PHYSICAL REVIEW D VOLUME 18, NUMBER 2 15 JULY 1978

Closed Regge eikonal formula for summing multiple-Reggeon-exchange contributions to the inclusive six-point function in the fragmentation region

N. S. Craigie
CERN-Geneva, Switzerland

K. J. M. Moriarty and J. H. Tabor
Department of Mathematics, Royal Holloway College, Englefield Green, Surrey, TW20 OEX, United Kingdom
(Received 24 November 1976)

We derive a closed Regge eikonal formula for summing the distinct types of multiple-Pomeron-exchange corrections to the Mueller-Regge contribution to the inclusive six-point functions $a+b\rightarrow c+X$ in the fragmentation regions. If one takes into account final-state absorption (i.e., Pomeron exchange between c and the missing-mass state X), as Reggeon renormalization to the basic triple-Regge term (Y graph), then using the renormalized Y graph as input, the formula accounts for all the absorptive corrections within a Mueller-Regge framework and essentially can provide a complete description of the inclusive distribution in the fragmentation region. The technique we use is a straightforward generalization of the functional derivative method introduced by Abarbanel and Itzykson for summing multiple-meson-exchange contributions to the four-point function. This generalization allows for a large class of possible nested ladders to be summed up to a closed expression within the eikonal approximation. The formula shows that one should expect the absorption corrections in the case of an inclusive process in the fragmentation region to be much stronger than one would naively expect from the corresponding study of two-body processes.

I. INTRODUCTION

For some time it has been thought that the Mueller-Regge pole model for inclusive distributions of the form $a+b\rightarrow c+X$ must have important Regge-cut corrections, particularly in the triple-Regge region. Such a conclusion arises from both purely theoretical[1] and phenomenological considerations.[2,3] From the phenomenological point of view it was shown in Ref. 2 that, for processes like $\gamma+p\rightarrow\pi^{\pm,0}+X$, in which spin and parity play an important role, the Mueller-Regge pole model [Fig. 1(a)] did not describe the data adequately, except for approximately reproducing the over-all normalization. Subsequently, it was argued in Ref. 3 that absorption corrections of the form shown in Fig. 1(b) could remedy the discrepancies; however, this required the strength of the cuts to be rather stronger than one might naively expect from the analysis of the corresponding exclusive processes $\gamma+p\rightarrow\pi^{\pm}+N$.[4] The formula derived in Ref. 3 has the simple form [Fig. 1(b)]

$$H(Q_{ac},Q_{ac},s_{ab},M_X^2)=\int\frac{d^2\vec{Q}_a}{(2\pi)^2}\frac{d^2\vec{Q}_{\bar a}}{(2\pi)^2}S(\vec{Q}_a)Y(Q_{ac}-Q_a,Q_{ac}-Q_{\bar a},s_{ab},M_X^2)S^*(\vec{Q}_{\bar a}),$$

where

$$S(\vec{Q})=(2\pi)^2\delta^{(2)}(\vec{Q})-Ce^{-\vec{Q}^2B(s)},\qquad(1.1)$$

where $\vec{Q}$ is a two-vector and Y is a triple-Regge pole term. [There is an error in Ref. 3 concerning the definition of C. This has been corrected in Ref. 15 and in Eq. (1.1) above.] However, Eq. (1.1) does not take into account final-state absorption or, in fact, all Pomeron-induced cuts of the rescattering type. In the case of strong absorption cuts for two-body processes like $a+b\rightarrow c+d$ it has been argued[5] that one should take into account all multiple-Pomeron-exchange contributions. This formidable task would be greatly simplified if the Regge eikonal model, first proposed by Frautschi and Margolis,[6] were a good approximation. The latter model received some respectability when it was shown[7] that it could be derived under certain reasonable assumptions by summing nested ladder diagrams in a ϕ^3 theory (Fig. 2).

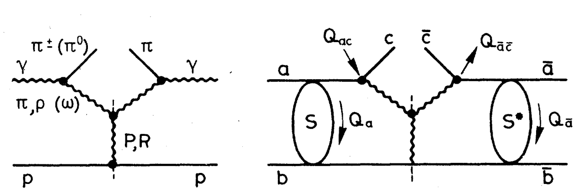

FIG. 1. (a) Mueller-Regge diagram for $\gamma+p\rightarrow\pi^{\pm,0}+X$. (b) Initial-state absorption of the triple-Regge contribution to $a+b\rightarrow c+X$.

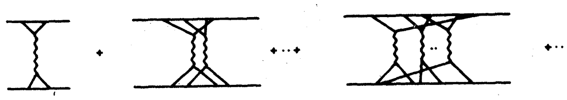

FIG. 2. Iteration of nested ladder diagrams leading to the Regge eikonal model.

The purpose of the present article is to show how one can derive in a similar way a Regge eikonal formula for summing all Pomeron-exchange rescattering corrections to the Mueller-Regge Y graph. The formula we arrive at is a simple generalization of Eq. (1.1) and has the form

$$H=\int d^2\vec{Q}_a d^2\vec{Q}_c d^2\vec{Q}_{\bar{a}} d^2\vec{Q}_{\bar{c}} S(\vec{Q}_a,\vec{Q}_c) Y(Q_{ac}-Q_a+Q_c, Q_{\bar{a}\bar{c}}-Q_{\bar{a}}+Q_{\bar{c}},\dots) S^*(\vec{Q}_{\bar{a}},\vec{Q}_{\bar{c}}),$$

where

$$S(\vec{Q}_a,\vec{Q}_c)=\int d^2\vec{B}_{ab} d^2\vec{B}_{cb} \exp(-i\vec{Q}_a\cdot\vec{B}_{ab}-i\vec{Q}_c\cdot\vec{B}_{cb})\exp(i\chi_{ab}[B_{ab}]+i\chi_{ac,b}[B_{ab},B_{cb}]+i\chi_{cb}[B_{cb}]). \tag{1.2}$$

This involves three distinct Regge eikonal phases and the impact parameters B_{ab} and B_{cb}, respectively. We shall use (1.2) to argue that one can expect the absorption corrections in the region $1>x_c\gg 0$ to be larger than one might expect from a simple comparison with the corresponding exclusive process $a+b\to c+d$. Further, if we argue that final-state absorption [Fig. 3(a)] can be taken into account by Pomeron renormalization of the Y graph,[8] then using the latter as input, Eq. (1.2) in principle accounts for all Regge-cut corrections in a Mueller-Regge description of $a+b\to c+X$ in the triple-Regge region. It is interesting to compare (1.2) with the recent works of Capella, Kaplan, and Tran Thanh Van[9] and Pumplin.[10] The former arrive essentially at (1.1) with

$$S(\vec{Q}_a,s_{ab})=\int d^2\vec{B}_{ab} e^{i\vec{Q}_a\cdot\vec{B}_{ab}} e^{i\chi_{ab}[B_{ab},s_{ab}]}, \tag{1.3}$$

while the latter obtains the formula

$$H=\int \frac{d^2\vec{Q}_c}{(2\pi)^2}\frac{d^2\vec{Q}_{\bar{c}}}{(2\pi)^2} S(\vec{Q}_c) Y S^*(\vec{Q}_{\bar{c}}), \tag{1.4}$$

with

$$S(\vec{Q}_c)=\int d^2\vec{B}_{cb} e^{i\vec{Q}_c\cdot\vec{B}_{cb}} e^{i\chi_{cb}[B_{cb},s_{cb}]}.$$

However, we do not agree with the derivation given in Ref. 10, since it appears to fail to take into account the nature of Regge exchange as opposed to elementary exchange.

In Sec. II we show how the algorithm, involving functional differentiation introduced by Abarbanel and Itzykson[11] in order to show how the sum of multimeson exchange graphs eikonalize, can be extended to the case of the exchange of nested ladder diagrams. In Sec. III we use the same method to derive the eikonal formula (1.2) for the inclusive six-point function. We conclude in Sec. IV with a short discussion. Some details are left to the Appendix.

II. CASE OF THE FOUR-POINT FUNCTION

In order to illustrate the method, and for later reference, we first consider the case of multiple Regge exchange in the four-point function (Fig. 2). We treat the Reggeons as ladders, or more generally as connected two-particle Green's functions [Fig. 4(a)], which we denote by the translational-invariant form

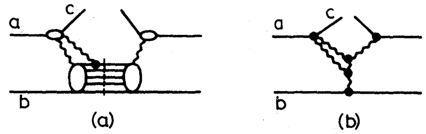

FIG. 3. (a) Final-state absorption corrections to $a+b\to c+X$ in the triple-Regge region. (b) Equivalent Reggeon renormalization graph.

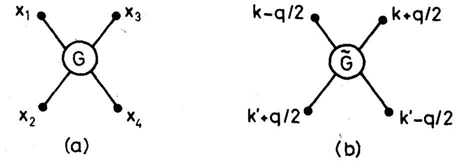

FIG. 4. (a) and (b) Two-particle connected Green's function.

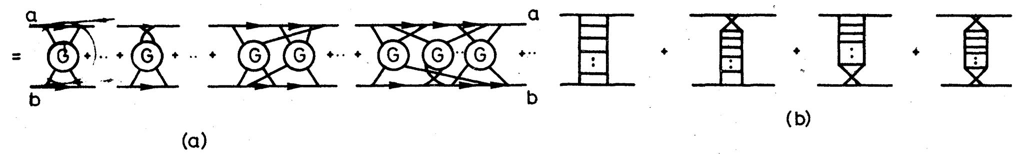

FIG. 5. (a) Complete s-channel iteration of nested ladder diagrams, represented here as two-particle connected Green's functions G. (b) The s-u terms building up the Regge eikonal phase.

$$G[x_1,x_2,x_3,x_4]=G(x_1-x_3,\tfrac{1}{2}(x_1+x_3)-\tfrac{1}{2}(x_2+x_4),x_2-x_4)+\text{Perms}(1\to 3,2\to 4), \tag{2.1}$$

where [referring to Fig. 4(b)]

$$G(x,y,x')=\int \frac{d^4k}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}\frac{d^4k'}{(2\pi)^4}\tilde{G}(k,q,k')e^{i(k\cdot x+q\cdot y+k'\cdot x')}. \tag{2.2}$$

One begins by introducing external sources A and B and one-particle Green's-function operators $G(A)$ and $G(B)$, respectively, where $G(A)=[p^2-m^2+i\epsilon-gA(x)]^{-1}$ with $[p^\mu,x^\nu]=ig^{\mu\nu}$. We can then write the sum of nested "ladder" diagrams [Fig. 5(a)] in terms of the following algorithm involving functional differentiation with respect to the sources

$$(2\pi)^4\delta^4(p_a+p_b-p_a'-p_b')T(s,t)=D\left(\frac{\delta}{\delta A},\frac{\delta}{\delta B}\right)\tau(p_a',p_a;A)\tau(p_b',p_b;B)\Big|_{A=B=0}, \tag{2.3}$$

where

$$\tau(p',p;A)=\lim_{\substack{p^2\to m^2\\ p'^2\to m^2}}(p^2-m^2)(p'^2-m^2)\langle p'|G(A)|p\rangle,$$

which can be expressed in the form[11]

$$\tau(p',p;A)=\langle p'|T\exp[ig\textstyle\int_0^\infty d\tau A(x-2\hat{p}\tau)]gA(x)|p\rangle, \tag{2.4}$$

and the functional differential operator is given by

$$D\left(\frac{\delta}{\delta A},\frac{\delta}{\delta B}\right)=\exp\left[\int\prod_{i=1}^4 d^4y_i\,\frac{\delta}{\delta A(y_1)}\frac{\delta}{\delta A(y_2)}G[y_1,y_2,y_3,y_4]\frac{\delta}{\delta B(y_3)}\frac{\delta}{\delta B(y_4)}\right], \tag{2.5}$$

$\tau(p',p;A)$ is a relativistic analog of a Lippmann-Schwinger scattering amplitude for a particle in the external potential A. Here the sources A and B are dummy variables, which generate vertices on the upper and lower lines (Fig. 4). After applying the functional differential operator (2.5), each such vertex is replaced by a vertex involving one leg of the Green's function (2.1). In this way this algorithm generates the complete set of Feynman graphs shown in Fig. 5(a). The eikonal approximation is achieved by assuming at high energies that the momentum flowing through the lines a and b in Fig. 5(a) suffer only small fluctuations as a result of their interaction via the exchange of Reggeons. This means that all the intermediate momenta in the expansion of $\tau(p',p;A)$ are strongly peaked around the average of the initial and final momenta p and p', respectively. Such a situation allows us to make a relativistic Glauber approximation, in which we replace the momentum operator $\hat{p}$ in the right-hand side of (2.4) by the c number $P=(p+p')/2$. Furthermore, we can drop the time ordering and replace (2.3) by

$$\begin{aligned}(2\pi)^4\delta^{(4)}&(p_a+p_b-p_{a'}-p_{b'})T(s,t)\\ &=D\left(\frac{\delta}{\delta A},\frac{\delta}{\delta B}\right)\int d^4x_a d^4x_b \exp[i(p_a-p_a')\cdot x_a+i(p_b-p_b')\cdot x_b]\\ &\quad\times\frac{\partial}{\partial\alpha_a}\frac{\partial}{\partial\alpha_b}\exp\left[ig\int_{\alpha_a}^\infty d\tau_a A(x_a-2P_a\tau_a)+ig\int_{\alpha_b}^\infty d\tau_b B(x_b-2P_b\tau_b)\right]\Big|_{\substack{A=B=0\\ \alpha_a=\alpha_b=0}},\end{aligned} \tag{2.6}$$

with $P_a=(p_a+p_a')/2$ and $P_b=(p_b+p_b')/2$.

It is now a simple matter to carry out the functional differentiation, and we obtain

$$(2\pi)^4\delta^{(4)}(p_a+p_b-p_a'-p_b')T(s,t)$$
$$=\int d^4x_a d^4x_b \exp[i(p_a-p_a')\circ x_a+i(p_b-p_b')\circ x_b]\frac{\partial}{i\partial\alpha_a}\frac{\partial}{i\partial\alpha_b}$$
$$\times\exp\left[g^4\int_{\alpha_a}^{\infty}d\tau_a d\tau_a'\int_{\alpha_b}^{\infty}d\tau_b d\tau_b' G(2P_a(\tau_a-\tau_a'),x_a-x_b-P_a(\tau_a+\tau_a')+P_b(\tau_b+\tau_b'),2P_b(\tau_b'-\tau_b))\right]\Bigg|_{\alpha_a=\alpha_b=0}. \tag{2.7}$$

The eikonal formula for $T(s,t)$ is obtained by expressing $y_{ab}=x_a-x_b$ in terms of the Sudakov variables

$$y_{ab}=B_{ab}+2\sigma P_a+2\sigma' P_b, \tag{2.8}$$

where $B_{ab}\circ P_a=B_{ab}\circ P_b=0$, and

$$\int d^4y_{ab}=\bar{s}\int_{-\infty}^{\infty}d\sigma d\sigma' d^2\vec{B}_{ab};\quad \bar{s}=4[(P_a\cdot P_b)^2-P_a{}^2P_b{}^2]^{1/2}.$$

After separating out the 4-momentum-conservation δ function and making the variable change

$$\eta=(\tau_a'-\tau_a),\quad \tau=(\tau_a+\tau_a')/2,\quad \eta'=(\tau_b'-\tau_b),\quad \tau'=(\tau_b+\tau_b')/2, \tag{2.9}$$

we arrive at[12]

$$T(s,t)=\bar{s}_{ab}\int_{-\infty}^{\infty}d\sigma d\sigma' d^2\vec{B}_{ab}e^{i\vec{Q}\cdot\vec{B}_{ab}}$$
$$\times\frac{\partial}{i\partial\sigma}\frac{\partial}{i\partial\sigma'}\exp\left[\int_{-\infty}^{\infty}d\eta d\eta'\int_{\sigma}^{\infty}d\tau\int_{\sigma'}^{\infty}d\tau' G(2P_a\eta,B_{ab}-2P_a\tau_a+2P_b\tau_b,2P_b\eta')\right], \tag{2.10}$$

with $Q=p_a-p_a'$.

It is a trivial matter to see that (2.10) can be written in the form

$$T(s,t)=\bar{s}_{ab}\int d^2\vec{B}_{ab}e^{-i\vec{Q}\cdot\vec{B}_{ab}}(1-e^{i\chi_{ab}[B_{ab},s_{ab}]}),$$

with

$$\chi_{ab}[B,s]=\int_{-\infty}^{\infty}d\eta d\tau d\eta' d\tau' G(2P_a\eta,B-2P_a\tau+2P_b\tau',2P_b\eta'). \tag{2.11}$$

Substituting expression (2.2) for G we obtain

$$\chi_{ab}[B,s]=\frac{1}{\bar{s}}\int\frac{d^2\vec{Q}}{(2\pi)^2}e^{i\vec{Q}\cdot\vec{B}}\int\frac{d^4k}{(2\pi)^4}\frac{d^4k'}{(2\pi)^4}2\pi\delta(2P_a\cdot k)2\pi\delta(2P_b\circ k')\tilde{G}(k,q,k'). \tag{2.12}$$

By noting[13] in the limit $s\to\infty$, $k^2/\sqrt{s}$, $t/\sqrt{s}\to 0$,

$$\frac{1}{(P-k)^2-m^2+i\epsilon}+\frac{1}{(P+k)^2-m^2+i\epsilon}\simeq\frac{1}{-2P\circ k+i\epsilon}+\frac{1}{2P\cdot k+i\epsilon}=2\pi\delta(2P\circ k), \tag{2.13}$$

we see that (2.12) corresponds to the Fourier-Bessel transform of the set of graphs in Fig. 5(b); hence for Regge exchange we obtain

$$\chi_{ab}[B,s]=\frac{1}{\bar{s}}\int\frac{d^2Q}{(2\pi)^2}e^{i\vec{Q}\cdot\vec{B}}\beta_a(-\vec{Q}^2)\beta_b(-\vec{Q}^2)(1+\tau e^{-i\pi\alpha(-\vec{Q}^2)})s^{\alpha(-\vec{Q}^2)}. \tag{2.14}$$

III. CASE OF THE TRIPLE-REGGE LIMIT OF AN INCLUSIVE DISTRIBUTION

For simplicity we consider the equal-mass case, and we begin by defining the Y graph in Fig. 6(a) through the expression

$$(2\pi)^4\delta^{(4)}(p_a+p_b-p_c-p_{\bar{a}}-p_{\bar{b}}+p_{\bar{c}})Y$$
$$=D_Y\lim_{p_\alpha{}^2\to m^2}\prod_\alpha(p_\alpha{}^2-m^2)\langle p_c|G(C)G_0^{-1}G(A)|p_a\rangle\langle p_{\bar{b}}|G(\bar{B})G_0^{-1}G(B)|p_b\rangle\langle p_{\bar{a}}|G(\bar{A})G_0^{-1}G(\bar{C})|p_{\bar{c}}\rangle\Big|_{\substack{A=B=C=0\\ \bar{A}=\bar{B}=\bar{C}=0}}, \tag{3.1}$$

where

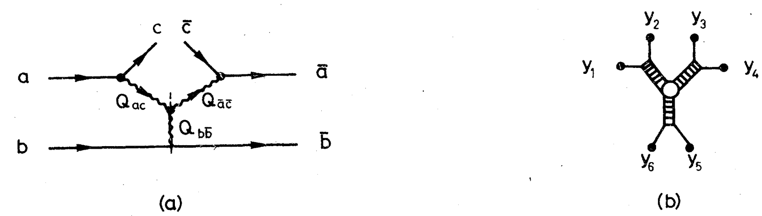

FIG. 6. (a) The triple-Regge pole Y graph for $a+b \to c+X$. (b) Green's function corresponding to a triple-Regge ladder diagram.

$$D_Y = \int \prod_{i=1}^{6} d^4y_i G^{(6)}[y_1, y_2, \ldots, y_6] \frac{\delta}{\delta A(y_1)} \frac{\delta}{\delta B(y_2)} \frac{\delta}{\delta C(y_3)} \frac{\delta}{\delta \bar{A}(y_4)} \frac{\delta}{\delta \bar{B}(y_5)} \frac{\delta}{\delta \bar{C}(y_6)} \,.$$

Here $G^{(6)}$ represents the Green's function for the triple-Reggeon ladder diagram shown in Fig. 6(b). (More generally one can use a double-Regge form.)

To generate the full set of nested ladders shown in Fig. 7, we start from

$$\begin{aligned} S_6 &= (2\pi)^4 \delta^{(4)}(p_a + p_b - p_c - p_{\bar a} - p_{\bar b} + p_{\bar c}) H \\ &= D^* D_Y D \lim_{p_\alpha^2 \to m^2} \prod (p_\alpha^2 - m^2) \langle p_c | G(C) G_0^{-1} G(A) | p_a \rangle \langle p_{\bar b} | G(\bar B) G_0^{-1} G(B) | p_b \rangle \langle p_{\bar a} | G(\bar A) G_0^{-1} G(\bar C) | p_{\bar c} \rangle \Big|_{\substack{A=B=C=0 \\ \bar A = \bar B = \bar C = 0}} , \end{aligned} \quad (3.2)$$

where

$$D = D_{ab}\left(\frac{\delta}{\delta A}, \frac{\delta}{\delta B}\right) D_{ac,b}\left(\frac{\delta}{\delta A}, \frac{\delta}{\delta C}, \frac{\delta}{\delta B}\right) D_{cb}\left(\frac{\delta}{\delta C}, \frac{\delta}{\delta B}\right),$$

$$D_{ac,b} = \exp\left[i \int \prod_{i=1}^{4} d^4y_i \frac{\delta}{\delta A(y_1)} \frac{\delta}{\delta C(y_2)} G[y_1, y_2, y_3, y_4] \frac{\delta}{\delta B(y_3)} \frac{\delta}{\delta B(y_4)} \right],$$

and $D_{ab}(\delta/\delta A, \delta/\delta B)$ and $D_{cb}(\delta/\delta C, \delta/\delta B)$ are as defined in Sec. II. Since the sources act only on single-particle states, we can use the completeness relation

$$\int \frac{d^4p}{(2\pi)^4} 2\pi \delta^+(p^2 - m^2) |p\rangle\langle p| = 1. \quad (3.3)$$

Further, the Reggeons in the Y graph carry away a large amount of longitudinal momentum from lines a and $\bar a$, so that nested diagrams of the form shown in Fig. 8 do not eikonalize, essentially because the intermediate propagators involved suffer drastic changes in momentum. We shall therefore collect these terms together and include them in the definition of the Y graph. The latter is characterized, in the target rest frame by a change of energy in the lines $a \to c$, $\Delta E \sim (1-x)E_a$, and consequently an interaction time $\Delta t \sim s/(1-x) \sim M_x^2$. Hence, for large missing mass the Y graph involves a rapid interaction, in contrast to the terms that eikonalize. It is thus reasonable to make this separation, since the physics of each piece is radically different and different approximations can be expected to be involved. The remaining summation in Fig. 7(b) can be rewritten in the following form, where we again make use of Eq. (2.4):

$$\begin{aligned} S_6 = D^* D \int \prod_{\alpha, \bar\alpha} \frac{d^4p'_\alpha}{(2\pi)^4} \frac{d^4p'_{\bar\alpha}}{(2\pi)^4} & 2\pi\delta^+(p'^2_\alpha - m^2) 2\pi\delta^+(p'^2_{\bar\alpha} - m^2)(2\pi)^4 \delta^{(4)}(p'_a + p'_b - p'_c - p'_{\bar a} - p'_{\bar b} + p'_{\bar c}) \\ & \times \tau^*_{\alpha_{\bar a}}(p_{\bar a}, p'_{\bar a}; \bar A) \tau^*_{\alpha_{\bar b}}(p_{\bar b}, p'_{\bar b}; \bar B) \tau^*_{\alpha_{\bar c}}(p_{\bar c}, p'_{\bar c}; \bar C) Y \tau_{\alpha_c}(p_c, p'_c; C) \tau_{\alpha_b}(p_b, p'_b; B) \tau_{\alpha_a}(p_a, p'_a; A) \Big|_{\substack{A=B=C=0 \\ \bar A = \bar B = \bar C = 0}} , \end{aligned} \quad (3.4)$$

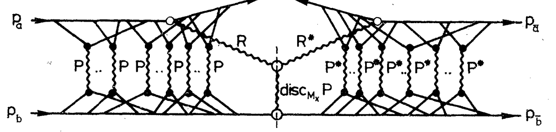

FIG. 7. The sum of all Pomeron-induced rescattering corrections to the Mueller-Regge Y graph. Here the Pomerons are treated as nested ladder diagrams.

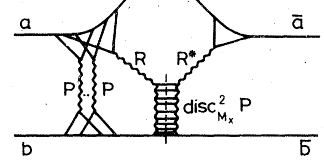

FIG. 8. Nested ladder diagrams that do not eikonalize in the case of the inclusive six-point function.

where, as before,

$$\tau(p'_\alpha, p_\alpha; A) = \langle p'_\alpha | T \exp[ig \int_0^\infty d\tau A(x - 2\hat{p}_\alpha \tau)] g A(x) | p_\alpha \rangle.$$

The eikonal approximation is made by replacing the operators $\hat{p}$ in the τ matrix elements by, respectively, $\hat{p}_\alpha \to p_\alpha$ ($\alpha = a, b, c$) and $\hat{p}_{\bar{\alpha}} \to p_{\bar{\alpha}}$ ($\bar{\alpha} = \bar{a}, \bar{b}, \bar{c}$) and by dropping the time ordering. The fact that we do not use the symmetric Glauber approximation is not likely to be important, since we are not dealing with large-angle effects. This simplifies the calculation. After these replacements we obtain

$$S_6 = \int \prod_{\alpha, \bar{\alpha}} \frac{d^4 p'_\alpha}{(2\pi)^4} \frac{d^4 p'_{\bar{\alpha}}}{(2\pi)^4} 2\pi \delta^+(p'^2_\alpha - m^2) 2\pi \delta^+(p'^2_{\bar{\alpha}} - m^2)(2\pi)^4 \delta^{(4)}(p_a + p_b - p_c - p_{\bar{a}} - p_{\bar{b}} + p_{\bar{c}})$$
$$\times D^* D \tau^*_{\alpha_{\bar{a}}}(p_{\bar{a}}, p'_{\bar{a}}; \bar{A}) \tau^*_{\alpha_{\bar{b}}}(p_{\bar{b}}, p'_{\bar{b}}; \bar{B}) \tau^*_{\alpha_{\bar{c}}}(p_{\bar{c}}, p'_{\bar{c}}; \bar{C}) Y$$
$$\times \tau_{\alpha_c}(p_c, p'_c; C) \tau_{\alpha_b}(p_b, p'_b; B) \tau_{\alpha_a}(p_a, p'_a; A) \Big|_{\substack{A=B=C=0 \\ \bar{A}=\bar{B}=\bar{C}=0}}, \tag{3.5}$$

where

$$\tau_\alpha(p', p; A) = \int d^4 x e^{i(p-p')\cdot x} \frac{\partial}{i\partial \alpha} \exp\left[ig \int_\alpha^\infty d\tau A(x - 2p\tau)\right].$$

After carrying out the functional differentiation we obtain the rather lengthy expression

$$S_6 = \int \prod_{\alpha, \bar{\alpha}} \frac{d^4 p'_\alpha}{(2\pi)^4} \frac{d^4 p'_{\bar{\alpha}}}{(2\pi)^4} 2\pi \delta^+(p'^2_\alpha - m^2) 2\pi \delta^+(p'^2_{\bar{\alpha}} - m^2)$$
$$\times \int \prod_{\beta, \bar{\beta}} d^4 x_\beta d^4 x_{\bar{\beta}} \exp[i(p_a - p'_a)\cdot x_a + i(p_b - p'_b)\cdot x_b + i(p'_c - p_c)\cdot x_c]$$
$$\times \exp[-i(p_{\bar{a}} - p'_{\bar{a}})\cdot x_{\bar{a}} - i(p_{\bar{b}} - p'_{\bar{b}})\cdot x_{\bar{b}} - i(p'_{\bar{c}} - p_{\bar{c}})\cdot x_{\bar{c}}]$$
$$\times \frac{\partial}{i\partial\alpha_a} \frac{\partial}{i\partial\alpha_b} \frac{\partial}{i\partial\alpha_c} \exp\left[\int_{\alpha_a}^\infty d\tau_a d\tau'_a \int_{\alpha_b}^\infty d\tau_b d\tau'_b G_{ab}\right.$$
$$\left. + \int_{\alpha_a}^\infty d\tau_a \int_{\alpha_c}^\infty d\tau_c \int_{\alpha_b}^\infty d\tau_b d\tau'_b G_{ac,b} + \int_{\alpha_c}^\infty d\tau_c d\tau'_c \int_{\alpha_b}^\infty d\tau_b d\tau'_b G_{cb}\right]$$
$$\times (2\pi)^4 \delta^{(4)}(p'_a + p'_b - p'_c - p'_{\bar{a}} - p'_{\bar{b}} + p'_{\bar{c}}) Y(Q'^2_{ac}, Q'^2_{\bar{a}\bar{c}}, Q'^2_{b\bar{b}}, s_{ab}, M_X^2)$$
$$\times \frac{\partial}{i\partial\alpha_{\bar{a}}} \frac{\partial}{i\partial\alpha_{\bar{b}}} \frac{\partial}{i\partial\alpha_{\bar{c}}} \exp\left[\int_{\alpha_{\bar{a}}}^\infty d\tau_{\bar{a}} d\tau'_{\bar{a}} \int_{\alpha_{\bar{b}}}^\infty d\tau_{\bar{b}} d\tau'_{\bar{b}} G^*_{\bar{a}\bar{b}}\right.$$
$$\left. + \int_{\alpha_{\bar{a}}}^\infty d\tau_{\bar{a}} \int_{\alpha_{\bar{c}}}^\infty d\tau_{\bar{c}} \int_{\alpha_{\bar{b}}}^\infty d\tau_{\bar{b}} d\tau'_{\bar{b}} G^*_{\bar{a}\bar{c},\bar{b}} + \int_{\alpha_{\bar{c}}}^\infty d\tau_{\bar{c}} d\tau'_{\bar{c}} \int_{\alpha_{\bar{b}}}^\infty d\tau_b d\tau'_{\bar{b}} G^*_{\bar{c}\bar{b}}\right], \tag{3.6}$$

with

$$G_{ab} = G(2p_a(\tau_a - \tau'_a), x_a - x_b - p_a(\tau_a + \tau'_a) + p_b(\tau_b + \tau'_b), 2p_b(\tau'_b - \tau_b)),$$
$$G_{ac,b} = G(x_a - 2p_a\tau_a - x_c + 2p_c\tau_c, \tfrac{1}{2}(x_a + x_c) - x_b - p_a\tau_a - p_c\tau_c + p_b(\tau_b + \tau'_b), 2p_b(\tau'_b - \tau_b)),$$
$$G_{cb} = G(2p_c(\tau'_c - \tau_c), x_c - x_b - p_c(\tau_c + \tau'_c) + p_b(\tau_b + \tau'_b), p_b(\tau'_b - \tau_b)),$$

similarly for $G^*_{\bar{a}\bar{b}}$, etc.,

$$Q'_{ac} = (p'_a - p'_c), \quad Q'_{\bar{a}\bar{c}} = (p'_{\bar{a}} - p'_{\bar{c}}), \text{ and } Q'_{b\bar{b}} = (p'_b - p'_{\bar{b}}).$$

We can considerably simplify (3.6) by making a series of variable changes beginning with the transformation

$$x_a, x_b, x_c \to y_{ab} = x_a - x_b, \quad y_{cb} = x_c - x_b, \quad x = \tfrac{1}{3}(x_a + x_b + x_c). \tag{3.7}$$

Similarly for the variables $x_{\bar{a}}$, $x_{\bar{b}}$, and $x_{\bar{c}}$, after which the x and $\bar{x}$ integrations can be immediately performed giving us the δ functions $\delta^4(p_a + p_b - p_c - p'_a - p'_b + p'_c)$ and $\delta^4(p'_{\bar{a}} + p'_{\bar{b}} - p'_{\bar{c}} - p_{\bar{a}} - p_{\bar{b}} + p_{\bar{c}})$, combining the latter with the δ function in Eq. (3.6) allows us to factor out the over-all four-momentum-conservation δ function $\delta^4(p_a + p_b - p_c - p_{\bar{a}} - p_{\bar{b}} + p_{\bar{c}})$.

We now make the Sudakov decompositions

$$y_{ab} = B_{ab} + 2\sigma_{ab} P_a + 2\sigma'_{ab} P_b, \quad y_{cb} = B_{cb} + 2\sigma_{cb} P_c + 2\sigma'_{cb} P_b, \tag{3.8}$$

with

$$B_{ab}\cdot p_a = B_{ab}\cdot p_b = B_{cb}\cdot p_c = B_{cb}\cdot P_b = 0,$$

and define the momentum-transfer variables

$$Q_a = (p_a - p_a') \text{ and } Q_c = -(p_c' - p_c), \tag{3.9}$$

and similarly for the barred variables.

After shifting the τ integrations in the same way as we did in the case of the four-point function, we can express Eq. (3.6) in the following form, in which we have factored out the over-all four-momentum δ functions:

$$\begin{aligned}H = (2\pi)^{-8} \int & d^4Q_a d^4Q_c d^4Q_{\bar a} d^4Q_{\bar c}\delta^+(p_a'^2 - m^2)\delta^+(p_b'^2 - m^2)\delta^+(p_c'^2 - m^2)\\ & \times \delta^+(p_{\bar a}'^2 - m^2)\delta^+(p_{\bar b}'^2 - m^2)\delta^+(p_{\bar c}'^2 - m^2) J_{ab}J_{cb}J_{\bar a\bar b}J_{\bar c\bar b}\\ & \times \int d^2\vec{B}_{ab} d^2\vec{B}_{cb} e^{i\vec{Q}_a\cdot\vec{B}_{ab}} e^{i\vec{Q}_c\cdot\vec{B}_{cb}} \int d^2\vec{B}_{\bar a\bar b} d^2\vec{B}_{\bar c\bar b} e^{-i\vec{Q}_{\bar a}\cdot\vec{B}_{\bar a\bar b}} e^{-i\vec{Q}_{\bar c}\cdot\vec{B}_{\bar c\bar b}}\\ & \times \int d\sigma_{ab} d\sigma'_{ab} d\sigma_{cb} d\sigma_{\bar a\bar b} d\sigma'_{\bar a\bar b} d\sigma_{\bar c\bar b} \int \frac{d\sigma}{2\pi} e^{2i\sigma(Q_c\cdot p_b)} \int \frac{d\sigma'}{2\pi} e^{2i\sigma'(Q_{\bar c}\cdot p_{\bar b})}\\ & \qquad \times \frac{\partial}{i\partial\sigma_{ab}}\frac{\partial}{i\partial\sigma'_{ab}}\frac{\partial}{i\partial\sigma_{cb}} \exp(V_{ab} + V^\sigma_{ac,b} + V_{cb}) Y(Q'_{ac}, Q'_{\bar a\bar c}, Q'_{b\bar b}, s_{ab}, M_X^2)\\ & \qquad \times \frac{\partial}{i\partial\sigma_{\bar a\bar b}}\frac{\partial}{i\partial\sigma'_{\bar a\bar b}}\frac{\partial}{i\partial\sigma_{\bar c\bar b}} \exp(V^*_{\bar a\bar b} + V^{\bar\sigma *}_{\bar a\bar c,\bar b} + V^*_{\bar c\bar b}),\end{aligned} \tag{3.10}$$

where

$$V_{ab} = \int_{\sigma_{ab}}^\infty d\tau_a d\tau_a' \int_{\sigma'_{ab}}^\infty d\tau_b d\tau_b' G(2p_a(\tau_a' - \tau_a), B_{ab} - p_a(\tau_a + \tau_a') + p_b(\tau_b + \tau_b'), 2p_b(\tau_b' - \tau_b)),$$

$$\begin{aligned}V^\sigma_{ac,b} = \int_{\sigma_{ab}}^\infty d\tau_a \int_{\sigma_{cb}}^\infty d\tau_c \int_{\sigma'_{ab}}^\infty d\tau_b d\tau_b' G(& B_{ab} - B_{cb} - 2p_a\tau_a + 2p_c\tau_c - 2\sigma p_b, \tfrac{1}{2}(B_{ab} + B_{cb}) - p_a\tau_a - p_c\tau_c + p_b(\tau_b + \tau_b')\\ & + 2\sigma p_b, 2p_b(\tau_b' - \tau_b)),\end{aligned}$$

$$V_{cb} = \int_{\sigma_{cb}}^\infty d\tau_c d\tau_c' \int_{\sigma'_{ab}}^\infty d\tau_b d\tau_b' G(2P_c(\tau_c' - \tau_c), B_{cb} - P_c(\tau_c + \tau_c') + 2P_b(\tau_b + \tau_b'), 2P_b(\tau_b' - \tau_b)),$$

and

$$\sigma = \sigma'_{ab} - \sigma'_{cb}, \quad \bar\sigma = \sigma'_{\bar a\bar b} - \sigma'_{\bar c\bar b}, \quad J_{\alpha\beta} = 4[(p_\alpha\cdot p_\beta)^2 - p_\alpha^2 p_\beta^2]^{1/2},$$

$$Q'_{ac} = p_a' - p_c' = Q_{ac} - Q_a + Q_c, \quad Q_{ac} = p_a - p_c, \quad Q'_{\bar a\bar c} = p_{\bar a}' - p_{\bar c}' = Q_{\bar a\bar c} - Q_{\bar a} + Q_{\bar c}, \quad Q_{\bar a\bar c} = p_{\bar a} - p_{\bar c},$$

$$Q'_{b\bar b} = p_{\bar b}' - p_b' = Q_{\bar a} + Q_{\bar c} - Q_a - Q_c.$$

From Eq. (3.10) we see that the σ_{ab}, σ'_{ab}, and σ_{cb} integrations can be immediately performed, which after allowing for the disconnected pieces leads to the eikonal factor

$$S^\sigma = \exp(i\chi_{ab} + i\chi^\sigma_{ac,b} + i\chi_{cb}), \tag{3.11}$$

where

$$i\chi_{ab} = V_{ab}(-\infty, -\infty), \quad i\chi^\sigma_{ac,b} = V^\sigma_{ac,b}(-\infty, -\infty, -\infty), \quad i\chi_{cb} = V_{cb}(-\infty, -\infty).$$

Finally, we decompose Q_a and Q_c into transverse and longitudinal parts, according to

$$Q_a = \frac{2p_b\cdot Q_a}{2p_a\cdot p_b} p_a + \frac{2p_a\cdot Q_a}{2p_a\cdot p_b} p_b + Q_{a\perp}, \qquad Q_c = \frac{2p_b\cdot Q_c}{2p_c\cdot p_b} p_c + \frac{2p_c\cdot Q_c}{2p_c\cdot p_b} p_b + Q_{c\perp}, \tag{3.12}$$

where $Q_\perp$ is a transverse four-vector and $Q_{a\perp}\cdot p_a = Q_{a\perp}\cdot p_b = 0$ and $Q_{c\perp}\cdot p_c = Q_{c\perp}\cdot p_b = 0$. We have already made use of the fact, for large s, that we can drop terms of order Q_a^2/s, etc.

Since we are solely interested in the eikonal approximation, we linearize the δ functions $\delta(2p_\alpha\cdot Q_\beta - Q_\beta^2)$ by replacing them by $\delta(2p_\alpha\cdot Q_\beta)$, which is consistent with the approximations we have already made and amounts to assuming only small four-momentum transfers are important, except at the Y graph. We thus arrive at the expression

$$H=(2\pi)^{-8}\int d^2\vec{Q}_a d^2\vec{Q}_c d^2\vec{Q}_{\bar{a}} d^2\vec{Q}_{\bar{c}}\exp(iQ_a\cdot B_{ab}+iQ_c\cdot B_{cb})\exp(-iQ_{\bar{a}}\cdot B_{\bar{a}\bar{b}}-iQ_{\bar{c}}\cdot B_{\bar{c}\bar{b}})$$
$$\times\int_{-\infty}^{\infty}d\alpha d\bar{\alpha}\int d(p_a\cdot Q_a)d(p_b\cdot Q_b)2\delta(2Q_a\cdot p_c)2\delta(2Q_a\cdot p_b-\alpha)$$
$$\times\int_{-\infty}^{\infty}d(p_c\cdot Q_c)d(p_b\cdot Q_b)\,2\delta(2p_c\cdot Q_c)2\delta(2p_b\cdot Q_c+\alpha)$$
$$\times\int_{-\infty}^{\infty}d(p_{\bar{a}}\cdot Q_{\bar{a}})d(p_{\bar{b}}\cdot Q_{\bar{a}})2\delta(2p_{\bar{a}}\cdot Q_{\bar{a}})2\delta(2p_{\bar{b}}\cdot Q_{\bar{a}}-\bar{\alpha})$$
$$\times\int_{-\infty}^{\infty}d(p_{\bar{c}}\cdot Q_{\bar{c}})d(p_{\bar{b}}\cdot Q_{\bar{c}})2\delta(2p_{\bar{c}}\cdot Q_{\bar{c}})2\delta(2p_{\bar{b}}\cdot Q_{\bar{c}}+\bar{\alpha})$$
$$\times\int_{-\infty}^{\infty}\frac{d\sigma}{2\pi}e^{i\alpha\sigma}\int_{-\infty}^{\infty}\frac{d\bar{\sigma}}{2\pi}e^{i\bar{\alpha}\bar{\sigma}}S^{\sigma}Y(Q'_{ac},Q'_{\bar{a}\bar{c}},Q'_{b\bar{b}},s_{ab},M_X^2)S^{\bar{\sigma}*}. \qquad (3.13)$$

In going from Eqs. (3.10) to (3.13), we notice that the Jacobians J_{ab}, J_{cb}, $J_{\bar{a}\bar{c}}$, and $J_{\bar{c}\bar{b}}$ cancel out. Equation (3.13) simplifies considerably if we note that at high energies Q'_{ac} and $Q'_{\bar{a}\bar{c}}$ become independent of α and $\bar{\alpha}$, respectively (see Appendix A). This means we can perform the integrations involving the δ functions $\delta(\sigma)$ and $\delta(\bar{\sigma})$.

Defining the variables $t_{ac}=(Q_{ac}-Q_a+Q_c)^2$, $\bar{t}_{ac}=(Q_{ac}-Q_{\bar{a}}+Q_{\bar{c}})^2$, $t_0=(Q_a+Q_c-Q_{\bar{a}}-Q_{\bar{c}})^2$, and $t=Q_{ac}{}^2=Q_{\bar{a}\bar{c}}{}^2$, we now consider only the forward missing-mass discontinuity for $s\to\infty$, where we can use

$$s_{cb}=-xs_{ab}\ \text{ and }\ M_X{}^2=(1-x)s_{ab},$$

and

$$p_c=xp_a+yp_b+p_{c\perp}\ \text{ with }\ y=\frac{m^2+\vec{p}_c{}^2}{xs}-\frac{xm^2}{s}. \qquad (3.14)$$

Then

$$t_{ac}=(2-x)m^2-\frac{m^2+(\vec{p}_c+x\vec{Q}_a+\vec{Q}_c)^2}{x},\quad \bar{t}_{ac}=(2-x)m^2-\frac{m^2+(\vec{p}_c+x\vec{Q}_{\bar{a}}+\vec{Q}_{\bar{c}})^2}{x},$$
$$t=(2-x)m^2-\frac{m^2+\vec{p}_c{}^2}{x},\quad t_0=-(\vec{Q}_a+\vec{Q}_c-\vec{Q}_{\bar{a}}-\vec{Q}_{\bar{c}})^2. \qquad (3.15)$$

Integrating over σ and $\bar{\sigma}$ in Eq. (3.13) leads to the following closed eikonal form, which has a simple convolution-like structure:

$$H[t,s_{ab},M_X{}^2]=\int\frac{d^2\vec{Q}_a}{(2\pi)^2}\frac{d^2\vec{Q}_c}{(2\pi)^2}\frac{d^2\vec{Q}_{\bar{a}}}{(2\pi)^2}\frac{d^2\vec{Q}_{\bar{c}}}{(2\pi)^2}S(\vec{Q}_a,\vec{Q}_c)Y(t_{ac},\bar{t}_{ac},t_0,s_{ab},M_X{}^2)S^*(\vec{Q}_{\bar{a}},\vec{Q}_{\bar{c}}),$$

where

$$S=\int d^2\vec{B}_{ab}d^2\vec{B}_{cb}\exp(i\vec{Q}_a\cdot\vec{B}_{ab}+i\vec{Q}_c\cdot\vec{B}_{cb})\exp\{i(\chi_{ab}[B_{ab}]+\chi_{ac,b}[B_{ab},B_{cb}]+\chi_{cb}[B_{cb}])\},$$

and (3.16)

$$S^*=\int d^2\vec{B}_{\bar{a}\bar{b}}d^2\vec{B}_{\bar{c}\bar{b}}\exp(-i\vec{Q}_{\bar{a}}\cdot\vec{B}_{\bar{a}\bar{b}}-i\vec{Q}_{\bar{c}}\cdot\vec{B}_{\bar{c}\bar{b}})\exp\{-i(\chi^*_{ab}[B_{\bar{a}\bar{b}}]+\chi^*_{ac,b}[B_{\bar{a}\bar{b}},B_{\bar{c}\bar{b}}]+\chi^*_{cb}[B_{\bar{c}\bar{b}}])\}.$$

The eikonal phases χ_{ab} were computed in Sec. II, and are given by

$$\chi_{ab}=\frac{1}{s_{ab}}\int\frac{d^2\vec{Q}_a}{(2\pi)^2}e^{-i\vec{Q}_a\cdot\vec{B}_{ab}}\beta_a(-\vec{Q}_a{}^2)\beta_b(-\vec{Q}_a{}^2)\xi_\alpha(-\vec{Q}_a{}^2)s_{ab}{}^{\alpha(-\vec{Q}_a{}^2)},$$

and (3.17)

$$\chi_{cb}=\frac{1}{s_{cb}}\int\frac{d^2\vec{Q}_c}{(2\pi)^2}e^{-i\vec{Q}_c\cdot\vec{B}_{cb}}\beta_c(-\vec{Q}_c{}^2)\beta_b(-\vec{Q}_c{}^2)\xi_\alpha(-\vec{Q}_c{}^2)s_{cb}{}^{\alpha(-\vec{Q}_c{}^2)},$$

$\chi_{ac,b}$ is more complicated and depends on both impact parameters. It is defined through the expression

$$\chi_{ac,b}=\int_{-\infty}^{\infty} d\tau_a d\tau_c d\tau_b d\tau_b' G(B_{ab}-B_{cb}-2p_a\tau_a+2p_c\tau_c,\tfrac{1}{2}(B_{ab}+B_{cb})-p_a\tau_a-p_c\tau_c+p_b(\tau_b+\tau_b'),2p_b(\tau_b'-\tau_b)). \quad (3.18)$$

In Appendix B we show that this can be cast in the following form:

$$\chi_{ac,b}=\frac{1}{s_{ab}}\int \frac{d^2\vec{Q}_a}{(2\pi)^2}\frac{d^2\vec{Q}_c}{(2\pi)^2}\exp[i(\vec{B}_{ab}+\vec{B}_{cb})\cdot(\vec{Q}_a-\vec{Q}_c)/2]\,\beta_b(-(\vec{Q}_a-\vec{Q}_c)^2)\xi_\alpha(-(\vec{Q}_a-\vec{Q}_c)^2)$$

$$\times s_{ab}{}^{\alpha(-(\vec{Q}_a-\vec{Q}_c)^2)}\{\exp[i(\vec{B}_{ab}-\vec{B}_{cb})\cdot(\vec{Q}_a+\vec{Q}_c)/2]F_{ac}(x,\vec{Q}_a,\vec{Q}_c)\}. \quad (3.19)$$

The function F_{ac} is related to the residue functions $\beta_a(t)$ and $\beta_c(t)$, however, its form depends on detailed dynamical considerations. For the purpose of computation one could use the exponential form

$$F_{ac}(x,\vec{Q}_a,\vec{Q}_c)=\left(\frac{x^{\alpha(t_{ac})}-x}{2(1-x^2)}\right)(\beta_a^0\beta_c^0)^{1/2}e^{b_a t_a/2}e^{b_c t_c/2}, \quad (3.20)$$

where

$$\beta_\alpha(t)=\beta_\alpha^0 e^{b_\alpha t};\quad \alpha=a,c;\quad t_{ac}=-(\vec{Q}_a-\vec{Q}_c)^2.$$

From (3.20) we see that the eikonal phase $\chi_{ac,b}$ is in fact small, because of the small slope of the Pomeron trajectory.

IV. DISCUSSION

It has been argued by Bartels and Kramer[18] that, in fact, the above sum of eikonal phases overcounts the contribution of exchanges in channel ab and cb in some sense. To resolve this problem one has to resort to a more detailed model, in which combinatoric questions can be answered. We have briefly examined a ϕ^3 theory in the weak-coupling limit following the calculation of Circuta and Sugar (see Ref. 7). One finds for inelastic processes that only the mixed eikonal phase enters (in the leading-logarithmic approximation), however, its weight is precisely that given by formula (3.11). The main point is that for the inelastic case there are 2^{n-1} more graphs at the n-ladder-exchange level. We strongly suspect the counting problem in general is more subtle than has previously been supposed.

From Eq. (3.16) we see that there are three distinct kinds of multiple-Pomeron-exchange corrections to the inclusive six-point function $a+b\to c+X$. Further, if we identify the Regge eikonal phases involved, with the corresponding ones for $a+b\to c+d$ (see Fig. 9), we see that the factor S defined in (3.16) contains both the initial- and final-state eikonal phases of the latter. However, for an inclusive distribution the final-state absorption corrections, shown in Fig. 3, should also play an important role, because we can expect their strength to grow with the multiplicity of the missing-mass state which rises like $\bar{n}_X=c\ln M_X{}^2$.[14] From the above consideration we conclude that the strength of Pomeron-cut corrections in the Mueller-Regge model to be strongly dependent on x and somewhat stronger than one might naively conclude from the analysis of the exclusive limit. This was indeed the conclusion of Ref. 3. It is interesting to note that using the parameters of Ref. 3 for the unpolarized distribution $\gamma+p\to\pi^\pm(x,p_\perp)+X$ as input, one obtains[15] a target asymmetry $\gamma+p^\uparrow\to\pi^\pm(x,p_\perp)+X$, which becomes appreciable for $p_\perp{}^2>0.2\ \mathrm{GeV}^2$. Further, the values obtained in Ref. 15 approximately agree with the preliminary data.[16] This should be contrasted with the Mueller-triple-Regge pole term, which predicts zero target asymmetry. The x dependence of the latter should be a good test of the detailed structure of the Regge-cut terms. An analysis[17] of reactions like $\pi p\to\Delta+X$ at high energies would be very useful in verifying the conclusions of Refs. 3 and 4, which rely on the available DESY data, the latter being at rather low energies.

ACKNOWLEDGMENTS

We wish to thank J. Bartels, A. Capella, I. G. Halliday, and G. Kramer for discussions of their work on the subject. We are also grateful to L. Caneschi for discussing the work with us. One of us (N.S.C.) wishes to thank Professor H. G. Eggleston and the Mathematics Department of the Royal Holloway College for the hospitality extended to him during the course of this work. (J.H.T.) wishes to thank the Science Research Council of Great Britain for support.

APPENDIX A

We can show Q'_{ac} is independent of α in Eq. (3.13) by noting that to leading order

$$2p_b\cdot Q_a=-2p_b\cdot Q_c=\alpha,$$

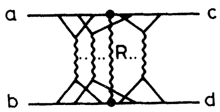

FIG. 9. Multiple-Pomeron-exchange corrections to the Regge pole exchange for $a+b \to c+X$.

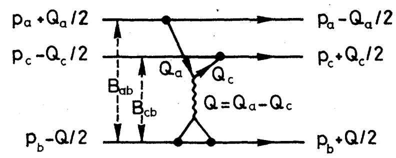

FIG. 10. Diagram involved in the Regge eikonal phase $\chi_{ac,b}$.

and

$$Q_a^\mu = \frac{\alpha}{s_{ab}} p_a^\mu + \frac{\vec{Q}_a^{\,2}}{s_{ab}} p_b^\mu + Q_{a\perp}^\mu, \quad Q_c^\mu = \frac{-\alpha}{s_{cb}} p_c^\mu + \frac{\vec{Q}_c^{\,2}}{s_{cb}} p_c^\mu + Q_{c\perp}^\mu, \tag{A1}$$

with

$$p_c^\mu = x p_a^\mu + y p_b^\mu + p_{c\perp}^\mu,$$
$$y = \frac{m^2 + \vec{p}_c^{\,2} - x m^2}{x s_{ab}}.$$

We see that

$$Q_{ac}'^\mu = \left(1 - x - \frac{\alpha}{s_{ab}} + \frac{\alpha x}{s_{cb}}\right) p_a^\mu + \left(-y - \frac{\vec{Q}_a^{\,2}}{s_{ab}} + \frac{\vec{Q}_c^{\,2}}{s_{cb}} + \frac{\alpha y}{s_{cb}}\right) p_b^\mu + (p_{c\perp}^\mu - Q_{a\perp}^\mu + Q_{c\perp}^\mu),$$

and since $s_{cb} = x s_{ab}$

$$Q_{ac}'^\mu = (1-x) p_a^\mu + y' p_b^\mu + (p_{c\perp}^\mu - Q_{a\perp}^\mu + Q_{c\perp}^\mu) + O(1/s). \tag{A2}$$

APPENDIX B

To calculate the mixed Regge eikonal phase $\chi_{ac,b}$ (Fig. 10), we proceed in much the same way as the calculation of χ_{ab} in Sec. II, starting from (3.18), namely,

$$\chi_{ac,b} = g^4 \int_{-\infty}^{\infty} d\tau_a d\tau_c d\tau_b d\tau_b' G(B_{ab} - B_{cb} - 2p_a\tau_a + 2p_c\tau_c, \tfrac{1}{2}(B_{ab} + B_{cb}) - p_a\tau_a - p_c\tau_c + p_b(\tau_b + \tau_b'), 2p_b(\tau_b' - \tau_b)). \tag{B1}$$

Using the Fourier representation of the Greens function G,

$$G(r, x, r') = \int \frac{d^4k}{(2\pi)^4} \frac{d^4Q}{(2\pi)^4} \frac{d^4k'}{(2\pi)^4} \tilde{G}(k, Q, k') e^{-ik\cdot r - iQ\cdot x - ik'\cdot r'}, \tag{B2}$$

we obtain the expression

$$\chi_{ac,b} = g^4 \int \frac{d^4Q_a}{(2\pi)^4} \frac{d^4Q_c}{(2\pi)^4} 2\pi\delta(2p_a \cdot Q_c) 2\pi\delta(2p_c \cdot Q_c) 2\pi\delta(2p_b \cdot (Q_a - Q_c))$$
$$\times \int \frac{d^4k}{(2\pi)^4} 2\pi\delta(2p_b \cdot k) \exp[i(B_{ab} + B_{cb}) \cdot (Q_a - Q_c)/2]$$
$$\times \exp[i(B_{ab} - B_{cb}) \cdot (Q_a + Q_c)/2] G\left(\frac{Q_a + Q_c}{2}, Q_a - Q_c, k\right). \tag{B3}$$

We now use

$$g^2 \int \frac{d^4k}{(2\pi)^4} 2\pi\delta(2p_b \cdot k) G((Q_a + Q_c)/2, Q_a - Q_c, k) = g^2 \Delta_F(Q_a^2) \Delta_F(Q_c^2) \beta_b(Q^2) [(Q_a \cdot p_b + Q_c \cdot p_b)^2]^{\alpha(Q^2)}, \tag{B4}$$

with $Q = Q_a - Q_c$ and

$$\Delta_F(k^2) = \int d\sigma\, \rho(\sigma)(k^2 - \sigma^2 + i\epsilon)^{-1}. \tag{B5}$$

Equation (B4) exhibits explicitly the propagators connecting lines a and c. In the case of χ_{ab} and χ_{cb} the latter go to making up the residue functions $\beta_a(Q_a^{\,2})$ and $\beta_c(Q_c^{\,2})$, respectively.

By defining the Sudakov variables

$$Q_a = 2\sigma_a p_a + 2\tau_a p_b + Q_{a\perp}, \quad Q_c = 2\sigma_c p_c + 2\tau_c p_b + Q_{c\perp}. \tag{B6}$$

We can express (B3) in the form

$$\chi_{ac,b} = \int \frac{d^2\vec{Q}_a}{(2\pi)^2}\frac{d^2\vec{Q}_c}{(2\pi)^2}\exp[i(\vec{B}_{ab}+\vec{B}_{cb})\cdot(\vec{Q}_a-\vec{Q}_c)/2]\exp[i(\vec{B}_{ab}-\vec{B}_{cb})\cdot(\vec{Q}_a+\vec{Q}_c)/2]$$
$$\times\frac{1}{2p_a\cdot p_b}\frac{1}{2p_c\cdot p_b}\int_{-\infty}^{\infty}d\sigma_a d\tau_a d\sigma_c d\tau_c\,\delta(2\tau_a p_a\cdot p_b)\delta(2\tau_c p_c\cdot p_b)$$
$$\times\delta(\sigma_a p_a\cdot p_b-\sigma_c p_c\cdot p_b)\beta_b((Q_a-Q_c)^2)$$
$$\times(\sigma_a s_{ab}+\sigma_c s_{bc})^{\alpha((Q_a-Q_c)^2)}\Delta_F(Q_a{}^2)\Delta_F(Q_c{}^2)g^2. \tag{B7}$$

By noting $\sigma_a p_a\cdot P_b-\sigma_c p_c\cdot p_b=0$, which implies $\sigma_a=-x\sigma_c=x\sigma$, we see that

$$Q_a{}^2\simeq x^2\sigma^2m^2-(1-x^2\sigma^2)\vec{Q}_a{}^2,\quad Q_c{}^2\simeq\sigma^2m^2-(1-\sigma^2)\vec{Q}_c{}^2,\quad (Q_a-Q_c)^2\simeq-(\vec{Q}_a-\vec{Q}_c)^2, \tag{B8}$$

and

$$\chi_{ac,b}=\frac{1}{s_{ab}}\int\frac{d^2\vec{Q}_a}{(2\pi)^2}\frac{d^2\vec{Q}_c}{(2\pi)^2}\exp[i(\vec{B}_{ab}+\vec{B}_{cb})\cdot(\vec{Q}_a-\vec{Q}_c)/2]\,\beta_b(-(\vec{Q}_a-\vec{Q}_c)^2)\xi_\alpha(s_{ab})^{\alpha(-(\vec{Q}_a-\vec{Q}_c)^2)}$$
$$\times\exp[i(\vec{B}_{ab}-\vec{B}_{cb})\cdot(\vec{Q}_a+\vec{Q}_c)]F_{ac}(x,\vec{Q}_a,\vec{Q}_c), \tag{B9}$$

where

$$F_{ac}(x,\vec{Q}_a,\vec{Q}_c)=g^2\int_{-\infty}^{\infty}\frac{d\sigma}{2\pi}\Delta_F(x^2\sigma^2m^2-(1-x^2\sigma^2)\vec{Q}_a{}^2)(x\sigma)^{\alpha(-(\vec{Q}_a-\vec{Q}_c)^2)}\Delta_F(\sigma^2m^2-(1-\sigma^2)\vec{Q}_c{}^2). \tag{B10}$$

The corresponding expression for the residue functions $\beta_a(t)$ and $\beta_c(t)$ is given by

$$\beta(-\vec{Q}^2)=\int\frac{d^2\vec{Q}'}{(2\pi)^2}\Delta_F(-\vec{Q}'^2)\Delta_F(-(\vec{Q}-\vec{Q}')^2). \tag{B11}$$

If we insert a propagator of the form $\Delta_F(k^2)=(k^2-m^2+i\epsilon)^{-1}$ into (B10), we obtain

$$F_{ac}(x,\vec{Q}_a,\vec{Q}_c)=g^2\int_{-\infty}^{\infty}\frac{d\sigma}{2\pi}\frac{1}{(1-x^2\sigma^2+i\epsilon)}\frac{1}{(1-\sigma^2+i\epsilon)}\frac{1}{(\vec{Q}_a{}^2+m^2)(\vec{Q}_c{}^2+m^2)}(x\sigma)^{\alpha(-(\vec{Q}_a-\vec{Q}_c)^2)}$$
$$=g^2\left[\frac{x^{\alpha(-(\vec{Q}_a-\vec{Q}_c)^2)}-x}{2(1-x^2)}\right]\Delta_F(-\vec{Q}_a{}^2)\Delta_F(-\vec{Q}_c{}^2). \tag{B12}$$

Equation (B12) suggests the form of F_{ac} proposed in Eq. (3.20).

---

[1]J. L. Cardy and A. R. White, Phys. Lett. 47B, 445 (1973); I. G. Halliday and C. T. Sachrajda, Phys. Rev. D 8, 3598 (1973); A. A. Migdal, A. M. Polyakov, and K. A. Ter-Martirosyan, Phys. Lett. 48B, 239 (1974); H. D. I. Abarbanel, J. Bartels, J. M. Bronzan, and D. Sidhu, Phys. Rev. D 12, 2459 (1975).

[2]N. S. Craigie, G. Kramer, and J. Körner, Nucl. Phys. B68, 509 (1974).

[3]N. S. Craigie and G. Kramer, Nucl. Phys. B75, 509 (1974).

[4]R. Worden, Nucl. Phys. B37, 253 (1972), and references therein.

[5]P. D. B. Collins, Phys. Rep. 1C, 103 (1971).

[6]S. Frautschi and B. Margolis, Nuovo Cimento 56A, 1155 (1968).

[7]S. J. Chang and T. M. Yan, Phys. Rev. Lett. 25, 1586 (1970); G. M. Cicuta and R. L. Sugar, Phys. Rev. D 3, 970 (1971).

[8]J. L. Cardy, R. L. Sugar, and A. R. White, Phys. Lett. 55B, 384 (1975); H. D. I. Abarbanel, J. Bartels, J. M. Bronzan, and D. Sidhu (Ref. 1).

[9]A. Capella, J. Kaplan, and J. Tran Thanh Van, Nucl. Phys. B105, 333 (1976).

[10]J. Pumplin, Phys. Rev. D 13, 1249 (1976).

[11]H. D. I. Abarbanel and C. Itzykson, Phys. Rev. Lett. 23, 53 (1969).

[12]After taking into account the $(1\to3)$, $(2\to4)$ permuted terms, we can extend the η and η' integrations from $-\infty$ to $+\infty$.

[13]The assumption $k^2/\sqrt{s}\to0$ is already implicit in making the eikonal approximation. Indeed the derivation of the eikonal model in Ref. 7 rests strongly on the linearization $2k\cdot p+k^2\simeq2k\cdot p$.

[14]In principle, this can be computed using the rules of Ref. 8.

[15]K. Ahmed, J. Körner, G. Kramer, and N. S. Craigie, Nucl. Phys. B108, 275 (1976).

[16]Private communication on the preliminary data, H. Genzel, DESY.

[17]K. J. M. Moriarty, J. H. Tabor, and A. Ungkitchanukit, Phys. Rev. D 16, 130 (1977).

[18]J. Bartels and G. Kramer, Nucl. Phys. B120, 96 (1977).