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Supersymmetry at high temperatures

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We investigate the properties of Green's functions in a spontaneously broken supersymmetric model at high temperatures. We show that, even at high temperatures, we do not get restoration of supersymmetry, at least in the one-loop approximation.

I. INTRODUCTION

Supersymmetry[1] is a rich theoretical concept which allows one to mix bosons and fermions in the same multiplet, which may have relevance for particle unification schemes. The theories that are most important are the ones which possess both supersymmetry and gauge invariance[2] simultaneously, because we believe that the electromagnetic and weak interactions can be understood as gauge theories. As in unified gauge theories, however, we would like supersymmetry to be spontaneously broken because fermions and bosons with degenerate masses do not occur in nature. One implements the idea of spontaneous breakdown of supersymmetry by adding a Fayet-Iliopoulos[3] term to the action. One then obtains spontaneous breakdown of either the gauge symmetry or the supersymmetry, depending on the signature of the term. The gauge symmetry and supersymmetry are so intertwined that when one is broken the other is preserved.

It has been pointed out in the literature that at high temperatures a symmetry that is spontaneously broken in ordinary gauge theories can be restored.[4] The theory undergoes a phase transition at a critical temperature which is reminiscent of the Meissner effect in ferromagnets. It has been demonstrated that the one-loop corrections at high temperatures wash away the local minima characterized by the classical solutions in such theories, giving us "restoration" of the symmetry.[5,6,7]

Similar investigations can be carried out in theories possessing both supersymmetry and gauge symmetry. They are interesting because of the special nature of symmetry breaking in such theories. In particular, it is quite natural to ask whether restrictions to a completely symmetric theory at very high temperatures select out a definite signature of the Fayet-Iliopoulos term. We show by explicit calculations and later through plausible arguments that a theory possessing both supersymmetry and gauge symmetry always displays spontaneous breakdown of supersymmetry at very high temperatures irrespective of the signature of the Fayet-Iliopoulos term.

In Sec. II, we study the problem of spontaneous symmetry breaking in the supersymmetric Higgs theory at zero temperature. We calculate the high-temperature effects in Sec. III and show in Sec. IV how supersymmetry gets broken at high temperatures. We emphasize here that the phenomena occur in the most general supersymmetric theory with a gauge invariance although we cannot say anything about the supergravity theories[8,9] because no fully renormalizable model exists.

II. SUPERSYMMETRIC HIGGS MODEL

In this section we study the supersymmetric Higgs model,[10] namely, a vector multiplet interacting with a left-handed chiral multiplet where all fields are massless. The Lagrangian contains the vector field A_μ, the Majorana spinor λ, the left-handed Dirac spinor ψ_L, and the complex scalar field ϕ. After elimination of the auxiliary fields the Lagrangian has the form

$$\mathcal{L} = -\tfrac{1}{4}F_{\mu\nu}{}^2 + \tfrac{1}{2}i\bar{\lambda}\partial\!\!\!/\lambda + i\bar{\psi}_L D\!\!\!\!/\,\psi_L + (D_\mu\phi)^\dagger(D_\mu\phi) - ie\sqrt{2}(\bar{\psi}_L\lambda\phi - \phi^\dagger\bar{\lambda}\psi_L) - \tfrac{1}{2}(\xi + e\phi^\dagger\phi)^2, \quad (2.1)$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \quad D_\mu\phi = \partial_\mu\phi - ieA_\mu\phi,$$

e is the electric charge, and ξ is the parity-violating parameter.

This Lagrangian is invariant under the following supersymmetry transformations:

$$\begin{aligned}
\delta\phi &= \bar{\epsilon}\psi_L, \\
\delta\psi_L &= -\tfrac{1}{2}i(1-\gamma_5)\partial\!\!\!/\phi\epsilon, \\
\delta\lambda &= \frac{1}{\sqrt{2}}\sigma^{\mu\nu}F_{\mu\nu}\epsilon + \frac{i}{\sqrt{2}}(\xi + e\phi^\dagger\phi)\gamma_5\epsilon, \\
\delta A_\mu &= \frac{i}{\sqrt{2}}\bar{\epsilon}\gamma_\mu\lambda.
\end{aligned} \quad (2.2)$$

Here ϵ is a constant Majorana spinor parameter.

The theory is also invariant under the following local gauge transformations:

$$\delta A_\mu = \frac{1}{e}\partial_\mu \alpha(x),$$
$$\delta\lambda = 0, \tag{2.3}$$
$$\delta\phi = -i\,\phi\,\alpha(x),$$
$$\delta\psi_L = -i\,\psi_L \alpha(x).$$

The scalar potential has the form

$$V(\phi) = \tfrac{1}{2}(\xi + e\phi^\dagger\phi)^2. \tag{2.4}$$

Without loss of generality we can choose $e > 0$. Then two cases arise, depending on whether $\xi < 0$ or $\xi > 0$.

Case I: $\xi < 0$. The potential has a minimum when $\langle|\phi|^2\rangle = -\xi/e$. If we decompose the complex scalar field into real fields and shift the fields by their classical values, it is easy to show that

$$\delta\lambda = \frac{1}{\sqrt{2}}\sigma^{\mu\nu}F_{\mu\nu}\epsilon + \frac{i}{\sqrt{2}}e\phi^\dagger\phi\gamma_5\epsilon. \tag{2.5}$$

That is, there are no constant terms in the supersymmetry transformation laws of the spinors. Supersymmetry, therefore, is still a good symmetry. However, it can be easily shown that the gauge symmetry is spontaneously broken. One of the scalar fields, the vector field, and the diagonalized spinors become massive. The Goldstone scalar is absorbed by imposing a gauge condition.

Case II: $\xi > 0$. The minimum for the potential occurs when the scalar field has zero vacuum expectation value. If we look at the transformation laws, we find that gauge symmetry is not broken. However, in the supersymmetry transformations we find

$$\delta\lambda = \frac{i}{\sqrt{2}}\xi\gamma_5\epsilon + \cdots, \tag{2.6}$$

where the dots stand for terms involving fields. Therefore, we note that supersymmetry is spontaneously broken and λ is the Goldstone spinor. This analysis tells us clearly how the symmetry breaking depends on the signature of the parameter ξ.

III. SUPERSYMMETRY AT HIGH TEMPERATURES

Finite-temperature calculations have been in the literature for a long time. The important observation is the fact that the finite-temperature Green's functions satisfy the same differential equation as the zero-temperature Green's function except that they satisfy periodic boundary conditions for imaginary times. Dolan and Jackiw and Weinberg have calculated symmetry behavior at high temperatures for nongauge and gauge theories. They have argued and shown that the only dominant higher-order contribution at high temperatures comes from the one-loop graphs for weak coupling. Weinberg has given an operator method for calculating one-loop effects for nongauge theories, whereas Dolan and Jackiw have developed a functional diagrammatic method. For our purpose, we follow the method of Dolan and Jackiw and direct the interested readers to Ref. 6 for details.

We start with the super-Higgs action of Sec. II, but now decompose the complex field into two real components:

$$\mathcal{L} = -\tfrac{1}{4}F_{\mu\nu}^2 + \tfrac{1}{2}i\bar\lambda\not\partial\lambda + i\bar\psi_L\not D\psi_L + \tfrac{1}{2}(\partial_\mu\phi_i)^2 + eA_\mu\epsilon_{ij}\phi_i\partial_\mu\phi_j - ie[\bar\psi_L\lambda(\phi_1 + i\phi_2) - \bar\lambda\psi_L(\phi_1 - i\phi_2)]$$
$$- \tfrac{1}{2}(\xi + e\phi_i^2)^2 + \tfrac{1}{2}e^2A_\mu^2\phi_i^2, \tag{3.1}$$
$$\phi \equiv (1/\sqrt{2})(\phi_1 + i\phi_2).$$

Let us assume that the classical solution occurs at $\langle\phi_a\rangle = \hat\phi_a$. Shifting the fields, $\phi_a \to \phi_a - \hat\phi_a$, we have

$$\mathcal{L} = -\tfrac{1}{4}F_{\mu\nu}^2 + \tfrac{1}{2}i\bar\lambda\not\partial\lambda + i\bar\psi_L\not D\psi_L + \tfrac{1}{2}(\partial_\mu\phi_i)^2 - eA_\mu\epsilon_{ij}\hat\phi_i\partial_\mu\phi_j + eA_\mu\epsilon_{ij}\phi_i\partial_\mu\phi_j + ie\bar\psi_L\lambda(\hat\phi_1 + i\hat\phi_2)$$
$$-ie\bar\lambda\psi_L(\hat\phi_1 - i\phi_2) - ie[\bar\psi_L\lambda(\phi_1 + i\phi_2) - \bar\lambda\psi_L(\phi_1 - i\phi_2)] - \tfrac{1}{2}(\xi + \tfrac{1}{2}e\phi_i^2 - e\phi_i\hat\phi_i + \tfrac{1}{2}e\hat\phi_i^2)^2 + \tfrac{1}{2}e^2A_\mu^2(\phi_i^2 - 2\phi_i\hat\phi_i + \hat\phi_i^2). \tag{3.2}$$

Keeping only the part of the action which is quadratic in fields, we obtain

$$\mathcal{L}_0(\hat\phi_i, \phi_i, \lambda, \psi_L, A_\mu) = -\tfrac{1}{4}F_{\mu\nu}^2 + \tfrac{1}{2}\bar\lambda\not\partial\lambda + i\bar\psi_L\not D\psi_L + \tfrac{1}{2}\partial_\mu\phi_i^2 - e\epsilon_{ij}\hat\phi_iA_\mu\partial_\mu\phi_j + ie\bar\psi_L\lambda(\hat\phi_1 + i\hat\phi_2)$$
$$- ie\bar\lambda\bar\psi_L(\hat\phi_1 - i\hat\phi_2) - \tfrac{1}{2}e^2\phi_i\phi_j\hat\phi_i\hat\phi_j - \tfrac{1}{2}e\xi\phi_i^2 - \tfrac{1}{4}e^2\hat\phi_i^2\phi_i^2 - \tfrac{1}{2}e^2A_\mu^2\hat\phi_i^2$$
$$= -\tfrac{1}{4}F_{\mu\nu}^2 + \tfrac{1}{2}m_V^2A_\mu^2 + \tfrac{1}{2}(\partial_\mu\phi_i)^2 - \tfrac{1}{2}\phi_iM_{ij}^2\phi_j - e\epsilon_{ij}\hat\phi_iA_\mu\partial_\mu\phi_j + \tfrac{1}{2}i\bar\lambda\not\partial\lambda + i\bar\psi_L\not\partial\psi_L + ie\bar\psi_L\lambda P - ie\bar\lambda\psi_LQ, \tag{3.3}$$

where we have defined

$$m_V^2 = e^2\hat\phi_i^2, \quad M_{ij}^2 = (e\xi + \tfrac{1}{2}e^2\hat\phi_k^2)\delta_{ij} + e^2\hat\phi_i\hat\phi_j, \quad P = \hat\phi_1 + i\hat\phi_2, \quad Q = \hat\phi_1 - i\hat\phi_2. \tag{3.4}$$

The one-loop contribution to the effective potential at a temperature β^{-1} is given by

$$V_1^\beta(\hat{\phi}^2) = i \ln \int \prod [d\phi]\, e^{iI}$$
$$= i \ln \int \prod [d\phi] \exp[i(I_{\text{boson}} + I_{\text{fermion}})], \tag{3.5}$$

where $I = \int d^4\chi \mathcal{L}$ and $[d\phi]$ stands generically for all fields.

We can, therefore, separate the one-loop contribution of bosons from fermions:

$$V_1^\beta = V_{1\,\text{boson}}^\beta + V_{1\,\text{fermion}}^\beta,$$
$$V_{1\,\text{boson}}^\beta = i \ln \int \prod [d\varphi_{\text{boson}}] \exp(iI_{\text{boson}}), \tag{3.6}$$
$$V_{1\,\text{fermion}}^\beta = i \ln \int \prod [d\varphi_{\text{fermion}}] \exp(iI_{\text{fermion}}).$$

Let us now evaluate the fermion contribution explicitly:

$$V_{1F}^\beta = i \ln \int d\lambda d\bar{\lambda} d\bar{\psi}_L d\psi_L \exp\left[i \int d^4\chi(\tfrac{1}{2} i \bar{\lambda}\partial\!\!\!/\lambda + i\bar{\psi}_L\partial\!\!\!/\psi_L + ie\bar{\psi}_L\lambda P - ie\bar{\lambda}\psi_L Q)\right]. \tag{3.7}$$

Noting that both λ and $\bar{\psi}_L$ are two-component spinors, the integral is easily evaluated to be

$$V_{1F}^\beta = \tfrac{1}{2} i \ln \det(\partial^2 + 2e^2\hat{\phi}_i^2), \tag{3.8}$$

which, following Ref. 6, one can write as

$$V_{1F}^\beta = \frac{2i}{-i\beta}\int_k \ln(\vec{k}^2 + 2e^2\hat{\phi}_i^2)$$
$$= -\frac{2}{\beta}\int \frac{d^3k}{(2\pi)^3}\sum_n \ln\left[-\frac{\pi^2(2n+1)^2}{\beta^2} - E^2(k,\hat{\phi})\right]$$
$$= -4\int \frac{d^3k}{(2\pi)^3}\left[\tfrac{1}{2}E(k,\hat{\phi}) + \frac{1}{\beta}\ln(1+e^{-\beta E})\right], \tag{3.9}$$

where $E^2 = \vec{k}^2 + 2e^2\hat{\phi}_i^2$. Thus, we see that there are two contributions—one independent of the temperature and the other depending on temperature:

$$V_{1F}^0 = -2\int \frac{d^3k}{(2\pi)^3} E(k,\hat{\phi}),$$
$$\bar{V}_{1F}^\beta = -\frac{4}{2\pi^2\beta^4}\int_0^\infty d\chi\, \chi^2 \ln\{1 + \exp[-(\chi^2 + \beta^2 m^2)]\}, \tag{3.10}$$
$$m^2 = 2e^2\hat{\phi}_i^2.$$

For small β, i.e., for very high temperatures, the second integral can be approximately evaluated and takes the form

$$\bar{V}_{1F}^\beta = -\frac{7\pi^2}{180\beta^4} + \frac{m^2}{12\beta^2} + \frac{m^4}{16\pi^2}\ln m^2\beta^2 + \frac{m^4 c}{16\pi^2} + O(m^6\beta^2).$$

Therefore,

$$\frac{\delta \bar{V}_{1F}^\beta}{\delta\hat{\phi}_i^2} = \frac{e^2}{6\beta^2} + \frac{e^4\hat{\phi}_i^2}{4\pi^2}\ln 2e^2\beta^2\hat{\phi}_i^2 + \frac{e^4\hat{\phi}_i^2}{4\pi^2} + \frac{e^4\hat{\phi}_i^2 c}{4\pi^2} + \cdots,$$

so that

$$\frac{\delta \bar{V}_{1F}^\beta}{\delta\hat{\phi}_i^2} > 0. \tag{3.11}$$

The bosonic one-loop contribution has already been discussed in Ref. 6, and for completeness we give the expression for the one-loop potential. If one works in the Lorentz gauge, then

$$V_{1B}^{\beta} = V_1^0 + \bar{V}_{1B}^{\beta},$$

$$V_{1B}^0 = \frac{1}{6}\left(m_1^4 \ln\frac{m_1^2}{m_2^2} + 3\mu^2 \ln\frac{\mu^2}{m^2} + R_1^4 \ln\frac{R_1^2}{m^2} + R_2^4 \ln\frac{R_2^2}{m^2} - 2e^2 m^2 \hat{\phi}_i^2 - \tfrac{15}{4} e^4 \hat{\phi}^4 + \alpha m_2^2 \mu^2 + a\hat{\phi}^4\right), \tag{3.12}$$

$$\bar{V}_{1B}^{\beta} = \frac{1}{2\pi^2\beta^4}\int_0^\infty d\chi\, \chi^2 (3\ln\{1 - \exp[-(\chi^2 + \beta^2\mu^2)^{1/2}]\} + \ln\{1 - \exp[-(\chi^2 + \beta^2 m_1^2)^{1/2}]\} + \ln\{1 - \exp[-(\chi^2 + \beta^2 R_1^2)^{1/2}]\}$$
$$+ \ln\{1 - \exp[-(\chi^2 + \beta^2 R_2^2)^{1/2}]\}),$$

where $\hat{\phi}^4 = (\hat{\phi}_i^2)^2$, $m^2 = e\xi$, $m_1^2 = m^2 + \frac{3}{2}e^2\hat{\phi}_i^2$, $m_2^2 = m^2 + \frac{1}{2}e^2\hat{\phi}^2$, and R_i^2's are roots of $x^2 - m_2^2 x + \alpha\mu^2 m_2^2$. It is not difficult to see that

$$\frac{\delta \bar{V}_{1B}^{\beta}}{\delta \hat{\phi}_i^2} > 0. \tag{3.13}$$

Thus,

$$\frac{\delta \bar{V}_{1B}^{\beta}}{\delta \hat{\phi}_i^2} + \frac{\delta \bar{V}_{1F}^{\beta}}{\delta \hat{\phi}_i^2} > 0,$$

so that

$$\frac{\delta \bar{V}_1^{\beta}}{\delta \hat{\phi}_i^2} > 0. \tag{3.14}$$

However, we know that the effective potential is given by

$$V^{\beta} = V^0 + \bar{V}_1^{\beta}, \tag{3.15}$$

where for simplicity we have assumed that the temperature-independent parts of the one-loop potential have been conventionally mass renormalized. Therefore, the minimum of the potential occurs when

$$\frac{\delta V^{\beta}}{\delta \hat{\phi}} = 2\hat{\varphi}\frac{\delta V^{\beta}}{\delta \hat{\phi}_i^2} = 0, \quad \hat{\phi} = (\hat{\phi}_i^2)^{1/2}. \tag{3.16}$$

The solutions to these equations are (1) $\hat{\phi} = 0$, $\delta V^{\beta}/\delta\hat{\phi} = 0$ for a physical nontrivial value of $\hat{\phi}$, in which case there is symmetry breaking and the solution $\hat{\varphi} = 0$ corresponds to a local maximum; and (2) $\delta V^{\beta}/\delta\hat{\phi} \neq 0$ for a physical nontrivial value of $\hat{\phi}$, in which case the minimum of the potential occurs at $\hat{\phi} = 0$.

Let us therefore analyze

$$\frac{\delta V^{\beta}}{\delta \hat{\phi}^2} = \frac{\delta V_0}{\delta \hat{\phi}^2} + \frac{\delta \bar{V}_1^{\beta}}{\delta \hat{\phi}^2}$$
$$= \frac{e\xi}{2} + \frac{\delta \bar{V}_1^{\beta}}{\delta \hat{\phi}^2} = 0. \tag{3.17}$$

Without loss of generality we can choose $e > 0$.

(a) $\xi > 0$. In this case we note that since $\delta\bar{V}_1^{\beta}/\delta\hat{\phi}^2 > 0$ the previous equation cannot be satisfied for a physical $\hat{\phi}$. Therefore, the minimum of the potential occurs at $\hat{\phi} = 0$. However, from our analysis of Sec. II we know that this corresponds to the case of spontaneous breakdown of supersymmetry. Therefore, although we start with a theory that has spontaneous breakdown of supersymmetry, it is not restored as we go to higher and higher temperatures.

(b) $\xi < 0$. In this case, Eq. (3.17) can have a solution, and from the structure of $\delta\bar{V}_1^{\beta}/\delta\hat{\phi}^2$ it turns out that the only solution is at $\hat{\phi} = 0$:

$$\left.\frac{\delta \bar{V}_1^{\beta}}{\delta \hat{\phi}^2}\right|_{\hat{\phi}=0} = \left.\frac{\delta \bar{V}_{1B}^{\beta}}{\delta \hat{\phi}^2}\right|_{\hat{\phi}=0} + \left.\frac{\delta \bar{V}_{1F}^{\beta}}{\delta \hat{\phi}^2}\right|_{\hat{\phi}=0}$$
$$= +\frac{1}{12\beta^2}\left(3e^2 + \frac{3e^2}{2} + \frac{3e^2}{6}\right) + \frac{2e^2}{12\beta^2}$$
$$= \frac{5e^2}{12\beta^2} + \frac{2e^2}{12\beta^2} = \frac{7e^2}{12\beta^2}.$$

Thus,

$$\frac{\delta \bar{V}_1^{\beta}}{\delta \hat{\phi}^2} + \frac{e\xi}{2} = 0,$$

$$\frac{7e^2}{12\beta^2} + \frac{e\xi}{2} = 0,$$

and hence

$$\beta_c = \left|\frac{7e}{6\xi}\right|^{1/2}.$$

Thus, although we started out with a theory that had spontaneous breakdown of gauge symmetry, it is restored at temperatures higher than β_c. However, an interesting thing has happened as a consequence, that is, since the potential now has a minimum at $\hat{\phi} = 0$, supersymmetry is broken spontaneously. Thus, we have demonstrated that no matter what is the signature of the parameter ξ at high temperatures supersymmetry is automatically broken.

IV. CONCLUSIONS

We now present a pictorial argument of why this should happen. The symmetry restoration in ordinary theories can be easily understood as follows. At low temperatures, the potential displays structures that determine the classical solutions. However, as the system is heated, energy is raised and whatever fine structures one notices at lower temperatures are completely washed away and the vacuum becomes symmetric. We can extend simi-

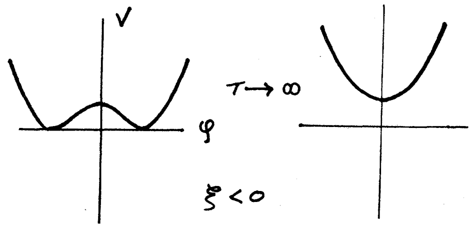

FIG. 1. Shape of potential at high temperatures $\xi < 0$.

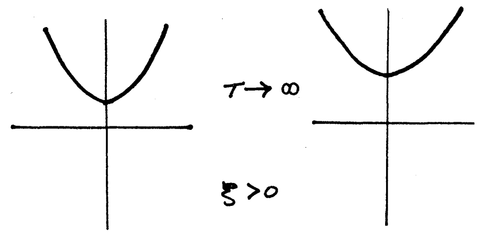

FIG. 2. Shape of potential at high temperature for $\xi > 0$.

lar observations to the case of supersymmetry also (see Fig. 1).

The potential has structure at low temperature. But the supersymmetric vacuums have lower energy, where gauge symmetry is broken. The local maximum possesses gauge symmetry, but violates supersymmetry. Therefore, as the temperature is raised the structure in the potential is wiped out and one has a gauge-symmetric vacuum which is not supersymmetric (see Fig. 1).

The potential has no structure. The minimum is at $\hat{\phi}=0$. This vacuum is gauge symmetric but is not supersymmetric. As the temperature is raised, the potential does not develop any structure and therefore the vacuum remains nonsupersymmetric (see Fig. 2).

What this analysis tells us is that if such theories are to be taken seriously, then one must learn to live with the fact that at high temperatures we do not have symmetry restoration. Whether this is good or bad is not clear because there is at least one other type of symmetry breaking that is not restored at high temperatures, namely, dynamical symmetry breaking. Yet dynamical symmetry breaking has continued to be an attractive idea. We cannot say anything about this question, in the case of supergravities, because no calculable model with symmetry breaking exists so far.

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