Charmed baryons, their mass splitting, and weight diagrams

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Sum rules relating the masses of the $1/2^+$ charmed and uncharmed baryons are derived using weight diagrams.

With the discovery of narrow resonances at 3.1 and 3.7 GeV[1] and with the introduction of theories with charmed quarks,[2] the search for the charmed baryons was begun.[3] It is therefore of interest to know any sum rules between the masses of the charmed and uncharmed baryons.

In a recent paper,[4] the present authors derived some relations between the masses of $\frac{3}{2}^+$ charmed and uncharmed baryons from weight diagrams. The purpose of the present note is to derive relations between the masses of $\frac{1}{2}^+$ charmed and uncharmed baryons using the same technique.

There are six SU(2) subgroups of SU(4), namely, I, U, V, L, P, and R spin.[5] The magnetic moments of hadrons are assumed to depend on L spin.[6] The values of L spin for quarks are given in Table I and for $\frac{1}{2}^+$ baryons in Table II. We plot two weight diagrams, one between I_3, Y and C (charm) (see Fig. 1), and the other between U_3, Q, and L_3 (see Fig. 2). Figure 1 gives the I-spin multiplets and Fig. 2 gives the U-spin multiplets.

The Hamiltonian of the hadrons is supposed to consist of the terms

$$H = H_{vs} + H_{ms} + H_{em} + H_{w} ,$$

where H_{vs} is the Hamiltonian of the very strong interactions, H_{ms} is the Hamiltonian of the medium-strong interactions, H_{em} refers to the electromagnetic interactions, and H_w refers to the weak interactions. It is assumed that H_{vs} is SU(4) invariant and H_w can be neglected.

Now H_{ms} conserves I spin and H_{em} conserves L spin. Thus we can write a parallelogram law.[4,7] This law gives good results for $\frac{3}{2}^+$ baryons. In the case of $\frac{1}{2}^+$ baryons it is difficult to apply the parallelogram law because the center is doubly occupied. For the parallelogram 1, 2, 3, and 11, the point 11 is doubly occupied and we have (the particle label stands for the mass of the particle)

$$n - p + \Sigma^+ - \Sigma^0 + \beta(\Sigma^0\Lambda) = 0 , \quad (1)$$

where $(\Sigma^0\Lambda)$ is the transition mass (i.e., the off-diagonal element of the mass operator) and β is a constant. Similarly, for the parallelogram 4, 5, 6, and 11 we have

$$\Xi^0 - \Xi^- + \Sigma^- - \Sigma^0 + \beta(\Sigma^0\Lambda) = 0 . \quad (2)$$

TABLE I. Quantum numbers of the quarks.

Label	I	I_3	U	U_3	L	L_3	C
u	$\frac{1}{2}$	$\frac{1}{2}$	0	0	$\frac{1}{2}$	$\frac{1}{2}$	0
d	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0
s	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	0
c	0	0	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	1

TABLE II. Quantum numbers of the $\frac{1}{2}^+$ baryons.

Label	Quark content	I	I_3	U	U_3	L	L_3	C
p	uud	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	1	0
n	udd	$\frac{1}{2}$	$-\frac{1}{2}$	1	1	$\frac{1}{2}$	$\frac{1}{2}$	0
Λ	$s(ud)_{anti}$	0	0	1	0	$\frac{1}{2}$	$\frac{1}{2}$	0
Σ^+	uus	1	1	$\frac{1}{2}$	$-\frac{1}{2}$	1	1	0
Σ^0	$s(ud)_{sym}$	1	0	1	0	$\frac{1}{2}$	$\frac{1}{2}$	0
Σ^-	dds	1	-1	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0
Ξ^0	uss	$\frac{1}{2}$	$\frac{1}{2}$	1	-1	$\frac{1}{2}$	$\frac{1}{2}$	0
Ξ^-	dss	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	0
C_1^{++}	cuu	1	1	0	0	$\frac{1}{2}$	$\frac{1}{2}$	1
C_1^+	$c(ud)_{sym}$	1	0	$\frac{1}{2}$	$\frac{1}{2}$	1	0	1
C_1^0	cdd	1	-1	1	1	$\frac{1}{2}$	$-\frac{1}{2}$	1
C_0^+	$c(ud)_{anti}$	0	0	$\frac{1}{2}$	$\frac{1}{2}$	1	0	1
S^+	$c(su)_{sym}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	1	0	1
S^0	$c(sd)_{sym}$	$\frac{1}{2}$	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	$-\frac{1}{2}$	1
A^+	$c(su)_{anti}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	1	0	1
A^0	$c(sd)_{anti}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	1
T^0	css	0	0	1	-1	$-\frac{1}{2}$	$-\frac{1}{2}$	1
X_u^{++}	ccu	$\frac{1}{2}$	$\frac{1}{2}$	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	2
X_d^+	ccd	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	-1	2
X_s^+	ccs	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	1	-1	2

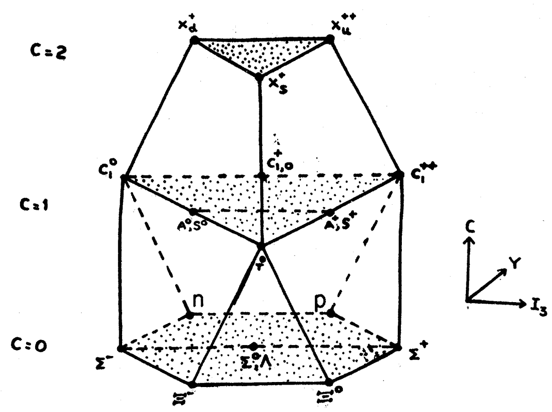

FIG. 1. The weight diagram with I_3, Y, and C as the three axes, giving the I-spin multiplets.

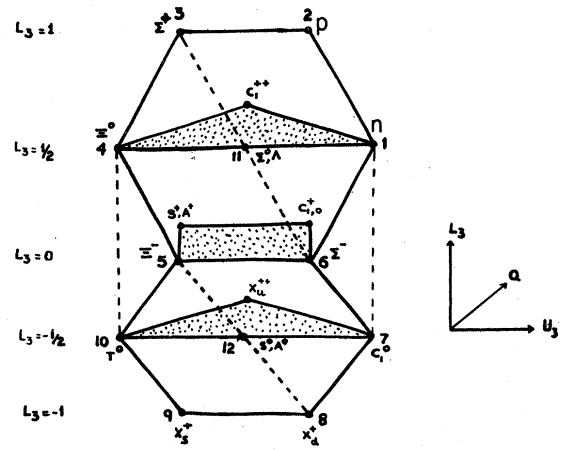

FIG. 2. The weight diagram between U_3, Q, and L_3.

Subtraction of (2) from (1) yields

$$n - p + \Sigma^+ - \Sigma^- + \Xi^- - \Xi^0 = 0 \ , \qquad (3)$$

which is the Coleman-Glashow relation.[8]

Applying the same technique to the parallelograms 7, 6, 5, 12 and 8, 9, 10, 12, we have

$$C_1^0 - \Sigma^- + \Xi^- - S^0 + \gamma(S^0A^0) = 0 \ , \qquad (4)$$

and

$$T^0 - X_s^+ + X_d^+ - S^0 + \gamma(S^0A^0) = 0 \ . \qquad (5)$$

Subtracting (5) from (4), we have

$$X_s^+ - X_d^+ - T^0 + C_1^0 - \Sigma^- + \Xi^- = 0 \ . \qquad (6)$$

Applying the parallelogram law to the parallelogram 1, 4, 10, and 7, we have

$$n + T^0 = C_1^0 + \Xi^0 \ . \qquad (7)$$

From (6) and (7) we get the following two sum rules for $\frac{1}{2}^+$ baryons:

$$T^0 - C_1^0 = \Xi^0 - n \ , \qquad (8)$$

$$X_s^+ - X_d^+ = \Sigma - n \ . \qquad (9)$$

In the literature there are a number of methods[9] for finding the mass splitting of charmed baryons, and a comparison shows that the above two sum rules have been derived by Okubo *et al.*[10] However, the advantage of the above technique is that it is more simple and efficacious in its applications.

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[1]J. J. Aubert *et al.*, Phys. Rev. Lett. 33, 1404 (1974); J.-E. Augustin *et al.*, *ibid.* 33, 1406 (1974).

[2]S. L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D 2, 1285 (1970); M. K. Gaillard, B. W. Lee, and J. L. Rosner, Rev. Mod. Phys. 47, 277 (1975).

[3]A. M. Boyarski *et al.*, Phys. Rev. Lett. 35, 195 (1975).

[4]Tej. K. Zadoo and M. Ahmad, Can J. Phys. 55, 783 (1977).

[5]M. Ahmad and Tej. K. Zadoo, Czech. J. Phys. B 27, 1337 (1977).

[6]In SU(3), the magnetic moment of a baryon depends on the charge and the U spin. In SU(4), L spin connects the quarks of the same charge and U spin; hence it can be safely assumed that the magnetic moment of the hadrons depends on the L spin.

[7]G. Feldman and P. T. Matthews, Ann. Phys. (N. Y.) 31, 469 (1965).

[8]S. Coleman and S. L. Glashow, Phys. Rev. Lett. 6, 423 (1961).

[9]V. Mathur, S. Okubo, and S. Borchardt, Phys. Rev. D 11, 2572 (1975); S. Okubo, *ibid.* 11, 3261 (1975); J. W. Moffat, *ibid.* 12, 288 (1975); A. W. Hendry and D. B. Lichtenberg, *ibid.* 12, 2756 (1975); A. J. Macfarlane, J. Phys. G 1, 601 (1975); M. Inose, R. Sugano, and T. Tsujimura, Prog. Theor. Phys. 53, 791 (1975).

[10]S. Okubo, V. Mathur, and S. Borchardt, Phys. Rev. Lett. 34, 236 (1975).