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Rescattering effects in three-particle final-state reactions

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The phenomenological aspect of single-pion production in $\pi^- p$ interaction is explored by considering the final-state interactions in a rescattering box-diagram approach. We predict the qualitative and the quantitative features expected from our model in angular and momentum distributions of final pions. The preliminary data available so far appear to support our theoretical predictions.

I. INTRODUCTION

Recently box diagrams have been extensively used either for "exotic"-exchange reactions[1-5] or for dynamical violation[6] of the Iizuka-Okubo-Zweig rule. For the exotic-exchange reactions, we have shown[1-3] that the main features of the observed production angular distributions, its strong energy dependence and the polarization of final-state baryons, are qualitatively as well as quantitatively reproduced by our box-diagram model without using any arbitrary parameter. The imaginary part of the invariant amplitude as calculated from our model surprisingly shows[7] Odorico zeros[8] at small values of t, and this is again supported by the experimental results. But the theoretical situations become a little more ambiguous if we consider the quasi-two-body final states; for example,

$$\pi^- + p \to \pi^+ + \Delta^-(1236)\,. \tag{1}$$

The forward peak in this exotic-exchange reaction may be due to kinematical reflections,[9] though later theoretical[10] and experimental[11] studies have revealed contradictory results. We have earlier shown[1] how far our model reproduces the experimental features for the above and analogous reactions. In order to explore the production mechanism for reaction (1) further, we think it worthwhile to extend our model for the reactions involving three-particle final state. In this paper we develop our model for the reaction

$$\pi^- + p \to \pi^+ + \pi^- + n\ , \tag{2}$$

and put the formulation in easy and accessible form for numerical computation so that the theoretical predictions can be compared with the experimental data.[12]

The phenomenological situation for the three-particle final-state reaction is not very clear.[13] The problem of the distribution or two-body information over these states has made it more ambiguous theoretically as well as experimentally. This becomes particularly more difficult when strong pairwise final-state interactions overlap in the final-state phase space. For example, we can consider the process (2). The Dalitz plots demonstrate the dominant presence of $\Delta^-(1236)$ production[14] although its production decreases at higher energies. These plots also demonstrate the overlap of $\rho^0(765)$ and Δ^- in the final state. In the intermediate-energy region (1.5–4 GeV/c) these features are very clearly marked. In this energy range, the available experimental data indicate very rapidly changing transition amplitudes. In this paper, our motivation is to show that these features could be explained by considering a rescattering diagram in which only the final-state interaction due to the ρ meson is considered. This intermediate state ($N+\rho$) arises because of the interaction between the incoming pion and the "cloud" pion, and finally these particles rescatter. In fact, such an interaction is feasible unless the incident beam energy is quite high so that the possibility of rescattering vanishes.

This problem has been considered in detail[15-18] in the energy range below the incident pion momentum of 1.6 GeV/c by many authors in the framework of the isobar model. Usually in this model one considers that the reaction proceeds through an intermediate state dominated by a two-particle resonance or an isobar which ultimately breaks up into its constituents, the main assumption being that this decay amplitude remains independent of the production process. If there are many isobars present in the intermediate state, in that case one simply adds the various amplitude for the process. Above the center-of-mass total energy of 2 GeV ($\simeq$1.6 GeV/c incident pion momentum), the departure from the predictions of the extended isobar technique strongly suggests that the collisions are peripheral and that the production angular distributions reveal sharp forward and/or backward peaks.

Here we recall the famous and the first application of peripheral[19] ideas of Chew and Low to the process (2). They pointed out that the π exchange contributes a pole to the physical amplitude in the variable corresponding to the squared four-momentum of the exchange particle. The pole occurs for the unphysical values of the concerned variable. This singularity is nearest to the physical region when the exchanged particle is the lightest, and then in the part of the physical region near the pole, the peripheral diagram gives the dominant contribution. Later, in order to stick to the exchange of lowest number of particles (one-particle exchange), the basic spirit of the Chew and Low hypothesis was stretched too far to accommodate the exchange of baryons[20] and baryon resonances. In this work, we would like to go back to the peripheral philosophy and assume that the singularities closest to the physical region give the dominant contributions. If one-meson exchange is not allowed, as is the case in (1), we would prefer to consider the exchange of two mesons rather than considering the exchange of a baryon. Here we wish to explore the consequence of this peripheral hypothesis for the process (2) in the intermediate-energy region above 1.5-GeV/c incident pion momentum. It should be mentioned here that such diagrams for the process (2) have also been studied by Goebel and Schnitzer[21] and also by Carruthers.[22] These calculations, however, use the formalism of a static model with pion-pion interactions and are valid at very low energy.

II. FORMULATION OF MODEL

Writing down the invariant amplitude for the process (2) as shown by Feynman diagram in Fig. 1, we get

$$T_4=\int\frac{d^4p'}{(2\pi)^4}\bar{u}(p_2)(-A+i\gamma\cdot QB)\frac{(-i\gamma\cdot p'+M_n)}{p'^2+M_n^2}\gamma_5 g_{pn\pi^-}u(p_1)$$
$$\times\frac{1}{(Q_1^2+m_\pi^2)}ig_{\rho\pi\pi}(q_1+Q_1)_\mu\frac{(\delta_{\mu\nu}+q'_\mu q'_\nu/m_\rho^2)}{q'^2+m_\rho^2}ig_{\rho\pi\pi}(q_2-Q_2)_\nu\frac{1}{Q_2^2+m_\pi^2}\,. \tag{3}$$

Assuming that the imaginary part of the amplitude dominates[23] over the real part, we find that the diagram becomes the rescattering diagram. Now we can determine[1-3] the integrations over the c.m. scattering angle and the azimuthal angle of intermediate particles, ρ and n, by taking the residue at the pole given by the propagator $1/(Q_2^2+m_\pi^2)$. Thus, finally, we get the amplitude as follows:

$$\operatorname{Im}T_4=\frac{i|\vec{q}'|}{32|\vec{q}_1|\,|\vec{q}'|^2|\vec{q}_2|}\frac{1}{W\sqrt{-\beta_1}}\bar{u}(p_2)(-A+i\gamma\cdot QB)(-i\gamma\cdot p'+M_n)\gamma_5 g_{pn\pi^-}u(p_1)$$
$$\times\, ig_{\rho\pi\pi}(q_1+Q_1)_\mu(\delta_{\mu\nu}+q'_\mu q'_\nu/m_\rho^2)ig_{\rho\pi\pi}(q_2-Q_2)_\nu\,, \tag{4}$$

where

$$\beta_1=1-\cos^2\theta-\alpha_1^2-\alpha_2^2+2\alpha_1\alpha_2\cos\theta\,,$$

$$\alpha_1=\frac{2q_{10}q'_0-m_\rho^2}{2|\vec{q}_1|\,|\vec{q}'|}\,,$$

$$\alpha_2=\frac{2q_{20}q'_0-m_\rho^2}{2|\vec{q}_2|\,|\vec{q}'|}\,,$$

and

$$\cos\theta=\frac{\vec{q}_2\cdot\vec{q}_1}{|\vec{q}_2|\,|\vec{q}_1|}\,.$$

Here W is the center-of-mass ($\vec{q}_1+\vec{p}_1=0$) total energy; $|\vec{q}_2|$, $|\vec{q}_1|$, and $|\vec{q}'|$ are the magnitudes of c.m. momenta of final, initial, and intermediate particles, respectively. Also A and B are invariant amplitudes in spin-space for the scattering $n\pi^-\to n\pi^-$ involved in diagram 1, and $Q=\frac{1}{2}(q_3+Q_2)$.

After the summation over the final-spin states and averaging over the initial-spin states, we get the expression for differential cross section as follows:

$$d\sigma=\int\frac{0.38935}{(2\pi)^5 8W|\vec{q}_1|}\times\sum|\operatorname{Im}T_4|^2\frac{d^3q_2}{q_{20}}\frac{d^3q_3}{q_{30}}\frac{d^3p_2}{p_{20}}\times\delta^4(q_2+q_3+p_2-q_1-p_1)\,, \tag{5}$$

where

$$\sum|\operatorname{Im}T_4|^2=\frac{g_{\rho\pi^+\pi^-}{}^4 g_{\pi^-pn}{}^2F_1F_2}{64|\vec{q}_1|^2|\vec{q}'|^2|\vec{q}_2|^2W^2(-\beta_1)}\,, \tag{6}$$

with

$$F_1=AA^*(M_n^2+p_1\cdot p')(M_n^2-p_2\cdot p')+BB^*(p_1\cdot p'+M_n^2)[2Q\cdot p_2\,Q\cdot p'-Q^2p_2\cdot p'-M_n^2Q^2]-2M_nA^*B[Q\cdot p_2+Q\cdot p'](M_n^2+p_1\cdot p')\,,$$

$$F_2=\left(q_2\cdot q_1+\frac{q_2\cdot q'\,q_1\cdot q'}{m_\rho^2}\right)^2.$$

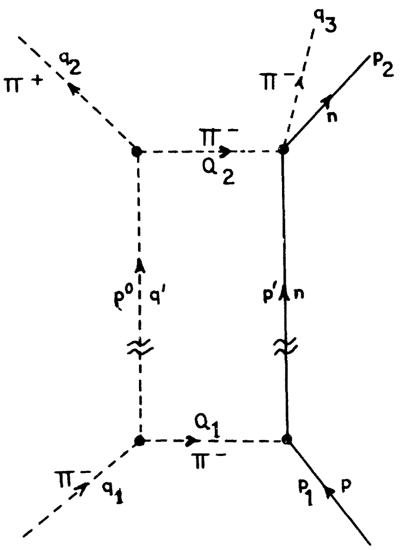

FIG. 1. Rescattering box diagram for the process $\pi^- p \to \pi^+ \pi^- n$.

Now in order to solve (5) further, we define[24] three center-of-mass systems as follows: (i) overall center-of-mass B system, that is, $\vec{q}_1^{\,B}+\vec{p}_1^{\,B}=0$; (ii) center-of-mass Q system of π^- and n in final state, that is, $\vec{q}_3^{\,Q}+\vec{p}_2^{\,Q}=0$; and (iii) center-of-mass P system of π^+ and n in final state, that is, $\vec{q}_2^{\,P}+\vec{p}_2^{\,P}=0$.

In the above expression (6), we can write the invariant amplitudes in terms of partial-wave amplitudes as follows:

$$\frac{A}{4\pi}=\frac{W+M_n}{p_{20}^Q+M_n}f_1-\frac{W-M_n}{p_{20}^Q-M_n}f_2\,, \tag{7}$$

$$\frac{B}{4\pi}=\frac{1}{p_{20}^Q+M_n}f_1+\frac{1}{(p_{20}^Q-M_n)}f_2\,,$$

and

$$f_1=\sum_{l=0}^{\infty}f_{l+}(\omega)P'_{l+1}(\cos\theta^Q)-\sum_{l=2}^{\infty}f_{l-}(\omega)P'_{l-1}(\cos\theta^Q)\,, \tag{8}$$

$$f_2=\sum_{l=1}^{\infty}\left[f_{l-}(\omega)-f_{l+}(\omega)\right]P'_l(\cos\theta^Q)\,,$$

with ω = total energy of the Q center-of-mass system and θ^Q is the scattering angle between two pions in this system. In Eq. (8) we approximate the dominant contribution coming from the $\Delta^-(1236)$ resonance and write

$$f_{1+}=\frac{1}{|\vec{p}_1|^Q}\,\frac{\frac{1}{2}\Gamma_P}{(\omega_R-\omega)-\frac{1}{2}i\Gamma_T}\,, \tag{9}$$

where ω_R is the resonance mass, and Γ_P, Γ_T are the partial and total decay widths, respectively.

The expressions for angular and momentum distributions for the differential cross section can be derived after carrying out[24] the integrations over the final-state phase space in (5). However, all the integrations cannot be done analytically. In the case of π^+ angular and momentum distributions, we require only one integration to be carried out numerically, whereas for π^- distributions, we have three integrations left for numerical computations. Fortunately, the values of the coupling constants are well known in this case. We take here $g_{\rho\pi\pi}^2/4\pi = 2.4$ and $g_{\pi NN}^2/4\pi=15$.

III. RESULTS AND DISCUSSION

The results of our calculations are shown in Figs. 2–5 together with the experimental data available so far. We find that our results show rapid energy dependence of the differential cross section and this is supported by the experiments. This amply shows that the production of $\Delta^-(1236)$ does not appreciably occur beyond the energy range we have considered and that at higher energies only ρ^0 production is possible. This trend is sup-

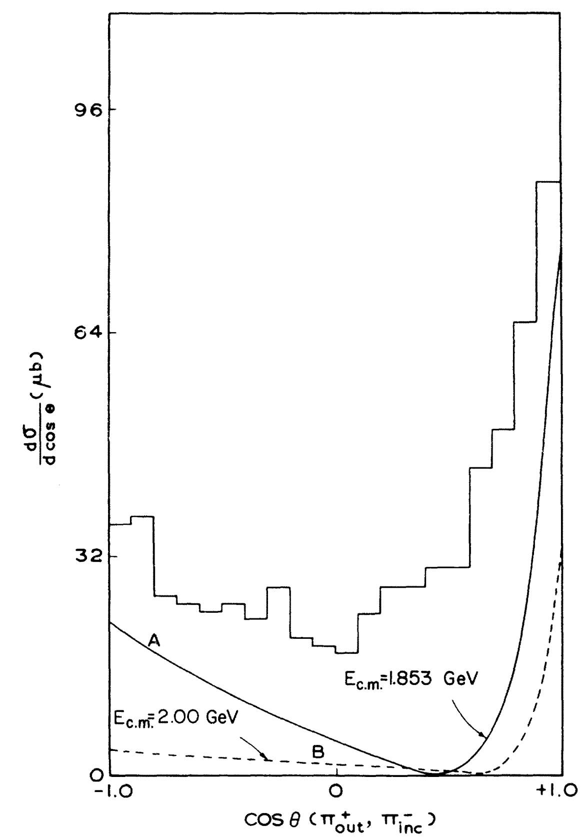

FIG. 2. Angular distributions for π^+ in the reaction (2). The experimental curve is taken from Brody *et al.* (Ref. 11) at $E_{\text{c.m.}}=1.853$ GeV.

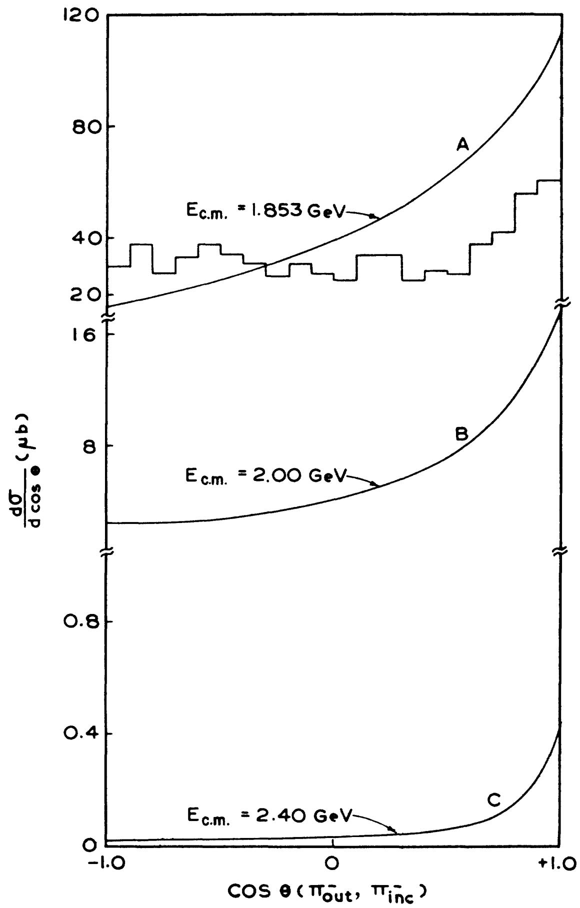

FIG. 3. Angular distribution for outgoing π^- in the reaction (2). The experimental curve is taken from Brody *et al*. (Ref. 11) at $E_{c.m.} = 1.853$ GeV.

ported by the experimental data also.

In Fig. 2 we have shown the result for π^+ production angular distribution as given by our model at center-of-mass total energies 1.853 and 2.0 GeV. The experimental data represent the histogram given by Brody *et al.*[12] at 1.853 GeV. We find that the forward and backward peaks are suitably reproduced by our model except for some background contribution which should always be present at this low energy. Seeing the dominance of the background contribution in the experimental distribution, it can alternatively be suggested that at this low energy, s-channel poles and resonances play a significant role in determining the amplitude. However, we should mention here that we are extracting the imaginary amplitude from the s-channel discontinuity. Thus the s-channel mechanism involved in our model calculation makes it equivalent to a model based on the superimposition of the s-channel resonances, and hence both need not be mutually exclusive.[3,25] We also notice that

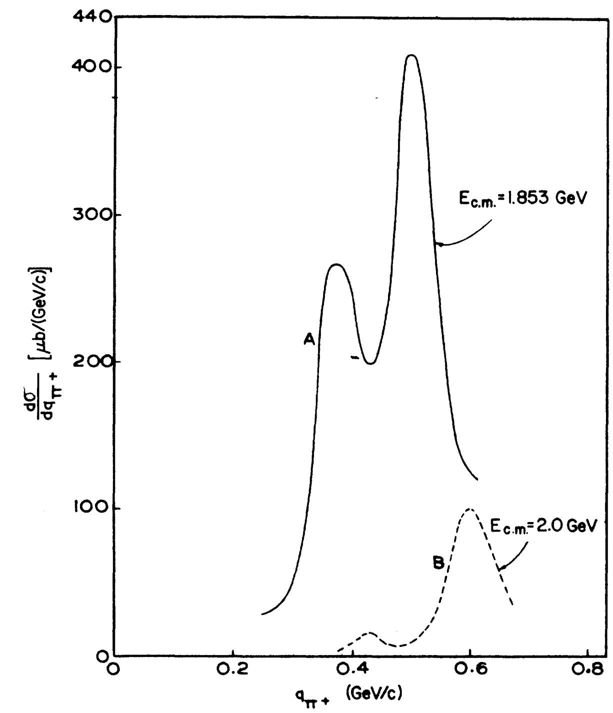

FIG. 4. Momentum distribution for final π^+ in the reaction (2).

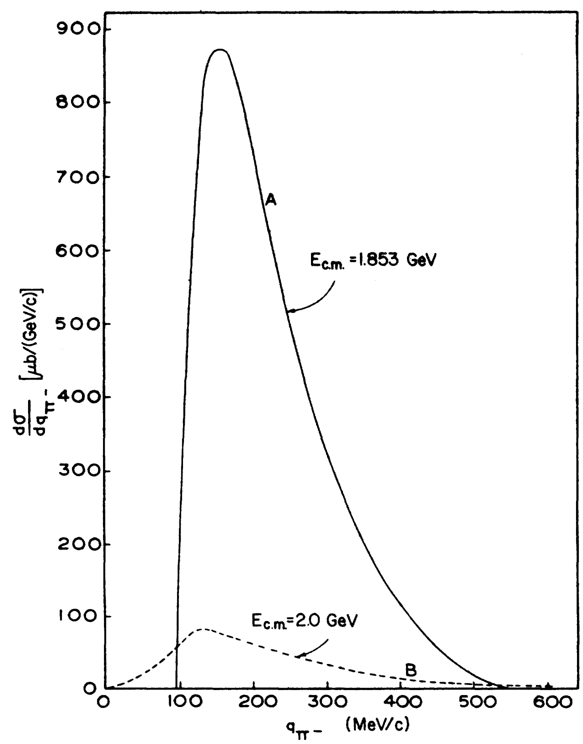

FIG. 5. Momentum distributions for final π^- in the reaction (2).

our result for the process (1) with Δ^- treated[1] as a stable particle remains essentially unchanged when we consider Δ^- as a $\pi^- n$ resonance with finite width. Thus, by considering Δ^- as a two-particle system, we get a consistency check to the hypothesis whether the effect is due to a rescattering mechanism or not.

In Fig. 3, we have demonstrated the result from our calculation for π^- angular distribution together with the experimental data of Brody *et al.* at 1.853 GeV. We have again found that the results are consistent with the experimental data. We have also predicted the results of our model at total center-of-mass energies of 2.0 and 2.4 GeV, in order to reveal the rapid energy dependence of the cross section involved in our model.

In Fig. 4, we have plotted the result of our calculation for π^+ momentum distribution at 1.853 and 2.0 GeV center-of-mass energies. No experimental result exists for π^+ momentum distributions at or above these energies. But the experimental results of Pickup *et al.*[26] at the lower energy of 1.798 GeV support the main features of the curves given by our model. However, we find that this energy is slightly above the threshold for $\rho^0 n$ production and, therefore, our model is not expected to work well at such energies.

In Fig. 5, we have predicted the result of our calculation for π^- momentum distribution at 1.853 and 2.0 GeV total center-of-mass energies. Again, we notice that the experimental results are not available, but the experiment by Pickup *et al.*[26] at $E_{\text{c.m.}} = 1.798$ GeV again supports the theoretical features of our calculation. Therefore, we feel encouraged to point out that the experiments for the process (2) should be carried out in order to test the qualitative and quantitative features predicted by us. This would essentially make clear the reaction mechanism involved in process (2).

We have tried to focus our attention mainly on two aspects of the problem, namely the angular and energy distributions of the final-state pions. It is because of the fact that energy or momentum distributions not only suggest the existence of strong final-state effects, but also the dominant angular-momentum state is often indicated, whereas the angular distributions reflect the production or reaction mechanism. The angular distributions are also to some extent model independent, since the energy-dependent part has been integrated out.

Attempts have been made[9] to explain the process (1) with the help of kinematical reflections where one considers the complete mass spectrum of the $(\pi\pi)$ state. The main differences between these mechanisms are that we have considered the topology of a box graph by including the final-state interaction explicity, and also the $(\pi\pi)$ state has been replaced by a ρ state in our calculation. Therefore, we get more rapid energy dependence of the cross section than what we expect from Berger's reflection mechanism.[9] In this connection, we can mention that the failures of Berger's mechanism become quite obvious for certain recently studied[10,11] processes like $p\bar{p} \to \bar{\Sigma}^+\Sigma^-$, $\pi^- p \to K^+ Y_1^{*-}(1385)$, etc. Thus, the two-meson-exchange mechanism appears to have a sound footing and our results provide crucial tests for the success of such mechanism.

An alternative explanation for the process (2) could be given by considering a sum of two Feynman diagrams: one with π^- exchange which involves $\pi\pi \to \pi\pi$ scattering dominated by the ρ formation, and the other with nucleon exchange which involves an interaction $\pi^- n \to \pi^- n$ dominated by Δ^- formation. These two diagrams when added coherently could explain the features for the process (2). However, the experimental data available so far focus two major problems: (i) the presence of a forward peak in π^+ production angular distributions which we cannot get if we consider nucleon exchange[27] for Δ^- production; (ii) the decay angular distributions[28] $\Delta^- \to n\pi^-$ in the plane perpendicular to its production reveal the structures usually shown by meson exchanges. We thus hope that in these circumstances it is worthwhile to plan a detailed calculation on the two-meson-exchange mechanism and to check its experimental validity.

In conclusion, we feel that the two-meson-exchange mechanism merits further study and more experimental data for the three-particle final state are needed to check the validity of this mechanism. This calculation thus lends further support to our previous finding[3] that we can ignore the u-channel baryon-exchange mechanism in the intermediate-energy region (i.e., below the incident meson momentum of 4 GeV/c) in constructing a scattering amplitude for any process.

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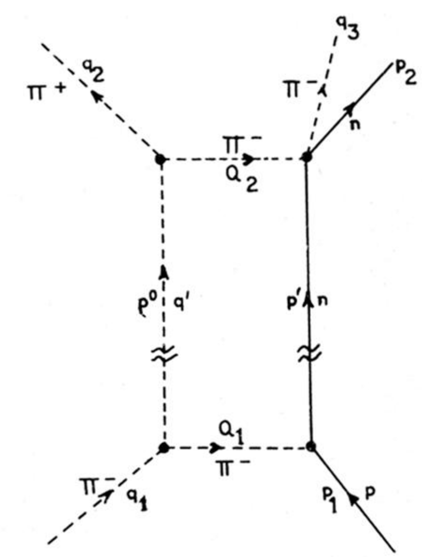

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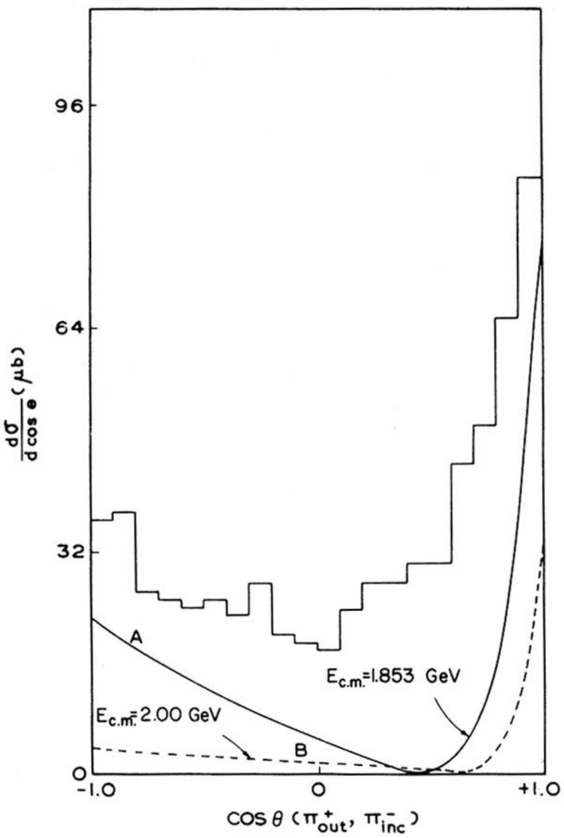

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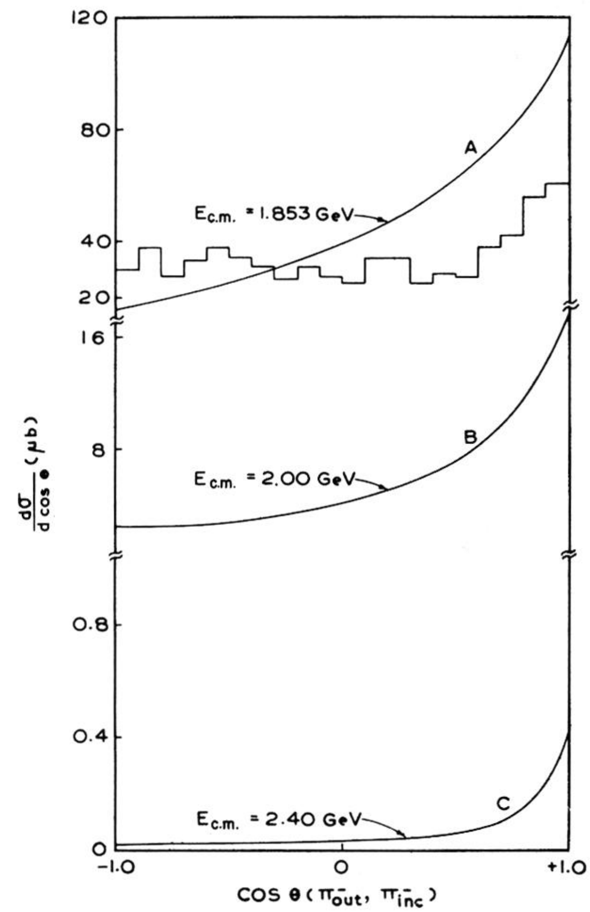

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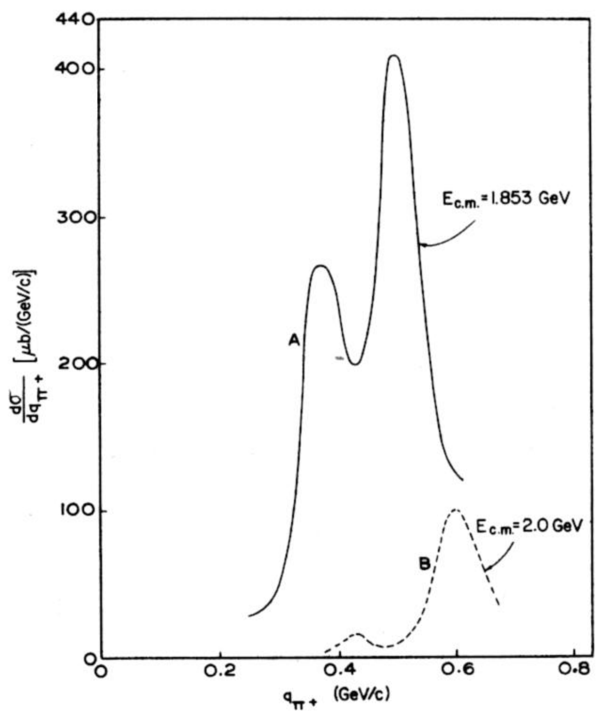

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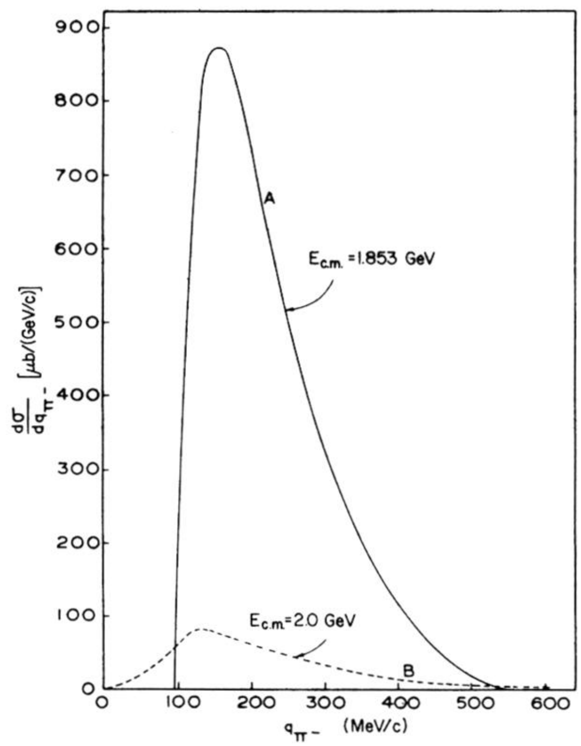

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