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Critical Feynman-Wilson gas: A model for multiparticle physics

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We develop a model in which hadron production in the true asymptotic region proceeds via the exchange of a factorizable singularity at $J = 1$, which implies a sensible meson spectrum. The rise of the hadronic total cross section and the inclusive plateau are ascribed to threshold effects of this mechanism, which is estimated to take effect at Fermilab energies. In the true asymptotic region the total cross section decreases like a small power of the rapidity, while fireball structure appears in the one-particle distribution. Both the exclusive (multiperipheral) and inclusive (Mueller) approaches are exploited. The discussion is in the language of statistical mechanics and our key assumptions are (i) existence of sensible thermodynamic limit, (ii) Koba-Nielsen-Olesen scaling, and (iii) factorization. We show that the nearest-neighbor interaction implied in the Feynman-Wilson "gas" by our factorizable singularity is responsible for its critical behavior at infinite rapidity.

I. INTRODUCTION

In a recent paper,[1] hereafter referred to as I, it was pointed out that the Koba-Nielsen-Olesen (KNO) scaling[2] assumption implies that when the true asymptotic region is reached, the Feynman-Wilson (FW) gas[3] undergoes a phase transition. The hadronic gas is looked at from the grand-canonical-ensemble point of view, and is assumed to be at constant temperature. As is the case with any classical system, one has to strictly consider the infinite-volume (infinite-total-rapidity) limit in order to reveal the mathematical characteristics of the phase transition, since at finite total rapidity the grand partition function is a polynomial, with positive coefficients, in the fugacity variable and the pressure cannot develop any singularities on the real fugacity axis.[4] Therefore, the infinite-rapidity limit considered here may become the source of undesired divergences. The way out proposed in I was to promote Yang and Lee's theorem[4] of classical statistical mechanics to a fundamental postulate in hadron physics and require that the FW gas have a finite and not identically vanishing analog pressure at infinite rapidity. This is certainly true in the context of a factorizable multiperipheral model with simple poles. The hadronic phase transition is, in general, of higher order. Results very similar to those obtained by the absorptive-model cutting rules[5] in Reggeon field theory[6] are found in I, unless the KNO scaling function decreases too rapidly to zero. In the latter case, the results of the simple two-component (multi-Regge plus simple diffraction) picture[7] are resurrected, namely a first-order phase transition and a logarithmic multiplicity growth.

In the present work we shall investigate the implications of the "macroscopic" properties of the FW gas found in I from grand-canonical-ensemble considerations, on the detailed form of the "microscopic interaction" between individual "molecules" of the hadronic gas.

We shall assume that the truly asymptotic diffractive contribution to hadronic collisions, which is responsible for the critical behavior of the FW gas, is built up by multiperipheral diagrams in which a factorizable singularity is exchanged. The structure of this singularity is simply related to the form of the interaction between FW gas nearest neighbors, which because of factorization are only allowed to interact. We then deal with a one-dimensional system with nearest-neighbor interactions only, hence the macroscopic information about the system, which is summarized by the asymptotic behavior of the KNO scaling function, uniquely determines the dynamics.[8] We find that the "potential energy" of a pair of FW gas nearest neighbors has a particularly simple and natural form, with a power-behaved hard core and a logarithmic long-range part responsible for the critical behavior of the hadronic gas, as revealed from macroscopic arguments in I. There emerges a well-defined model for asymptotic hadron physics in the longitudinal region. In particular, we find that before critical phenomena do strictly appear, the symmetry possessed by the lowest-potential-energy state of an FW gas molecule in the potential well of its nearest neighbors is spontaneously broken, hence condensation phenomena (clustering) appear in the FW gas at finite rapidity. The two-body interaction we found, supplemented by s-channel unitarity, requires the existence of a complicated Pomeron singularity with intercept at exactly unity, which is accompanied by an infinite series of complex-conjugate pairs of daughters. We predict that in the true asymptotic region the hadronic total cross sections slowly decrease to zero, while fireball structure appears in the one-particle inclusive distribution.

The plan of this paper is as follows: In Sec. II we formulate the FW gas analogy, develop the macroscopic description of the hadronic gas in the grand canonical ensemble and summarize some of the results in I for further reference. In Sec. III, from the leading asymptotic behavior of the KNO scaling function together with finite thermodynamic limit and factorization assumptions, we determine the form of the potential acting between FW-gas nearest neighbors. Its relevance to the meson spectrum is investigated in Sec. IV, while in Sec. V we present our solution to the problem of hadron physics in the asymptotic longitudinal region. In Sec. VI we look at the FW gas from the inclusive point of view and check the self-consistency of the scheme, proving that both the macroscopic and microscopic approaches lead to identical behaviors for the moments c_p. Finally, our conclusions are given in Sec. VII.

II. FEYNMAN-WILSON GAS ANALOGY AND KNO SCALING

The configurational partition function of a one-dimensional classical system of n particles confined in a "box" of total length L may be written as

$$Z_n(L)=\frac{1}{n!}\int_0^L\cdots\int_0^L dx_1\cdots dx_n \exp\left[-\beta\sum_{i>j=0}^{n}V(x_i,x_j)\right], \tag{2.1}$$

where $V(x_i,x_j)$ is the potential energy of the pair of particles at positions x_i and x_j, and $\beta=(kT)^{-1}$. The factorial accounts for the correct Boltzmann counting, and since classical particles are distinguishable, it disappears when we integrate over the subregion $0\leq x_1\leq x_2\leq\cdots\leq x_n\leq L$. We imagine the end particles to be rigidly placed at positions $x_0=0$ and $x_{n+1}=L$ but allowed to interact freely with the rest of the particles in the system. On the other hand, the n-particle production cross section in the most general multiperipheral model, where the total rapidity available is Y and all transverse dimensions are integrated over, may be written as

$$\frac{\sigma_n(Y)}{2\pi}=\frac{e^{-2Y}}{(16\pi^3)^{n+1}}\int_0^Y\cdots\int_0^Y dy_1\cdots dy_n\prod_{i>j=0}^{n}\tau^2(y_i,y_j), \tag{2.2}$$

where $\tau(y_i,y_j)$ is an effective propagator corresponding to the object exchanged between particles with rapidities y_i and y_j. The target and projectile have rapidities $y_0=0$ and $y_{n+1}=Y$, respectively. Equations (2.1) and (2.2) reveal the formal mathematical analogy[3] ($x_i\leftrightarrow y_i$, $L\leftrightarrow Y$) between a one-dimensional classical system of material points and a general multiperipheral model for hadron production where the transverse variables are averaged. Of course, the analogy may be extended to include many-body forces as well.[8] In the following, we assume the FW gas to be at constant temperature, and set $\beta\equiv1$.[8]

We define the grand partition (generating) function by

$$Q(z,Y)=\sum_{n=0}^{\infty}z^n\sigma_n(Y)/\sigma_t(Y), \tag{2.3}$$

where we have introduced the parameter z which is the fugacity of classical statistical mechanics. Our normalization is such that $Q(1,Y)=1$. Next comes one of our most crucial assumptions, namely that the hadronic gas obeys KNO scaling:

$$\langle n\rangle\sigma_n(Y)/\sigma_t(Y)=\psi(x),\quad x=n/\langle n\rangle. \tag{2.4}$$

Hence, at large rapidity we obtain the following integral representation for the grand partition function, in terms of the KNO scaling function $\psi(x)$:

$$Q(w)=\int_0^\infty e^{wx}\psi(x)dx,\quad w=\langle n\rangle\ln z. \tag{2.5}$$

We next define the quantity

$$\tilde{P}(z,Y)=\frac{1}{Y}\ln Q(w), \tag{2.6}$$

and as explained in I, we require that in the thermodynamic limit $Y\to\infty$, it converges uniformly to the finite and not identically vanishing analog pressure $P(z)$. This is the fundamental requirement which gives physical content to the formal mathematical analogy expressed by (2.1) and (2.2). In the context of a factorizable multiperipheral model with internal coupling constant g_{MP}, (2.3) may be written as

$$\sigma_t(\tilde{g},Y)=Q(\tilde{g}^2/g_{\text{MP}}^2,Y)\sigma_t(g_{\text{MP}},Y), \tag{2.3$'$}$$

where $\tilde{g}^2=zg_{\text{MP}}^2$ is an "unphysical" coupling constant for $z\neq1$. We thus see that the requirement of finite thermodynamic limit in this model is equivalent to the requirement of Regge-behaved total cross section when the coupling constant is continued from its physical value g_{MP}, and the pressure is related to the continued Regge intercept, $P(z)=\alpha(z)-\alpha(z=1)$.

For further reference, we next list some of the results obtained in I which we shall heavily rely upon. Without restricting the generality, the KNO scaling function is written as

$$\psi(x)=g(x)\exp[-f(x)], \tag{2.7}$$

where the function $g(x)$ is bounded by a power of x, and $f(x)$ must increase faster than x for large x, in order that the grand partition function we constructed converges for $z>1$. It can then easily be seen[1] that in the $Y\to\infty$ limit the *asymptotic behavior of $\psi(x)$ only, uniquely determines the (macroscopic) analog pressure* $P(z)$ of the FW gas, as a function of the fugacity. Specifying the asymptotic behavior of $f(x)$ for large x to be

$$f(x)=\alpha x^{\kappa}, \tag{2.8}$$

where α is a positive constant and $\kappa>1$, it is obtained in I for large Y that

$$\tilde{P}(z,Y)=\begin{cases}\frac{1}{Y}\left[-(\text{const})\ln|w|+O(1)\right] & \text{if } z<1,\\ \frac{1}{Y}\left[\alpha(\kappa-1)\left(\frac{w}{\alpha\kappa}\right)^{\nu}+O(\ln w)\right] & \text{if } z>1,\end{cases} \tag{2.9}$$

where $\nu=\kappa/(\kappa-1)$. In fact, the result $\tilde{P}(z<1, Y\to\infty)=0$ is independent of the particular asymptotic behavior of $\psi(x)$. Thus, requiring that the thermodynamic limit does exist and also that $P(z)\neq 0$, we obtain a definite prediction for the asymptotic behavior of the average hadronic multiplicity, namely

$$\langle n\rangle=cY^{1-\eta}, \tag{2.10}$$

where c is any positive constant and $\eta=1/\kappa$, hence $0<\eta<1$. Moreover, for the analog pressure we have

$$P(z)=\begin{cases}0 & \text{if } z<1,\\ \left(\frac{1}{b}\ln z\right)^{\nu} & \text{if } z>1\end{cases}. \tag{2.11}$$

The constant b is defined by

$$b=\frac{\kappa}{\kappa-1}[a(\kappa-1)]^{1/\kappa}, \tag{2.12}$$

where

$$a=\alpha c^{-\kappa}. \tag{2.13}$$

The result (2.11) shows explicitly that the pressure of the FW gas is a continuous and nondecreasing function of z, but since it is manifestly nonanalytic at $z=1$ and $\nu>1$, the hadronic system undergoes a higher-order phase transition, which is a λ transition unless ν is an integer. When $\kappa\to\infty$, a first-order phase transition is obtained, while $\langle n\rangle=cY$. If instead of (2.8) one considers the more general asymptotic behavior

$$f(x)=\alpha x^{\kappa}(\ln x)^{\lambda}, \tag{2.14}$$

then the behavior (2.10) of the multiplicity is modified by a power of $\ln Y$, but the pressure (2.11), and therefore the critical behavior of the hadronic gas, remains unchanged. If $f(x)$ increases for large x faster than any power of x, or if $\psi(x)$ has an explicit cutoff, one obtains the above results with $\kappa\to\infty$ $(\eta=0)$. For the large-order moments of the multiplicity distribution, it is found that

$$c_p\equiv\langle n^p\rangle/\langle n\rangle^p\propto(\text{const})^p\times p^{\eta p}. \tag{2.15}$$

All these results are checked in I to be consistent with the results obtained with the absorptive-model cutting rules[5] from Reggeon field theories.[6] In particular, the quantity $1/\kappa$ is identified with the critical exponent η of Reggeon field theories and is essentially the only parameter to appear in all our considerations in the rest of this paper.

Finally, we want to throw some more light onto the analog significance of the exponent η. We consider the ensemble average

$$N(z,Y)=z\frac{\partial}{\partial z}\ln Q(w), \tag{2.16}$$

where $N(1,Y)\equiv\langle n\rangle$ is the average hadronic multiplicity, and define the quantity

$$\tilde{\rho}(z,Y)=\frac{N(z,Y)}{Y}=\frac{\partial}{\partial\ln z}\tilde{P}(z,Y), \tag{2.17}$$

which in the $Y\to\infty$ limit becomes the analog density $\rho(z)$ of the FW gas. Eliminating z between (2.9) and (2.17), we obtain the equation of state of the hadronic gas, which for $Y\to\infty$ reads

$$\rho=\frac{\nu}{b}P^{1/\kappa}. \tag{2.18}$$

Since the critical values of pressure and density of the FW gas are both vanishing $[\tilde{P}(z\to1, Y\to\infty)=0,\ \tilde{\rho}(z\to1, Y\to\infty)=0]$ from relation (2.18) we see that the exponent κ is naturally identified with the critical index δ defined in statistical mechanics.[4] This index governs the relation between the "order parameter" and the "ordering field" near the critical point. E.g., for a gas-liquid transition, we have

$$\rho-\rho_c\propto(P-P_c)^{1/\delta}, \tag{2.18$'$}$$

where ρ_c and P_c are the critical values of density and pressure. For a ferromagnetic transition, the same relation reads

$$M\propto B^{1/\delta}, \tag{2.18$''$}$$

where B is the (external) magnetic field and M is the (spontaneous) magnetization.

III. TWO-BODY ANALOG POTENTIAL IN THE HADRONIC GAS

In the preceding section, we briefly reviewed the implications of KNO scaling together with the requirement for a sensible thermodynamic limit on the macroscopic properties, namely average multiplicity, pressure, and density of the hadronic gas. We have seen that the appearance of a phase transition will characterize the macroscopic behavior of hadron physics at infinite total rapidity.

We now want to throw some light on the microscopic form of the interaction among the FW gas molecules, which gives rise to the critical behavior revealed macroscopically. We shall assume factorization in the multiperipheral ladders which build up the asymptotic diffractive component of hadron physics we will be concerned with. From Eqs. (2.1) and (2.2) this means that only nearest-neighbor interactions will be allowed in the hadronic gas. On the other hand, since we are dealing with a one-dimensional system, it is well known from classical statistical mechanics[9] that we should not expect it to exhibit critical behavior, unless the intermolecular potential is of long range. Hence, it is no longer instructive to think of a one-dimensional "gas" as the analog of longitudinal hadron production, since long-range interactions between nearest neighbors only do not make proper sense in a classical gas (but for simplicity we shall continue to speak of the FW "gas"). We may still think at the classical level of a one-dimensional system of material points connected with light strings, which will be required to have peculiar mechanical properties. The end points of the system are rigidly placed at positions 0 and Y and form the walls of the container of the system.

Under these circumstances, the configurational partition function (2.1) of the system may be expressed as

$$Z_n(Y)=\int_0^Y dy_n e^{-V(Y-y_n)}\int_0^{y_n} dy_{n-1}e^{-V(y_n-y_{n-1})}\cdots\int_0^{y_2} dy_1 e^{-V(y_2-y_1)}e^{-V(y_1)}\,. \tag{3.1}$$

Thus the mathematical analogy between (i) a factorizable multiperipheral model in which only longitudinal phase space is available and (ii) a one-dimensional classical system with nearest-neighbor interactions only is realized by the identifications

$$\sigma_n(Y)/2\pi g^2 \longrightarrow Z_n(Y)\,,\quad g=G_{\rm MP}{}^2/g_{\rm MP} \tag{3.2}$$

$$g_{\rm MP}{}^2 e^{-2y}\tau^2(y)/16\pi^3 \longrightarrow \exp[-V(y)]\,, \tag{3.3}$$

where $\tau(y)$ is the effective factorizable propagator relevant to asymptotic multiperipheralism and we have explicitly introduced the external ($G_{\rm MP}$) and internal ($g_{\rm MP}$) coupling constants appropriate for the multiperipheral graph. We obviously have

$$\sigma_0(Y)/2\pi g^2 \equiv \sigma_{\rm el}(Y)/2\pi g^2 = \exp[-V(Y)]\,. \tag{3.4}$$

Now, Eq. (3.1) is simply an iterated Laplace convolution and it is equivalent to[9]

$$\int_0^\infty e^{-sY}Z_n(Y)dY=K^{n+1}(s)\,, \tag{3.5}$$

where the Laplace transform $K(s)$ of (3.3) has been introduced, i.e.,

$$\int_0^\infty dy\,\exp[-sy-V(y)]=K(s)\,. \tag{3.6}$$

The parameter s is known from statistical mechanics[9] to be directly connected to the pressure of the system: $s=\beta P=P$. In classical statistical mechanics, given the intermolecular potential in the system under consideration, one calculates its partition function from (3.5) and (3.6), hence, all of its properties follow. Here we shall follow the reverse process; in fact, given *some* information about the partition function of the FW gas through the asymptotic behavior of the KNO function, we shall *completely* determine the form of the two-body potential $V(y)$. It is because we deal with a *one-dimensional* system with *nearest-neighbor interactions* only (factorization) that the macroscopic properties of the system (described by the asymptotic behavior of the KNO scaling function) uniquely define the dynamics. Let us also point out that it is not *a priori* certain that only nearest-neighbor interactions can give rise to a *prescribed* critical behavior of a classical system. In fact, it is not certain that *any* sort of potential interactions can result in it at all.

We shall presently show, however, that (i) factorization, together with (ii) the requirement that hadron physics possesses a sensible thermodynamic limit, and (iii) the asymptotic behavior of the KNO scaling function are enough to determine uniquely and unambiguously a very natural two-body potential which does gives rise to the critical behavior of the hadronic gas found in I.

We now start the calculation of the analog potential $V(y)$ by collecting together all the pieces of information we have about the structure of the partition function of the FW gas, namely:

(i) We have KNO scaling (2.4), and the KNO scaling function has the form (2.7), with $f(x)$ bounded by a finite power of x, since otherwise we have to deal with an extreme and rather uninteresting case as explained in Sec. II.

(ii) For the total cross section we shall assume the behavior

$$\sigma_t(Y)=u(Y)\exp[-h(Y)]\,, \tag{3.7}$$

where

$$\lim_{Y\to\infty} h(Y)=0\,, \tag{3.8}$$

and $u(Y)$ is bounded by a power of Y for large Y.

(iii) The average multiplicity is also bounded by a power of the rapidity, in fact $\langle n\rangle < Y$ in order to have a finite thermodynamic limit as found in I. Consequently, the partition function of the FW gas may be written as

$$Z_n(Y)=\frac{\sigma_t(Y)}{2\pi g^2\langle n\rangle}\psi(x) = \frac{u(Y)g(x)}{2\pi g^2\langle n\rangle}\exp[-f(x)-h(Y)], \qquad (3.9)$$

or, more compactly,

$$Z_n(Y)=G_n(Y)\exp[-F_n(Y)], \qquad (3.9')$$

with an obvious definition for the functions $G_n(Y)$ and $F_n(Y)$ which are then both bounded by powers of both n and Y.

We next estimate the kernel $K(s)$ with steepest-descent approximation to the integral in (3.5). The result is

$$K^{n+1}(s)=\left(\frac{2\pi}{F_n''(Y_0)}\right)^{1/2}\left\{\sum_m \frac{G_n^{(2m)}(Y_0)}{(2m)!![F_n''(Y_0)]^m}\right\} \times\exp[-sY_0-F_n(Y_0)], \qquad (3.10)$$

where the saddle point Y_0 is defined by

$$s=-F_n'(Y_0). \qquad (3.11)$$

Considering the $n\to\infty$ limit, the kernel $K(s)$ is found to be independent of $g(x)$ and $u(Y)$:

$$K(s)=\lim_{n\to\infty}\exp\left[-\frac{sY_0+F_n(Y_0)}{n}\right]. \qquad (3.12)$$

It is clear from this relation that an arbitrary asymptotic behavior of the KNO scaling function is not in general consistent with factorization, in the sense that the right-hand side of (3.12) is not independent of n for an arbitrary function $F_n(Y)$.

We now consider explicitly the asymptotic behavior (2.8) of $f(x)$ which allows for thermodynamic limit if the average multiplicity has the asymptotic behavior (2.10). By virtue of (2.8) and (2.10), relation (3.11) becomes

$$[s+h'(Y_0)]Y_0^{\kappa}=(\kappa-1)an^{\kappa}, \qquad (3.11')$$

where the constant a has been defined in (2.13). This shows that $Y_0\to\infty$ for $n\to\infty$; hence, because of (3.8), the saddle point Y_0 is determined independently of the particular form of the total cross section in the $n\to\infty$ limit:

$$Y_0=[(\kappa-1)a/s]^{1/\kappa}n. \qquad (3.13)$$

We note that at the position of the saddle point we have for large n

$$\langle n\rangle \sim Y_0^{1-\eta}\sim n^{1-\eta}\ll n. \qquad (3.14)$$

This means that the asymptotic behavior of $\psi(x)$ we considered is indeed only relevant to the calculation of $K(s)$ in the $n\to\infty$ limit, which is then unambiguously determined to be

$$K(s)=\exp(-bs^{1-\eta}). \qquad (3.15)$$

The constant b has been defined in (2.12). If $Z_n(Y)\propto Y^{\lambda}x^{\delta}e^{-\alpha x^{\kappa}}$ (λ,δ constants), the result (3.15) may be checked by direct integration of (3.5), to be exact, in the $n\to\infty$ limit, for $\eta=\frac{1}{2}$ or $s=0$. As already discussed, the fact that $K(s)$ was found independent of n means that the asymptotic behavior (2.8) of the KNO scaling function is indeed consistent with factorization. Note that the condition for "mechanical stability" of the FW gas, namely $\partial P/\partial Y<0$ which indeed follows from the equation of state (2.18), guarantees that the second derivative of the exponent in the integrand of (3.5) with respect to Y is negative, which is necessary in order that the steepest-descent approximation be sensible.

To conclude this discussion, it is worth noting that the relation[7,8,9]

$$\frac{1}{z}=\int_0^{\infty}dy\, e^{-Py-V(y)}, \qquad (3.6')$$

which holds quite generally in one-dimensional systems with nearest-neighbor interactions and uniquely interrelates the macroscopic pressure $P=P(z)$ of the system to the underlying dynamics, is equivalent to our Eqs. (3.6) and (3.15). Indeed, inverting the expression (2.11) for the pressure of the FW gas which was determined by the asymptotic behavior of $\psi(x)$ only, we obtain

$$\frac{1}{z}=\exp(-bP^{1-\eta})\equiv K(P). \qquad (3.15')$$

This result is not surprising, since (3.6′) is a general relation, but it shows the consistency between our macroscopic and microscopic considerations. In fact, we could have worked from the beginning with (3.6′) and (3.15′), instead of (3.5) and (3.6), in order to determine the two-body potential $V(y)$.

Once we have determined, one way or another, the structure of the kernel $K(s)$, we may now invert the Laplace transform (3.6) in order to determine the two-body "potential energy" $V(y)$. We have

$$\exp[-V(y)]=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}ds\,\exp(sy-bs^{1-\eta}). \qquad (3.16)$$

The behavior of (3.16) for small y can be found by the standard stationary-phase approximation technique. We choose a contour of integration which passes through the point of stationary phase s_0 of the integrand ($c=s_0$), which lies on the real s axis and is defined by the relation

$$y-(1-\eta)bs_0^{-\eta}=0. \qquad (3.17)$$

We find

$$\exp[-V(y)]_{y\to 0}=[\kappa(\kappa-1)a/2\pi]^{1/2}y^{-(\kappa+1)/2} \times\exp(-a/y^{\kappa-1}), \qquad (3.18)$$

which means that the potential has a "hard core" of the form $V(y) \sim y^{1-\kappa}$ $(y \to 0)$, which also provides the threshold behavior to the elastic cross section. Note that the small y behavior (3.18) is continued for all y if $\kappa = 2$.

We next convert (3.16) into an integral along the cut of the integrand, i.e.,

$$\exp[-V(y)] = \frac{1}{\pi}\int_0^\infty ds\, e^{-sy}\exp(-bs^\mu \cos\mu\pi) \times \sin(bs^\mu \sin\mu\pi)\,, \qquad (3.19)$$

where $\mu = 1-\eta$. Hence, expanding the last two factors in the integrand of this representation in a power series in s^μ, we obtain the following series representation of the potential:

$$\exp[-V(y)] = \frac{1}{\pi}\sum_{l=1}^{\infty}\frac{(-1)^{l+1}b^l \sin l\mu\pi}{l!}\frac{\Gamma(l\mu+1)}{y^{l\mu+1}}\,, \qquad (3.20)$$

which is convergent for all y. The behavior of (3.20) for $y \to \infty$, i.e., $V(y) \sim (2-\eta)\ln y$ shows the form of the long-range part of the potential which is responsible for the phase transition of the one-dimensional system. It also shows the asymptotic behavior of the elastic cross section $\sigma_{el} \sim Y^{-2+\eta}$.

In Fig. 1 we show the result of a numerical evaluation of the potential and of the elastic cross section, using the exact series representation (3.20). The particular numerical values of the parameters we have chosen will be justified in the next section. In the same figure, the stationary-phase approximation (3.18) is also shown as a dotted curve. Although it does not have the correct asymptotic behavior, (3.18) provides a good numerical estimation of the potential for rather large rapidities.

Let us finally remark that had we started with the more general asymptotic behavior (2.14) of the KNO scaling function, instead of (2.8), we would have been led, as remarked in the preceding section, to the same pressure (2.11) and relation (3.15′); hence, the potential (3.20) would remain unchanged.

IV. MESON SPECTRUM

From the asymptotic behavior (2.7) and (2.8) of the KNO scaling function and the asymptotic behavior (2.10) of the average multiplicity, which in turn was dictated by the requirement of the existence of thermodynamic limit in the theory, we were able in the preceding section to determine uniquely and unambiguously the structure of the analog potential which is relevant to nearest-neighbor interactions in the FW gas. In order to carry over to high-energy physics the techniques of classical statistical mechanics, we had to consider the limit of the production of an infinite number of hadrons. But since we assume that hadrons are produced from an infinite factorizable multi-

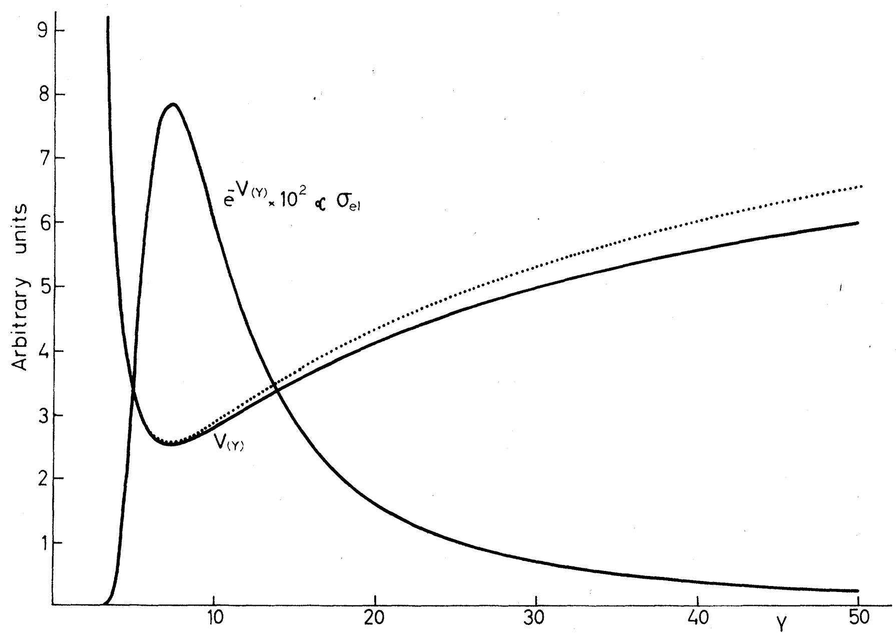

FIG. 1. Two-body analog potential (3.20) acting between Feynman-Wilson gas nearest neighbors for $\eta = \frac{1}{4}$. The parameter a is chosen, so that the model gives the correct pion intercept, $\mathrm{Re}\,\alpha_\pi(0) = -0.02$ as explained in Sec. IV. The approximate closed formula (3.18) is also shown as a dotted curve. The quantity $\exp[-V(y)]$ is proportional to the elastic cross section.

peripheral chain, the structure of the propagator that our analog two-body potential implies is in fact relevant to the production of any finite number of hadrons. In the rest of this paper, we consider a factorizable multiperipheral model in which only longitudinal phase space is available and an object with structure $\exp[-V(y)]$ is iterated in the multiperipheral ladder.

We shall show in this section that the potential (3.20) implies rich and interesting pole structure in the t-channel elastic partial-wave amplitudes. Denoting by $A(Y)$ the imaginary part of the asymptotic s-channel forward elastic amplitude, s-channel unitarity at $t=0$ in our model, namely

$$\sum_{n=0}^{\infty}\sigma_n(Y)=\sigma_t(Y)=e^{-Y}A(Y)\,,$$

is equivalent to the integral equation

$$A(Y)=2\pi g^2\exp[Y-V(Y)]+\int_0^Y dy\,\exp[y-V(y)]A(Y-y)\,. \tag{4.1}$$

This equation is of convolution type, and since at high energies the Froissart-Gribov projection reduces to a Laplace-transform, for the t-channel elastic partial-wave amplitudes at $t=0$, we readily obtain

$$A(j)=2\pi g^2[\exp b(j-1)^{1-\eta}-1]^{-1}\,, \tag{4.2}$$

where we used the result (3.15) for the Laplace transform of $\exp[-V(y)]$. This expression is explicitly seen to have poles at $j=j_m$, $m=0,1,2,\ldots$, with

$$\mathrm{Re}\,j_m=1+\left(\frac{2\pi m}{b}\right)^{\nu}\cos\frac{\nu\pi}{2}\,,\qquad \mathrm{Im}\,j_m=\pm\left(\frac{2\pi m}{b}\right)^{\nu}\sin\frac{\nu\pi}{2}\,. \tag{4.3}$$

That is, in general, they lie in complex-conjugate positions on the j plane, except for the mother ($m=0$) singularity which is always real and lies at exactly unity. This is our Pomeron, which is then seen to have a complicated cut structure and to pass from exactly unity. All the daughters ($m=1,2,\ldots$, become real when

$$\eta=\tfrac{1}{2},\tfrac{3}{4},\tfrac{5}{6},\ldots\,, \tag{4.4}$$

while they acquire maximal imaginary part, and all fall on the $\mathrm{Re}\,j=1$ line on the j plane when

$$\eta=0,\tfrac{2}{3},\tfrac{4}{5},\tfrac{6}{7},\ldots\,. \tag{4.5}$$

We are very much tempted to interpret these poles as the intercepts of Regge trajectories $j_m\equiv\alpha_m(0)$, associated with physical particles which should manifest themselves as first-sheet poles on the j plane. Assuming that extrapolation of the high-ranking members of (4.3) to physical masses will not move them very much on the j plane—which is a reasonable assumption at least for the pion—and requiring that the poles (4.3) themselves lie on the first sheet, for the critical exponent η we obtain the bound

$$\eta\leq\tfrac{1}{2}\,. \tag{4.6}$$

In order that all first-sheet poles satisfy the Froissart bound one by one, i.e., $\mathrm{Re}\,\alpha_m(0)\leq 1$ for $m=1,2,\ldots$, we must have

$$0\leq\eta\leq\tfrac{1}{2}\,. \tag{4.7}$$

Since $0<\eta<1$ is *a priori* required in I in order that the grand partition function of the hadronic system exists for $z>1$, we see that all first-sheet poles do respect the Froissart bound individually. This is a good situation because, although the sum of the contributions of all of the poles (4.3) to the s-channel elastic amplitude obeys the Froissart bound, as we shall show in the next section, we may speculate that introduction of internal symmetries into our scheme will allow the exchange of only a few of the poles (4.3) for a particular process, since the residues of those poles not consistent with quantum number conservation are expected to vanish.

Let us now look closer at the structure of the meson spectrum predicted by (4.3). The intercept of the first daughter is related to that of the mth by

$$[1-\mathrm{Re}\,\alpha_m(0)]/[1-\mathrm{Re}\,\alpha_1(0)]=m^{\nu}\,. \tag{4.8}$$

Since it is an experimental fact that meson trajectories are almost equidistant, we see at this stage that the critical exponent η cannot be close to unity. On the other hand, it is very crucial to have a good candidate for the pion pole which is very close to $t=0$, and we do not need to extrapolate our scheme very much in order to reach it. Fifteen years of experience with few- and many-body reactions teaches us that no meson intercept lies close to unity.[10] In fact, we think that the estimation

$$\mathrm{Re}\,\alpha_1(0)\lesssim 0.6 \tag{4.9}$$

is quite justifiable. Hence, if $\mathrm{Re}\,\alpha_m(0)\simeq 0$ for some m, from (4.8) we obtain

$$m^{\nu}\lesssim 2.5\,. \tag{4.10}$$

This means that the pion may only be the second daughter, whence from (4.10) we obtain

$$\eta\lesssim 0.25\,. \tag{4.11}$$

Although values of η close to zero are favored by the meson spectrum, since they lead to almost equidistant trajectory intercepts (4.3), they also suggest that the KNO scaling function is too rapidly decreasing with x (for $x\gtrsim 1$) to be consistent with the existing Fermilab data. Instead, if we choose $\eta=\frac{1}{4}$, the upper bound, and if we require $\mathrm{Re}\,\alpha_2(0)=-0.02$ (the pion), we obtain from (4.3), $b=7.36$, hence from (2.12), $a=310$. With these values of the parameters, it is apparent from Fig. 1 that the threshold for the onset of this mechanism is at about three units of rapidity. This is just at the right place if we decide to ascribe the rise of the total hadronic cross section and of the inclusive plateau at Fermilab and CERN ISR to the threshold effects of this asymptotic mechanism. We will come back to this point in the next section.

We note that the spectrum (4.3) appears highly degenerate. Our first daughter intercept should possibly be identified with the ρ-A_2-ω-f-ϕ-f' trajectory intercept, while A_1, η, and B should all possibly lie on the second (pion) daughter. For $\eta=\frac{1}{4}$ and $a=310$ [which means $\mathrm{Re}\,\alpha_2(0)=-0.02$], we obtain $\mathrm{Re}\,\alpha_1(0)=0.59$ and also $\mathrm{Re}\,\alpha_3(0)=-0.75(\eta', \delta, S^*?)$, $\mathrm{Re}\,\alpha_4(0)=1.57$ (ϵ?). In Fig. 2 we illustrate these results. Of course, these numerical estimations of the degenerate intercepts should not be taken too literally. What is important and should be concluded from this discussion is the correlation of the truly asymptotic behavior of hadron physics with the properties of the existing meson spectrum, through the introduction of the critical exponent η.

One could perhaps worry that even $\kappa=4$ is too large a value to be consistent with the existing data on the KNO scaling function. But remember that it is only asymptotically ($Y\to\infty$) that we require KNO scaling with (2.8) as the large x behavior of the KNO scaling function. In fact, we shall show in the next section that, in the context of this model, at nonasymptotic rapidities, the quantity $\langle n\rangle\sigma_n(Y)/\sigma_t(Y)$ decreases for large x much slower than is apparent from its asymptotic ($Y\to\infty$) behavior (2.8).

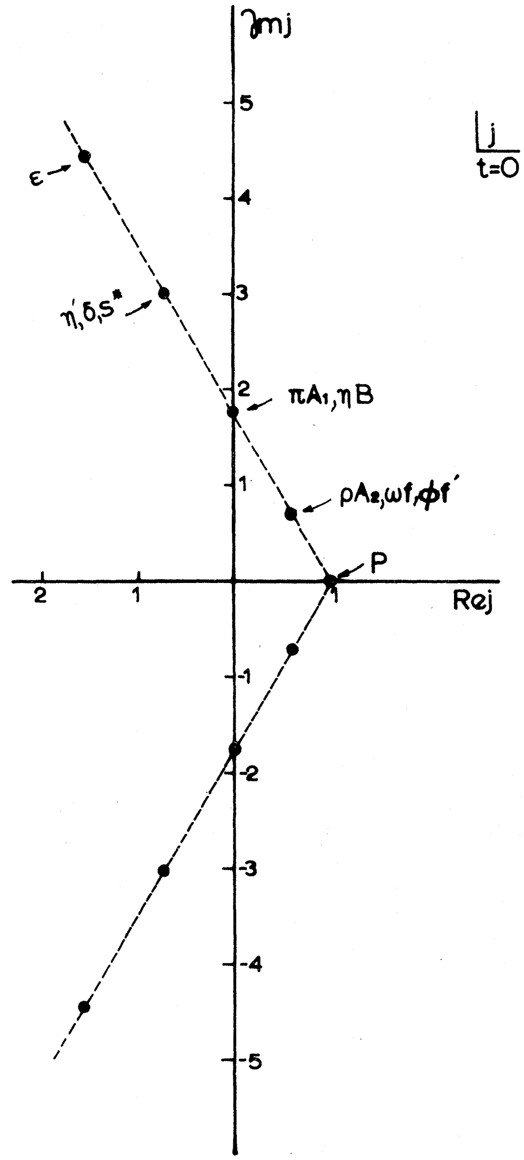

FIG. 2. Positions of the poles (4.3) in the t-channel elastic partial wave amplitude for $t=0$, $\eta=\frac{1}{4}$, and $\mathrm{Re}\,\alpha_\pi(0)=-0.02$. Their conjectured identification with the intercepts of degenerate physical meson trajectories is shown.

V. TRULY ASYMPTOTIC HADRON PHYSICS

Having shown that the truly asymptotic diffractive component of hadron physics, which we uncovered by considering the critical behavior of the FW gas at infinite rapidity, implies a meson spectrum with reasonable properties, we now turn to calculate the contribution of this component, to some quantities relevant to longitudinal asymptotic physics. We have seen that (i) the assumption of the existence of thermodynamic limit and (ii) the asymptotic ($n\gg\langle n\rangle$) behavior of the KNO scaling function completely determine the macroscopic properties (pressure, density, average multiplicity) of the FW gas. Moreover, since we assume (iii) factorization and the hadronic gas lives in one dimension, its macroscopic properties uniquely determine the dynamics. Hence, in this model the physics for *large n* completely determines the physics for *all n*. In particular, starting from (3.20), we shall determine in this section the behavior of $\sigma_t(Y)$ and $g(x)$, which although present in (3.9) did not affect the determination of the kernel $K(s)$ in the $n\to\infty$ limit.

n-particle production and total cross sections

In the framework of our factorizable multiperipheral model in which no transverse dimensions are accessible and the object $\exp[-V(y)]$ is iterated in the multiperipheral ladder, we have to invert the transform (3.5) with the expression (3.15) of the kernel. We have

$$\frac{\sigma_n(Y)}{2\pi g^2}=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} ds\, \exp[sY-(n+1)bs^{\mu}]\,. \quad (5.1)$$

Similarly to (3.18), the $Y\rightarrow 0$ (threshold) behavior of (5.1) for fixed n is found by the standard stationary-phase approximation technique to be

$$\sigma_n(Y)/2\pi g^2=[\kappa(\kappa-1)a/2\pi]^{1/2}(n+1)^{\kappa/2}Y^{-(\kappa+1)/2}\times\exp[-a(n+1)^{\kappa}/Y^{\kappa-1}]\,. \quad (5.2)$$

This result is exact for all Y and n if $\eta=\frac{1}{2}$. From (5.2) we see that the hierarchy of the n-particle production thresholds Y_n is qualitatively given by $Y_n\propto(n+1)^{\nu}$ $(n=0,1,2,\ldots)$. Note that for fixed Y, (5.2) also gives the large-n behavior of $\sigma_n(Y)$.

Similarly to (3.20), we obtain from (5.1) the exact series representation of $\sigma_n(Y)$,

$$\frac{\sigma_n(Y)}{2\pi g^2}=\frac{1}{\pi}\sum_{l=1}^{\infty}\frac{(-1)^{l+1}(n+1)^l b^l \sin l\mu\pi}{l!}\times\frac{\Gamma(l\mu+1)}{Y^{l\mu+1}}\,, \quad (5.3)$$

which is convergent for all Y and n. In Fig. 3 we illustrate the results of a numerical evaluation of $\sigma_n(Y)$ for $\eta=\frac{1}{4}$ and $b=7.36$. For completeness we also include the elastic $(n=0)$ cross section in this figure. We see that inelastic channels via this mechanism open up at considerably large rapidities, and for $Y>20$ they all add up to give a smooth total cross section, which is also illustrated in Fig. 3. The asymptotic behavior of σ_t may be immediately found by Laplace-transforming the leading singularity of (4.2) at $j=1$ to be

$$\sigma_t\sim Y^{-\eta}\,. \quad (5.4)$$

We thus see that this new asymptotic mechanism for hadron production we have revealed imitates strongly, but on a considerably lengthened rapidity scale, most of the properties of hadron production at low rapidity via the well-established mechanisms. One can possibly argue that the rise of the hadronic total cross sections observed at Fermilab and ISR is just the low-energy side of a broad local maximum which reflects the threshold effects for the onset of a new mechanism for hadron production which alone survives in the true asymptotic region, where we predict that total hadronic cross sections are slowly decreasing to zero. Of course, in this picture for $Y\lesssim 3$ only the usual meson-exchange mechanisms build up the total hadronic cross section. It is worth recalling that the idea of ascribing the rise of the pp total cross section at ISR to threshold effects is by no means a new one. In particular, attempts have been made[11] to explain this rise as a threshold effect of the onset of baryon-antibaryon production. To conclude this discussion we note that our scheme is free of the Finkelstein-Kajantie problem.[12] In fact, the unitarity iteration of the object $\exp[-V(y)]$ which represents the elastic cross section gives back a total cross section of the form $y^{-\eta}$. The logarithmic terms in $\exp[-V(y)]$ are such that they are not promoted to a power of the energy; we deal with a stable Pomeron at $j=1$.

One-particle exclusive rapidity distribution

The potential energy of an FW gas molecule trapped in the potential well of total length (rapidity) Y, which is formed by its nearest neighbors, is simply given as

$$V_1(y;Y)=V(\tfrac{1}{2}Y+y)+V(\tfrac{1}{2}Y-y)\,. \quad (5.5)$$

The microscopic origin of the phase transition in

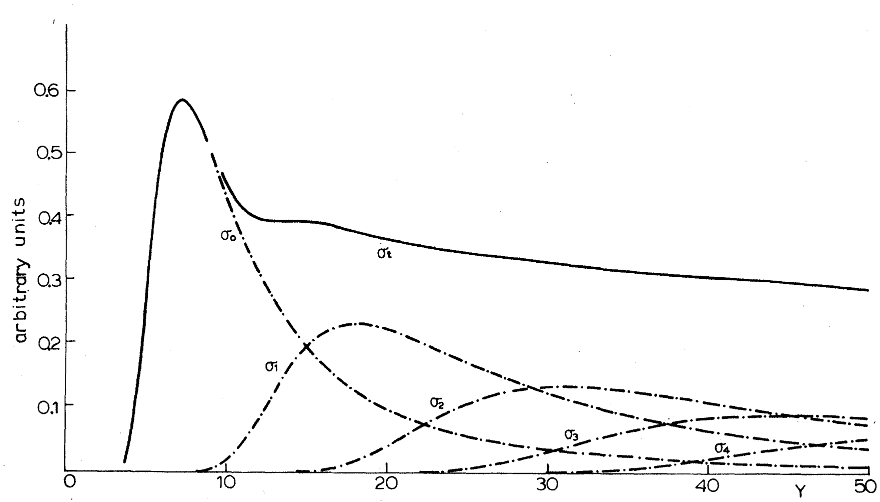

FIG. 3. n-particle production cross sections (5.3) together with the total cross section for $\eta=\frac{1}{4}$ and $\mathrm{Re}\,\alpha_\pi(0)=-0.02$.

the one-dimensional FW system must be searched in the spontaneous symmetry-breaking property possessed by (5.5). Since

$$V(y) \sim \ln y \ (y \to \infty), \quad V(y) \sim y^{1-\kappa} \ (y \to 0), \tag{5.6}$$

it follows that the stationary point of $V_1(y;Y)$ at $y=0$ changes character for a finite value Y_c of the total rapidity, which is given as a solution of the equation

$$\left. \frac{\partial^2 V_1(y;Y)}{\partial y^2} \right|_{\substack{y=0 \\ Y=Y_c}} = 0, \tag{5.7}$$

and from a local minimum it becomes a local maximum. Conditions (5.6) guarantee that (5.7) does have a solution $Y_c = Y_c(\kappa, a)$. For example, using the approximate closed formula (3.18) for the potential, we easily find

$$Y_c \simeq 2[2\kappa(\kappa-1)a/(\kappa+1)]^{1/(\kappa+1)}. \tag{5.8}$$

Hence, there always exists a critical value Y_c of the total rapidity such that if $Y \le Y_c$, one minimum potential energy position exists at $y=0$, leading to pionization in the one-particle exclusive rapidity distribution

$$\frac{1}{2\pi g^2} \frac{d\sigma_1^{ex}}{dy} = \exp[-V_1(y;Y)]. \tag{5.9}$$

But if $Y > Y_c$, two symmetric minima appear in $V_1(y;Y)$ and fragmentation phenomena show up in the one-particle distribution. We are thus faced with a spontaneous-symmetry-breaking phenomenon, which in the language of physical chemistry means that condensation phenomena (which may be called clustering phenomena in high-energy physics) will manifest themselves in the FW gas, after a finite value Y_c of the total rapidity is reached, before the phase transition actually takes place at $Y \to \infty$. With the favorite values of our parameters, we find $Y_c \simeq 22.8$ units. Figure 4 illustrates the above results at two values of the total rapidity, below and above its critical value.

KNO scaling

Recalling the asymptotic expressions (2.10) and (5.4) for the average multiplicity and total cross section, we readily obtain from (5.3)

$$\frac{\langle n \rangle}{\sigma_t(Y)} \sigma_n(Y) \underset{Y \to \infty}{\sim} \frac{1}{\pi} \sum_{l=1}^{\infty} \frac{(-1)^{l+1}(cb)^l \sin l\mu\pi}{l!} \Gamma(l\mu+1) x^l, \tag{5.10}$$

where at large Y, $(n+1)/\langle n \rangle \simeq n/\langle n \rangle = x$. That is, although we started assuming KNO scaling essen-

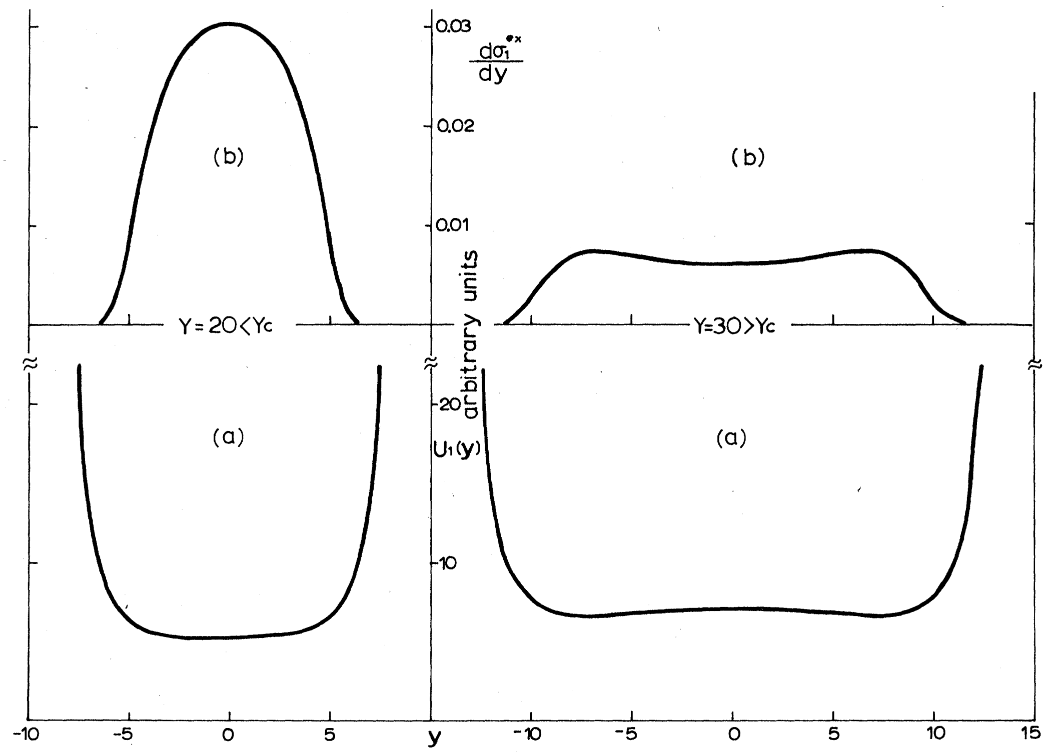

FIG. 4. (a) Potential energy of a Feynman-Wilson gas molecule in the potential well formed by its nearest neighbors and (b) one-particle exclusive rapidity distributions, for total rapidities 20 and 30 units. For $\eta = \frac{1}{4}$ and $\mathrm{Re}\,\alpha_\pi(0) = -0.02$, the critical value of the total rapidity is 22.8 units.

tially at large x,[1] our factorizable model implies KNO scaling for all x. For $x\to 0$ the KNO scaling function behaves like $\psi(x)\sim x$. In order to find its behavior at large x, we remember that (3.18) is the small-y behavior of the series (3.20); hence, recalling relation (2.13), we immediately find from (5.10)

$$\psi(x)\sim x^{\kappa/2}\exp(-\alpha x^{\kappa}) . \tag{5.11}$$

That is, we obtained the exponential piece of $\psi(x)$ which we started with, plus the behavior of the function $g(x)$ which was irrelevant to the calculation of the potential on which our scheme is based. The asymptotic ($Y\to\infty$) expression (5.11) with $\kappa=4$ is too rapidly decreasing with x for $x\gtrsim 1$ in order to be consistent with the existing Fermilab data, although it has a maximum at $x\simeq 1$. But KNO is only an extreme asymptotic property in our model. In fact, in Fig. 5 we illustrate the quantity $\langle n\rangle\sigma_n/\sigma_t$ as a function of x at various rapidities, we well as the asymptotic expression (5.11), as well clearly seen that at nonasymptotic rapidities the quantity $\langle n\rangle\sigma_n/\sigma_t$ decreases with x (for $x\gtrsim 1$) much slower than its asymptotic ($Y\to\infty$) behavior (5.11) suggests.

VI. MUELLER APPROACH

We now turn to the implications of the potential (3.20) for the Mueller graph. We have

$$\langle n(n-1)\cdots(n-p+1)\rangle\sigma_t(Y)=p!\int_0^Y dy_p\int_0^{y_p}dy_{p-1}\cdots\int_0^{y_2}dy_1\frac{d^p\sigma^{in}}{dy_1dy_2\cdots dy_p} . \tag{6.1}$$

Summing up the multiperipheral diagrams which build up the Mueller graph, we obtain

$$\frac{d^p\sigma^{in}}{dy_1dy_2\cdots dy_p}=\left(\frac{g_{\rm ML}}{G_{\rm ML}{}^2}\right)^p\sigma_t(Y-y_p)\sigma_t(y_p-y_{p-1})\cdots\sigma_t(y_2-y_1)\sigma_t(y_1) , \tag{6.2}$$

where the internal ($g_{\rm ML}$) and external ($G_{\rm ML}$) coupling constants in the Mueller graph are related to the coupling constants $g_{\rm MP}$ and $G_{\rm MP}$ of the multiperipheral graph by

$$g_{\rm ML}/G_{\rm ML}{}^2=(g_{\rm MP}/G_{\rm MP}{}^2)^2/2\pi . \tag{6.3}$$

Thus (6.1) becomes an iterated Laplace convolution and, similarly to (3.5) and (3.6), we obtain as equivalent to (6.1) and (6.2) the equations

$$\Lambda^{p+1}(s)=\int_0^\infty dY\,e^{-sY}\langle n(n-1)\cdots(n-p+1)\rangle\sigma_t(Y)\Big/p!\left(\frac{g_{\rm ML}}{G_{\rm ML}{}^2}\right)^p , \tag{6.4}$$

where

$$\Lambda(s)=\int_0^\infty dy\,e^{-sy}\sigma_t(y) . \tag{6.5}$$

It is worth noting that these equations, with the asymptotic behavior (5.4) of the total cross section, lead[13] to the behavior (2.15) of the large-p moments obtained in I from macroscopic considerations in the FW gas, assuming KNO scaling. Conversely, and similarly to our procedure in Sec. III in determining the potential (3.20), assuming the structure (2.15) for the moments c_p, we may determine from (6.4) and (6.5) the behavior of σ_t which they imply. Indeed, assuming the general structure (3.7) and (3.8) for σ_t, with standard steepest-descent approximation to the integral in (6.4) in the $p\to\infty$ limit, we find

$$\Lambda(s)=(\text{const})/s^{1-\eta} , \tag{6.6}$$

independently of the particular form of the total cross section. As discussed below (3.15), the fact that $\Lambda(s)$ was found from (6.4) to be independent of p means that the behavior (2.15) of the moments c_p is consistent with factorization. The result (6.6) may be checked to be exact, in the $p\to\infty$ limit, by direct integration of (6.4), if $\sigma_t(Y)$ is assumed to be an arbitrary power of Y. Inversion of (6.5) with the kernel (6.6) leads us back to the asymptotic behavior (5.4) of the total cross section, and this may serve as another test of the consistency between the macroscopic and microscopic approaches to the FW gas statistical mechanics.

The above discussion shows that the system we have considered in this work is equivalent to a hadronic world in which $\sigma_t(Y)\sim Y^{-\eta}$ for $Y\to\infty$ ($0<\eta<1$) and the multihadron production is generated by factorizable Mueller graphs. In particular, owing to factorization in the Mueller graphs, the critical phenomenon which was found to dominate the physics of hadrons for $Y\to\infty$ can be described by a pole-cut interaction in an appropriate complex plane. In fact, the generating function (2.3) satisfies the following integral equation, obtained by Bardeen and Peccei,[14] which is a direct consequence of Mueller factorization,

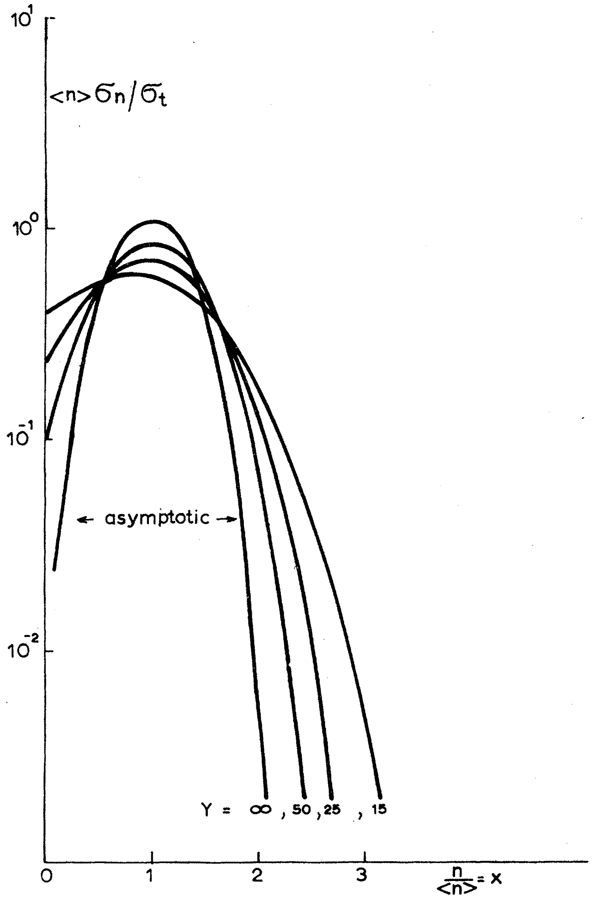

FIG. 5. The quantity $\langle n\rangle \sigma_n/\sigma_t$ as a function of $x=n/\langle n\rangle$ at various values of the total rapidity Y, for $\eta=\frac{1}{4}$ and $\mathrm{Re}\,\alpha_\pi(0)=-0.02$.

and in our notation and normalization reads:

$$\sigma_t(Y)Q(z,Y)=\sigma_t(Y)+g_{\mathrm{ML}}G_{\mathrm{ML}}{}^{-2}(z-1)\times\int_0^Y dy\,\sigma_t(y)\sigma_t(Y-y)Q(z,Y-y)\,. \tag{6.7}$$

The solution to this equation is given by

$$\sigma_t(Y)Q(z,Y)=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\frac{e^{\theta Y}d\theta}{\Lambda^{-1}(\theta)-g_{\mathrm{ML}}G_{\mathrm{ML}}{}^{-2}(z-1)}\,. \tag{6.8}$$

Since the total cross section satisfies the multiperipheral integral equation (4.1), recalling the Laplace transform (3.15) of $\exp[-V(y)]$, we readily obtain

$$\Lambda(\theta)=2\pi g^2[\exp(b\,\theta^{1-\eta})-1]^{-1}\,, \tag{6.9}$$

and because of (6.3), relation (6.8) becomes

$$\frac{\sigma_t(Y)}{2\pi g^2}Q(z,Y)=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\frac{e^{\theta Y}d\theta}{\exp(b\,\theta^{1-\eta})-z}\,. \tag{6.8$'$}$$

This relation may also be obtained by combining (2.3) with (5.1). The integrand in (6.8′) has a real moving pole for $z>1$ which dominates $Q(z,Y)$ for $Y\to\infty$, and it is identified with the pressure (2.11) in this region. In fact, (6.8′) for large Y gives (2.6) which is the definition of the analog pressure. If $z<1$, the dominant singularity is the fixed branch point at $\theta=0$, and, therefore, the pressure vanishes for all $z<1$. The pole and the branch point at $\theta=0$ exchange their role when they collide at $z=1$, and this mechanism is the complex-θ-plane realization of the phase transition in

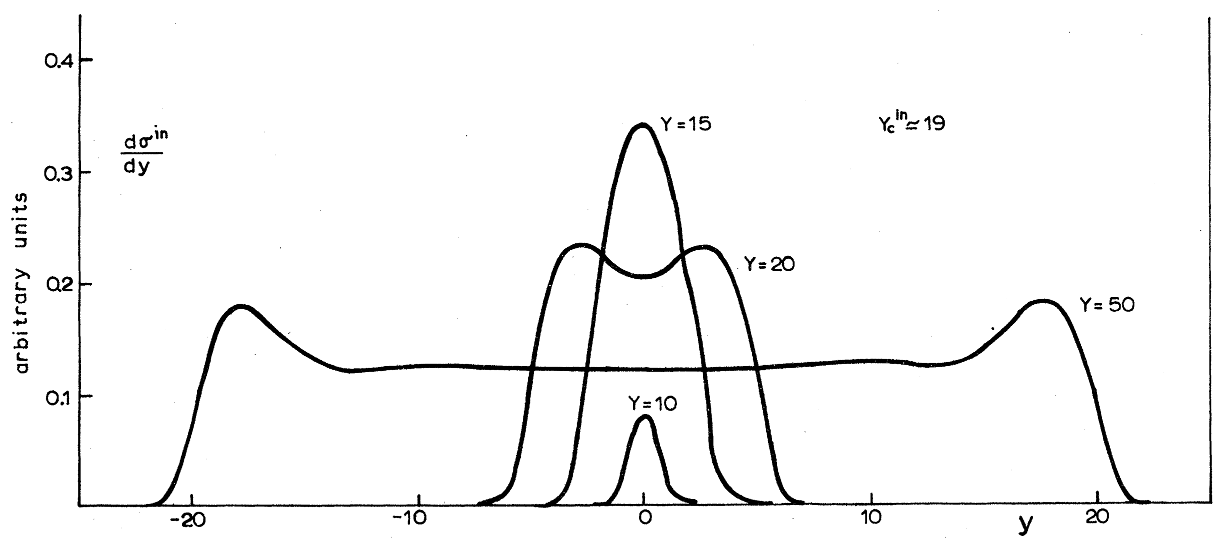

FIG. 6. One-particle inclusive rapidity distribution (6.10) for $\eta=\frac{1}{4}$ and $\mathrm{Re}\,\alpha_\pi(0)=-0.02$ at various values of the total rapidity below and above its critical value (at about 19 units).

the FW gas under consideration. A byproduct of this discussion could be the determination, through the singularity structure of (6.8′), of the coupling constants in the Mueller graph, in terms of the coupling constants of the multiperipheral graph. Indeed, requiring that the pole in (6.8′) for $z > 1$ coincide with the pressure (2.11), relation (6.3) follows. Let us remark that although (6.9) is a steepest-descent result, all the conclusions which are based on it may be checked to be exact for z close to unity, where only the asymptotic part (5.4) of the total cross section is relevant, by direct integration of (6.5).

Finally, we calculate the one-particle inclusive rapidity distribution, in the center of mass:

$$\left(\frac{g_{\mathrm{ML}}}{G_{\mathrm{ML}}{}^2}\right)^{-1}\frac{d\sigma^{in}}{dy}=\sigma_t(\tfrac{1}{2}Y+y)\sigma_t(\tfrac{1}{2}Y-y)\,. \qquad (6.10)$$

It is illustrated in Fig. 6 for the favorable values of the parameters. The threshold effects in σ_{e1} (hard core) provide the walls of the rapidity distribution. We have similar phenomena as in the exclusive distribution, but more pronounced and at lower critical rapidity, at about 19 units for the favorable values of the parameters. When $Y < 19$ we have pionization and the center of the plateau rises to a maximum value. The rise of the inclusive plateau for $pp \rightarrow \text{hadron} + X$, recently confirmed[15] at ISR, may be attributed to the setup of this mechanism. When the critical rapidity is reached, the bump in σ_{e1} reflects itself into a clear fireball structure on $d\sigma^{in}/dy$. This structure remains near the walls at higher rapidities, leaving a flat plateau in the central region, while secondary fireball effects appear as well. Note that these predictions are similar, although at much larger rapidities, to those of the fireball models designed to explain old cosmic-ray data above ISR energies.[16] At asymptotic rapidities we only have approximate Feynman scaling, with a slowly decreasing plateau height.

VII. CONCLUSIONS

We have argued that the critical behavior of the FW gas at infinite rapidity reveals the existence of a new asymptotic diffractive component for hadron production which takes effect at Fermilab energies. Assuming this component to be built up by multiperipheral diagrams in which a new factorizable singularity is exchanged and guided by the critical behavior of the FW gas, which in turn is necessitated by KNO scaling and finite thermodynamic limit assumptions, we have determined the form of the potential interaction between FW gas nearest neighbors which, because of factorization, are only allowed to interact. We have shown that a factorizable long-range interaction can indeed result to the critical behavior of the FW gas, and in order to determine it with the techniques of classical statistical mechanics we had to consider the limit of the production of an infinitely large number of hadrons. But once only nearest-neighbor interactions are important, this very same potential is relevant to the production of any number of hadrons at sufficiently high energies. We thus obtained a solution to the truly asymptotic diffractive component of hadron production, the relevance of which to longitudinal high-energy physics was investigated. We have also shown that an equivalent approach to the FW gas statistical mechanics may be formulated starting from the inclusive point of view.

Our main points may be summarized as follows:

(i) The critical behavior of the FW gas at infinite rapidity requires the existence of a meson spectrum with daughter structure. The mother (Pomeron) singularity is real and lies at exactly unity, while all daughters appear in complex-conjugate positions and as far as the critical exponent η satisfies the bound $\eta < \frac{1}{2}$, they all lie on the first sheet and have intercepts below unity.

(ii) Our mechanism takes effect at Fermilab energies and may possibly be responsible for the rise of the hadronic total cross sections and of the inclusive plateau there. After all threshold effects have settled, a smooth total cross section is established which in the true asymptotic region behaves like $\sigma_t \sim Y^{-\eta}$.

(iii) The exponent η has the meaning of the critical index $1/\delta$ of statistical mechanics as revealed by the relation $p \sim \rho^{\eta}$ near the critical point of the FW gas.

(iv) In the true asymptotic region we have KNO scaling. The average multiplicity behaves like $\langle n\rangle \sim Y^{1-\eta}$ while the large order moments of the distribution like $c_p \sim (\text{const})^p \times p^{\eta p}$. As long as we have factorization, this behavior of c_p is equivalent to the asymptotic behavior $\sigma_t \sim Y^{-\eta}$ of the total cross section.

(v) At moderate rapidity we have pionization in the one-particle distribution, but after a critical rapidity is reached fireball structure appears. We have only approximate Feynman scaling and the height of the inclusive plateau decreases slowly in the true asymptotic region.

In subsequent work we intend to look more closely at the possible phenomenological implications of these ideas, and to include transverse dimensions in the formalism.

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