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Quantum decay process of metastable vacuum states in SU(2) Yang-Mills theory: A probability theoretical point of view

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The problem of vacuum-tunneling phenomena such as the quantum decay process of metastable vacuum states in SU(2) Yang-Mills theory is investigated from a probability theoretical point of view. This is done by adopting the stochastic quantization procedure to quantize the Yang-Mills field in the $A_0 = 0$ gauge. The mechanism of vacuum tunneling can be illustrated within the realm of the stochastic quantization. It is shown that the quantized vacuum field configuration, which manifests the decay process of metastable vacuum states, is a Euclidean-Markov field of Gaussian type. The validity of the Euclidean path-integral description of vacuum-tunneling phenomena is also shown from the probability theoretical point of view. Passing to the semiclassical limit, the concept of an instanton is justified as a classical Euclidean Yang-Mills field which manifests the most probable tunneling path.

I. INTRODUCTION

In the recent non-Abelian gauge field theory we encounter a problem of vacuum instability such as the quantum decay process of metastable vacuum states. Classically there exist a number of topologically inequivalent vacuum states characterized by different Pontryagin indices. In quantum field theory they are rendered unstable by the tunnel effect; they are metastable vacuum states.

Introducing a WKB approximation in Feynman's path-integral quantization procedure, a Euclidean technique has been developed to solve the problem and also to clarify the structure of the gauge theory vacuum: Beginning with Polyakov's work,[1] several authors[2-4] have investigated a Euclidean path integral description of vacuum-tunneling phenomena in non-Abelian gauge theory. They suggested that a Euclidean path integral

$$\int \exp(-S_E[A]+\text{gauge-fixing term})\,\delta A\ , \quad (1.1)$$

$$S_E[A]=\frac{1}{4}\sum_{a=1}^{3}\sum_{\mu,\nu=0}^{3}\int F^a_{\mu\nu}F^a_{\mu\nu}\,d^4x\ , \quad (1.2)$$

provides a powerful mathematical tool for exploring the structure of the vacuum state of a quantized SU(2) Yang-Mills field. A vacuum-to-vacuum transition amplitude of the quantized Yang-Mills field, which is defined in physical space-time (i.e., Minkowski space), was assumed to be given by the Euclidean path integral (1.1). Classical Euclidean Yang-Mills fields, which minimize the Euclidean action (1.2), were introduced as an indication of tunneling phenomena between metastable vacuum states.

Although the application of such a Euclidean technique to the problem of vacuum instability has been put into practice successfully,[5-10] it has not been carried out to make the quantum-theoretical foundation of the Euclidean technique concrete. Moreover, a physical interpretation of the Euclidean path-integral description of vacuum-tunneling phenomena seems unclear. Much work is needed to clarify the origin of the Euclidean formulation within the framework of quantum field theory.

In the present paper, adopting the stochastic quantization procedure to quantize the SU(2) Yang-Mills field in the $A_0=0$ gauge, we derive the Euclidean formulation from a probability theoretical point of view. The mechanism of vacuum tunneling is shown to be illustrated precisely also from this point of view.

In Sec. II we sketch the stochastic quantization of the Yang-Mills field working in the $A_0=0$ gauge. Section III is devoted to exploring the structure of the vacuum state of the quantized Yang-Mills field, to deriving the Euclidean formulation, and to proving the validity of the Euclidean path-integral description of vacuum-tunneling phenomena from a probability theoretical point of view. In Sec. IV it is shown that the quantized vacuum field configuration, which manifests the decay process of metastable vacuum states, is a Euclidean-Markov field of Gaussian type. In Sec. V we justify the concept of an instanton as a classical Euclidean Yang-Mills field which manifests the most probable tunneling path. We give a brief summary and discussion in Sec. VI. An analysis of the quantum decay process of metastable vacuum states and a calculation of the decay rate for a spatially homogeneous σ model are given in the Appendix.

II. STOCHASTIC QUANTIZATION OF THE YANG-MILLS FIELD

Consider an SU(2) Yang-Mills field A_μ^a described by a Lagrangian

$$L=\frac{1}{4}\sum_{a=1}^{3}\ \sum_{\mu,\nu=0}^{3}{}'\int F_{\mu\nu}^a F_{\mu\nu}^a\, d^3x, \tag{2.1}$$

where

$$F_{\mu\nu}^a=\partial_\mu A_\nu^a-\partial_\nu A_\mu^a+\sum_{b,c=1}^{3}\epsilon^{abc}A_\mu^b A_\nu^c \tag{2.2}$$

is a field strength tensor. (Latin letters a, b, c denote isovector indices and i, j, k denote space indices. Greek letters μ, ν denote space-time indices, and the terms in the primed sum for μ, $\nu=1$, 2, 3 are taken with opposite sign.) In terms of field variables

$$E_i^a=F_{0i}^a,\quad B_i^a=\frac{1}{2}\sum_{k,j=1}^{3}\epsilon_{ijk}F_{jk}^a \tag{2.3}$$

the Lagrangian (2.1) can be written as

$$L=\frac{1}{2}\sum_{i,a=1}^{3}\int(E_i^aE_i^a-B_i^aB_i^a)d^3x\ . \tag{2.4}$$

Hereafter, to avoid the complexity of the Coulomb gauge,[11] we shall work in the $A_0=0$ gauge. Then E_i^a becomes identical with $\partial_0A_i^a=\dot{A}_i^a$, and B_i^a does not contain $\dot{A}_i^a$. In this gauge the A_i^a's with i, $a=1$, 2, 3 are dynamical variables of the Yang-Mills field. The Lagrangian (2.1) becomes

$$L=\frac{1}{2}\sum_{i,a=1}^{3}\int(\dot{A}_i^a\dot{A}_i^a-B_i^aB_i^a)d^3x, \tag{2.5}$$

which gives us the following equation of motion for A_i^a:

$$\ddot{A}_i^a=-\frac{\delta}{\delta A_i^a}\frac{1}{2}\sum_{j,b=1}^{3}\int B_j^bB_j^b\,d^3x\ . \tag{2.6}$$

To quantize the Yang-Mills field, it is convenient to adopt Nelson's stochastic quantization procedure.[12] This is because not only the structure of the vacuum state but also the mechanism of the vacuum tunneling of the quantized Yang-Mills field can be illustrated within the realm of the stochastic quantization.[13]

To make our discussion mathematically rigorous, we need to use the nonstandard analysis for the definitions of functional derivatives and functional integrals as we did in a previous paper.[14] In the present paper, however, we do not make use of the terminologies of the nonstandard analysis explicitly.

Following the previous paper,[14] we shall sketch the procedure for the stochastic quantization of the Yang-Mills field in what follows.

Let $^*\epsilon(R^3)$ be a totality of isovector and three-vector valued functions defined on R^3. According to the stochastic quantization procedure, a quantized Yang-Mills field in the $A_0=0$ gauge is a diffusion process $A_i^a(t)$ in the infinite-dimensional function space $^*\epsilon(R^3)$.[14] (Hereafter, for simplicity, we do not indicate the $\vec{x}$ dependence of the field variables explicitly.) The diffusion process is assumed to be a solution of a stochastic differential equation

$$dA_i^a(t)=U_i^a(\vec{A}(t),t)\,dt+dW_i^a(t)\ , \tag{2.7}$$

where $U_i^a(\cdot,t)$ denotes a time-dependent transformation in $^*\epsilon(R^3)$ which will be related with the state functional, $W_i^a(t)$ is a Wiener process in $^*\epsilon(R^3)$ with a variance parameter $\frac{1}{2}$ and $\vec{A}$ is an abbreviation for $\{A_i^a\}_{i,a=1}^3$. The probability distribution of $A_i^a(t)$ is given by a functional integral

$$\mathrm{Prob}[A_i^a(t)\in\mathfrak{M}]=\int_{\mathfrak{M}}P[\vec{A};t]\prod_{i,a=1}^{3}\delta A_i^a, \tag{2.8}$$

where $\mathfrak{M}$ is a measurable subset of $^*\epsilon(R^3)$, and $P[\vec{A};t]$ is a probability density functional of $A_i^a(t)$ which satisfies the Fokker-Planck equation

$$\frac{\partial}{\partial t}P[\vec{A};t]=-\sum_{i,a=1}^{3}\int d^3x\frac{\delta}{\delta A_i^a}\{U_i^a(\vec{A},t)P[\vec{A};t]\}+\frac{1}{2}\sum_{i,a=1}^{3}\int d^3x\frac{\delta^2}{\delta A_i^a\delta A_i^a}P[\vec{A};t]\ . \tag{2.9}$$

The transformation $U_i^a(\cdot,t)$ should be determined in such a way that the Yang-Mills field equation (2.6) is valid with the substitution

$$\ddot{A}_i^a(t,\vec{x})\to\frac{1}{2}(DD^*+D^*D)A_i^a(t)\ ,\qquad A_i^a(t,\vec{x})\to A_i^a(t)\ . \tag{2.10}$$

D and D^* are the mean forward derivative and the mean backward derivative defined as

$$DF[\vec{A}(t);t]=\lim_{h\to0^+}E\{F[\vec{A}(t+h);t+h]-F[\vec{A}(t);t]\,|\,\vec{A}(t)\}$$
$$=\left(\frac{\partial}{\partial t}+\sum_{i,a=1}^{3}\int d^3x\,U_i^a(\vec{A},t)\frac{\delta}{\delta A_i^a}+\frac{1}{2}\sum_{i,a=1}^{3}\int d^3x\frac{\delta^2}{\delta A_i^a\delta A_i^a}\right)F[\vec{A}(t);t]\ , \tag{2.11}$$

$$D^*F[\vec{A}(t);t]=\lim_{h\to 0^+}\frac{1}{h}E\{F[\vec{A}(t);t]-F[\vec{A}(t-h);t-h]\,|\,\vec{A}(t)\}$$
$$=\left(\frac{\partial}{\partial t}+\sum_{i,a=1}^{3}\int d^3x\,V_i^a(\vec{A},t)\frac{\delta}{\delta A_i^a}-\tfrac{1}{2}\sum_{i,a=1}^{3}\int d^3x\,\frac{\delta^2}{\delta A_i^a\delta A_i^a}\right)F[\vec{A}(t);t] \tag{2.12}$$

for any smooth functional $F[\vec{A};t]$, where $V_i^a(\vec{A},t)=U_i^a(\vec{A},t)-\delta\ln P[\vec{A};t]/\delta A_i^a$ and $E\{\circ|\vec{A}(t)\}$ denotes the conditional expectation value with respect to $A_i^a(t)$. Namely, we assume that the following generalized Yang-Mills field equation holds:

$$\tfrac{1}{2}(DD^*+D^*D)A_i^a(t)=-\frac{\delta}{\delta A_i^a(t)}\tfrac{1}{2}\sum_{j,b=1}^{3}\int B_j^bB_j^b\,d^3x\ . \tag{2.13}$$

The left-hand side of Eq. (2.13) can be manipulated as

$$\tfrac{1}{2}\Bigg[\frac{\partial}{\partial t}V_i^a(\vec{A}(t),t)+\sum_{j,b=1}^{3}\int d^3y\,U_j^b(\vec{A}(t,\vec{y}),t)\frac{\delta}{\delta A_j^b(t,\vec{y})}V_i^a(\vec{A}(t),t)$$
$$+\tfrac{1}{2}\sum_{j,b=1}^{3}\int d^3y\,\frac{\delta^2}{\delta A_j^b(t,\vec{y})\delta A_j^b(t,\vec{y})}V_i^a(\vec{A}(t),t)+\frac{\partial}{\partial t}U_i^a(\vec{A}(t),t)$$
$$+\sum_{j,b=1}^{3}\int d^3y\,V_j^b(\vec{A}(t,\vec{y}),t)\frac{\delta}{\delta A_j^b(t,\vec{y})}U_i^a(\vec{A}(t),t)-\tfrac{1}{2}\sum_{j,b=1}^{3}\int d^3y\,\frac{\delta^2}{\delta A_j^b(t,\vec{y})\delta A_j^b(t,\vec{y})}U_i^a(\vec{A}(t),t)\Bigg]. \tag{2.14}$$

It is worthwhile to note that the Fokker-Planck equation (2.9) and the generalized Yang-Mills field equation (2.13) with given initial conditions, say $P[\vec{A};0]=P_0[\vec{A}]$ and $U_i^a(\vec{A},0)=U_{0i}^a(\vec{A})$, are sufficient to characterize the quantized Yang-Mills field $A_i^a(t)$ completely.

To clarify the relation between the stochastic quantization and the conventional canonical one, it is convenient to make the following additional assumption on the transformations $U_i^a(\cdot,t)$ and $V_i^a(\circ,t)$:

$$\tfrac{1}{2}[U_i^a(\vec{A},t)+V_i^a(\vec{A},t)]=\frac{\delta}{\delta A_i^a}S[\vec{A};t]\,, \tag{2.15}$$

where $S[\cdot;t]$ is a functional on $^*\epsilon(R^3)$. Then, introducing the state functional

$$\Omega[\vec{A};t]=P[\vec{A};t]^{1/2}\exp(iS[\vec{A};t])\,, \tag{2.16}$$

we can convert two equations, (2.9) and (2.13), for two real quantities, $P[\vec{A};t]$ and $U_i^a(\vec{A},t)$, to a single equation for one complex quantity, $\Omega[\vec{A};t]$. The equation is nothing but the Schrödinger equation

$$i\frac{\partial}{\partial t}\Omega[\vec{A};t]=\tfrac{1}{2}\sum_{j,b=1}^{3}\int d^3x\left(-\frac{\delta^2}{\delta A_j^b\delta A_j^b}+B_j^bB_j^b\right)\times\Omega[\vec{A};t]. \tag{2.17}$$

Equation (2.17) with a given initial condition $\Omega[\vec{A};0]=\Omega_0[\vec{A}]$, which is equivalent to the ones $P[\vec{A};0]=P_0[\vec{A}]$ and $U_i^a(\vec{A},0)=U_{0i}^a(\vec{A})$, describes the behavior of the quantized Yang-Mills field $A_i^a(t)$ completely.

Thus the stochastic quantization is shown to provide the same Schrödinger representation as the canonical one. The only typical difference between the two quantization procedures is that the stochastic one teaches us that the behavior of the quantized Yang-Mills field in the vacuum state $\Omega[\vec{A};t]=\Omega[\vec{A}]\exp(-iE_0t)$ is a diffusion process $A_i^a(t)$ in $^*\epsilon(R^3)$. We shall investigate the structure of the vacuum state by making use of a probability theoretical framework of the stochastic quantization in the following section.

III. VACUUM-STATE STRUCTURE OF THE QUANTIZED YANG-MILLS FIELD

As we are much interested in the topological structure of the vacuum state in SU(2) Yang-Mills theory, throughout this section it is convenient to adopt a matrix notation as well as the isovector notation flexibly. We define

$$A_j=\sum_{a=1}^{3}A_j^a\sigma^a/2i\in \mathrm{SU}(2)\,, \tag{3.1}$$

where the σ's are the Pauli spin matrices.

Removing the infinite zero-point energy, we define a quantum-theoretical vacuum state of the Yang-Mills theory by a vacuum-state functional (wave functional) $\Omega[\vec{A}]$ which satisfies the Schrödinger equation

$$\sum_{i,a=1}^{3}\int d^3x\left(-\frac{1}{2}\frac{\delta^2}{\delta A_i^a\delta A_i^a}+\tfrac{1}{2}B_i^aB_i^a\right)\Omega[\vec{A}]=0\ . \tag{3.2}$$

What we are going to study, in the following, is a vacuum-state structure of the quantized Yang-Mills field characterized by the Schrödinger equation (3.2).

In classical field theory, vacuum states of the Yang-Mills theory are classical field configurations $A_i^{\prime a}$'s with zero potential energy,

$$\frac{1}{2}\sum_{i,a=1}^{3}\int B_i^a(\vec{A}')B_i^a(\vec{A}')d^3x=0\ . \tag{3.3}$$

They are pure gauge fields

$$A_i'=g^{-1}\partial_i g\in \mathrm{SU}(2), \tag{3.4}$$

where the g's are unitary matrices such that

$$\lim_{|\vec{x}|\to\infty} g(\vec{x})=I\ .$$

Before proceeding with further considerations, it seems worthwhile to note that two pure gauge fields which can be joined with each other continuously in the manifold of SU(2) should be understood as the same classical vacuum state.[10] Namely, classical vacuum states of the Yang-Mills theory consist of an infinite number of homotopy classes[15] of SU(2):

$$[A_i'=g^{-1}\partial_i g],\ \ [A_i''=g'^{-1}\partial_i g'],\cdots \tag{3.5}$$

where $[\cdot]$ denotes a homotopy class to which the pure gauge field inside the brackets belongs.

In order to classify the homotopy classes, it is convenient to introduce the Pontryagin index[10]

$$q=-\frac{1}{24\pi^2}\sum_{i,j,k,=1}^{3}\int \mathrm{Tr}(A_iA_jA_k)d^3x\ . \tag{3.6}$$

For pure gauge fields, q is an integer. Then the classical vacuum states (3.5) can be rearranged as

$$\{[{}^qA_i={}^qg^{-1}\partial_i\,{}^qg]\}_{q=-\infty}^{\infty}\ , \tag{3.7}$$

where ${}^qA_i$ is a pure gauge field with Pontryagin index q.

In quantum field theory the classical vacuum states (3.7) are rendered unstable by the tunnel effect; they are metastable vacuum states.

Let us investigate the vacuum-tunneling phenomena between metastable vacuum states ${}^qA_i$'s from a probability theoretical point of view.

In the conventional framework of quantum field theory, the wave functional $\Omega[\vec{A}]$ does not demonstrate the detail of the vacuum tunneling. One can describe the tunneling behavior of the quantized Yang-Mills field only in the semiclassical limit, i.e., within the realm of the WKB approximation. In the probability theoretical framework of the stochastic quantization, on the contrary, the behavior of the quantized Yang-Mills field in the vacuum state $\Omega[\vec{A}]$ is known to be a diffusion process $A_i^a(t)$ in $*\epsilon(R^3)$ as we have seen in the preceding section. From Eqs. (2.7) and (2.15) we find that $A_i^a(t)$ is a solution of the stochastic differential equation

$$dA_i^a(t)=U_i^a(\vec{A}(t))\,dt+dW_i^a(t)\ , \tag{3.8}$$

where the transformation $U_i^a(\cdot)$ is related to the vacuum-state wave functional $\Omega[\vec{A}]$ by

$$U_i^a(\vec{A})=\frac{\delta}{\delta A_i^a}\ln\Omega[\vec{A}]\ . \tag{3.9}$$

Therefore, a transition probability law of the diffusion process $A_i^a(t)$ manifests the tunneling process of the quantized vacuum field configuration between metastable vacuum states ${}^qA_i$'s. The transition probability law is given by an elementary solution $Q[\vec{A};s|\vec{A}';u]$ of the Fokker-Planck equation (2.9).[13] Namely, we have

$$\frac{\partial}{\partial s}Q=-\sum_{i,a=1}^{3}\int d^3x\frac{\delta}{\delta A_i^a}[U_i^a(\vec{A})Q]+\frac{1}{2}\sum_{i,a=1}^{3}\int d^3x\frac{\delta^2}{\delta A_i^a\delta A_i^a}Q \tag{3.10}$$

and

$$\lim_{s\to u+}Q[\vec{A};s|\vec{A}';u]=\delta[\vec{A}-\vec{A}'], \tag{3.11}$$

where $\delta[\vec{A}-\vec{A}']$ denotes a delta functional on $*\epsilon(R^3)$.

To illustrate the mechanism of vacuum tunneling, one needs to solve the Cauchy problem, Eqs. (3.10) and (3.11). This can be done by introducing a relative transition law $F[\vec{A};s|\vec{A}';u]$ by

$$Q[\vec{A};s|\vec{A}';u]=\Omega[\vec{A}]F[\vec{A};s|\vec{A}';u]\Omega[\vec{A}']^{-1}\ . \tag{3.12}$$

Equation (3.10) is transformed into its self-adjoint from

$$-\frac{\partial}{\partial s}F=\sum_{i,a=1}^{3}\int d^3x\left(-\frac{1}{2}\frac{\delta^2}{\delta A_i^a\delta A_i^a}+\frac{1}{2}B_i^aB_i^a\right)F\ , \tag{3.13}$$

and the initial condition (3.11) into

$$\lim_{s\to u+}F[\vec{A};s|\vec{A}';u]=\delta[\vec{A}-\vec{A}'] \tag{3.14}$$

by the substitution (3.12). Equation (3.13) is nothing but a Euclidean analog of the Schrödinger equation (2.17). The Feynman-Kac formula[16] asserts that a solution of the Cauchy problem Eqs. (3.13) and (3.14) is given by a Wiener integral

$$F[\vec{A};s|\vec{A}';u]=\int_{\Gamma(\vec{A}_s|\vec{A}'_u)} \exp\left[-\tfrac{1}{2}\sum_{i,a=1}^{3}\int_u^s dt\int d^3x\, B_i^a(\vec{A}(t))B_i^a(\vec{A}(t))\right]d\mu_w(\vec{A}(t)). \quad (3.15)$$

Here μ_w denotes a Wiener measure with a variance parameter $\frac{1}{2}$ and the region of integration $\Gamma(\vec{A}_s|\vec{A}'_u)$ is a totality of continuous paths $\vec{A}(t)$'s in $*\epsilon(R^3)$ such that $\vec{A}(s)=\vec{A}$ and $\vec{A}(u)=\vec{A}'$ hold. Therefore, from Eq. (3.12), we find that the transition probability law of the diffusion process $A_i^a(t)$ has the following expression:

$$Q[\vec{A};s|\vec{A}';u]=(\Omega[\vec{A}]/\Omega[\vec{A}'])\int_{\Gamma(\vec{A}_s|\vec{A}'_u)}\exp\left[-\frac{1}{2}\sum_{i,a=1}^{3}\int_u^s dt\int d^3x\, B_i^a(\vec{A}(t))B_i^a(\vec{A}(t))\right]d\mu_w(\vec{A}(t)). \quad (3.16)$$

A tunneling probability of the quantized vacuum field configuration between metastable vacuum states ${}^qA_i$ and ${}^pA_i$, $q\neq p$, from a remote past to a remote future is $Q[{}^p\vec{A};\infty|{}^q\vec{A};-\infty]$. Noticing that $Q[{}^p\vec{A};\infty|{}^q\vec{A};-\infty]$ would coincide with the conventional expression of the tunneling probability, i.e., the ratio $|\Omega[{}^p\vec{A}]/\Omega[{}^q\vec{A}]|^2$, we find a vacuum-tunneling amplitude of the quantized Yang-Mills field in the $A_0=0$ gauge to be

$$\begin{aligned}\mathrm{Amp}[{}^p\vec{A};\infty|{}^q\vec{A};-\infty]&=\Omega[{}^p\vec{A}]/\Omega[{}^q\vec{A}]\\ &=\int_{\Gamma({}^p\vec{A}_\infty|{}^q\vec{A}_{-\infty})}\exp\left[-\tfrac{1}{2}\sum_{i,a=1}^{3}\int_{-\infty}^{\infty}dt\int d^3x\, B_i^a(\vec{A}(t))B_i^a(\vec{A}(t))\right]d\mu_w(\vec{A}(t)). \end{aligned} \quad (3.17)$$

If we introduce a functional path-integral expression of the Wiener measure[13]

$$d\mu_w(\vec{A}(t))=N\exp\left[-\tfrac{1}{2}\sum_{i,a=1}^{3}\int_{-\infty}^{\infty}dt\int d^3x\,\dot{A}_i^a(t)\dot{A}_i^a(t)\right]\delta\vec{A}(t), \quad (3.18)$$

where $\delta\vec{A}(t)$ means taking a functional path-integral and N is a normalization constant, Eq. (3.17) becomes

$$\mathrm{Amp}[{}^p\vec{A};\infty|{}^q\vec{A};-\infty]=N\int_{{}^q\vec{A}}^{{}^p\vec{A}}\exp\left\{-\sum_{i,a=1}^{3}\int_{-\infty}^{\infty}dt\int d^3x[\tfrac{1}{2}\dot{A}_i^a(t)\dot{A}_i^a(t)+\tfrac{1}{2}B_i^a(\vec{A}(t))B_i^a(\vec{A}(t))]\right\}\delta\vec{A}(t). \quad (3.19)$$

In terms of the field strength tensor $F_{\mu\nu}^a$, this can be written as

$$\mathrm{Amp}[{}^p\vec{A};\infty|{}^q\vec{A};-\infty]=N\int_{{}^q\vec{A}}^{{}^p\vec{A}}\exp\left(-\tfrac{1}{4}\sum_{a=1}^{3}\sum_{\mu,\nu=0}^{4}\int F_{\mu\nu}^aF_{\mu\nu}^a d^4x\right)\delta\vec{A}(t,\vec{x}), \quad (3.20)$$

which provides a Euclidean path-integral description of vacuum-tunneling phenomena in the $A_0=0$ gauge.

Thus the validity of the Euclidean path-integral description of the vacuum-tunneling phenomena in SU(2) Yang-Mills theory has been proved from the probability theoretical point of view.

IV. QUANTUM DECAY PROCESS OF METASTABLE VACUUM STATES

To illustrate the mechanism of the vacuum tunneling more clearly, we shall investigate the quantum decay process of metastable vacuum states by solving the stochastic differential equation (3.8) explicitly. It is convenient to rewrite Eq. (3.8) in terms of the white noise,[17] obtaining

$$\dot{A}_i^a(t,\vec{x})=U_i^a(\vec{A}(t,\vec{x}))+Z_i^a(t,\vec{x}), \quad (4.1)$$

where $Z_i^a(t,\vec{x})=\dot{W}_i^a(t,\vec{x})$ denotes a white noise with mean 0 and covariance

$$E\{Z_i^a(t,\vec{x})Z_j^b(u,\vec{y})\}=\delta^{ab}\delta_{ij}\delta(t-u)\delta^3(\vec{x}-\vec{y}),\quad 1\leq a,b,i,j\leq 3. \quad (4.2)$$

This is simply because one can consider the problem of the quantum decay process in a concrete mathematical framework of distribution theory.

As the transformation $U_i^a(\cdot)$ is related with the vacuum-state wave functional $\Omega[\vec{A}]$ by Eq. (3.9), first of all, we have to construct a physically relevant vacuum state which is invariant under gauge transformations. We know it to be a coherent superposition of Gaussian functionals peaked around each metastable vacuum state ${}^qA_i(\vec{x})$.[2,4,10] Such a gauge-invariant vacuum-state wave functional is parametrized by an angle θ,

$$\Omega_\theta[\vec{A}]=\sum_{q=-\infty}^{\infty}e^{iq\theta}\Phi_\omega[\vec{A}-{}^q\vec{A}] \quad (4.3)$$

with

$$\Phi_\omega[\vec{A}]=\exp\left[-\sum_{i,a=1}^{3}\int A_i^a(\vec{x})\omega A_i^a(\vec{x})\,d^3x/2\right], \tag{4.4}$$

where ω is a positive linear operator chosen in a way that $\Phi_\omega[\vec{A}-{}^q\vec{A}]$ is a local solution of the Schrödinger equation (3.2), i.e., $\omega=(-\sum_{i=1}^{3}\partial_i^2)^{1/2}$. The gauge-invariant vacuum state of the quantized Yang-Mills field is a Bloch state (4.3).

To describe a quantum decay process of the metastable vacuum state ${}^qA_i(\vec{x})$, we shall approximate Eq. (4.3) by taking only a Gaussian functional peaked around ${}^qA_i(\vec{x})$ into account, obtaining

$$\Omega_\theta[\vec{A}]\simeq e^{iq\theta}\Phi_\omega[\vec{A}-{}^q\vec{A}]. \tag{4.5}$$

Then Eq. (4.1) becomes

$$\dot{A}_i^a(t,\vec{x})=\omega\{A_i^a(t,\vec{x})-{}^qA_i^a(\vec{x})\}+Z_i^a(t,\vec{x}), \tag{4.6}$$

which is a linear inhomogeneous stochastic differential equation of white-noise type.[18] A solution of Eq. (4.6) under the initial condition $A_i^a(0,\vec{x})={}^qA_i^a(\vec{x})$ manifests the quantum decay process of the metastable vacuum state ${}^qA_i(\vec{x})$.

Such a solution of Eq. (4.6) can be obtained by introducing a contraction semigroup on $*\epsilon(R^3)$ (Ref. 19),

$$T(t)=\exp(-\omega t)\delta^{ab}\delta_{ij},\ t\geq 0. \tag{4.7}$$

Namely,

$$A_i^a(t,\vec{x})=T(0)^qA_i^a(\vec{x})+\int_0^t T(t-u)Z_i^a(u,\vec{x})du \tag{4.8}$$

solves Eq. (4.6) with the initial condition.[18] Equation (4.8) uniquely determines a distribution-valued Gaussian process $A_i^a(t,\vec{x})$ with mean

$$E\{A_i^a(t,\vec{x})\}=T(0)^qA_i^a(\vec{x}) \tag{4.9}$$

and covariance

$$E\left(\sum_{i,a=1}^{3}\int[A_i^a(x)-E\{A_i^a(x)\}]f_i^a(x)\,d^4x\sum_{j,b=1}^{3}\int[A_j^b(y)-E\{A_j^b(y)\}]h_j^b(y)d^4y\right)=\sum_{i,a=1}^{3}\int f_i^a(x)(-\Delta_4)^{-1}h_i^a(x)d^4x, \tag{4.10}$$

where f_i^a and h_i^a are arbitrary test functions, $\Delta_4=\sum_{\mu=0}^{4}\partial_\mu^2$ is a four-dimensional Laplacian, and we have made an abbreviation $x=(t,\vec{x})$ and $d^4x=dt\ d^3x$. This is nothing but a Euclidean-Markov field[20] of Gaussian type.[18]

Thus we have found, within the realm of the stochastic quantization, that the quantum decay process of the metastable vacuum state ${}^qA_i(\vec{x})$ can be represented by the Euclidean-Markov field (4.8). Needless to say, the integral which appears in the right-hand side of Eq. (4.10) is of an infrared-divergent nature. In other words, an object which manifests the quantum decay process of the metastable vacuum state has a long-range correlation such as the Coulomb gas. We may be allowed to consider the object as a quantum field theoretical version of the instanton which was originally introduced as an indication of the vacuum-tunneling phenomena.[1] The Euclidean-Markov field (4.8) may play an important role in quark confinement as was suggested by Polyakov.[1]

V. MOST PROBABLE TUNNELING PATH AND INSTANTON

In the present section we shall justify the concept of instanton (or pseudoparticle) from the probability theoretical point of view.

We found the vacuum-tunneling amplitude of the quantized Yang-Mills field in the $A_0=0$ gauge to be given by the Wiener integral (3.17). This can be written also in a functional path-integral form (3.19). However, as the functional path-integral expression of the Wiener measure (3.18) has only a formal meaning, we can no longer utilize Eq. (3.19) to derive a rigorous probability theoretical characterization of the instanton.

Let us start with the transition probability law of the quantized Yang-Mills field $A_i^a(t,\vec{x})$ in the vacuum state (3.16). It is convenient to approximate the Wiener integral in Eq. (3.16) by taking only an n-triple functional integral into account

$$\int_{\Gamma(\vec{A}_s|\vec{A}'_u)}d\mu_w(\vec{A}(t))\simeq\gamma\int\exp\left[-\frac{\|\vec{A}-\vec{A}_n\|^2}{2(s-t_n)}\right]\cdots\exp\left[-\frac{\|\vec{A}_1-\vec{A}'\|^2}{2(t_1-u)}\right]\delta\vec{A}_n\cdots\delta\vec{A}_1 \tag{5.1}$$

with $s>t_n>\cdots>t_1>u$,[21] where

$$\|\vec{A}\|^2=\sum_{i,a=1}^{3}\int A_i^a(\vec{x})A_i^a(\vec{x})d^3x$$

is a norm on $*\epsilon(R^3)$ (Ref. 14) and γ is an infinitesimal constant. Then Eq. (3.16) becomes

$$Q[\vec{A};s|\vec{A}';u]\simeq(\Omega[\vec{A}]/\Omega[\vec{A}'])\gamma\int\exp\left[-\frac{\|\vec{A}-\vec{A}_n\|^2}{2(s-t_n)}\right]\cdots\exp\left[-\frac{\|\vec{A}_1-\vec{A}'\|^2}{2(t_1-u)}\right]$$
$$\times\exp\left\{-\tfrac{1}{2}\left[\left\|\vec{B}\left(\frac{\vec{A}+\vec{A}_n}{2}\right)\right\|^2(s-t_n)+\cdots+\left\|\vec{B}\left(\frac{\vec{A}_1+\vec{A}}{2}\right)\right\|^2(t_1-u)\right]\right\}\delta\vec{A}_n\cdots\delta\vec{A}_1$$
$$\simeq(\Omega[\vec{A}]/\Omega[\vec{A}'])\gamma\int\exp\left\{-\tfrac{1}{2}\left[\left(\frac{\|\vec{A}-\vec{A}_n\|}{s-t_n}\right)^2+\left\|\vec{B}\left(\frac{\vec{A}+\vec{A}_n}{2}\right)\right\|^2\right](s-t_n)-\cdots\right.$$
$$\left.-\tfrac{1}{2}\left[\left(\frac{\|\vec{A}_1-\vec{A}'\|}{t_1-u}\right)^2+\left\|\vec{B}\left(\frac{\vec{A}_1+\vec{A}'}{2}\right)\right\|^2\right](t_1-u)\right\}\delta\vec{A}_n\cdots\delta\vec{A}_1\,. \tag{5.2}$$

Now what is left for us to do is to replace each functional integration in Eq. (5.2) by taking the maximum value in the exponent, noting the fact that the most probable value of a Gaussian distribution might dominate the Gaussian integral, obtaining

$$Q[\vec{A};s|\vec{A}';u]\simeq(\Omega[\vec{A}]/\Omega[\vec{A}'])\gamma\exp\left\{-\tfrac{1}{2}\left[\left(\frac{\|\vec{A}-\vec{A}_n\|}{s-t_n}\right)^2+\left\|\vec{B}\left(\frac{\vec{A}+\vec{A}_n}{2}\right)\right\|^2\right](s-t_n)-\cdots\right.$$
$$\left.-\tfrac{1}{2}\left[\left(\frac{\|\vec{A}_1-\vec{A}'\|}{t_1-u}\right)^2+\left\|\vec{B}\left(\frac{\vec{A}_1+\vec{A}'}{2}\right)\right\|^2\right](t_1-u)\right\}_{\max}, \tag{5.3}$$

where $[\cdot]_{\max}$ means to take a maximum value.[21] Passing to the limit $n\to\infty$, we finally obtain an approximative expression of the transition probability law of the quantized vacuum field configuration as follows[13]:

$$Q[\vec{A};s|\vec{A}';u]\simeq(\Omega[\vec{A}]/\Omega[\vec{A}'])\gamma\exp\left[-\sum_{i,a=1}^{3}\int_u^s dt\int d^3x\,(\tfrac{1}{2}\dot{A}_i^a\dot{A}_i^a+\tfrac{1}{2}B_i^aB_i^a)\right]_{\max}. \tag{5.4}$$

Correspondingly the vacuum-tunneling amplitude (3.17) has an approximative expression,

$$\mathrm{Amp}[{}^p\vec{A};\infty|{}^q\vec{A};-\infty]$$
$$\simeq\gamma\exp\left[-\sum_{i,a=1}^{3}\int_{-\infty}^{\infty}dt\int d^3x\,(\tfrac{1}{2}\dot{A}_i^a\dot{A}_i^a+\tfrac{1}{2}B_i^aB_i^a)\right]_{\max}. \tag{5.5}$$

This is the well-known WKB prescription.

Let us introduce a notion of the most probable tunneling path $\bar{A}_i^a(t,\vec{x})$. It is a classical Euclidean Yang-Mills field which minimizes the Euclidean action

$$S_E[\vec{A}]=\sum_{i,a=1}^{3}\int(\tfrac{1}{2}\dot{A}_i^a\dot{A}_i^a+\tfrac{1}{2}B_i^aB_i^a)dt\,d^3x$$
$$=\tfrac{1}{4}\sum_{\mu,\nu=0}^{3}\int F_{\mu\nu}^aF_{\mu\nu}^a\,d^4x \tag{5.6}$$

under the boundary conditions

$$\lim_{t\to\infty}\bar{A}_i^a(t,\vec{x})={}^pA_i^a(\vec{x})\,, \tag{5.7}$$

$$\lim_{t\to\infty}\bar{A}_i^a(t,\vec{x})={}^qA_i^a(\vec{x})\,. \tag{5.8}$$

Then Eq. (5.5) becomes

$$\mathrm{Amp}[{}^pA;\infty|{}^qA;-\infty]$$
$$\simeq\gamma\exp\left[-\sum_{i,a=1}^{3}\int(\tfrac{1}{2}\dot{\bar{A}}_i^a\dot{\bar{A}}_i^a+\tfrac{1}{2}\bar{B}_i^a\bar{B}_i^a)dt\,d^3x\right]. \tag{5.9}$$

Thus we have found that the instanton, associated with the Euclidean action-minimum classical field configuration, manifests the most probable tunneling path of the quantized Yang-Mills field between metastable vacuum states.

VI. SUMMARY AND DISCUSSIONS

We have presented a probability theoretical approach with the vacuum-tunneling phenomena in SU(2) Yang-Mills theory by making use of Nelson's stochastic quantization procedure. From this probability theoretical point of view we conclude that the vacuum tunneling can be described by the Euclidean formulation. Namely, the tunneling amplitude of the quantized Yang-Mills field, in the vacuum state, between metastable vacuum states characterized by different Pontryagin indices is given by the Euclidean path integral (3.19). Heretofore the Euclidean path-integral description of the vacuum-tunneling phenomena was proposed only because one knew from experience in quantum mechanics that such a Euclidean (imaginary time) formulation can be used to obtain the WKB prescription of the tunneling phenomena.[22,23] In the present paper we have derived the Euclidean formulation rigorously from the probability theoretical point of view working with quantum field theory. The quantized vacuum field configuration, which manifests the quantum decay process of metastable vacuum states, has been also shown to be a Euclidean-Markov field of Gaussian type. Classical Euclidean Yang-Mills fields which minimize the Euclidean action, i.e., instanton, have been shown to represent the most probable tunneling path between metastable vacuum states.

It is worthwhile to notice that the deviation of the Yang-Mills field (4.8) which manifests the decay process of metastable vacuum states, $D_i^a(t,\vec{x}) = A_i^a(t,\vec{x}) - E\{A_i^a(t,\vec{x})\}$, forms a free Euclidean-Markov field.[20] That is, $D_i^a(t,\vec{x})$ is a Gaussian random field on the Euclidean space-time R^4 with mean 0 and covariance $(-\Delta_4)^{-1}$. Such a random field with a long-range correlation has been supposed to play an important role in quark confinement by its screening effect.[1]

We may be allowed to emphasize that the probability theoretical approach developed in the present paper can be used in clarifying the mechanism of quark confinement if it is applied to the case of the SU(3) gauge group. An application to the analyses of extended models of hadron, e.g., the string model and the bag model, as suggested by Haba and Lukierski,[24,25] also seems to be worth considering.

We have worked in the noncovariant $A_0 = 0$ gauge. The covariant Lorentz gauge, $\sum_{\mu=0}^{3\prime}\partial_\mu A_\mu = 0$, forces us to introduce a concept of negative probability. However, this is beyond the scope of our probability theoretical point of view.

We have ignored couplings of fermions to the Yang-Mills field. To see the effect of the presence of fermion fields to the gauge field, we investigate a spatially homogeneous σ model and calculate the decay rate of metastable vacuum states from the probability theoretical point of view in the Appendix.

APPENDIX: ANALYSIS OF A SPATIALLY HOMOGENEOUS σ MODEL

In this appendix we present a simple model calculation of the decay rate of a metastable vacuum states. Consider a σ model described by the Lagrangian density

$$\mathcal{L} = \tilde{\psi}\left(i\sum_{\mu=0}^{3}\gamma_\mu\partial_\mu - g\sigma\right)\psi + \tfrac{1}{2}\sum_{\mu=0}^{3\prime}(\partial_\mu\sigma)^2 - \frac{\lambda}{4}(\sigma^2 - v^2)^2 . \tag{A1}$$

where g, λ, and v are constants and γ's Dirac matrices, respectively. The Dirac spinor ψ and the real scalar σ represent a fermion field and a boson field interacting with each other through the Yukawa coupling scheme (A1). Field equations obtained from (A1) are

$$\left(i\sum_{\mu=0}^{3}\gamma_\mu\partial_\mu - g\sigma\right)\psi = 0 , \tag{A2}$$

$$\sum_{\mu=0}^{3\prime}\partial_\mu{}^2\sigma + \lambda\sigma(\sigma^2 - v^2) = -g\tilde{\psi}\psi . \tag{A3}$$

We shall restrict the discussion to the spatially homogeneous case in which $\psi(t,\vec{x}) = \psi(t)$ and $\sigma(t,\vec{x}) = \sigma(t)$ hold. Then Eqs. (A2) and (A3) become

$$i\dot{\psi}(t) = g\gamma_0\sigma(t)\psi(t) , \tag{A4}$$

$$\ddot{\sigma}(t) = -\lambda\sigma(t)[\sigma(t)^2 - v^2] - g\tilde{\psi}(t)\psi(t) . \tag{A5}$$

Equation (A4) can be solved in the product integral form[26]

$$\psi(t) = \prod_{s=0}^{t}[1 - ig\gamma_0\sigma(s)\,ds]\psi(0) , \tag{A6}$$

or a more familiar T-product form

$$\psi(t) = T\exp\left[-ig\gamma_0\int_0^t \sigma(s)\,ds\right]\psi(0) , \tag{A7}$$

which yields a conservation law for the fermion number

$$\tilde{\psi}(t)\psi(t) = \tilde{\psi}(0)\psi(0) . \tag{A8}$$

Thus we find that the spatially homogeneous σ model is nothing but an anharmonic oscillator described by the equation of motion

$$\ddot{\sigma}(t) = -\lambda\sigma(t)[\sigma(t)^2 - v^2] - g\tilde{\psi}(0)\psi(0) \tag{A9}$$

(see Ref. 9).

A quantum theoretical vacuum state of such an anharmonic oscillator as (A9) is given by the wave function $u(\sigma) \in L_2(R)$ which satisfies the Schrödinger equation

$$\left[-\frac{1}{2}\frac{\partial^2}{\partial\sigma^2} + \frac{\lambda}{4}(\sigma^2 - v^2)^2 + g\tilde{\psi}(0)\psi(0)\sigma\right]u(\sigma) = Eu(\sigma) . \tag{A10}$$

The potential $V(\sigma) = \lambda(\sigma^2 - v^2)^2/4 + g\tilde{\psi}(0)\psi(0)\sigma$ is bounded from below and has an absolute minimum $\sigma = -v$ and a relative minimum $\sigma = v$. In classical mechanics, the two minima $\sigma = \pm v$ represent different stable vacuum states. In quantum mechanics, however, the relative minimum $\sigma=v$ is rendered unstable by the tunnel effect. The field configuration $\sigma=v$ corresponds to a metastable vacuum state.

Now we shall calculate the decay rate of the metastable vacuum within the realm of our probability theoretical formulation. The quantized motion of the vacuum field configuration is expressed as a diffusion process $\sigma=\Sigma(t)$ described by the stochastic differential equation

$$d\Sigma(t)=a(\Sigma(t))dt+dW(t)\,, \tag{A11}$$

where $a(\sigma)=\partial \ln u(\sigma)/\partial\sigma$, and $W(t)$ is a Wiener process with a variance parameter $\frac{1}{2}$. The diffusion process $\Sigma(t)$ has a stationary probability distribution $u(\sigma)^2$. The decay rate of the metastable vacuum state can be calculated by evaluating the transition probability law of the diffusion process $\Sigma(t)$.

The transition probability law $p(\sigma,t|\sigma',t')$, $t>t'$, is known to be an elementary solution of the Fokker-Planck equation

$$\frac{\partial}{\partial t}p=-\frac{\partial}{\partial\sigma}[a(\sigma)p]+\frac{1}{2}\frac{\partial^2}{\partial\sigma^2}p\,, \tag{A12}$$

which can be transformed into a self-adjoint equation

$$-\frac{\partial}{\partial t}f=\left[-\frac{1}{2}\frac{\partial^2}{\partial\sigma^2}+v(\sigma)-E\right]f \tag{A13}$$

for the relative transition probability law

$$f(\sigma,t|\sigma',t')=u(\sigma)^{-1}p(\sigma,t|\sigma',t')u(\sigma')\,.$$

The decay rate of the metastable vacuum $\sigma=v$ is given by investigating an asymptotic behavior of the transition probability law $p(-v,t|v,0)$ for large t.[27] Since the elementary solution of Eq. (A12) has an asymptotic expression

$$f(-v,t|v,0)=u(-v)u(v)+u_1(-v)u_1(v)\times\exp[-(E_1-E)t]\,, \tag{A14}$$

and also Eq. (A13),

$$p(-v,t|v,0)\simeq u(-v)^2\left\{1+\frac{u_1(-v)u_1(v)}{u(-v)u(v)}\times\exp[-(E_1-E)t]\right\}, \tag{A15}$$

the decay rate is found to be (E_1-E),[9] where E_1 denotes the energy eigenvalue of the first excited state and $v_1(\sigma)\in L_2(R)$ the eigenfunction

$$\left[-\frac{1}{2}\frac{\partial^2}{\partial\sigma^2}+V(\sigma)\right]u_1(\sigma)=E_1u_1(\sigma),\quad E_1>E\,. \tag{A16}$$

The level splitting, i.e., the decay rate, can be computed with the use of the WKB prescription,[28] obtaining

$$E_1-E\simeq V(\xi_1)-V(\xi)\,, \tag{A17}$$

where ξ and ξ_1 are the real roots of

$$\int_\eta^\xi [V(\xi)-V(\sigma)]^{1/2}d\sigma=\frac{\pi}{2\sqrt{2}},\quad \eta=V^{-1}[V(\xi)]\neq\xi \tag{A18}$$

and

$$\int_{\eta_1}^{\xi_1} [V(\xi_1)-V(\sigma)]^{1/2}d\sigma=\frac{3\pi}{2\sqrt{2}},\quad \eta_1=V^{-1}[V(\xi_1)]\neq\xi_1\,, \tag{A19}$$

respectively.

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