Note on low-energy proton Compton scattering

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The unpolarized differential cross section for spin-1/2 Compton scattering is shown to be completely specified up to terms of the sixth power of the photon frequency by the mass, charge, anomalous magnetic moment, and dynamic electromagnetic polarizabilities of the target and four other sturcture dependent constants. The two polarizabilities and the four new constants are expressed in terms of the threshold values and threshold derivatives of the invariant amplitudes of the process. Their parts which can be expected to satisfy fixed-$t = 0$ unsubtracted dispersion relations are evaluated in terms of single-pion photoproduction data. The extraction of the proton polarizabilities from low-energy elastic γ-proton scattering is discussed in the light of the results obtained.

I. INTRODUCTION

Recently there has been a renewed interest in both theoretical[1-3] and experimental[4] study of the so-called dynamic electric and magnetic polarizabilities[5] of the nucleons. These two structure-dependent constants α and β (describing the scattering produced by time-varying induced dipoles driven by the electric and magnetic fields of the photon), together with the mass m, charge e, and anomalous magnetic moment λ of the target, determine rigorously to lowest order in electromagnetism the low-energy behavior of the unpolarized differential cross section up to terms of the fourth power in the photon frequency ω according to the following formula[6]:

$$\frac{d\sigma}{d\Omega}=\left(\frac{d\sigma}{d\Omega}\right)_P-\left(\frac{e^2}{4\pi m}\right)\omega^2[\alpha(1+z^2)+2\beta z]\,[1-3(\omega/m)(1-z)]+O(\omega^4)\,, \tag{1}$$

where ω is the incident photon energy in the laboratory system, $z=\cos\theta_{\text{lab}}$, θ_{lab} is the scattering angle, and the first term in Eq. (1) represents the Powell cross section[7] for the scattering of γ quanta on a structureless spin $\frac{1}{2}$ particle of anomalous magnetic moment λ:

$$\left(\frac{d\sigma}{d\Omega}\right)_P=\frac{1}{2}\left(\frac{e^2}{4\pi m}\right)^2\left\{\frac{(1+z^2)}{[1+(\omega/m)(1-z)]^2}+\left(\frac{\omega}{m}\right)^2\frac{[(1-z)^2+f(z)]}{[1+(\omega/m)(1-z)]^3}\right\}\,, \tag{2}$$

$$\begin{aligned} f(z)&=a_0+a_1z+a_2z^2\,,\\ a_0&=2\lambda+\tfrac{9}{2}\lambda^2+3\lambda^3+\tfrac{3}{4}\lambda^4\,,\\ a_1&=-4\lambda-5\lambda^2-2\lambda^3\,,\\ a_2&=2\lambda+\tfrac{1}{2}\lambda^2-\lambda^3-\tfrac{1}{4}\lambda^4\,. \end{aligned} \tag{3}$$

The first experimental measurement of the parameters α and β for the proton was done by Goldansky *et al.*[8] from an analysis of the elastic γ-p scattering (at photon lab energy $\omega = 55$ MeV) with a formula for $d\sigma/d\Omega$ which contains powers of ω only up to ω^2, inclusive [e.g., disregarding the last term in the second set of square brackets in Eq. (1)]. In their recent experimental analysis Baranov *et al.*[4] used Eq. (1) to find α and β from better data in the ω region from 80 to 110 MeV and lab scattering angles of $\theta=90°$ and $150°$, assuming, of course, that the contribution of the neglected terms of $O(\omega^4)$ in Eq. (1) is very small in the energy region considered. However, it is known that the π^0-meson pole contribution to the differential cross section [which appears in order $O(\omega^4)$ and higher] is quite significant at low energies owing to the smallness of the denominator μ^2-t (μ is pion mass, t is the four-momentum transfer squared). Therefore, in a subsequent paper[9] Baranov, Filkov, and Shtarkov performed a new extraction of α and β based on the following formula for $d\sigma/d\Omega$ which explicitly includes the π^0-meson pole contribution to the cross section:

$$\frac{d\sigma}{d\Omega}=\left(\frac{d\sigma}{d\Omega}\right)_P-\left(\frac{e^2}{4\pi m}\right)\omega^2[\alpha(1+z^2)+2\beta z]\left[1-3\,\frac{\omega}{m}\,(1-z)\right]+\frac{2}{m^2}\left(\frac{\omega}{\mu}\right)^2\frac{(1-z)}{[1+(\omega/m)(1-z)]^3}\,B_\pi(B_\pi+E)+O(\omega^4)\,, \tag{4}$$

where

$$B_\pi = \frac{\mu}{16\pi} g_{\pi NN} \frac{F_\pi t}{\mu^2 - t} \ ,$$

$$E = \left(\frac{e^2}{4\pi}\right) \frac{\mu}{2m} [(1-z) + \lambda^2 + \lambda(3-z)] \ , \qquad (5)$$

$$t = -2\omega^2(1-z)/[1+(\omega/m)(1-z)] \ ,$$

$g_{\pi NN}$ is the πNN coupling constant, and F_π is the $\pi^0 \to 2\gamma$ coupling.

The results of Ref. 9, obtained with the assumption that the remaining terms of $O(\omega^4)$ are small with respect to those already taken into account, show that the combination $\alpha - \beta$ is particularly sensitive to the contribution of the π^0 meson pole. The numbers found for α and β are consequently substantially changed with respect to those given in Ref. 4 and the final conclusion which emerges is that in order to settle the question new measurements of the energy dependence of the γ-p scattering (for large angles) in the region 20–100 MeV, with better energy resolution, are needed.[10]

For the time being, in this context it becomes useful for both practical and theoretical reasons to have at hand a low-energy expansion of $d\sigma/d\Omega$ coming from general principles and which goes to higher orders in the frequency ω. The purpose of this note is to provide a low-energy theorem for $d\sigma/d\Omega$ valid up to the sixth order in ω and to discuss its possible usefulness in the extraction of the electric and magnetic generalized polarizabilities from low-energy Compton-scattering data. As will be shown below (Sec. II), the term of order ω^4 in $d\sigma/d\Omega$ can be written down at the expense of introducing four new structure-dependent constants which in turn will also fix the ω^5 contribution. The identification of the new constants entering the $O(\omega^6)$ low-energy theorems obtained in this paper is given in terms of the threshold values and threshold derivatives of the invariant amplitudes of Bardeen and Tung.[11]

In Sec. III the four new structure-dependent constants which fix the ω^4 and ω^5 contributions to the unpolarized differential cross section are theoretically evaluated by means of a dispersion treatment in terms of single-pion photoproduction data and a certain model for the exchanges in the annihilation channel. The calculation should be quite reliable owing to the increased convergence of the dispersion representations, the needed quantities being mainly expressed in terms of derivatives of the scattering amplitudes at threshold. The resulting formula of the differential cross section is compared with the accurate data from Ref. 4 and the difference $\alpha - \beta$ is extracted keeping $\alpha + \beta$ fixed in terms of the total cross section for photoabsorption on protons. The analysis shows that the inclusion of the ω^4 and ω^5 terms is compulsory at energies as high as 80–110 MeV and large angles, e.g., in the actual domain of the experimental measurements presented in Ref. 4.

II. DERIVATION OF THE RESULTS

The easiest way to obtain low-energy theorems[12] for observable quantities in Compton scattering on spin-$\frac{1}{2}$ targets is to express the object of interest in terms of six independent invariant amplitudes known to be free of any kinematical singularities and zeros and to expand the resulting expression in the photon energy at fixed scattering angle. We choose to work with the amplitudes A_i of Ref. 11 which do have the above-mentioned property. In terms of these amplitudes the unpolarized differential cross section for spin-$\frac{1}{2}$ Compton scattering in the laboratory frame has the following form:

$$\frac{d\sigma}{d\Omega} = \left(\frac{\omega'}{\omega}\right)^2 \frac{1}{32} \frac{1}{64\pi^2 m^2} \{ t[(-2t^2+8m^2 t)|A_1|^2 - 2m^2 t^2 |A_2|^2 - (64\nu^2 + 8m^2 t)|A_3|^2 + (16\nu)(mt\,\mathrm{Re}A_2 A_3^* - t\,\mathrm{Re}A_1 A_3^*)]$$
$$+ (4\nu^2 - \tfrac{1}{4}t^2 + m^2 t)[8(4\nu^2 + \tfrac{1}{4}t^2 + m^2 t)|A_4|^2 + (4\nu^2 - \tfrac{1}{4}t^2 + m^2 t)(2m^2 - \tfrac{1}{2}t)|A_5|^2$$
$$+ (8\nu^2 + \tfrac{1}{2}t^2 - 2m^2 t)|A_6|^2 + 8m(4\nu^2 - \tfrac{1}{4}t^2 + m^2 t)\mathrm{Re}A_4 A_5^* + 16\nu t\,\mathrm{Re}A_4 A_6^*]\} \ , \qquad (6)$$

where s, t, u are the usual Mandelstam variables, $\nu = \frac{1}{4}(s-u)$, and ω' is the energy of the emergent photon.

Separating the single nucleon s- and u-channel Born poles of the amplitudes A_i according to

$$A_i(s,t,u) = A_i^B(s,t,u) + A_i^C(s,t,u) \ , \qquad (7)$$

using the following developments for the continuum parts of the amplitudes A_i^C (considered as functions of ν, t), which take into account their s-u crossing symmetry

$$A_i^C(\nu,t) = \sum_{l,m=0}^{\infty} C_i^{l,m} \nu^{2l} t^m, \quad i = 1,2,4,5 \ , \qquad (8a)$$

$$A_i^C(\nu,t)=\nu\sum_{l,m=0}^{\infty}C_i^{l,m}\nu^{2l}t^m,\quad i=3,6\ , \tag{8b}$$

and noting the kinematical relations Eq. (5) and

$$\nu=\frac{m\omega}{2}\,\frac{[2+(\omega/m)(1-z)]}{[1+(\omega/m)(1-z)]}\ ,$$

$$\omega'=\frac{\omega}{[1+(\omega/m)(1-z)]}\ ,$$

one gets in a straightforward manner the desired low-energy theorem for $d\sigma/d\Omega$ holding up to terms of order ω^6:

$$\frac{d\sigma}{d\Omega}=\left(\frac{d\sigma}{d\Omega}\right)_P+\left(\frac{d\sigma}{d\Omega}\right)_{(\alpha,\beta)}-\frac{1}{4}\,\frac{e^2}{64\pi^2m^2}\left[1-4\left(\frac{\omega}{m}\right)(1-z)\right]\left(\frac{\omega}{m}\right)^4[A+Bz+Cz^2+Dz^3]+O(\omega^6)\ , \tag{9}$$

where

$$\begin{aligned}\left(\frac{d\sigma}{d\Omega}\right)_{(\alpha,\beta)}=&-\frac{m}{2}\,\frac{e^2}{4\pi}\left[1-3\left(\frac{\omega}{m}\right)(1-z)+6\left(\frac{\omega}{m}\right)^2(1-z)^2-10\left(\frac{\omega}{m}\right)^3(1-z)^3\right]\left(\frac{\omega}{m}\right)^2[(1-z)^2(\alpha-\beta)+(1+z)^2(\alpha+\beta)]\\&-\frac{m}{4}\,\frac{e^2}{4\pi}\left[1-4\left(\frac{\omega}{m}\right)(1-z)\right]\left(\frac{\omega}{m}\right)^4\left\{\frac{(1-z)^2}{2}[(2-2\lambda-\lambda^2)-z(2+2\lambda+\lambda^2)](\alpha-\beta)\right.\\&\qquad-\frac{4\pi m^3}{e^2}(1-z)^2(\alpha-\beta)^2-\frac{(1+z)^2(1-z)}{2}(\lambda^2+2\lambda)(\alpha+\beta)\\&\qquad\left.-\frac{4\pi m^3}{e^2}(1+z)^2(\alpha+\beta)^2\right\}\end{aligned} \tag{10}$$

and

$$\begin{aligned}A=&(4m^4)\{(2m^3)[C_1^{1,0}+m(C_4^{1,0}+\tfrac{1}{2}mC_5^{1,0})]-(4m)[C_1^{0,1}+m(C_4^{0,1}+\tfrac{1}{2}mC_5^{0,1})]\\&+(1+3\lambda+\lambda^2)C_2^{0,0}+(8\lambda+3\lambda^2)mC_3^{0,0}+(2+3\lambda+2\lambda^2)C_4^{0,0}-\tfrac{1}{2}(3\lambda^2+4\lambda+4)m^2C_6^{0,0}\}\ ,\\B=&(4m^4)\{-2(2m^3)[C_1^{1,0}-m(C_4^{1,0}+\tfrac{1}{2}mC_5^{1,0})]+(4m)[3C_1^{0,1}-m(C_4^{0,1}+\tfrac{1}{2}mC_5^{0,1})]\\&-(3+7\lambda+2\lambda^2)C_2^{0,0}-(8\lambda+2\lambda^2)mC_3^{0,0}-(2+\lambda)C_4^{0,0}-\lambda^2m^2C_6^{0,0}\}\ ,\\C=&(4m^4)\{(2m^3)[C_1^{1,0}+m(C_4^{1,0}+\tfrac{1}{2}mC_5^{1,0})]-(4m)[3C_1^{0,1}-m(C_4^{0,1}+\tfrac{1}{2}mC_5^{0,1})]\\&+(3+5\lambda+\lambda^2)C_2^{0,0}-\lambda^2mC_3^{0,0}-(2+3\lambda+2\lambda^2)C_4^{0,0}+\tfrac{1}{2}(\lambda^2+4\lambda+4)m^2C_6^{0,0}\}\ ,\\D=&(4m^4)\{(4m)[C_1^{0,1}+m(C_4^{0,1}+\tfrac{1}{2}mC_5^{0,1})]-(1+\lambda)C_2^{0,0}+(2+\lambda)C_4^{0,0}\}\ .\end{aligned} \tag{11}$$

The dynamic (generalized) electric and magnetic polarizabilities of the target, α and β, are identified as

$$\alpha=\frac{1}{8\pi}\,[C_1^{0,0}+m(C_4^{0,0}+\tfrac{1}{2}mC_5^{0,0})]\ , \tag{12}$$

$$\beta=\frac{1}{8\pi}\,[-C_1^{0,0}+m(C_4^{0,0}+\tfrac{1}{2}mC_5^{0,0})]\ . \tag{13}$$

Equation (9) up to terms of order ω^4 confirms the result of Ref. 6, and in addition gives the next ω^4 and ω^5 terms of the expansion by means of the four constants A, B, C, and D. As seen from Eq. (9), no new structure-dependent constants appear in the terms of odd power in ω (e.g., of order ω^3 or ω^5) with respect to those already fixing the corresponding previous even order (e.g., ω^2 or ω^4). This fact, which is a simple consequence of the s-u crossing symmetry [as can be easily shown using Eqs. (6) and (8)], is valid in general: the unpolarized differential cross section is determined to the order ω^{2n+1} inclusively in terms of the same constants which have already established it in the previous even order ω^{2n}.

Now we will pay particular attention to the contribution of the π^0-meson pole to the differential cross section, because it could be essential for the practical extraction of the polarizabilities α and β from low-energy data. The radius of convergence of the development of the π^0 pole term in powers of ω (at fixed angle) being quite small [as can be easily seen looking at the location of this pole in the context of the analyticity structure (in energy at fixed angle) of the relevant amplitude], it is preferable to keep the contribution of the π^0 pole to the differential cross section (in the low-energy region which is of interest) in its original form, that is, undeveloped in powers of ω. The π^0-meson pole appears only in the invariant amplitude A_2 and we will now separate it explicitly, modifying Eqs. (7) and (8a) for A_2 as follows:

$$A_2=A_2^B+\frac{2}{m}\,\frac{F_\pi g_{\pi NN}}{\mu^2-t}+\bar{A}_2^C\ , \tag{14}$$

$$\bar{A}_2^C(\nu,t)=\sum_{l,m=0}^{\infty}\bar{C}_2^{l,m}\nu^{2l}t^m\ . \tag{15}$$

Then, with the explicit inclusion of the π^0-meson pole contribution, the low-energy expansion Eq. (9) is replaced by

$$\begin{aligned}\frac{d\sigma}{d\Omega}&=\left(\frac{d\sigma}{d\Omega}\right)_P+\left(\frac{d\sigma}{d\Omega}\right)_{(\alpha,\beta)}\\&+\frac{2}{m^2}\left(\frac{\omega}{\mu}\right)^2\frac{(1-z)}{[1+(\omega/m)(1-z)]^3}B_\pi(B_\pi+E)\\&-\frac{1}{4}\frac{e^2}{64\pi^2m^2}\left[1-4\left(\frac{\omega}{m}\right)(1-z)\right]\left(\frac{\omega}{m}\right)^4\\&\times[\bar{A}+\bar{B}z+\bar{C}z^2+\bar{D}z^3]+O(\omega^6)\ ,\end{aligned} \tag{9'}$$

where the first two terms of the above formula are those given by Eqs. (2) and (10), and the third term is the one which appeared in Eq. (4), and the quantities $\bar{A}$, $\bar{B}$, $\bar{C}$, and $\bar{D}$ are obtained from the corresponding A, B, C, and D of Eqs. (11) through the replacements

$$\begin{aligned}&C_2^{0,0}\to\bar{C}_2^{0,0}=C_2^{0,0}-\frac{2}{m}\frac{F_\pi g_{\pi NN}}{\mu^2}\ ,\\&C_2^{1,0}\to\bar{C}_2^{1,0}=C_2^{1,0}\ ,\\&C_2^{0,1}\to\bar{C}_2^{0,1}=C_2^{0,1}-\frac{2}{m}\frac{F_\pi g_{\pi NN}}{\mu^4}\ .\end{aligned} \tag{16}$$

III. NUMERICAL ANALYSIS AND DISCUSSION OF THE RESULTS

In this note we presented a framework for a detailed investigation of the low-energy elastic γ scattering on hadronic spin-$\frac{1}{2}$ targets which takes into account correctly the higher-order low-energy theorems valid for this process. The dynamic electric and magnetic polarizabilities α and β as well as the other structure-dependent constants A, B, C, D or $\bar{A}$, $\bar{B}$, $\bar{C}$, and $\bar{D}$, on one side, can be extracted from a careful experimental low-energy analysis (new measurements at energies as low as 20–30 MeV, in spite of their difficulty, are badly needed) and, on the other side, can be theoretically computed using, for instance, dispersion relations. The Compton-scattering process on spin-$\frac{1}{2}$ targets is determined up to terms of $O(\omega^6)$ by the six threshold values of the continuum parts of the invariant amplitudes A_i,

$$C_i^{0,0}=A_i^C(\nu=0,\ t=0),\quad i=1,2,4,5$$

$$C_i^{0,0}=\frac{A_i^C}{\nu}(\nu=0,\ t=0),\quad i=3,6$$

the six threshold values of the derivatives of A_i^C with respect to ν^2,

$$C_i^{1,0}=\left.\frac{\partial A_i^C}{\partial\nu^2}\right|_{\nu=0,t=0},\quad i=1,2,4,5$$

$$C_i^{1,0}=\left.\frac{\partial}{\partial\nu^2}\left(\frac{A_i^C}{\nu}\right)\right|_{\nu=0,\ t=0},\quad i=3,6$$

and the six threshold values of the derivatives of A_i^C with respect to t,

$$C_i^{0,1}=\left.\frac{\partial A_i^C}{\partial t}\right|_{\nu=0,\ t=0},\quad i=1,2,4,5$$

$$C_i^{0,1}=\frac{\partial}{\partial t}\left(\frac{A_i^C}{\nu}\right)_{\nu=0,\ t=0},\quad i=3,6\ .$$

Among the constants $C_i^{l,m}$ ($l,m=0,1$) which specify the spin-$\frac{1}{2}$ Compton effect up to terms of the sixth power of ω, many of them ($C_{3,4,6}^{0,0}$; $C_{1,2,3,4,5,6}^{1,0}$; $C_{3,4,5,6}^{0,1}$) may be quite well evaluated in terms of meson photoproduction data by means of fixed $t=0$ dispersion relations, while for the other ($C_{1,2}^{0,0}$; $C_{1,2}^{0,1}$), in view of the probable bad high-energy asymptotic behavior of the corresponding invariant amplitudes, the consideration of the exchanges in the annihilation channel (by means of an adequate dispersion treatment) may be necessary.

In the following we shall evaluate numerically the quantities $\bar{A}$, $\bar{B}$, $\bar{C}$, and $\bar{D}$ [according to Eqs. (11) and (16)] and shall compare the resulting values of the unpolarized differential cross section as given by Eq. (9′) with the experimental determinations of Ref. 4. The combination $\alpha+\beta$ will be taken as known from the analysis of Damashek and Gilman[13] in terms of the total photoabsorption cross section $\sigma_{\gamma p}^T$:

$$\begin{aligned}\alpha+\beta&=\frac{1}{2\pi^2}\int_{\omega_{\text{threshold}}}\frac{d\omega}{\omega^2}\sigma_{\gamma p}^T(\omega)\\&=(14.1\pm0.3)\times10^{-4}\,\text{fm}^3\ .\end{aligned} \tag{17}$$

The difference $\alpha-\beta$ will then be obtained by least-squares fits to the data of Ref. 4. The relative importance of the different contributions entering Eq. (9′) will then be discussed.

To compute the coefficients $C_i^{0,0}$, $C_i^{1,0}$, and $C_i^{0,1}$ which determine $\bar{A}$, $\bar{B}$, $\bar{C}$, and $\bar{D}$ we start from the following dispersion representation for the six invariant amplitudes:

$$\begin{aligned}A_i(s,t)&=A_i^B(s,t)+\frac{1}{\pi}\int_{(m+\mu)^2}^{\infty}ds'\left(\frac{1}{s'-s}-\frac{1}{s'-u}\right)A_i^{(s)}(s',t)\quad(i=3,6)\ ,\\A_i(s,t)&=A_i^B(s,t)+\frac{1}{\pi}\int_{(m+\mu)^2}^{\infty}ds'\left(\frac{1}{s'-s}+\frac{1}{s'-u}\right)A_i^{(s)}(s',t)\quad(i=4,5)\ ,\end{aligned} \tag{18}$$

$$A_2(s,t)+\frac{1}{m}A_3(s,t)=A_2^B(s,t)+\frac{1}{m}A_3^B(s,t)+\frac{2}{m}\frac{F_\pi g_{\pi NN}}{\mu^2-t}$$
$$+\left[\frac{1}{\pi}\int_{4\mu^2}^{\infty}dt'\,\frac{A_2^{(t)}(t',u=m^2)+1/mA_3^{(t)}(t',u=m^2)}{t'-t}\right.$$
$$\left.+\frac{1}{\pi}\int_{(m+\mu)^2}^{\infty}ds'\,\frac{A_2^{(s)}(s',u=m^2)+1/mA_3(s',u=m^2)}{s'-m^2+t}\right]$$
$$+\frac{(u-m^2)}{\pi}\left[\int_{(m+\mu)^2}^{\infty}ds'\,\frac{A_2^{(s)}(s',t)-1/mA_3^{(s)}(s',t)}{(s'-m^2)(s'-u)}\right.$$
$$\left.-\int_{(m+\mu)^2}^{\infty}ds'\,\frac{A_2^{(s)}(s',t)+1/mA_3^{(s)}(s',t)}{(s'-m^2+t)(s'-s)}\right]\ , \qquad (19)$$

$$A_1(s,t)-A_3(s,t)=A_1^B(s,t)-A_3^B(s,t)+(4\pi)(\alpha-\beta)$$
$$+t\left[\frac{1}{\pi}\int_{4\mu^2}^{\infty}dt'\,\frac{A_1^{(t)}(t',u=m^2)-A_3^{(t)}(t',u=m^2)}{t'(t'-t)}\right.$$
$$\left.-\frac{1}{\pi}\int_{(m+\mu)^2}^{\infty}ds'\,\frac{A_1^{(s)}(s',u=m^2)-A_3^{(s)}(s',u=m^2)}{(s'-m^2)(s'-m^2+t)}\right]$$
$$+\frac{(u-m^2)}{\pi}\left[\int_{(m+\mu)^2}^{\infty}ds'\,\frac{A_1^{(s)}(s',t)+A_3^{(s)}(s',t)}{(s'-m^2)(s'-u)}\right.$$
$$\left.-\int_{(m+\mu)^2}^{\infty}ds'\,\frac{A_1^{(s)}(s',t)-A_3^{(s)}(s',t)}{(s'-m^2+t)(s'-s)}\right]\ . \qquad (20)$$

In other words, we use fixed-t unsubtracted dispersion relations for $A_{3,4,5,6}$ [Eqs. (18)], while for the combinations $[A_2+(1/m)A_3]$ and (A_1-A_3) we write fixed-t dispersion relations subtracted at $u=m^2$. The subtraction functions have been expressed respectively through an unsubtracted dispersion relation in the t variable [the π^0-pole term plus the term in the first set of square brackets in Eq. (19)] and a dispersion relation in the t variable subtracted at $t=0$ [the term with $(\alpha-\beta)$ and the term in the first set of square brackets in Eq. (20)].

The sign of F_π in Eq. (19) is chosen in agreement with Lapidus and Kuang-Chao,[14] i.e., $g_{\pi NN}F_\pi<0$. We have taken

$$g_{\pi NN}{}^2/4\pi=14.5\ ,$$

$$\tau_{\pi^0\to 2\gamma}=64\pi/\mu^3F_\pi{}^2=0.85\times10^{-16}\ \text{sec}\ .$$

The superscripts s and t in Eqs. (18), (19), and (20) denote the corresponding absorptive parts of the amplitudes in the direct and annihilation channels. Working in the two-particle unitarity approximation, we keep in $A_i^{(s)}$ only πN intermediate states and use the pion photoproduction multipoles of Ref. 15 for the low-energy region (ω from 180 MeV up to 250 MeV) and of Ref. 16 for ω from 250 MeV up to 1210 MeV. Only waves with $J=\frac{1}{2}$ and $\frac{3}{2}$ are retained. The s-channel absorptive parts appearing in the subtraction functions from Eqs. (19) and (20) at $u=m^2$ have been calculated in terms of photoproduction multipoles without any kind of extrapolations, so disregarding complications due to the unphysical region of certain kinematical variables. This approximation is not expected to introduce large errors. In terms of tabulated quantities (see also for definitions Refs. 3 and 16) the needed coefficients are

$$C_2^{0,0}=\frac{2F_\pi g_{\pi NN}}{m\mu^2}-\frac{16\times10^{-4}}{m}\int_{(m+\mu)^2}^{\infty}ds'\,\frac{s'}{(s'-m^2)^4}\Bigg\{2\sqrt{s'}\Big[(2|A_{S\,1/2}^{V3}|^2+3|A_{S\,1/2}^{p}|^2-2|A_{P\,1/2}^{V3}|^2-3|A_{P\,1/2}^{p}|^2)$$
$$-2\frac{(s'+2m^2)}{(s'-m^2)}(2|A_{P\,3/2}^{V3}|^2+3|A_{P\,3/2}^{p}|^2-2|A_{D\,3/2}^{V3}|^2-3|A_{D\,3/2}^{p}|^2)\Big]$$
$$+\frac{3s'\sqrt{s'}}{(s'-m^2)}(2|B_{P\,3/2}^{V3}|^2+3|B_{P\,3/2}^{p}|^2-2|B_{D\,3/2}^{V3}|^2-3|B_{D\,3/2}^{p}|^2)\Bigg\}\ , \qquad (21a)$$

$$C_1^{1,0}=64\times10^{-4}\int_{(m+\mu)^2}^{\infty}ds'\,\frac{s'}{(s'-m^2)^6}\Big\{2\sqrt{s'}\big[(2|A^{V3}_{S\,1/2}|^2+3|A^{p}_{S\,1/2}|^2-2|A^{V3}_{P\,1/2}|^2-3|A^{p}_{P\,1/2}|^2)$$
$$+4(2|A^{V3}_{P\,3/2}|^2+3|A^{p}_{P\,3/2}|^2-2|A^{V3}_{D\,3/2}|^2-3|A^{p}_{D\,3/2}|^2)\big]$$
$$+12(s'/m)\mathrm{Re}(2B^{V3*}_{P\,3/2}A^{V3}_{P\,3/2}+3B^{p*}_{P\,3/2}A^{p}_{P\,3/2}-2B^{V3*}_{D\,3/2}A^{V3}_{D\,3/2}-3B^{p*}_{D\,3/2}A^{p}_{D\,3/2})\Big\}\,, \tag{21b}$$

$$C_4^{1,0}+\tfrac{1}{2}mC_5^{1,0}=64\times10^{-4}\int_{(m+\mu)^2}^{\infty}ds'\,\frac{s'}{(s'-m^2)^6}\Big\{2\big[(2|A^{V3}_{S\,1/2}|^2+3|A^{p}_{S\,1/2}|^2+2|A^{V3}_{P\,1/2}|^2+3|A^{p}_{P\,1/2}|^2)$$
$$+2(2|A^{V3}_{P\,3/2}|^2+3|A^{p}_{P\,3/2}|^2+2|A^{V3}_{D\,3/2}|^2+3|A^{p}_{D\,3/2}|^2)\big]$$
$$+3(2|B^{V3}_{P\,3/2}|^2+3|B^{p}_{P\,3/2}|^2+2|B^{V3}_{D\,3/2}|^2+3|B^{p}_{D\,3/2}|^2)\Big\}\,, \tag{21c}$$

$$C_4^{0,0}=16\times10^{-4}\int_{(m+\mu)^2}^{\infty}ds'\,\frac{s'}{(s'-m^2)^4}\Big\{2\big[(2|A^{V3}_{S\,1/2}|^2+3|A^{p}_{S\,1/2}|^2+2|A^{V3}_{P\,1/2}|^2+3|A^{p}_{P\,1/2}|^2)$$
$$+2(2|A^{V3}_{P\,3/2}|^2+3|A^{p}_{P\,3/2}|^2+2|A^{V3}_{D\,3/2}|^2+3|A^{p}_{D\,3/2}|^2)\big]$$
$$+\frac{24m\sqrt{s'}}{(s'-m^2)}\mathrm{Re}(2B^{V3*}_{P\,3/2}A^{V3}_{P\,3/2}+3B^{p*}_{P\,3/2}A^{p}_{P\,3/2}+2B^{V3*}_{D\,3/2}A^{V3}_{D\,3/2}+3B^{p*}_{D\,3/2}A^{p}_{D\,3/2})$$
$$+\frac{3(s'+3m^2)}{(s'-m^2)}(2|B^{V3}_{P\,3/2}|^2+3|B^{p}_{P\,3/2}|^2+2|B^{V3}_{D\,3/2}|^2+3|B^{p}_{D\,3/2}|^2)\Big\}\,, \tag{21d}$$

$$C_3^{0,0}=32\times10^{-4}\int_{(m+\mu)^2}^{\infty}ds'\,\frac{s'}{(s'-m^2)^5}\Big\{-2\sqrt{s'}\big[(2|A^{V3}_{S\,1/2}|^2+3|A^{p}_{S\,1/2}|^2-2|A^{V3}_{P\,1/2}|^2-3|A^{p}_{P\,1/2}|^2)$$
$$+4(2|A^{V3}_{P\,3/2}|^2+3|A^{p}_{P\,3/2}|^2-2|A^{V3}_{D\,3/2}|^2-3|A^{p}_{D\,3/2}|^2)\Big\}\,, \tag{21e}$$

$$C_6^{0,0}=64\times10^{-4}\int_{(m+\mu)^2}^{\infty}\frac{ds'}{(s'-m^2)^5}\Big\{-2\big[(2|A^{V3}_{S\,1/2}|^2+3|A^{p}_{S\,1/2}|^2+2|A^{V3}_{P\,1/2}|^2+3|A^{p}_{P\,1/2}|^2)$$
$$+2(2|A^{V3}_{P\,3/2}|^2+3|A^{p}_{P\,3/2}|^2+2|A^{V3}_{D\,3/2}|^2+3|A^{p}_{D\,3/2}|^2)\big]$$
$$+3(2|B^{V3}_{P\,3/2}|^2+3|B^{p}_{P\,3/2}|^2+2|B^{V3}_{D\,3/2}|^2+3|B^{p}_{D\,3/2}|^2)\Big\}\,, \tag{21f}$$

$$C_4^{0,1}+\tfrac{1}{2}mC_5^{0,1}=8\times10^{-4}\int_{(m+\mu)^2}^{\infty}ds'\,\frac{s'}{(s'-m^2)^6}\big[-2(s'-m^2)(2|A^{V3}_{S\,1/2}|^2+3|A^{p}_{S\,1/2}|^2+2|A^{V3}_{P\,1/2}|^2+3|A^{p}_{P\,1/2}|^2)$$
$$+(20s'+4m^2)(2|A^{V3}_{P\,3/2}|^2+3|A^{V3}_{P\,3/2}|^2+2|A^{V3}_{D\,3/2}|^2+3|A^{p}_{D\,3/2}|^2)$$
$$+24m\sqrt{s'}\,\mathrm{Re}(2B^{V3*}_{P\,3/2}A^{V3}_{P\,3/2}+3B^{p*}_{P\,3/2}A^{p}_{P\,3/2}$$
$$+2B^{V3*}_{D\,3/2}A^{V3}_{D\,3/2}+3B^{p*}_{D\,3/2}A^{p}_{D\,3/2})$$
$$+(9m^2-3s')(2|B^{V3}_{P\,3/2}|^2+3|B^{p}_{P\,3/2}|^2+2|B^{V3}_{D\,3/2}|^2+3|B^{p}_{D\,3/2}|^2)\big]\,, \tag{21g}$$

$$C_1^{0,1}=C_1^{0,1}(t\text{ channel})-\tfrac{1}{4}C_3^{0,0}$$
$$-8\times10^{-4}\int_{(m+\mu)^2}^{\infty}ds'\,\frac{s'}{(s'-m^2)^5}\Big\{4\sqrt{s'}\Big[(2|A^{V3}_{S\,1/2}|^2+3|A^{p}_{S\,1/2}|^2-2|A^{V3}_{P\,1/2}|^2-3|A^{p}_{P\,1/2}|^2)$$
$$-2\,\frac{(s'+2m^2)}{(s'-m^2)}(2|A^{V3}_{P\,3/2}|^2+3|A^{p}_{P\,3/2}|^2-2|A^{V3}_{D\,3/2}|^2-3|A^{p}_{D\,3/2}|^2)\Big]$$
$$-6\,\frac{s'\sqrt{s'}}{(s'-m^2)}(2|B^{V3}_{P\,3/2}|^2+3|B^{p}_{P\,3/2}|^2-2|B^{V3}_{D\,3/2}|^2-3|B^{p}_{D\,3/2}|^2)$$
$$-\frac{24ms'}{(s'-m^2)}\mathrm{Re}(2B^{V3*}_{P\,3/2}A^{V3}_{P\,3/2}+3B^{p*}_{P\,3/2}A^{p}_{P\,3/2}-2B^{V3*}_{D\,3/2}A^{V3}_{D\,3/2}-3B^{p*}_{D\,3/2}A^{p}_{D\,3/2})\Big\}\,, \tag{21h}$$

As seen from the above relations, among the coefficients C_i entering $\bar{A}$, $\bar{B}$, $\bar{C}$, and $\bar{D}$ only the constant $C_1^{0,1}$ is affected by the (less known) annihilation-channel contributions, through the term $C_1^{0,1}$ (t channel).

Performing the numerical integrations and col-

TABLE I. The calculated values [with $\alpha+\beta$ and $\alpha-\beta$ given, respectively, by Eqs. (17) and (23)] of the different contributions on the right-hand side of Eq. (9′). $(d\sigma/d\Omega)_{\pi^0}$ stands for the π^0-pole contribution [the third term in the right-hand side of Eq. (9′)]; $(d\sigma/d\Omega)_{(\bar{A},\bar{B},\bar{C},\bar{D})}$ stands for the last term in Eq. (9′), and we recall that $(d\sigma/d\Omega)_P$ and $(d\sigma/d\Omega)_{(\alpha,\beta)}$ [the first and second term in Eq. (9′)] are given, respectively, by Eqs. (2) and (10). To put better in evidence the role played by the ω^4 (and ω^5) contributions, we have also separately given the ω^2, ω^3 $[(d\sigma/d\Omega)^{(2,3)}_{(\alpha,\beta)}]$ and ω^4, ω^5 $[(d\sigma/d\Omega)^{(4,5)}_{(\alpha,\beta)}]$ terms of the piece $(d\sigma/d\Omega)_{(\alpha,\beta)}$ containing the generalized electric and magnetic polarizabilities. $(d\sigma/d\Omega)_{\text{th}}$ denotes the total calculated value of the differential cross section from Eq. (9′). All cross-section values are given in units of 10^{-32} cm^2/sr.

θ_{lab}	ω (MeV)	$\left(\frac{d\sigma}{d\Omega}\right)_P$	$\left(\frac{d\sigma}{d\Omega}\right)^{(2,3)}_{(\alpha,\beta)}$	$\left(\frac{d\sigma}{d\Omega}\right)^{(4,5)}_{(\alpha,\beta)}$	$\left(\frac{d\sigma}{d\Omega}\right)_{(\alpha,\beta)}$	$\left(\frac{d\sigma}{d\Omega}\right)_{\pi^0}$	$\left(\frac{d\sigma}{d\Omega}\right)_{(\bar{A},\bar{B},\bar{C},\bar{D})}$	$\left(\frac{d\sigma}{d\Omega}\right)_{\text{th}}$	$\left(\frac{d\sigma}{d\Omega}\right)_{\text{exp}}$
90°	80.9	1.300	−0.284	0.008	−0.276	0.049	−0.031	1.042	1.15±0.06
	85.4	1.321	−0.310	0.010	−0.300	0.057	−0.037	1.041	1.09±0.04
	109.9	1.455	−0.458	0.017	−0.441	0.115	−0.085	1.044	1.03±0.06
150°	81.9	1.949	−0.368	−0.063	−0.431	0.130	−0.100	1.548	1.44±0.12
	86.3	1.963	−0.388	−0.078	−0.465	0.148	−0.112	1.534	1.37±0.10
	106.7	2.039	−0.444	−0.188	−0.632	0.240	−0.124	1.523	1.60±0.08
	111.1	2.058	−0.446	−0.222	−0.668	0.261	−0.113	1.538	1.44±0.06

lecting the results, one finally has

$$\begin{aligned}(\mu/m)^4\bar{A}&=-0.1108-\gamma\ ,\\ (\mu/m)^4\bar{B}&=0.3462+3\gamma\ ,\\ (\mu/m)^4\bar{C}&=-0.0491-3\gamma\ ,\\ (\mu/m)^4\bar{D}&=0.0256+\gamma\ ,\end{aligned}\tag{22}$$

where $(m/\mu)^4\gamma\equiv 16m^5C_1^{0,1}$ (t channel). As the quantity γ is highly model-dependent, we shall first extract $\alpha-\beta$ using Eqs. (22) and Eq. (17) in a two-parameter [$\alpha-\beta$ and γ] least-squares fit of the differential cross section [as given by Eq. (9′)] to the seven experimental points of Ref. 4, e.g., to the values given in the last column of Table I. The optimal value found for $\alpha-\beta$ and γ are

$$\begin{aligned}\alpha-\beta&=15.53\times10^{-4}\ \text{fm}^3\ ,\\ \gamma&=-0.3787\ .\end{aligned}\tag{23}$$

In Table I we have listed separately the calculated values of the different contributions from the right-hand side of Eq. (9′) so that the reader can easily judge their relative importance.

We now come back to the theoretical estimation of the parameter γ containing the continuum t-channel contributions. We evaluate the needed absorptive parts in the t channel, keeping only $\pi\pi$ intermediate states and taking the amplitudes $N\bar{N}\to\pi\pi$ and $\pi\pi\to\gamma\gamma$ in Born approximation (that is, nucleon pole terms alone for the covariant matrix element of $N\bar{N}\to\pi\pi$, and pole terms plus the constants dictated by the existing low-energy theorem for the reaction $\pi\pi\to\gamma\gamma$). In this model, first used by Holliday[17] in his analysis of nucleon Compton scattering, γ is given by

$$\begin{aligned}(m/\mu)^4\frac{\gamma}{16m^5}&=C_1^{0,1}(t\text{ channel})\\ &=-\frac{1}{\pi}\int_{4\mu^2}^{\infty}dt'\,\frac{f(t')}{t'^2}\ ,\end{aligned}\tag{24}$$

with

$$f(t')=\left(\frac{e^2}{4\pi}\,\frac{g_{\pi NN}{}^2}{4\pi}\,\frac{\mu^2}{m}\right)\frac{16\pi}{t'^2}\left\{\ln\left(\frac{\sqrt{t'}+(t'-4\mu^2)^{1/2}}{\sqrt{t'}-(t'-4\mu^2)^{1/2}}\right)+\frac{\rho}{(4-\rho^2)^{1/2}}\,\text{arc tan}\left(\frac{[t'(t'-4\mu^2)m^2\mu^2(4-\rho^2)]^{1/2}}{\mu^2t'}\right)\right.$$
$$\left.-\frac{1}{2}\left(\frac{t'-\mu^2}{4m^2}\right)\frac{\varphi(t')}{(t'/4m^2-\frac{1}{4}\rho^2+\frac{1}{2}\rho)^{1/2}}\right\}\ ,\tag{25}$$

where

$$\rho=(\mu/m)\ ,\tag{26}$$

$$\varphi(t')=\begin{cases}\dfrac{4m}{(2\mu m+\mu^2-t')^{1/2}}\,\text{arc tan}\left(\dfrac{[(t'-4\mu^2)(t'-\mu^2+2m\mu)(2m\mu+\mu^2-t')]^{1/2}}{\sqrt{t'}(t'-3\mu^2)}\right) & \text{if } (2m\mu+\mu^2-t')>0\ ,\\[2ex] \dfrac{4m[(t'-4\mu^2)(t'-\mu^2+2m\mu)]^{1/2}}{\sqrt{t'}(t'-3\mu^2)} & \text{if } (2m\mu+\mu^2-t')=0\ ,\\[2ex] \dfrac{2m}{(t'-\mu^2-2m\mu)^{1/2}}\ln\left(\dfrac{\sqrt{t'}(t'-3\mu^2)+[(t'-4\mu^2)(t'-\mu^2+2m\mu)(t'-2m\mu-\mu^2)]^{1/2}}{\sqrt{t'}(t'-3\mu^2)-[(t'-4\mu^2)(t'-\mu^2+2m\mu)(t'-2m\mu-\mu^2)]^{1/2}}\right) & \text{if } (2m\mu+\mu^2-t')<0\ .\end{cases}$$

We note that in Eq. (24) we have corrected a general sign error present in Ref. 17 which persisted for a long time in the literature and which affected also those parts of Ref. 3 related to the evaluation of the annihilation-channel contributions to the proton polarizabilities. (We are much indebted to Bernabeu and Tarrach, who pointed out to us that the $\pi\pi$ contribution to α-β in Ref. 3 should come with an opposite sign.) The origin of this sign change should be traced back to the expression of the $\pi\pi$ contribution to the t-channel absorptive part of the T matrix for the $\gamma p \to \gamma p$ process given in Eq. (4.9) of Ref. 17. A direct calculation in perturbation theory of the matrix element $\langle\pi\pi|(\gamma_\mu \partial/\partial x_\mu + m)\psi(x)|p\rangle$ appearing in Eq. (4.8) of Ref. 17 shows that the general sign of the t-channel contributions should be reversed. It can, however, be shown[18] that a more detailed investigation of the t-channel 2π exchanges which incorporates the $I=J=0$ $\pi\pi$ phase shift through an N/D procedure could restore the final conclusion of Refs. 3 and 10 (e.g., that the annihilation channel could contribute to $\alpha-\beta$ with a positive quantity such that the negative amount coming from the u- and s-channel singularities be balanced and hence $\alpha-\beta$ still be finally positive).

In this paper we limit ourselves to the crude estimation of γ as given by Eqs. (24)–(26). The result of the numerical integration is

$$\gamma = (\mu/m)^4(16m^5)C_1^{0,1}(t \text{ channel}) = -0.103 \ , \qquad (27)$$

to be compared with the result for this quantity given by the two-parameter fit done previously [Eq. (23)]. Using this calculated value of γ in a one-parameter least-squares fit to the differential cross section (Eq. 9′) to the same seven experimental points of Ref. 4 one finds that the optimal value of $\alpha-\beta$ is, this time,

$$\alpha-\beta = 19.41\times10^{-4}\,\text{fm}^3 \ , \qquad (28)$$

which is not too far above the number given by the first of Eqs. (23).

The numbers for $\alpha-\beta$ from Eqs. (23) and (28) suggest a great positive value for $\alpha-\beta$. This represents one of the main conclusions of the analysis given in this section. It is true that it is partly model dependent, because the coefficients $\bar{A}$, $\bar{B}$, $\bar{C}$, and $\bar{D}$ were calculated in a dispersion approach. When more accurate experimental points [especially at lower energies (20–80 MeV) and large angles] will accumulate, it perhaps could become possible to determine all the six low-energy parameters α, β, $\bar{A}$, $\bar{B}$, $\bar{C}$, and $\bar{D}$ directly from the data, and a comparison with the calculated values will then furnish interesting checks for the theory. We hope that the numerical calculations presented in this section can at least be regarded as a support for the assertion that taking into account the next photon-frequency order ω^4 (and ω^5) in the low-energy expansion of the proton-Compton-scattering differential cross section, especially at (under the pion photoproduction threshold) energies which might still be too high to keep only ω^2 (and ω^3) terms, makes nontrivial sense.

Concerning the number found for $\alpha-\beta$ [Eq. (23)], it should not be forgotten that (i) it has been deduced from the results of only one experiment (Ref. 4), (ii) even though, owing to the good convergence properties of the dispersion representation the model calculation of $\bar{A}$, $\bar{B}$, $\bar{C}$, and $\bar{D}$ should be quite reliable, one could still lose the needed precision required by a trustworthy extraction, and (iii) the numerical result is sensibly dependent on the sign of the $\pi^0 \to 2\gamma$ amplitude (taken by us as in Refs. 14), in agreement with the common attitude presently adopted on this subject (see also Refs. 9 and 10). In connection with point (ii), we note that a more detailed investigation which, in the context of a new dispersion analysis of the low-energy proton Compton effect, goes beyond the consideration of a few terms in a simple ω-power expansion, is in progress, and the results will be published elsewhere.[21]

With respect to the new structure-dependent constants entering the higher-than-the-usual-second-order low-energy theorem, a nonrelativistic interpretation would be desirable. Here we shall only show by means of simple classical electrodynamics considerations how the scattering due to an induced quadrupole polarizability of the proton (κ) will appear in the ω^4 order as a part of the D coefficient in Eq. (9). To compute the effect of the electric quadrupole induced polarizability we express the Compton cross section through the energy flow $S(\theta)$ of the scattered wave at the distance R and the energy density $H=E^2/4\pi$ (E is the electric field) of the incoming plane wave

$$\frac{d\sigma}{d\Omega} = 4\pi R^2 \frac{S(\theta)}{E^2} \ .$$

In the case of electric dipole and quadrupole radiation there is the following contribution to the Poynting vector, coming from the dipole-quadrupole interference term[19]:

$$\vec{S} = \frac{1}{4\pi R^2}\frac{1}{6}\{-(\ddot{\vec{d}}\times\vec{n})\times[(\dddot{\vec{Q}}\times\vec{n})\times\vec{n}] + [(\ddot{\vec{d}}\times\vec{n})\times\vec{n}]\times(\dddot{\vec{Q}}\times\vec{n})\} \ ,$$

where $\vec{n}$ is a unit vector in the photon direction,

$\vec{d}=e_0\vec{r}$ ($\vec{d}=\vec{E}e_0{}^2/m$), $e_0=(1/\sqrt{4\pi})e$, and $Q_i=\sum_j Q_{ij}n_j$ is the induced quadrupole moment

$$Q_{ij}=\frac{\kappa}{2}\left[\frac{1}{2}\left(\frac{\partial E_i}{\partial x_j}+\frac{\partial E_j}{\partial x_i}\right)-\frac{1}{3}\delta_{ij}\,\nabla\vec{E}\right],$$

with κ the static induced electric quadrupole polarizability of the proton. (This parameter has been theoretically estimated in Ref. 20.) Working with the plane wave $\vec{E}=\vec{E}_0\exp[i(\vec{k}\cdot\vec{r}-\omega t)]$, neglecting powers higher than ω^4, and averaging over polarizations, one finally finds

$$\left(\frac{d\sigma}{d\Omega}\right)_\kappa=-\frac{e^2}{4\pi}\,\frac{\kappa}{12}\,m^3\left(\frac{\omega}{m}\right)^4\cos^3\theta\ .$$

So, a certain part of the ω^4 scattering appearing through the D coefficient in the expansion given by Eq. (9) can be viewed as due to the induced electric quadrupole polarizability of the system.

We conclude this note recalling that Eqs. (9) or (9′) represent a full exploitation of the relativistic- (and gauge-) invariance requirements insofar as the model-independent specification of the unpolarized differential cross section for the Compton effect on spin-$\frac{1}{2}$ hadronic targets up to the sixth power of the photon frequency is concerned.

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