Comments and Addenda

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Diagonalization of the Weyl tensor

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The gravitational superenergy flux can usually be reduced to zero by a suitable Lorentz transformation. The latter is derived here explicitly.

The following problem was recently posed by Wheeler[1]: Find the Lorentz transformation diagonalizing $\mathcal{E}$ and $\mathcal{B}$ (the two 3×3 symmetric traceless matrices representing the Weyl tensor). This problem can easily be solved by noting that the complex matrix $\mathcal{E}+i\mathcal{B}$ transforms under the Lorentz group according to[2,3]

$$\mathcal{E}'+i\mathcal{B}'=M^{-1}(\mathcal{E}+i\mathcal{B})M,$$

where M is a complex orthogonal 3×3 matrix. The latter is related in a well-known way[4] to the Lorentz-transformation matrix, e.g.,

$$M^1_{1'}=L^2_{2'}L^3_{3'}-L^3_{2'}L^2_{3'}+iL^0_{2'}L^1_{3'}-iL^1_{2'}L^0_{3'}.$$

To diagonalize $\mathcal{E}+i\mathcal{B}$, we need only the following steps:

(1) Solve the characteristic equation

$$|\mathcal{E}+i\mathcal{B}-\lambda I|=0.$$

This is a cubic equation for λ; see Ref. 2, Eqs. (9–11).

(2) In the generic case, the three eigenvalues are different and the linear equations

$$(\mathcal{E}+i\mathcal{B})M=\lambda M$$

have a unique solution, up to a sign. [We want M to be orthogonal, so we must normalize it by $M^TM=I$ and $\mathrm{Det}(M)=1$.]

(3) Express M in terms of complex Euler angles.[5]

(4) Reconstruct L from their real and imaginary parts.[5]

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[2] A. Peres, Nuovo Cimento 18, 36 (1960).
[3] A. Peres, J. Math. Mech. 11, 61 (1962).
[4] A. Einstein and W. Mayer, Sitzungsber. Preuss. Akad. Wiss. 522 (1932).
[5] F. Halbwachs, P. Hillion, and J. P. Vigier, Ann. Inst. H. Poincaré 16, 115 (1959).