhaven, 1971 (unpublished).

[11]M. Kac, Nucl. Phys. B62, 902 (1973).

[12]V. P. Sukhatme, Phys. Rev. Lett. 38, 124 (1977).

[13]We used the indications on preliminary data from Fermilab: J. Orear *et al.* (private communication).

[14]H. DeKerret *et al.*, Phys. Lett. 68B, 374 (1977).

[15]B. Schrempp and F. Schrempp, Phys. Lett. 70B, 88 (1977); CERN Report No. TH. 2319 (unpublished).

[16]R. B. Blanckenbecler, S. J. Brodsky, and J. F. Gunion, Phys. Lett. 39B, 649 (1972).

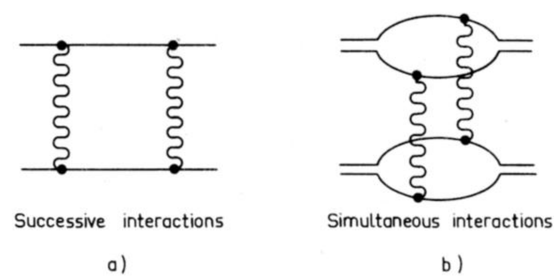

FIG. 1. (a) Two successive interactions—no contribution to the two-Reggeon cut. (b) Two simultaneous interactions.

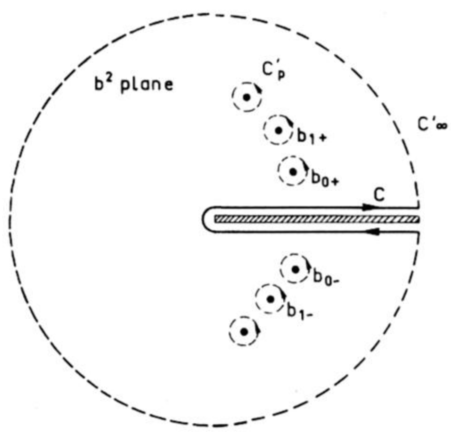

FIG. 2. The complex-impact-parameter plane and the deformation of the integration contour. Continuous line: the original contour C. Dashed line: the pieces of the deformed contour C'. C_p' corresponds to the contour at infinity.

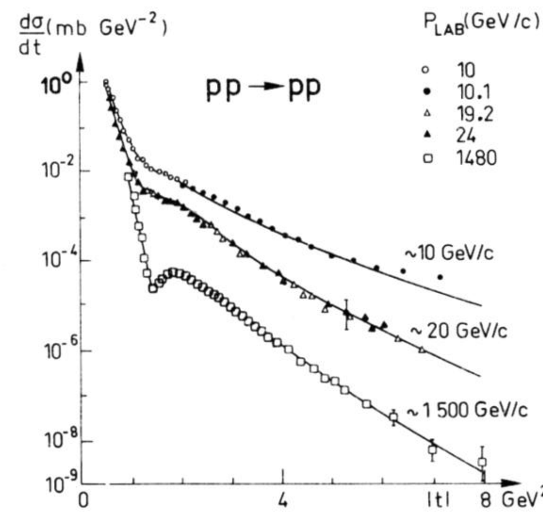

FIG. 3. Results of the fits with the approximation formula: *pp* scattering. The data are from Refs. 9 (PS range) and 2 (ISR range).

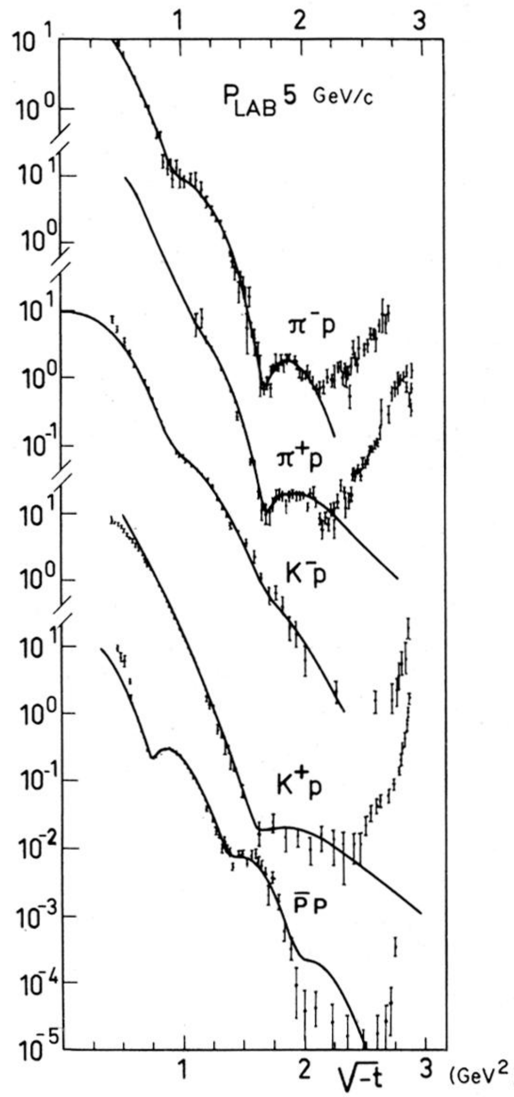

FIG. 4. Results of the fits with the approximation formula: elastic reactions at 5 GeV/*c*. The data are from Ref. 11.

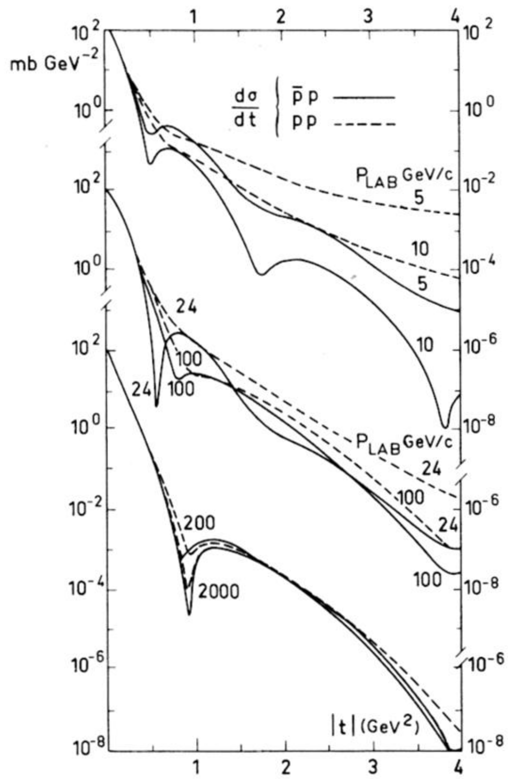

FIG. 5. Qualitative description and predictions for $\bar{p}p$ and pp scattering at various energies in a model with exchange-degenerate Regge poles.

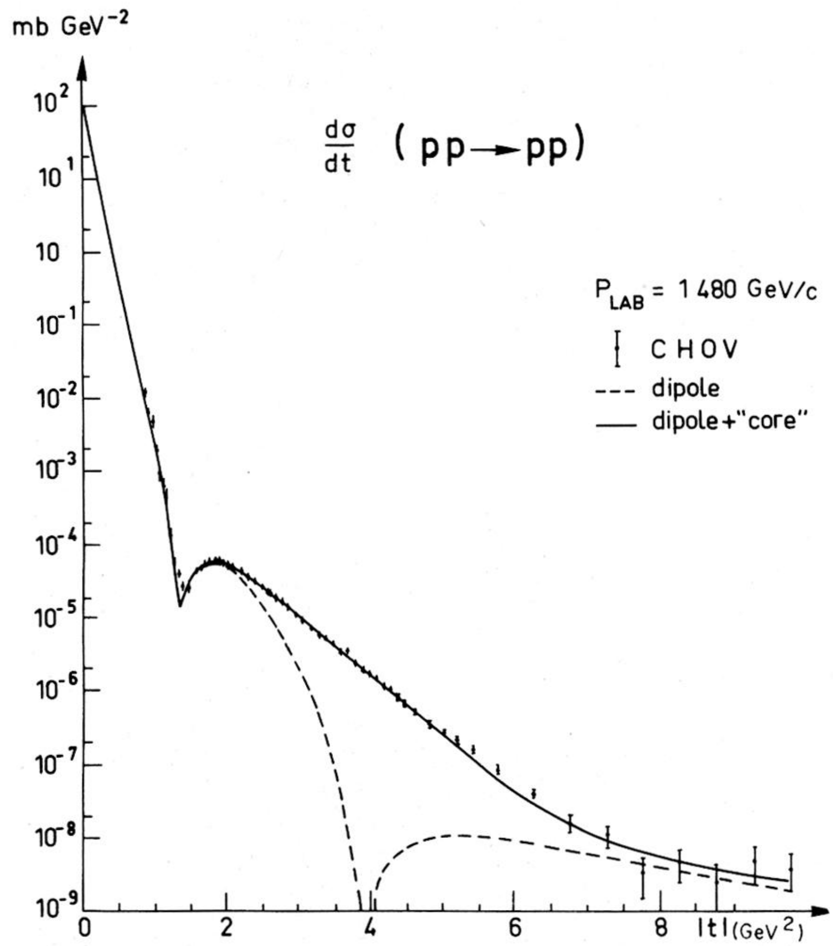

FIG. 6. Comparison of an eikonal model with (continuous line) and without (dashed line) an additive real contribution. The data are from Ref. 12.