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Comments and Addenda

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Gauge-theory predictions for deep-inelastic polarized-electron–nucleon scattering asymmetries

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Asymmetries between the deep-inelastic scattering of right- and left-handed electrons by nucleons predicted by three distinct models of weak and electromagnetic interactions are compared. We find that even a left-right–symmetric model, for which neutral currents conserve parity in lowest order, predicts sizeable asymmetries as a result of radiative corrections. Asymmetry measurements are contrasted with atomic-physics parity-violation searches and the relevance of our results for an ongoing experiment is discussed.

Although the "standard" Weinberg-Salam SU(2) $\times$U(1) model[1] of weak and electromagnetic interactions is in excellent agreement with the worldwide neutral-current data from neutrino-hadron scattering, its prediction for the electron's neutral-current interaction seems to disagree with the essentially null results of several searches for parity violation in bismuth.[2-4] To accommodate these findings, a variety of alternative models have been advocated which preserve the successful properties of the "standard" model but suppress the magnitude of parity violation in bismuth. Suppression is *naturally* accomplished by making the electron's neutral-current coupling pure vector [the example we consider is an SU(2)$\times$U(1) vectorlike (or "hybrid") model[4]] or by having all neutral currents naturally conserve parity in lowest order, as in a recent left-right–symmetric $SU_L(2)\times SU_R(2)\times U(1)$ model.[5] Forthcoming experiments designed to detect parity violation in other atomic transitions as well as measurements of asymmetries between the scattering of right-handed and left-handed electrons should clarify things.[4,6,7] They will either vindicate the "standard" model or perhaps justify some alternative.

In a previous publication,[4] we pointed out that even models which were designed to naturally suppress parity violation in bismuth may actually predict, as a result of radiative corrections, much larger parity-violating effects than naive estimates indicate. Our intention in this paper is to apply the results of those calculations to deep-inelastic polarized-electron–nucleon asymmetries. The motivation behind this application is our anticipation that an ongoing experiment[6] at SLAC which is designed to measure deep-inelastic asymmetries on a deuterium target will yield results at the level of gauge-theory predictions in the near future. As we shall see, the outcome of this experiment will have important implications for the viability of various models and the validity of the bismuth-experiment results.[2]

Our format will be to give general formulas for the asymmetry parameter in the case of deuterium and proton targets, employing our previous parametrization[4,7] of the electron-quark parity-violating interaction. This parametrization emphasizes an important distinction between deep-inelastic asymmetries and atomic parity violation due to the respective incoherence and coherence of these processes. Then we list the predictions of the "standard" SU(2)$\times$U(1) Weinberg-Salam model,[1] the vectorlike (or "hybrid") model,[4] and a particular left-right–symmetric $SU_L(2)$ $\times SU_R(2)\times U(1)$ model[5] which predicts no asymmetry *in lowest order*. Our results include one-loop radiative corrections for all quantities that naturally vanish in lowest order. (In the case of nonvanishing quantities, we expect radiative corrections to make fairly small modifications.[7]) For the left-right model these corrections completely determine the sign and magnitude of the asymmetry. Finally, the predictions of these three models are compared and their relevance for a presently running experiment[6] on deuterium is discussed.

DEEP-INELASTIC ASYMMETRIES

The parity-violating (PV) neutral-current interaction between an electron and up or down quark is conveniently parametrized by the effective

Hamiltonian[4,7,8]

$$H_{\mathrm{PV}}=\frac{G_F}{\sqrt{2}}\,(C_{1u}\bar{e}\gamma_\mu\gamma_5 e\bar{u}\gamma^\mu u+C_{2u}\bar{e}\gamma_\mu e\bar{u}\gamma^\mu\gamma_5 u+C_{1d}\bar{e}\gamma_\mu\gamma_5 e\bar{d}\gamma^\mu d+C_{2d}\bar{e}\gamma_\mu e\bar{d}\gamma^\mu\gamma_5 d),\quad (1)$$

where G_F is the Fermi constant and $C_{1u}, C_{2u}, C_{1d}, C_{2d}$ are constants determined by the specific model considered. In terms of these C's, the deep-inelastic asymmetry [$A\equiv(d\sigma_R-d\sigma_L)/(d\sigma_R+d\sigma_L)$ where $d\sigma_R$ and $d\sigma_L$ are the differential scattering cross sections of right- and left-handed electrons] for an isoscalar target such as deuterium is given (for $x\gtrsim 0.2$) by[9]

$$A_{\mathrm{D}}(x,y)\simeq\frac{-3G_F\,|q^2|}{5\sqrt{2}\,\pi\alpha}\left[(C_{1u}-\tfrac{1}{2}C_{1d})+(C_{2u}-\tfrac{1}{2}C_{2d})\,\frac{1-(1-y)^2}{1+(1-y)^2}\right]\ \text{(deuterium target)},\quad (2)$$

where $x=-q^2/2M_p(E_i-E_f)$ and $y=(E_i-E_f)/E_i$. Similarly, for a proton target[9] at $x=\frac{1}{3}$,

$$A_p(x=\tfrac{1}{3},y)\simeq\frac{-2G_F\,|q^2|}{3\sqrt{2}\,\pi\alpha}\left[(C_{1u}-\tfrac{1}{4}C_{1d})+(C_{2u}-\tfrac{1}{4}C_{2d})\,\frac{1-(1-y)^2}{1+(1-y)^2}\right]\ \text{(proton target)}.\quad (3)$$

From the formulas in (2) and (3), the asymmetry predictions of any model can be discerned once its C's are known.

At this point it is instructive to compare the result in (2) with predictions for parity violation in heavy atoms, using the same parametrization. For heavy atoms such as bismuth where coherence effects are important, the dominant parity-violating effects are directly proportional to the so-called weak charge Q_W,

$$Q_W(Z,A)\equiv 2[C_{1u}(A+Z)+C_{1d}(2A-Z)],\quad (4)$$

where Z and A are the atomic and mass numbers. Notice the difference between (4) and (2). In (4), C_{1u} and C_{1d} enter with the *same* sign, whereas in (2) they have *opposite* signs because deep-inelastic scattering is an incoherent process. This distinction has important implications for gauge-theory model precitions. For example, in the case of bismuth, $Q_W({}^{209}_{83}\mathrm{Bi})=584C_{1u}+670C_{1d}$, so it is possible that a "conspiracy" model in which C_{1u} and C_{1d} are of $O(1)$ but $C_{1u}\simeq-1.15C_{1d}$ [for this case $Q_W(\mathrm{Bi})\approx 0$] is responsible for *accidentally* small parity violation in bismuth. From (2) we then see that such a model must predict a relatively large asymmetry in deuterium. Indeed a very large asymmetry in deuterium would force us to accept one of the following: (1) The null results of the bismuth experiments[2] and/or their associated atomic physics calculations are wrong. (2) A "conspiracy" model is correct.

This difference between (4) and (2) also has important implications for the effect of radiative corrections on asymmetries as we shall see. We now list the values of C_{1u}, C_{1d}, C_{2u}, and C_{2d} for three popular models.

"Standard" SU(2) × U(1) Weinberg-Salam model

The parity-violation constants for the "standard" model are[1,4,7]

$$C_{1u}=\tfrac{1}{2}(1-\tfrac{8}{3}\sin^2\theta_W),\quad C_{2u}=\tfrac{1}{2}(1-4\sin^2\theta_W),$$
$$C_{1d}=-\tfrac{1}{2}(1-\tfrac{4}{3}\sin^2\theta_W),\quad C_{2d}=-\tfrac{1}{2}(1-4\sin^2\theta_W),\quad (5)$$

where θ_W is the weak mixing angle.

SU(2) × U(1) vectorlike (or "hybrid") model

If the right-handed component of the electron field e_R is made a member of an isodoublet rather than an isosinglet, then the electron's coupling to the neutral vector boson Z is $\sim Z_\mu\bar{e}\gamma^\mu e$, pure vector. We call this modified SU(2) × U(1) model the vectorlike model. (The neutrino, up-quark, and down-quark interactions are the *same* as in the "standard" model.) For this model we found[4,7,8]

$$C_{1u}=\frac{\alpha}{\pi}\left[\frac{3}{8\sin^2\theta_W}+\cos 2\theta_W\ln(M_Z{}^2/m^2)+(\cot 2\theta_W)^2(\cos 2\theta_W-\tfrac{1}{4})\right],$$
$$C_{1d}=\frac{\alpha}{\pi}\left[\frac{3}{8\sin^2\theta_W}+\tfrac{1}{2}\cos 2\theta_W\ln(M_Z{}^2/m^2)+\tfrac{1}{2}(\cot 2\theta_W)^2(\cos 2\theta_W+\tfrac{1}{2})\right],\quad (6)$$
$$C_{2u}=\cos 2\theta_W,\quad C_{2d}=-\cos 2\theta_W,$$

where M_Z is the neutral-vector-boson mass and m is a typical hadronic mass scale ~1–2 GeV.

$\mathrm{SU}_L(2)\times\mathrm{SU}_R(2)\times\mathrm{U}(1)$ left-right model

For the particular left-right model of Ref. 5, we found[4,7,8,10]

$$C_{1u}=\frac{\alpha}{\pi}\left(\tfrac{14}{3}\ln M_R/M_L+\frac{1}{\sin^2\xi}\right),$$
$$C_{1d}=-\frac{\alpha}{\pi}\left(\tfrac{7}{3}\ln M_R/M_L+\frac{1}{4\sin^2\xi}\right),$$
$$C_{2u}=\frac{\alpha}{\pi}\left(7\ln M_R/M_L+\frac{1}{\sin^2\xi}+\tfrac{2}{9}\ln M_L/m\right),\quad (7)$$
$$C_{2d}=-\frac{\alpha}{\pi}\left(7\ln M_R/M_L+\frac{1}{4\sin^2\xi}+\tfrac{4}{9}\ln M_L/m\right),$$

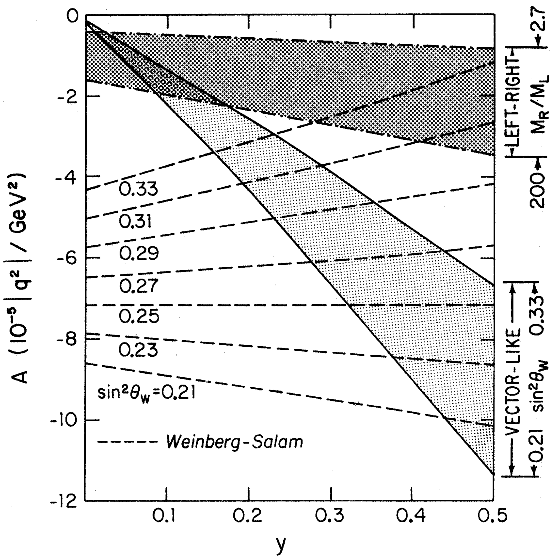

FIG. 1. The asymmetry, $A(x,y)$, for deep-inelastic polarized-electron–deuterium scattering in units of $10^{-5}|q^2|/\text{GeV}^2$ plotted as a function of y. Predictions are given for the Weinberg-Salam model and vectorlike model using $0.21 \leq \sin^2\theta_W \leq 0.33$ and for the left-right model using $2.7 \leq M_R/M_L \leq 200$.

where M_R and M_L are the masses of the right and left charged vector bosons and ξ is a mixing angle which approximately satisfies $\sin^2\xi \simeq 2\sin^2\theta_W$. All of our results in (7) are proportional to α/π because they arise from one-loop radiative corrections.

COMPARISON OF MODELS

Illustrated in Fig. 1 are the asymmetry predictions of the three models in the case of a deuterium target for $0 \leq y \leq 0.5$. [The results for a proton target are similar to these and easily found from (3).] These plots were obtained using (2), (5), (6), and (7) with the specific values $\ln M_Z/m \simeq \ln M_L/m \simeq 4$, $\sin^2\xi = 0.5$ while allowing $\sin^2\theta_W$ and M_R/M_L to vary.

The "standard" Weinberg-Salam model predictions are plotted in Fig. 1 for values of $\sin^2\theta_w$ in the domain,[11] 0.21 to 0.33. Notice that the asymmetry exhibits a very sensitive dependence on $\sin^2\theta_W$ and *increases* in magnitude as $\sin^2\theta_W$ decreases (for the domain we consider). This is to be contrasted with the predicted optical rotation in bismuth which *decreases* in magnitude as $\sin^2\theta_W$ decreases [for the "standard" model, $Q_W(\text{Bi}) = -43 - 332\sin^2\theta_W$].

A band of vectorlike model predictions is given in Fig. 1 for $0.21 \leq \sin^2\theta_W \leq 0.33$. Notice that the range of asymmetry values is not as large as in the "standard" model. Radiative corrections increase the asymmetry magnitude; however, they are not as important for this process as they are for predicting the optical rotation in bismuth.[4,7] The reason is that C_{1u} and C_{1d} have the same sign for this model. Furthermore, in bismuth, radiative corrections almost completely determine the sign and magnitude of the optical rotation predicted by the vectorlike model.

A separate band of predictions for the left-right model is also given in Fig. 1. As can be seen from (7), the asymmetry is not very sensitive to the mixing angle ξ, so we have used $\sin^2\xi = 0.5$ throughout. Instead, it depends mainly on the size of M_R/M_L. Our band is for the range

$$2.7 \leq M_R/M_L \leq 200\,, \tag{8}$$

where 2.7 is a phenomenological lower bound[12] and 200 is an upper bound that we impose based on the most recent bismuth result[2] combined with our previous calculations.[4,7] (For $M_R/M_L > 200$ the left-right model actually predicts too large a value for the optical rotation in bismuth.) For this model, radiative corrections are extremely important; they determine the sign and magnitude of the asymmetry. Their effect is enhanced because C_{1u} and C_{2u} are opposite in sign from C_{1d} and C_{2d} [see Eq. (7) and compare with (2)].

DISCUSSION

Asymmetry measurements in deep-inelastic polarized electron-deuterium scattering at the level of gauge theory predictions will become available in the near future.[6] We briefly discuss our results and their relevance for this experiment.

Note that for the domain considered in Fig. 1, all three models predict negative values for $A_D(x,y)$. Also, the left-right model predicts a considerably larger asymmetry than the naive estimate $\sim 20\alpha/\pi$ (in the units of Fig. 1) one might have expected.

How will the deuterium experiment distinguish between these models? To see how, we consider the specific case $E_i = 19.4$ GeV, $\theta = 4°$ (angle between $\vec{p}_i, \vec{p}_f$) at the median point $y = 0.25$ which is relevant for the ongoing SLAC experiment.[6] For these definite values, we find (see Ref. 13) using $q^2 = -4E_i^2(1-y)\sin^2(\theta/2)$ the following allowed asymmetry regions (for $0.21 \leq \sin^2\theta_W \leq 0.33$, $2.7 \leq M_R/M_L \leq 200$):

$$\begin{aligned} &-13.0\times10^{-5} \leq A_D(0.15, 0.25) \leq -4.57\times10^{-5} \\ &\quad(\text{Weinberg-Salam model}), \\ &-6.23\times10^{-5} \leq A_D(0.15, 0.25) \leq -3.65\times10^{-5} \\ &\quad(\text{vectorlike model}), \\ &-3.23\times10^{-5} \leq A_D(0.15, 0.25) \leq -0.79\times10^{-5} \\ &\quad(\text{left-right model}), \end{aligned} \tag{9}$$

If the measured value of $A_D(0.15, 0.25)$ falls in the overlap region $-(3-6)\times 10^{-5}$ of (9), then a careful measurement of the y distribution would be necessary to distinguish these models. In the case of the Weinberg-Salam model,

$$A_D(0.15, 0.25) \simeq -2.77\times 10^{-4}(1 - 2.53 \sin^2\theta_W),$$

so it could in principle accommodate the very wide range

$$-27\times 10^{-5} \lesssim A_D(0.15, 0.25) \lesssim 42\times 10^{-5}$$

if we eliminate the constraint $0.21 \leqslant \sin^2\theta_W \leqslant 0.33$. However, values for $\sin^2\theta_W$ considerably outside of this domain would have to be reconciled with the findings of neutrino experiments.[11] The vectorlike model could accommodate a measurement somewhat outside of its range in (9) if we unconstrain $\sin^2\theta_W$, but not by so very much. We can state rather definitely that an experimental result outside of the left-right model's predicted range in (9) would immediately deal that model a fatal blow.

Note added in proof. After we submitted this work for publication, the results of the SLAC experiment mentioned in Ref. 6 were announced [C. Prescott *et al.*, Phys. Lett. 77B, 347 (1978)]. They found $A_D(x=0.2, y=0.21)/|q^2| = (-9.5\pm 1.6)\times 10^{-5}$. This measured asymmetry is consistent with the "standard" Weinberg-Salam model for $\sin^2\theta_W = 0.20\pm 0.03$ and lends strong support towards its credibility. The "hybrid" model can accommodate this large asymmetry only if we allow $\sin^2\theta_W \approx 0.01$ so that the radiative corrections in Eq. (6) become large; however, such a small value is inconsistent with neutrino scattering experiments and is therefore presumably ruled out. The left-right model discussed in our text can also accommodate this asymmetry if $M_R/M_L \simeq 10^{10}$; however, $Q_W(\text{Bi})$ is then predicted to be $\approx +62$ which disagrees with all the bismuth experiments.[2,3]

ACKNOWLEDGMENT

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[3] L. Barkov and M. Zolotorev, Zh. Eksp. Teor. Fiz. Pis'ma Red. 27, 379 (1978) [JETP Lett. 26, 379 (1978)] find an optical rotation in bismuth equal to 1.1 ± 0.3 times the Weinberg-Salam prediction for $\sin^2\theta_W = \frac{1}{4}$. The contradiction between this finding and Ref. 2 needs to be resolved.

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[8] In employing an effective Hamiltonian, we assume $-q^2$, $M_p E_i \ll M^2$, where M is a typical intermediate vector boson mass. We note that although the radiative corrections given in (6) and (7) were obtained ignoring external momenta, their inclusion does not significantly alter our results. The main effect is to replace m^2 by $\sim -q^2$ or $M_p E_i$ in the log terms. [Other neglected modifications are of order $(\alpha/\pi)(q^2/M^2)$ or $(\alpha/\pi)(M_p E_i/M^2)$.] Our most important one-loop contributions, the $\ln(M_R/M_L)$ terms in (7), are unmodified.

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[10] Q. Shafi and Ch. Wetterich, Phys. Lett. 69B, 464 (1977).

[11] The world average for $\sin^2\theta_W$ from neutrino data has been coming down toward 0.25. Here we consider 0.21 to 0.33 a viable domain.

[12] M. A. B. Bég *et al.*, Phys. Rev. Lett. 38, 1252 (1977).

[13] For the values $E_i = 19.4$ GeV, $\theta = 4°$, $y = 0.25$, we are at $x = 0.15$ which means that (2) is modified by antiquark and strange quark contributions to the asymmetry (Ref. 9). Using the approximate quark distributions (for deuterium) at $x = 0.15$, $f_{\bar u} \simeq f_{\bar d} \simeq 1.6 f_{\bar s} \simeq 1.6 f_s \simeq 0.088 f_u \simeq 0.088 f_d$ and $C_{1s} = C_{1d}$, we find the following modifications to (2): $C_{1d} \to 1.1 C_{1d}$, $C_{2u} \to 0.84 C_{2u}$, $C_{2d} \to 0.84 C_{2d}$ and the overall asymmetry is multiplied by 0.98. These are the quantities used to determine (9).