Baryon number of the universe

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(Received 9 June 1978)

We consider the possibility that the observed particle-antiparticle imbalance in the universe is due to baryon-number, C, and CP nonconservation. We make general observations and describe a framework for making quantitative estimates.

I. INTRODUCTION

Evidence exists[1] that the universe contains many more particles than antiparticles. A quantitative measure of this particle excess is given by the number of baryons within a unit thermal cell of size $R=T^{-1}$. Such a cell contains a single black-body photon.[2] In current cosmological theories it is a box, expanding according to[2]

$$R^{-1}\frac{d}{dt}R(t)\approx\left(\frac{8\pi}{3}G\rho\right)^{1/2}. \tag{1.1}$$

In the very early universe there was approximately 1 of every species of particle within a unit cell. However, the unit cell today contains only 10^{-9} baryons and essentially no antibaryons. If baryon number is conserved then the unit cell has always contained a baryon number of order 10^{-9}.

One cannot rule out the possibility that the universe was created with net baryon number and no explanation is needed. However, to quote Einstein: "If that's the way God made the world then I don't want to have anything to do with Him." In fact modern theories of particle interactions suggest that baryon number is not strictly conserved.[3,4] If this is true then today's baryon number is as much dependent on dynamical processes as on initial conditions. Indeed Yoshimura[5] has made the exciting suggestion that baryon-number violation can combine with CP noninvariance to produce a calculable net baryon number even though the universe was initially baryon neutral. Yoshimura has also made estimates[5] which indicate that this may be quantitatively plausible.

There are three interesting reasons to believe that baryon number is not exactly conserved:

(1) Black holes can swallow baryons.[6]

(2) Quantum-mechanical baryon-number violations have been discovered by '*t* Hooft in the *standard* Weinberg-Salam theory.[4]

(3) Superunified theories of strong, electromagnetic, and weak interactions naturally violate baryon number at superhigh energy.[3]

Although baryon-number violations are minute at ordinary energy, in cases (2) and (3) they may become significant at sufficiently high temperature.

Baryon-number violations is not enough to create an excess of baryons. The process itself must be particle-antiparticle asymmetric.[5] Otherwise the sign of the effect will be random and cancel in different cells. In this case the total baryon excess would be of the order of the square root of the total number of photons. However, the total number of photons in the observed universe is $\sim 10^{88}$ and the baryon number is $\sim 10^{79}$.

The required particle-antiparticle asymmetry is known to exist. Indeed charge conjugation is maximally violated in ordinary weak interactions. Were this the only asymmetry, CP invariance would destroy any possible effect because total baryon number changes sign under CP as well as C. Luckily CP violations are known to exist.[7]

CPT invariance also imposes a very interesting constraint on the expansion rate of the universe. As we shall see, CPT invariance ensures vanishing baryon density in thermal equilibrium. Therefore the expansion rate must remain rapid enough to prevent the baryon-number-violating forces from coming to equilibrium.

In this paper we will discuss how baryon-number, C, and CP nonconservation can conspire with the early Hubble expansion to produce an observable baryon excess.

As we shall see, the baryon excess may originate at or close to the very earliest times, $\sim 10^{-42}$ sec. At that time the temperature, energy density, and local space-time curvature are assumed to be of order unity in units of the Planck mass. The metric in Planck units is of the Robertson-Walker type[2]

$$(ds)^2=dt^2-R(t)^2dx_idx_i\,,$$

where $R(t)\sim 1$ at the Planck time $t=1$.

Let us follow the evolution of a single unit coordinate cell of dimensions $\Delta x_i = 1$. At the earliest of times it is a cube of unit volume (10^{-100} cm^3) in Planck units. We will assume that quantum fluctuations and gravitational interactions between gravitons and matter rapidly bring the universe to equilibrium at a temperature of unity. It follows that our unit cell initially contains about one elementary particle of each species. In current unified theories this means ~100 particles (photons, leptons, gravitons, intermediate bosons, quarks, vector gluons, Higgs bosons, superheavy bosons,...).

As the unit cell evolves it expands and cools. The process is not too different from the slow expansion of a box containing radiation. As in this case, the entropy within the cell is not significantly changed during the expansion. Roughly speaking this implies that the number of particles within that cell is the same today as it was at creation. Of course, by now, the only particles left are photons, neutrinos, and any excess protons and electrons. The others all annihilated or decayed when the temperature decreased below their mass.

The excess, expressed as a baryon number in the unit coordinate cell, is a number of order

$$n_B = \frac{N_B}{N_\gamma} n_\gamma \,,$$

where $N_B/N_\gamma \approx 10^{-9}$ and n_γ is the number of photons in the unit cell today. Assuming it is of the order of the number of elementary particle types, we must account for 10^{-7} baryons per box.

The estimates made in later sections for the baryon excess are too uncertain to be taken seriously. In addition to particle physics uncertainties, the properties of the initial conditions at creation are unknown and can influence the result. Our estimates are made for the most pessimistic case which we call "chaotic initial conditions." Such an initial condition is described by a density matrix ρ which is diagonal in baryon number and symmetric under the interchange of baryons and antibaryons. It is the sort of initial condition which would describe equilibrium if the earliest interactions respected baryon-number, C and CP invariance.

II. *CPT* AND EQUILIBRIUM

It is self-evident that if C or CP are symmetries of the equations of motion then no global baryon excess can result from baryon-number-violating processes. To illustrate the constraints imposed by CPT in an expanding universe we discuss some examples.

Consider a complex scalar field $\phi(x)$ in an expanding universe described by the metric

$$(ds)^2 = (dt)^2 - R(t)^2 (d\vec{x})^2 \,. \tag{2.1}$$

The action for this model is taken to be

$$s = \int d^4x \sqrt{-g}\,[g^{\mu\nu}\partial_\mu \phi \partial_\nu \phi^* - V(\phi)] \,, \tag{2.2}$$

where

$$V(\phi) = \lambda(\phi\phi^*)^n (\phi + \phi^*)(\alpha\phi^3 + \alpha^*\phi^{*3}) \tag{2.3}$$

and α is a complex phase. The baryon current density is

$$B_\mu = \sqrt{-g}\, i\phi \overleftrightarrow{\partial}_\mu \phi^* \,. \tag{2.4}$$

Note that $V(\phi)$ violates baryon-number conservation, C invariance ($\phi \to \phi^*$), and CP invariance.

The Hamiltonian for this model is

$$H(t) = \int d^3x \left[\frac{\pi\pi^*}{R^3(t)} + R(t)|\nabla\phi|^2 + R^3(t)\,V(\phi)\right] . \tag{2.5}$$

This Hamiltonian is invariant under the following CPT transformation[8]:

$$\phi(x) \to \phi(-x) \,, \tag{2.6}$$

$$\pi(x) \to -\pi(-x) \,. \tag{2.7}$$

The baryon number

$$B_\mu(x) = \begin{cases} i(\phi\pi - \pi^*\phi^*), & \mu = 0 \\ \sqrt{-g}\, i\phi \overleftrightarrow{\nabla}\phi^*, & \mu = i \end{cases} \tag{2.8}$$

changes sign under (2.6) and (2.7).

The CPT transformation is a symmetry of the spectrum of the instantaneous Hamiltonian but not of the equation of motion because of the explicit time dependence of H.

Now consider the case where the universe expands so slowly that at every instant it is in thermal equilibrium with respect to the instantaneous Hamiltonian $H(t)$. The density matrix at time t is

$$\rho(t) = \exp[-\beta(t)H(t)] \,. \tag{2.9}$$

Since CPT conjugate states carry equal energy but opposite baryon charge B the expectation value of B vanishes,

$$\langle B \rangle = \mathrm{Tr}(e^{-\beta(t)H(t)}\hat{B}) = 0 \,. \tag{2.10}$$

Therefore the only hope of generating baryon excess is for the baryon-number-violating interactions to remain out of thermal equilibrium. This implies that the rate of expansion of the universe has to be faster than the baryon-number-violating reaction rates.

Now we will discuss a second model to illustrate the possibility of baryon-number generation if we are out of equilibrium.

Consider a time-independent Hamiltonian $H = H_0 + V$. H_0 is baryon-number, C, and CP conserving and V is a small perturbation which violates these quantum numbers. Suppose that at time $t=0$ the system is in thermal equilibrium with respect to the Hamiltonian H_0,

$$\rho(0)=e^{-\beta H_0}. \tag{2.11}$$

Under the action of the full Hamiltonian the density matrix at time t has evolved to

$$\rho(t)=e^{-iHt}e^{-\beta H_0}e^{+iHt}.$$

The mean baryon number is

$$\langle B(t)\rangle=\mathrm{Tr}[e^{-\beta H_0}\hat{B}(t)]/\mathrm{Tr}\rho\,, \tag{2.12}$$

where

$$\hat{B}(t)=e^{iHt}\hat{B}e^{-iHt}. \tag{2.13}$$

The CPT invariance of H and H_0 implies that $\langle B(t)\rangle$ is an odd function of time

$$\begin{aligned}\langle B(t)\rangle &= \mathrm{Tr}e^{-\beta H_0}\hat{B}(t) && (2.14)\\ &= \mathrm{Tr}[\theta e^{-\beta H_0}\theta^{-1}\theta\hat{B}(t)\theta^{-1}]\\ &= \mathrm{Tr}\{e^{-\beta H_0}[-\hat{B}(-t)]\}\\ &= -\mathrm{Tr}[e^{-\beta H_0}\hat{B}(-t)]\\ &= -\langle B(-t)\rangle\,, && (2.15)\end{aligned}$$

where $\theta=CPT$. This antisymmetry of $\langle B\rangle$ with time is the *only* constraint implied by CPT.

Interesting information can also be extracted by looking at the rate of change of B,

$$\langle\dot{B}(t)\rangle=i\,\mathrm{Tr}\{e^{-iHt}[e^{-\beta H_0},V]e^{iHt}\hat{B}\}. \tag{2.16}$$

If we approximate e^{-iHt} by e^{-iH_0t} then $\langle\dot{B}\rangle$ must vanish since $[B,H_0]=0$. This implies $\langle\dot{B}\rangle$ is at least second order in V and first order in time, $\langle\dot{B}\rangle\sim t$. But since $\langle B\rangle$ is an odd function of t, $\langle\dot{B}\rangle$ must be even and cannot be of order t. It follows that $\langle\dot{B}\rangle$ is at least second order in t and $\langle B\rangle$ is third order:

$$\langle B(t)\rangle\sim t^3. \tag{2.17}$$

That baryon-number excess vanishes to first order in V is to be expected. The nontrivial part of the time translation operator U is anti-Hermitian to first order. Therefore amplitudes changing B by opposite amounts have equal magnitude and cancel. The relation $\langle B\rangle\sim t^3$ shows that baryon excess builds up slowly in the beginning.

In this example, a period of time will elapse during which $\langle B\rangle$ is not zero. Eventually the interactions in V will restore the system to true thermal equilibrium with vanishing $\langle B\rangle$. If, however, the baryon-number-violating force is switched off after a finite time the system will retain a finite net baryon excess.

The process of early expansion can disturb thermal equilibrium and lead to a temporary excess. If the universe expands and cools sufficiently rapidly the baryon-number-violating forces may not have time to come back to equilibrium. This is especially true if the reaction rates for these processes are rapidly falling with decreasing temperature. In order to estimate if this is so we consider the quantity $\dot{R}/R$ which measures the rate of expansion of the universe. The condition for equilibrium is

$$\frac{\dot{R}}{R}<\text{reaction rate}. \tag{2.18}$$

The expansion rate in the radiation-dominated epoch is given by

$$\frac{\dot{R}}{R}\approx T^2, \tag{2.19}$$

where the temperature T and time are in units $c=\hbar=G=1$.

The dependence of the reaction rate on temperature can be obtained from dimensional considerations. For example, in a renormalizable theory with all mass scales much lower than T the reaction rate must be proportional to T. This is because coupling constants in renormalizable theories are dimensionless. Accordingly the condition for equilibrium is

$$T^2<T \tag{2.20}$$

or

$$T<1. \tag{2.21}$$

Therefore the condition for thermal equilibrium in renormalizable theories is increasingly satisfied as the universe cools. This continues as long as explicit masses can be ignored. From these arguments it is easy to see that ordinary strong electromagnetic and weak interactions are in thermal equilibrium from superhigh temperatures ($\sim10^{15}$ GeV) down to ordinary temperatures (~1 GeV).

In superunified theories baryon-number-violating processes are effectively nonrenormalizable Fermi interactions below energies $\sim10^{18}$ GeV. This energy corresponds to the mass $\tilde{M}$ of the superheavy bosons which mediate the process. The effective Fermi coupling constant is

$$\tilde{G}\approx\frac{\alpha}{M^2}\sim10^{-38}\ \mathrm{GeV}^{-2}. \tag{2.22}$$

The reaction rate is proportional to $\tilde{G}^2$ and by dimensional arguments is

$$(\text{reaction rate})\approx\tilde{G}^2T^5.$$

The condition for equilibrium becomes

$$T^2<G^2T^5\ \text{(in Planck units)}$$

or

$$T > \left(\frac{\tilde{M}^4}{\alpha^2}\right). \tag{2.23}$$

For $\tilde{M} \sim M_{\text{Planck}}$ it is unlikely that the baryon-number-violating forces were ever in equilibrium.

Note that the baryon-number violations are of order α at temperatures $\sim 10^{18}$ GeV. Effectively we are in a situation where these interactions are switched on for a brief time interval and are then switched off. These considerations indicate that the possibility of generating baryon excess is viable.

III. MODELS WITH BARYON-NUMBER VIOLATION

By a unified theory[3] we mean a theory in which the strong, weak, and electromagnetic gauge invariance are embedded in a simple unifying group. Such theories involve a single coupling constant of the order of the electric charge. Both leptons and quarks appear in the same multiplets. Therefore quarks can turn into leptons by the emission of vector bosons called $\tilde{W}$. For example, in the SU_5 theory of Georgi and Glashow the process shown in Fig. 1 is possible. This process implies that a proton can decay into a positron and photons.

In order to suppress the decay of the proton, the mass of the $\tilde{W}$ must be made large. Consistency with the empirical bounds on the lifetime of the proton requires

$$\tilde{M} > 10^{15}\ \text{GeV}. \tag{3.1}$$

We will assume $\tilde{M}$ is approximately the Planck mass and set it equal to unity. This assumption simplifies our discussion.

At energies below $\tilde{M}$ the baryon-number-violating processes are effectively described by four-Fermi interactions. The coupling constant is approximately

$$G = \frac{\alpha}{\tilde{M}^2} = \alpha \tag{3.2}$$

in Planck units. The baryon-number-changing interactions obviously are unimportant for temperatures very much smaller than $\tilde{M}$.

The other ingredient needed for baryon excess

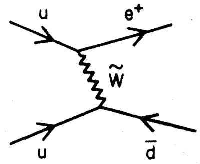

FIG. 1. Baryon-number-violating process occurring in the SU_5's unified theory.

is CP violation.[7] In principle the observed CP violation could arise spontaneously[9] or from explicit asymmetry of the Lagrangian.[10] If it arises spontaneously then it disappears at temperatures well above 1 TeV. In this case the CP and baryon processes cannot combine to yield an excess.

We will assume that a CP violation, perhaps unrelated to observed CP violation, exists at the superheavy scale. We might suppose that this breaking is also spontaneous. However, in this case it could not be effective in producing an excess. The reason is because the radius of an event horizon is very small at the time when the baryon excess is produced. This means that uncorrelated domains of different CP directions must occur with small spatial extent. Within these domains the baryon excess will have opposite sign and therefore cancel. Thus we must have an explicit CP violation in the part of the Lagrangian which is relevant at superheavy scales. This does not exclude the idea[9] that the observed CP violation is spontaneous.

For definiteness we will assume explicit four-Fermi vertices which break both CP and baryon-number conservation.

A second source of baryon-number violation has been discovered in the standard Weinberg-Salam theory. In this model the baryon-number violation is of purely quantum-mechanical origin.[4] There exists a discrete infinity of classical degenerate vacuums[11] labeled by the "winding number" n. Quantum-mechanical transitions between these classical vacuums can occur by tunneling through an energy barrier. These events are called instantons. The physics is analogous to tunneling between the minima of a periodic potential. As 't Hooft first noted,[4] each instanton event is accompanied by a change in baryon number. A change in lepton number also occurs in order to compensate the electric charge. The tunneling amplitued at zero temperature is proportional to[4]

$$e^{-8\pi^2/g^2},$$

which is of the order of 10^{-93}. At very high temperatures $T > 250$ GeV two qualitatively new things happen. First, the Higgs vacuum expectation value goes away.[12] Second, there exists a lot of thermal energy available. This can be used to overcome the potential barrier.

To estimate the importance of this effect we must compare the barrier height with the available thermal energy. Consider an instanton of space-time radius ρ. For temperatures $\gg 250$ GeV the expectation value of the Higgs potential vanishes and the action of an instanton is roughly what it would be for pure Yang-Mills theory:

$$\text{action} = 8\pi^2/g^2 . \tag{3.3}$$

The tunneling barrier is estimated by dividing this action by the duration of the event ρ,

$$V = 8\pi^2/g^2\rho . \tag{3.4}$$

[We remind the reader that Eq. (3.4) only applies above the transition temperature for the Higgs field to disappear.]

Equation (3.4) suggests that we can always lower the barrier as small as we like by considering arbitrarily large instantons. This is not so. The reason is that a tunneling event is a coherent process in which the instanton density $F_{\mu\nu}\tilde{F}_{\mu\nu}$ is of a definite sign over the size of the tunneling region. Thus ρ cannot exceed the coherence length which is given by the Debye screening length in the gauge-field plasma.[13] This is given by the plasmon Compton wavelength which for pure Yang-Mills theory is

$$\lambda_{\text{plasma}} \approx \left(\frac{gT}{\sqrt{6}}\right)^{-1} = \rho_{\max} . \tag{3.5}$$

The thermal energy within such a volume is $\sim(\pi^2/2)\rho_{\max}{}^3 T^4 \sim (\pi^2/2)T^4\lambda_P{}^3$. The condition that this thermal energy overcomes the barrier V is

$$\frac{\pi^2}{2}T^4\lambda_P{}^3 > \frac{8\pi^2}{g^2}\frac{1}{\lambda_P}$$

or (3.6)

$$(18 - 8g^2)\frac{\pi^2}{g^4} \geq 0.$$

This appears to be satisfied for the coupling constants characteristic of weak-electromagnetic theories.

These crude estimates only suggest the possibility that baryon-number-violating interactions are not suppressed at $T > 250$ GeV. Quantitative calculations are needed to decide the importance of this effect. In particular, the effects of fermions will probably suppress the tunneling. For the remainder of this paper we will ignore this quantum-mechanical source of baryon-number violation, although it is possible for it to seriously alter the results of this paper.

IV. BARYON GENERATION MECHANISM IN FIELD THEORY

In this section we will describe field-theoretic methods for computing the baryon-number excess in an expanding universe. For definiteness we will consider a model in which both baryon and CP violation are mediated by superheavy bosons of mass $\sim M_{\text{Planck}}$. In practice this means that these interactions are described as four-Fermi couplings.

We are going to consider a field theory in an expanding universe described by the metric

$$ds^2 = (dt)^2 - R(t)^2(d\vec{x})^2 \tag{4.1}$$

$$= (dt)^2 - t\,(d\vec{x})^2 . \tag{4.2}$$

The choice $R = \sqrt{t}$ is appropriate to a radiation-dominated epoch. We will illustrate such a system by considering a scalar field with action

$$S = \int d^4x\sqrt{-g}\left[g^{\mu\nu}\frac{\partial\phi}{\partial x^\mu}\frac{\partial\phi^*}{\partial x^\nu} + V(\phi)\right] . \tag{4.3}$$

Now the metric in Eq. (4.2) is of the conformally flat type meaning that by a change of variables it can be brought to the form

$$ds^2 = \rho^2(x)[(dx_0)^2 - (d\vec{x})^2] . \tag{4.4}$$

In particular, if we change variables from t to $\tau = (2t)^{1/2}$ then

$$ds^2 = \tau^2(d\tau^2 - d\vec{x}^2) . \tag{4.5}$$

Now the reader can verify that if the field ϕ is replaced by

$$s = \rho^{-1}\phi , \tag{4.6}$$

then the free part of the Lagrangian becomes

$$S = \int d^3x\,d\tau\left[\left(\frac{ds}{d\tau}\right)^2 - (\nabla s)^2\right] + \text{pure divergence} . \tag{4.7}$$

Furthermore, if a renormalizable ϕ^4 interaction is present in V then it is replaced by s^4. If on the other hand nonrenormalizable terms such as ϕ^{4+2n} are present they are replaced by

$$V(s) = \frac{s^{4+2n}}{\tau^{2n}} . \tag{4.8}$$

Thus, in the new time coordinate, the free and renormalizable terms in the action take their flat-space form and appear to be τ independent. The nonrenormalizable terms appear time dependent with rapidly falling coefficients.

Similar results hold for more general theories. If we consider the usual type of theory containing scalar spinor and vector fields ϕ, ψ, A_μ and define conformal fields by

$$\phi \to \rho^{-1}\phi , \quad \psi \to \rho^{-3/2}\psi , \quad A_\mu \to A_\mu , \tag{4.9}$$

then the free and renormalizable terms take their flat-space form. The nonrenormalizable Fermi couplings are replaced by their flat-space counterparts times the factor $1/\tau^2$. Thus the form that the action for our model takes is

$$S=\int d^3x\,d\tau\left(L_0+\frac{1}{\tau^2}L_I\right), \tag{4.10}$$

where L_0 is a renormalizable τ-independent Lagrangian containing all the usual interactions and L_I is a four-Fermi coupling containing the superheavy mediated effects.

We will make two cautionary remarks before proceeding to study baryon-number excess generation. The first is that the flat-space form for renormalizable theories ignores mass effects. Since we only use if for very high temperatures this is no problem. The second remark concerns ultraviolet divergences. The above analysis was purely classical and fails when renormalization is accounted for. However, because the unified coupling is small at the Planck length, the failure only involves very weakly varying logarithms. In fact, these effects would show up as logarithms of τ multiplying the renormalizable interactions. They are completely unimportant for our problem.

Let us now return to the baryon excess problem. We write the Hamiltonian resulting from Eq. (4.10) as

$$H=H_0+V(\tau), \tag{4.11}$$

where H_0 is baryon-number and CP conserving. $V(\tau)$ contains the violating terms and scales like τ^{-2}.

Suppose the initial density matrix at the Planck time $\tau=1$ is given by $\rho(1)$. The expectation value of the baryon number at this time is

$$\langle B(1)\rangle=\mathrm{Tr}\rho(1)\hat{B}. \tag{4.12}$$

At a later time τ the value of $\langle B\rangle$ is

$$\begin{aligned}\langle B(\tau)\rangle&=\mathrm{Tr}\rho(1)U^\dagger(\tau)\hat{B}U(\tau)\\&=\mathrm{Tr}\,U(\tau)\rho(1)U^\dagger(\tau)\hat{B},\end{aligned} \tag{4.13}$$

where $U(\tau)$ is the time translation operator from $\tau=1$ to τ.

For the case that $V(\tau)$ is τ independent (renormalizable interactions) we may immediately conclude that as $\tau\to\infty$ $\langle B\rangle\to 0$. This is because a field theory with time-independent Hamiltonian will eventually come to thermal equilibrium and we have seen that CPT ensures $B=0$ in this case.

On the other hand, if $V(\tau)\to 0$ fast enough we can use ordinary perturbation theory in V to compute the baryon-number excess as $\tau\to\infty$. To do this we use the standard interaction-picture formalism to obtain

$$\begin{aligned}&U(\tau)=U_0(\tau)U_V(\tau),\quad U_0(\tau)=e^{-iH_0(\tau-1)},\\&U_V(\tau)=T\exp\left[-i\int_1^\tau V_I(\tau')d\tau'\right],\\&V_I(\tau)=U_0^\dagger(\tau)V(\tau)U_0(\tau).\end{aligned} \tag{4.14}$$

Thus using $[B,U_0]=0$,

$$\langle B(\tau)\rangle=\mathrm{Tr}\rho(1)U_V^\dagger(\tau)\hat{B}U_V(\tau). \tag{4.15}$$

Graphical rules are derived in Appendix A for the evaluation of (4.15). The following features emerge from analysis of these rules:

(1) For the case $V(\tau)\sim 1/\tau^2$ each order has a finite limit as $\tau\to\infty$. These limits give an order-by-order expansion of the final baryon-number excess.

(2) The first order in which a nonvanishing excess occurs depends on certain features of $\rho(1)$. In particular, if $[\rho(1),B]=0$ then the first order vanishes.

(3) If in addition to $\rho(1)$ being diagonal in baryon number it is CP symmetric then the second order also vanishes. Thus in the case of initially chaotic conditions, baryon-number excess is a third-order effect. Thus, since we suppose [see Eq. (3.2)] that $V\sim\alpha$, baryon-number excess will be $\sim\alpha^3$ for an initially chaotic ρ.

We are currently constructing Feynman rules for the evaluation of Eq. (4.15). These rules will be applied to some unified models in a future paper.

V. SCALAR TOY MODEL

Consider the model introduced in Sec. II [Eq. (2.2)]. In conformal coordinates the action becomes

$$S=\int d^3x\,d\tau\left[\left|\frac{\partial\phi}{\partial\tau}\right|^2-|\nabla\phi|^2-\frac{\lambda}{\tau^{2n}}(\phi\phi^*)^n(\phi+\phi^*)\times(\alpha\phi^3+\alpha^*\phi^{*3})-g(\phi\phi^*)^2\right], \tag{5.1}$$

where we have added the renormalizable term $g\phi^4$ to represent all the renormalizable interactions. In this section we will make some very crude approximations which reduce the system to a single degree of freedom.

First we shall assume that the initial density matrix is in thermal equilibrium at a temperature ~ 1. If we ignore the small ($\sim\alpha$) nonrenormalizable couplings then the system will remain in equilibrium at this temperature for all τ. (Note that in transforming to the original coordinates the temperature becomes $1/\tau$ since it scales like energy.) Thus the average value of $|\phi|$ will remain constant of order unity. Indeed the first simplification will be to replace $|\phi|$ by unity.

The other drastic simplification will be to focus on a single unit coordinate cell over which ϕ will be assumed spatially constant. Setting $\phi=e^{i\theta}$ we obtain a system described by the Lagrangian

$$\mathcal{L}=\left(\frac{d\theta}{d\tau}\right)^2-\tau^{-2n}V(\theta)\,. \tag{5.2}$$

The baryon number of a unit cell is given by Eq. (2.4):

$$B=R^3(t)i\phi\overleftrightarrow{\partial}_t\phi^*$$
$$=2\frac{d\theta}{d\tau}\,. \tag{5.3}$$

Equation (5.2) describes a pendulum in a time-dependent unsymmetric potential and Eq. (5.3) says that the baryon number of a single cell is given by its angular velocity. The *CPT* invariance of the original instantaneous Hamiltonian corresponds to the time-reversal invariance of the pendulum.

The approximation of ignoring the interaction of neighboring cells is surely too severe to correctly describe the high-temperature nonequilibrium properties of the subsystem. In particular, it is impossible for the single pendulum to relax to thermal equilibrium if it is disturbed. For example, if the pendulum is given a hard "clockwise" swing it will forever continue to rotate so that $\dot\theta \not\to 0$. But in thermal equilibrium $\langle\dot\theta\rangle=0$ by the same arguments which we used to prove $\langle B\rangle=0$.

By ignoring the surrounding heat bath we have eliminated the possibility of dissipation. A simple method for incorporating it is to introduce a dissipative damping term into the equation of motion. Thus we write the equation of motion

$$\frac{d^2\theta}{d\tau^2}+\tau^{-2n}\frac{\partial V}{\partial\theta}+f(\tau)\frac{d\theta}{d\tau}=0\,. \tag{5.4}$$

The computation of friction coefficients in nonequilibrium statistical mechanics typically involves the computation of the absorptive (imaginary) part of some thermal Green's function.[14] That is to say, we calculate the width of some excitation which propagates in the medium.

In the case of electrical resistance we calculate the absorptive part of the plasmon propagator.[14] In our case, a nonzero baryon charge must dissipate as equilibrium is restored. Accordingly we must compute the width of the charge-carrying excitation described by the field ϕ due to baryon-number-violating processes. In the model field theory with interaction $V(\phi)=\lambda(\phi^*\phi)^n(\phi+\phi^*)(\alpha\phi^3+\alpha^*\phi^{*3})$ the relevant width is described by graphs shown in Fig. 2. Dimensional arguments require the temperature-dependent width to be

$$\gamma(T)\approx\lambda^2T^{4n+1}\,. \tag{5.5}$$

Thus if the number of baryons in the unit cell

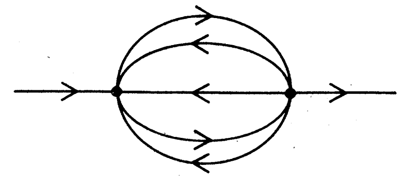

FIG. 2. Graph contributing to baryon dissipation.

is B, the number lost by dissipation is

$$\left(\frac{d}{dt}B\right)_{\rm dis}=-B\gamma=-B\lambda^2T^{4n+1} \tag{5.6}$$

or

$$\frac{dB}{dt}_{\rm dis}=-\frac{\lambda^2B}{\tau^{4n}}\,. \tag{5.7}$$

Recalling that B is identified with $d\theta/d\tau$ we interpret Eq. (5.7) to mean that the coefficient f in Eq. (5.4) is λ^2/τ^{4n}:

$$\frac{d^2\theta}{d\tau^2}+\tau^{-2n}\frac{\partial V}{\partial\theta}+\frac{\lambda^2}{\tau^{4n}}\frac{d\theta}{d\tau}=0\,. \tag{5.8}$$

Equation (5.8) defines the toy model.

To see how the toy model can lead to an asymmetric distribution of baryons and antibaryons consider a $V(\theta)$ which looks like Fig. 3, i.e., it has no point of reflection symmetry. Now suppose the initial probability density in θ and $\dot\theta$ is uniform in θ and symmetric under $\dot\theta\to-\dot\theta$. We observe that a particle has a large probability to get a small kick to the left and a small probability for a large kick to the right. Thus the probability distribution becomes asymmetric. However, to first order in time no average change in $\dot\theta$ occurs. This is because the average force $\partial V/\partial\theta$ vanishes for a uniform distribution in θ. In fact $\langle\dot\theta\rangle$ only becomes nonzero in order τ^5. Furthermore, the first nonvanishing order in V is third order.

If the universe were a nonexpanding box at fixed temperature then no net baryon excess could be maintained at long times. Indeed the toy model is consistent with this. In a nonexpanding universe the form of the toy model is

$$\frac{d^2\theta}{dt^2}+\lambda T^{2n+2}\frac{\partial V}{\partial\theta}+\lambda^2T^{4n+1}\frac{d\theta}{dt}=0\,. \tag{5.9}$$

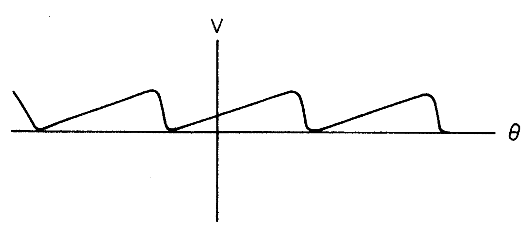

FIG. 3. A potential which violated $V(\theta)$.

Let us suppose after a long time that the baryon number $T^2 d\theta/dt$ is constant. Then

$$\lambda T^{2n+2}\frac{\partial V}{\partial\theta}+\lambda^2 T^{2+n+1}\dot{\theta}=0\,. \tag{5.10}$$

Integrating this over a period and using the periodicity of $V(\theta)$ we see that the baryon number has to vanish. Note that both the periodicity of the potential and the existence of the friction term are important in reaching this conclusion.

Now we find the conditions that will allow a nonvanishing baryon number at large times. Multiplying Eq. (5.8) by τ^{2n} and integrating over a period we obtain

$$\oint \tau^{2n}\frac{dK}{d\tau}\,d\tau=-2\lambda^2\oint\frac{K}{\tau^{2n}}\,d\tau, \tag{5.11}$$

where $K=\frac{1}{2}(d\theta/d\tau)^2$. After a long time, this equation effectively becomes

$$\frac{dK}{d\tau}=-2\lambda^2\frac{K}{\tau^{4n}}\;. \tag{5.12}$$

For the renormalizable case $n=0$ we see that baryon number is exponentially damped. This agrees with our previous expectations. For $n>\frac{1}{4}$ this equation has solutions for which the baryon number tends to a constant. Thus we see that for nonrenormalizable theories the friction term can be neglected, and baryon-number excess occurs as $\tau\to\infty$.

VI. CONCLUDING REMARKS

In this paper we have argued that a baryon-number excess may be produced in an expanding universe even though the initial conditions are symmetric. For the case of unified theories the excess is developed at times of order 10^{-40} sec while the temperature is comparable to the Planck mass. An admittedly oversimplified model yields a small number of baryons per unit cell of the order α^3.

The conclusion that the effect is $\sim\alpha^3$ does not appear to be general. It is a consequence of replacing the superheavy interactions by four-Fermi interactions. While this helps us visualize the process, it is not entirely consistent. This is because the main action occurs at energies of order $\tilde{M}$ and not much lower energies. Therefore it is important to open up the "black box" hiding the superheavy-boson exchange. As far as we can tell there are then order-α^2 effects. This is somewhat too large empirically but we must keep in mind that there are effects which we ignored which decrease N_B/N_γ. We have treated the universe expansion as if it were a reversible process with respect to the ordinary interactions. In fact there are possible sources of irreversibility which can heat up the system.[15] Eventually this heat must appear as photons.

Unfortunately this optimistic picture which emerges in unified theories may be drastically changed if the baryon-number-violating tunneling events are really important. The point is that the rates for these processes are of the renormalizable type for $T>250$ GeV. Thus they can allow the system to return to equilibrium and may wash out any excess which developed at superhigh temperature.

Of course as the temperature goes below 250 GeV the tunneling processes also go out of equilibrium. In principle the observed baryon-number excess could be attributed to this final stage of baryon-number violation. In this case the number of baryons in the universe is independent of the initial conditions and the details of the particular unified model.

ACKNOWLEDGMENTS

We would like to thank R. Wagoner for many critical insights without which we could not have written this paper. We are also indebted to E. Liang, D. Sciama, and G. Steigman for interesting comments. One of us (S. D.) would like to thank Sid Drell and Dirk Walecka for hospitality at SLAC and Stanford University. This work was supported in part by the National Science Foundation under Contract No. 76-16992.

APPENDIX A

Graphical rules for computing $\langle B(\tau)\rangle$. Consider a theory of fermions interacting with baryon-number-, C- and CP-violating four-Fermi forces. The Hamiltonian of this theory in the expanding universe in terms of the conformal coordinates is

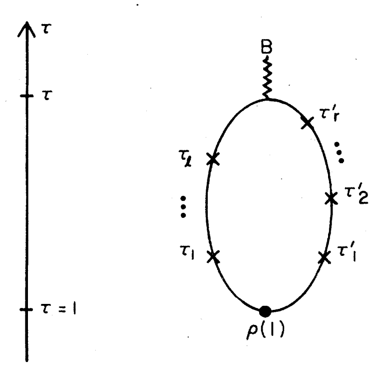

FIG. 4. Graphical notations. Solid lines represent propagating state vectors. Crosses represent the action of V. The balck dot represents the initial density matrix and the wavy line represents the measurement of baryon number.

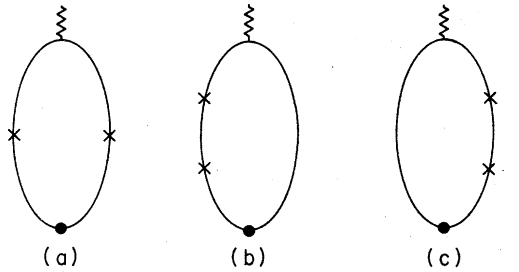

FIG. 5. The second-order contributions to $\langle B(\tau)\rangle$.

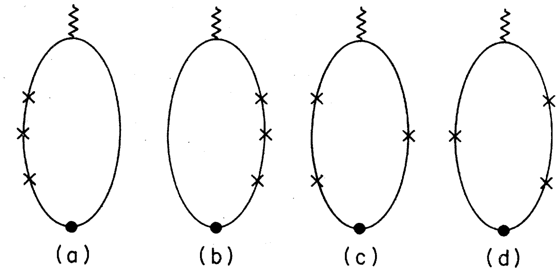

FIG. 6. The third-order contributions to $\langle B(\tau)\rangle$.

of the form

$$H = H_0 + V(\tau).$$

The baryon-number-violating piece $V(\tau)$ is of the form

$$V(\tau) = \frac{\alpha}{\tau^2}\int d^3x(\bar{\psi}\Gamma\psi)^2 \equiv \frac{v}{\tau^2}.$$

The graphical rules for the evaluation of $\langle B(\tau)\rangle$ can be deduced from the expression

$$\langle B(\tau)\rangle = \mathrm{Tr}\rho(1)U^\dagger_{V_I}(\tau)U^\dagger_{H_0}(\tau)\hat{B}U_{H_0}(\tau)U_{V_I}(\tau), \quad \text{(A1)}$$

where

$$U_{H_0}(\tau) = T\exp\left[-i\int_1^\tau H_0(\tau')d\tau'\right] = e^{-iH_0(\tau-1)},$$

$$U_{V_I}(\tau) = T\exp\left[-i\int_1^\tau V_I(\tau')d\tau'\right], \quad \text{(A2)}$$

$$V_I(\tau) = U^\dagger_{H_0}(\tau)V(\tau)U_{H_0}(\tau).$$

Since H_0 conserves baryon number, expression (A1) simplifies to

$$\langle B(\tau)\rangle = \mathrm{Tr}\rho(1)U^\dagger_{V_I}(\tau)\hat{B}U_{V_I}(\tau). \quad \text{(A3)}$$

The graphical rules for the evaluation of this quantity are the following:

(1) Draw the closed loop shown in Fig. 4 in order $l+r$.

(2) For each cross on the right write $ie^{iH_0(\tau'-1)}\times ve^{-iH_0(\tau'-1)}$. For each cross on the left write $-ie^{iH_0(\tau-1)}ve^{-iH_0(\tau-1)}$.

(3) Write down the terms indicated in Fig. 4 in anticlockwise order and take the trace.

(4) Carry out the time integrations with weight $1/\tau^2$. Respect time ordering.

Do the same for the $l+r+1$ graphs appearing in order $l+r$. Note that the lines in Fig. 4 are not particle lines. They represent propagation of states.

APPENDIX B

Here we will show explicitly that for the model discussed in Appendix A the second-order contributions to $\langle B(\tau)\rangle$ vanish. We shall label each state solely by its baryon number $|n\rangle$. The CPT conjugate state will be denoted by $|-n\rangle$, and by CPT invariance, $\rho_n(1) = \rho_{-n}(1)$. Since $[B,\rho(1)]=0$, $\rho_{nm}(1) \equiv \rho_{nm} = \rho_n\delta_{nm}$. Since B is CPT odd, $B_{-n} = -B_n$. The second-order contributions to $\langle B(\tau)\rangle$ arise from the graphs of Fig. 5. The contribution of graph (a) is

$$i(-i)\int_1^\tau \frac{d\tau_1}{\tau_1^2}\int_1^\tau \frac{d\tau_2}{\tau_2^2}\rho_n e^{i\epsilon_n\tau_1}v_{nm}e^{-i\epsilon_m\tau_1}B_m e^{i\epsilon_m\tau_2} \times v_{mn}e^{-i\epsilon_n\tau_2} = 0.$$

In deriving this we used the CPT invariance of the Hamiltonian H,

$$\epsilon_n = +\epsilon_{-n} \text{ and } |v_{nm}|^2 = |v_{-m,-n}|^2.$$

The contribution of graph (b) is

$$(-i)^2\int_1^\tau \frac{d\tau_2}{\tau_2^2}\int_1^{\tau_2}\frac{d\tau_1}{\tau_1^2}\rho_n B_n e^{i\epsilon_n\tau_2}v_{nm}e^{-i\epsilon_m\tau_2}e^{i\epsilon_m\tau_1} \times v_{mn}e^{-i\epsilon_n\tau_1} = 0.$$

This vanishes for the same reason with graph (a). The vanishing of the second-order contribution to $\langle B(\tau)\rangle$ is not a general feature of all models. It only happens because the explicit time dependence of $V(\tau)$ can be factored out.

APPENDIX C

In this appendix we write down the third-order contributions to $\langle B(\tau)\rangle$ for the model of Appendix A. The graphs contributing are those of Fig. 6.

Graph (a) contributes

$$(-i)^3\int_1^\tau\frac{d\tau_3}{\tau_3^2}\int_1^{\tau_3}\frac{d\tau_2}{\tau_2^2}\int_1^{\tau_2}\frac{d\tau_1}{\tau_1^2}\rho_n B_n e^{i\epsilon_n\tau_3}v_{nm}e^{-i\epsilon_m(\tau_3-\tau_2)}v_{me}e^{-i\epsilon_e(\tau_2-\tau_1)}v_{en}e^{-i\epsilon_n\tau_1}$$

$$= (-i)^3\int_1^\tau\frac{d\tau_3}{\tau_3^2}e^{i\tau_3(\epsilon_n-\epsilon_m)}\int_1^{\tau_3}\frac{d\tau_2}{\tau_2^2}e^{i\tau_2(\epsilon_m-\epsilon_e)}\int_1^{\tau_2}\frac{d\tau_1}{\tau_1^2}e^{i\tau_1(\epsilon_e-\epsilon_n)}\rho_n B_n v_{nm}v_{me}v_{en}.$$

Graph (b) contributes

$$i^3\int_1^{\tau}\frac{d\tau_3}{\tau_2^2}\int_1^{\tau_3}\frac{d\tau_2}{\tau_2^2}\int_1^{\tau_2}\frac{d\tau_1}{\tau_1^2}\rho_n B_n e^{i\epsilon_n\tau_1}v_{nm}e^{-i\epsilon_m(\tau_1-\tau_2)}v_{me}e^{-\epsilon_e(\tau_2-\tau_3)}v_{en}e^{-i\epsilon_n\tau_3}$$

$$=i\int_1^{\tau}\frac{d\tau_3}{\tau_3^2}e^{i\tau_3(\epsilon_e-\epsilon_n)}\int_1^{\tau_3}\frac{d\tau_2}{\tau_2^3}e^{i\tau_2(\epsilon_m-\epsilon_e)}\int_1^{\tau_2}\frac{d\tau_1}{\tau_1^2}e^{i\tau_1(\epsilon_n-\epsilon_m)}\rho_n B_n v_{nm}v_{me}v_{en}.$$

Graph (b), of course, is just the complex conjugate of graph (a).

Graph (c) yields

$$(-i)^2 i\int_1^{\tau}\frac{d\tau_1'}{\tau_1'^2}e^{i(\epsilon_n-\epsilon_m)\tau_1'}\int_1^{\tau}\frac{d\tau_2}{\tau_2^2}e^{i(\epsilon_m-\epsilon_e)\tau_2}\int_1^{\tau_2}\frac{d\tau_1}{\tau_1^2}e^{i(\epsilon_e-\epsilon_n)\tau}\rho_n v_{nm}\beta_m v_{me}v_{en}.$$

Graph (d) yields the complex conjugate of (c),

$$i^2(-i)\int_1^{\tau}\frac{d\tau_2'}{\tau_2'^2}e^{i\tau_2'(\epsilon_m-\epsilon_e)}\int_1^{\tau_2'}\frac{d\tau_1'}{\tau_1'^2}e^{i\tau_1'(\epsilon_n-\epsilon_m)}\int_1^{\tau}\frac{d\tau_1}{\tau_1^2}e^{i\tau_1(\epsilon_e-\epsilon_n)}\rho_n v_{nm}v_{me}B_e v_{en}.$$

These expressions do not vanish in general. They, of course, vanish if we assume C- or CP-invariant matrix elements for v.

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