Bose-Einstein correlations and determination of fireball dimension in hadron collisions

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(Received 23 February 1977)

We investigate assumptions necessary for determination of the interaction region fireball in hadron collisions from recently observed correlations between pions of the same charge. We show, in analogy to quantum optics, that this is possible when the emission from fireball is partially incoherent. This leads to a density matrix for produced pions which embodies several models previously used. A simple formula is derived, relating space-time distribution of the emission region to the experimentally measured like-particle correlations at small momentum difference.

I. INTRODUCTION

There have been several reports[1] on the observations of positive correlations of like pions in hadron-hadron collisions. Namely, there is an excess of pairs of pions of the same charge emitted with small four-momentum difference as compared with analogous pairs of opposite charge. This effect does not seem to be of dynamical origin, as there are no doubly charged resonances. Other explanations, such as three-body decays of resonances or kinematical effects, also seem to be ruled out.[2] Those correlations are most naturally interpreted as the effect of Bose-Einstein statistics,[3] in analogy to the Goldhaber effect.[4] In fact Grishin, Kopylov, and Podgoretsky pointed out that such correlations should be observable in hadron collisions. Using definite models, these authors showed that this effect could be used for the determination of space-time dimensions of the interaction region. It is our purpose here to investigate the assumptions underlying the production process necessary to make such an interpretation possible. It appears that the production process has to be of a statistical nature and the interaction region has to behave as a chaotic source. Similar sources are widely used in quantum optics,[5] and the Glauber representation[6] for the density matrix of produced pions is most natural. Therefore we see the measurement of like-meson interference as a hadronic analog of the Hanbury Brown and Twiss experiment.[7] The coherent-state representation for the pion field was used previously (see, for instance, Ref. 8) but the space-time interpretation was not given. In Sec. II we review the concept of coherence as used in optics. We also introduce there the coherent-state representation. The description of the interaction region in hadron collision as a chaotic source is presented in Sec. III, and its properties are discussed in Sec. IV.

II. COHERENCE AND COHERENT STATES

Let us recall that in classical optics one introduces, for the description of coherence phenomena, the function of mutual coherence which is defined as the statistical average of the (electric) fields,[5] i.e.,

$$\Gamma(\vec{r}, t; \vec{r}', t') = \langle V^*(\vec{r}, t) V(\vec{r}', t')\rangle \ . \tag{1}$$

Thus defined, the function Γ obeys the wave equation with respect to $\vec{r}, t$ as well as with respect to $\vec{r}', t'$,

$$\left(\Delta - \frac{\partial^2}{\partial t^2}\right)\Gamma(\vec{r}, t; \vec{r}', t') = 0 \ . \tag{2}$$

One can use Green's theorem to generate an integral representation for Γ at arbitrary points just in terms of its values on some surface which can be the surface of the light-emitting source,

$$\tilde{\Gamma}(\vec{r}, \vec{r}'; \nu) = \int_S d\vec{s} \times \int_S d\vec{s}' \frac{\partial G}{\partial \vec{n}}(\vec{r}, \vec{s}) \frac{\partial G}{\partial \vec{n}'}(\vec{r}', \vec{s}') \tilde{\Gamma}(\vec{s}, \vec{s}'; \nu) \ , \tag{3}$$

where G is the appropriate Green's function and $\tilde{\Gamma}$ is the coherence function for a single frequency

$$\Gamma(\vec{r}, t; \vec{r}', t) = \int_0^\infty e^{2\pi i \nu (t' - t)} \tilde{\Gamma}(\vec{r}, \vec{r}'; \nu) d\nu \ . \tag{4}$$

Notice that even if different points of the source emit light independently,

$$\tilde{\Gamma}(\vec{s}, \vec{s}'; \nu) = I(\vec{s}, \nu) \delta_{(S)}(\vec{s} - \vec{s}') \ , \tag{5}$$

the mutual-coherence function is not proportional to the delta function in other points of space

$$\Gamma(\vec{r}, \vec{r}'; \vec{r}) = \int_S d\vec{s} \frac{\partial G}{\partial \vec{n}}(\vec{r}, \vec{s}) \frac{\partial G}{\partial \vec{n}}(\vec{r}', \vec{s}) I(\vec{s}, \nu)$$

$$\neq \delta^3(\vec{r} - \vec{r}') . \tag{6}$$

In other words, the field, which is statistically independent for different points of the source, requires partial spatial coherence after propagation. In the wave zone, formula (6) takes the following form:

$$\tilde{\Gamma}(\vec{r}, \vec{r}', \nu) = \frac{1}{R^2} \int_S \exp\left[-i \frac{\nu}{R} \vec{s}(\vec{r} - \vec{r}')\right] I(\vec{s}, \nu) d\vec{s} , \tag{7}$$

where $R \gg |\vec{r} - \vec{r}'|$ is the distance of observation points from the source. This formula shows that under the assumption of independent emission, the coherence function is simply the Fourier transform of the emission intensity at the radiating surface. This property is the basis for the interferometric determination of stellar diameters. Quantum description of chaotic sources shows[6] that the correlations among the photon-counting distributions are also expressible in terms of geometric properties of the source. For more detailed treatment of those topics, see for instance Ref. 5.

Coherence properties in quantum theory are most easily expressible in terms of coherent states. Let us denote by $a^\dagger(\vec{k})$ and $a(\vec{k})$ creation and annihilation operators of a particle of momentum $\vec{k}$, with the following comutation relations:

$$[a(\vec{k}), a^\dagger(\vec{k})] = \delta_3(\vec{k} - \vec{k}') . \tag{8}$$

We consider for simplicity scalar isospin = 0 particles. By definition, the coherent state $|\alpha\rangle$ is the eigenstate of the annihilation operator:

$$a(\vec{k}) | \alpha\rangle = \alpha(\vec{k}) | \alpha\rangle , \tag{9}$$

where $\alpha(\vec{k})$ is an arbitrary complex function. The state $|\alpha\rangle$ can be written in the form

$$|\alpha\rangle = \exp\left[-\tfrac{1}{2} \int |\alpha(\vec{k})|^2 d^3k\right] \times \exp\left[\int \alpha(\vec{k}) a^\dagger(\vec{k}) d^3k\right] |0\rangle , \tag{10}$$

where $|0\rangle$ is the vacuum defined by

$$a(\vec{k}) |0\rangle = 0 . \tag{11}$$

We denote $\omega = (\mu^2 + \vec{k}^2)^{1/2}$ the pion energy, where μ is the pion mass. States $|\alpha\rangle$ are normalized to unity but are not orthogonal

$$\langle \alpha | \beta \rangle = \exp\left[-\tfrac{1}{2} \int | \alpha(\vec{k}) - \beta(\vec{k}) |^2 d^3k\right]. \tag{12}$$

The probability density of finding n particles with momenta $\vec{k}_1, \ldots, \vec{k}_n$ is

$$|\langle 0 | a(\vec{k}_1) \cdots a(\vec{k}_n) | \alpha \rangle|^2 / n! = |\alpha(\vec{k}_1)|^2 \cdots |\alpha(\vec{k}_n)|^2 e^{-\bar{n}} / n! , \tag{13}$$

where the average number $\bar{n}$ in the state $|\alpha\rangle$ is

$$\bar{n} = \langle \alpha | \int d_3k a^\dagger(\vec{k}) a(\vec{k}) | \alpha \rangle = \int d^3k |\alpha(\vec{k})|^2 . \tag{14}$$

The multiplicity distribution in the state $|\alpha\rangle$ is given by the Poisson formula

$$P(n) = \int d^3k_1 \cdots d^3k_n |\langle 0 | a(\vec{k}_1) \cdots a(\vec{k}_n) | \alpha \rangle|^2 / n! = \frac{\bar{n}^n}{n!} e^{-\bar{n}} . \tag{15}$$

Although not orthogonal, coherent states form a complete set:

$$1 = \int \prod_k \frac{d^2\alpha_k}{\pi} |\{\alpha_k\}\rangle\langle\{\alpha_k\}| , \tag{16}$$

where $|\{\alpha_k\}\rangle = |\alpha\rangle$,

$$d^2\alpha_k = d(\mathrm{Re}\,\alpha_k) d(\mathrm{Im}\,\alpha_k) . \tag{17}$$

The continuous product in (16) should be understood as a functional integration, or we can think of our system as enclosed in a large box.

All the density matrices can be written in a form diagonal in the coherent-state representation[5]

$$\hat{\rho} = \int P(\{\alpha_k\}) |\{\alpha_k\}\rangle\langle\{\alpha_k\}| \prod_k \frac{d^2\alpha_k}{\pi} , \tag{18}$$

where the weigth function P is, in general, a distribution. For a pure coherent state $|\gamma\rangle$, the weight function is simply a product of delta functions:

$$P(\{\alpha_k\}) = \prod_k \delta^2(\alpha_k - \gamma_k) . \tag{19}$$

It is worth noting that a classical radiating current $j(\vec{r}, t)$ produces a field in a coherent state[6] $|\gamma\rangle$,

$$\gamma_{(\vec{k})} = \frac{1}{(2\nu)^{1/2}} \int \frac{d^3r\,dt}{(2\pi)^{3/2}} \exp[i(\omega t - \vec{k} \cdot \vec{r})] j(\vec{r}, t) . \tag{20}$$

One of the nice properties of the coherent-state representation is the composition rule for several radiating currents. Given two currents producing separately coherent states $|\beta\rangle$ and $|\gamma\rangle$, the state produced by both currents taken together is simply $|\beta+\gamma\rangle$. For the weight functions we have then

$$P(\{\alpha_k\})=\int P_1(\{\beta_k\})P_2(\{\gamma_k\}) \times\prod_k \delta^2(\alpha_k-\beta_k-\gamma_k)\frac{d^2\beta_k}{\pi}\frac{d^2\gamma_k}{\pi} \quad . \quad (21)$$

III. INCOHERENT-EMISSION MODEL

We have seen in the previous section that, in optics the shape of the source could be deduced from the correlation functions if we assume that the radiation from different points of the radiating source is statistically independent. We will construct here a density matrix for a source ("fireball") independently radiating pions. The source can be taken as the interaction region in the hadron-hadron collision. More specifically, let us assume that the fireball can radiate pions at point $\vec{r}$ and time t with probability $\rho(\vec{r},t)$.

In general, in the "elementary" act of emission many pions can be produced. For simplicity we assume here that they are produced in the coherent state $|\{\gamma_k(\vec{r},t)\}\rangle$. The function $\rho(\vec{r},t)$ is the probability density of elementary acts of emission. Our conclusions are to a large extent independent of the assumptions concerning the elementary act of emission, provided that the radius of the elementary source is smaller than the characteristic radius of the fireball. We can think of the elementary act of emission as the radiation produced by a small classical source concentrated around $\bar{r}_0$ and t:

$$j(\bar{r},t;\bar{r}_0,t_0)=j_0(\bar{r}-\bar{r}_0,t-t_0)\,, \quad (22)$$

i.e., all the elementary acts are expressible in terms of a unique function j_0. Using Eq. (20), the function $\gamma_k(\vec{r},t)$ can be expressed in terms of the Fourier transform of j_0:

$$\gamma_k(\vec{r},t)=\exp[i(\omega(\vec{k})t-\vec{r}\cdot\vec{k})]\,\gamma_0(\vec{k})\ , \quad (23)$$

where

$$\gamma_0(\vec{k})=\frac{1}{(2\omega)^{1/2}}\int\frac{d^3r\,dt}{(2\pi)^{3/2}}\,e^{i(\omega t-\vec{k}\cdot\vec{r})}j_0(\vec{r},t)\ . \quad (24)$$

The probability of the emission at the point $x=(t,\vec{r})$ is $\rho(x)\Delta x$, and the probability of no emission is $1-\rho(x)\Delta x$. The weight function for this case has the form

$$P_x(\{\alpha_k\})=[1-\rho(x)\Delta x]\prod_k\delta^2(\alpha_k) +\rho(x)\Delta x\prod_k\delta^2(\alpha_k-\gamma_k(x))\ . \quad (25)$$

We now have to add the contributions from all points x. According to (21), the weight function for the whole process is

$$P(\{\alpha_k\})=\int\left[\prod_k\delta^2\left(\alpha_k-\sum_k\beta_k(x)\right)\prod_x P_x(\{\beta_k(x)\})\right] \times\prod_x\prod_k d^2\beta_k(x)\ . \quad (26)$$

It is convenient to introduce the Fourier transform of $P(\{\alpha_k\})$

$$\Xi(\{\lambda_k\})=\int P(\{\alpha_k\})\exp\left[i\sum_k\tfrac{1}{2}(\lambda_k\alpha_k^*+\lambda_k^*\alpha_k)\right] \times\prod_k d^2\alpha_k\ , \quad (27)$$

where the asterisk indicates complex conjugation. In our case, we have

$$\Xi(\{\lambda_k\})=\prod_x Z_x(\{\lambda_k\})\ , \quad (28)$$

where

$$Z_x(\{\lambda_k\})=\int P_x(\{\beta_k\})\exp\left[i\sum_k\tfrac{1}{2}(\lambda_k\beta_k^*+\lambda_k^*\beta_k)\right] \times\prod_k d^2\beta_k\ . \quad (29)$$

Using (25) we find

$$Z_x(\{\lambda_k\})=1+\rho(x)\Delta x \times\left\{\exp\left[i\sum_k\tfrac{1}{2}[\lambda_k^*\gamma_k(x)+\lambda_k\gamma_k^*(x)]\right]-1\right\}. \quad (30)$$

Taking the logarithm of both sides of (28) and using (30) we find

$$\ln\Xi(\{\lambda_k\})=\int d^4x\,\rho(x) \times\left\{\exp\left[i\sum_k\tfrac{1}{2}[\lambda_k^*\gamma_k(x)+\lambda_k\gamma_k^*(x)]\right]-1\right\}. \quad (31)$$

This exact result can be further simplified if the average number of particles produced by the fireball is much larger than the average number of particles produced in the elementary act. In this case we can expand the right-hand side of (31) keeping only terms linear and quadratic in λ_k.[6] We finally obtain

$$\Xi(\{\lambda_k\})=\exp\left\{\frac{i}{2}\sum_k \Gamma(k)(\lambda_k+\lambda_k^*)-\tfrac{1}{8}\sum_{kk'}\left[G^{(+)}(k,k')(\lambda_k\lambda_{k'}+\lambda_k^*\lambda_{k'}^*)+2G^{(-)}(k,k')\lambda_k\lambda_{k'}^*\right]\right\}, \quad (32)$$

where

$$\Gamma(k)=\gamma_0(k)\tilde{\rho}_0(k) , \quad (33)$$

$$G^{(\pm)}(k,k')=\gamma_0(k)\gamma_0(k')\tilde{\rho}(k\pm k') . \quad (34)$$

We have introduced the four-dimensional Fourier transform of $\rho(x)$:

$$\tilde{\rho}(k)=\int d^4x e^{ik\cdot x}\rho(x) , \quad (35)$$

$$\tilde{\rho}_0(k)=\tilde{\rho}(\vec{k}, \omega=(\mu^2+\vec{k}^2)^{1/2}) . \quad (36)$$

Formula (32) is our basic result, as it defines the pion density matrix in our model. Actually, it is never necessary to compute the weight function $P(\{\alpha_k\})$ as its Fourier transform Ξ is the generating function for inclusive cross sections.

IV. CONSEQUENCES OF THE MODEL

We begin the discussion of our model by looking at the one- and two-particle inclusive cross sections. One can easily check that the n-particle inclusive cross section can be expressed in terms of

$$\frac{d\sigma_n^{\text{incl}}}{d_3q_1\cdots d_3q_n}=\left(\frac{-\partial^2}{\partial\lambda_{q_1}\partial\lambda_{q_1}^*}\right)\times\cdots\times\left(\frac{-\partial^2}{\partial\lambda_{q_1}\partial\lambda_{q_n}^*}\right)\Xi(\{\lambda_k\})\bigg|_{\{\lambda_k=0\}} . \quad (37)$$

In particular, we find for the one-particle inclusive cross section

$$\frac{d\sigma_1^{\text{incl}}}{dq}=\tfrac{1}{4}[G^{(-)}(q,q)+\Gamma^2(q)]$$

$$=\gamma_0{}^2(k)[1+\tilde{\rho}_0{}^2(q)]/4 . \quad (38)$$

If we assume that the radius of the elementary act of emission is much smaller than the fireball dimension, the Fourier transform $\gamma_0(k)$ is a slowly varying function compared with $\tilde{\rho}_0(q)$. Thus the inclusive one-particle cross section for center-of-mass-system (c.m.s.) small momentum is determined only by $\tilde{\rho}_0(k)$, i.e., by the fireball shape. Only for large c.m.s. momentum can we see fine details of the production mechanism— the effects of elementary acts of emission. Looking at a two-particle cross section we find

$$\frac{d\sigma_2^{\text{incl}}}{d^3qd^3k}=\frac{d\sigma_1^{\text{incl}}}{d^3q}\frac{d\sigma_2^{\text{incl}}}{d^3k}+\tfrac{1}{16}\{2\Gamma(q)\Gamma(k)[G^{(+)}(q,k)+G^{(-)}(q,k)]+G^{(+)}(q,k)^2+G^{(-)}(q,k)^2\} . \quad (39)$$

We see that the model indeed gives nontrivial two-particle correlations, expressed by the second term in (39). From the definitions of $\Gamma(k)$ and $G^{(\pm)}(k,q)$ [Eqs. (33)–(37)] we see that they are most pronounced for $\vec{k}\approx\vec{q}$. The dominant term is $G^-(q,k)^2$, the other terms being smaller by at least a factor of $e^{-2\tau^2\mu^2}$, where τ is a characteristic time of interaction. The correlation function

$$R(q,k)=\frac{d\sigma_2^{\text{incl}}}{d^3qd^3k}\bigg/\left(\frac{d\sigma_1^{\text{incl}}}{d^3q}\frac{d\sigma_1^{\text{incl}}}{d^3k}\right) \quad (40)$$

does not depend on the structure of the elementary act of emission. It depends only on the fireball density distribution $\rho(\vec{r},t)$. For τ sufficiently long to make $\tilde{\rho}(k+q)$ and $\tilde{\rho}_0(k)$ negligible we find a particularly simple expression:

$$R(\vec{q},\vec{k})=1+\frac{1}{\tilde{\rho}(0)^2}\tilde{\rho}^2(\omega_q-\omega_k,\vec{q}-\vec{k}) . \quad (41)$$

Formula (42) provides a direct link between the experimentally measured function R and the space-time distribution of the emission region. Its physical content is the following: the range of correlations in energy $(\omega_q-\omega_k)$ is inversely proportional to the duration τ_0 of emission, while the three-momentom correlation range is inversely proportional to the radius r_0. Note that R can never exceed 2 (reached at $\vec{q}=\vec{k}$); in the case of partial incoherence it may be less; full coherence implies $R=1$ (no correlations).

Recent results of Biswas *et al.*[9] indicate that

$$R=1+A\exp[-B(\vec{q}-\vec{k})^2] , \quad (42)$$

with

$$A=0.8\pm0.1 \quad (43)$$

and

$$B=(11.2\pm2.4)\ \text{GeV}^{-2} . \quad (44)$$

Formula (42) predicts

$$A=1 , \quad (45)$$

in fair agreement with (43). From (A4) we deduce

$$(\langle r^2\rangle)^{1/2}=(1.64\pm0.18)\ \text{fm} . \quad (46)$$

The fact that (42) depends only on the momentum difference suggests that

$$\tau_0 \ll r_0/c \approx 1.64 \text{ fm}/c \ . \tag{47}$$

To find τ_0, experimentally one should carefully measure the dependence of R not only on $(\vec{q}-\vec{k})$ but also on $(\vec{q}+\vec{k})$. On the other hand, analysis performed by Para[12] leads to different conclusions concerning the interaction time. This high-statistics experiment was interpreted in terms of formulas provided by Kopylov.[3] In our language it describes a source which is a disk of radius r_0, having a duration τ_0. Para[12] finds for $\pi^- $-$p$ interactions

$$r_0 = (1.85 \pm 0.15) \text{ fm} \ , \tag{48}$$

$$c\tau_0 = (1.0 \pm 0.2) \text{ fm} \ . \tag{49}$$

Production is found to be totally incoherent,

$$R = 2 \tag{50}$$

for $\omega_q - \omega_k = 0$, $\vec{q} - \vec{k} = 0$.

The general expression (32) yields, under additional assumptions, some of the models used previously. Neglecting incoherence effects, i.e., the part quadratic in λ_k, we recover the uncorrelated-jet model[10] which gives the Poisson multiplicity distribution. In this model, inclusive cross sections (39) are factorable; in particular, the correlation function R [Eq. (40)] is equal to 1 and does not depend on the momentum of pions, in spite of the fact that this model takes properly into account the Bose-Einstein statistics of pions. Therefore we see that the positive correlation of like pions does not follow automatically from Bose-Einstein statistics, but is instead *evidence for (partial) incoherence of the pion field.*

Kopylov *et al.*[3] presented several specific models for purely incoherent radiation, i.e., neglecting linear terms in (33), in the context of two-particle correlations.

Karczmarczuk,[11] also neglecting the coherent part and assuming, for the incoherent part, simple Gaussian dependence in difference of rapidities (short-range order in rapidities) has shown that the resulting multiplicity distribution is given by a Polyá distribution, which is indeed broader than a Poisson distribution and has been applied to describe proton collisions.[12] Botke *et al.*[8] noted early the advantages of using a coherent-state representation for the pion field and constructed on this basis several eikonal-type models which have exact s-channel unitarity, emphasizing the fact that the pion field should not be taken as purely coherent.

V. CONCLUSIONS

We have shown that like-pion correlations are the effect of the incoherence of the pion field produced in hadron-hadron collisions. In analogy to quantum optics, such an incoherent field arises naturally if we assume statistical emission from an extended source. For this case we have constructed explicitly the density matrix for pions. Generating the function for the density matrix has two parts, corresponding to purely coherent and purely incoherent radiation. The incoherent (chaotic) part gives rise to the like-pion correlations and allows for the determination of spatial extension of the production region: the two-particle correlation function is approximately given by the Fourier transform of the source density distribution. Our formalism can be easily extended to include impact-parameter dependence and the effects of isospin, topics which we are now investigating.

ACKNOWLEDGMENT

This research was supported by the National Science Foundation under Grant No. GF 36217.

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