Spectrum-generating SU(4) in particle physics. II. Electromagnetic decays of vector mesons

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The decay rates for the electromagnetic decays of vector mesons are derived within the spectrum-generating SU(4) approach. Radiative as well as leptonic decays of vector mesons can be derived from one theoretical assumption and given in terms of three reduced matrix elements. The implication of the experimental value $\Gamma(\rho\to\pi\gamma) = 35\pm 10$ keV for the form of the electromagnetic current operator is discussed. Two alternatives have been considered: (1) The electromagnetic current operator in SU(3) is given by the Gell-Mann–Nishijima formula; then the experimental value $\Gamma(\rho\to\pi\gamma) = 35\pm 10$ keV cannot be obtained. (2) The experimental value of $\Gamma(\rho\to\pi\gamma)$ is taken to determine the value of the SU(3)-scalar term in the electromagnetic current operator. The resulting ansatz for the electromagnetic current operator is compatible with the experimental values for $\Gamma(\eta'\to\rho\gamma)/\Gamma(\eta'\to\omega\gamma)$.

I. INTRODUCTION

If the groups SU(2), SU(3), SU(4), etc., are treated as symmetry groups,

$$[P_\mu, \mathrm{SU}(4)] = 0\,, \tag{1.1}$$

where P_μ is the momentum operator in the hadron space, then one uses the Wigner-Eckart theorem to write the matrix elements of a regular tensor operator T^β in the form

$$\langle \alpha' p' | T^\beta | p\alpha\rangle = \sum_\gamma C(\alpha, \beta\alpha')_\gamma \langle m_{\alpha'} p' \| T \| p m_\alpha\rangle_\gamma\,, \tag{1.2}$$

where $C(\alpha, \beta, \alpha')_\gamma$ are SU(4) Clebsch-Gordan coefficients and $\langle m_{\alpha'} p' \| T\| p m_\alpha\rangle_\gamma$ are SU(4)-invariant reduced matrix elements. Expressions like (1.2) may be fulfilled (i.e., $\langle m_{\alpha}, p' \| T \| p m_\alpha\rangle_\gamma$ may be invariant) only to the extent to which the mass dependence m_α upon the internal quantum numbers α can be neglected. Such a neglect of the mass differences within a multiplet is obviously unjustified for SU(4). Already for SU(3) the variation of the "reduced matrix element" $\langle m_{\alpha'} p' \| T \| p m_\alpha\rangle_\gamma$ with α and α' may be as large as the variation of the Clebsch-Gordan coefficient $C(\alpha, \beta, \alpha')$ with α and α'; for SU(4) the variation of $\langle m_{\alpha'} p' \| T \| p m_\alpha\rangle_\gamma$ may be much larger than the variation of $C(\alpha, \beta, \alpha')$.

In the spectrum-generating-group approach,[1,2] the group that classifies particles with different masses in a multiplet, which we will call $\mathrm{SU}(4)_E$, is assumed to fulfill instead of (1.1) the relation

$$[\hat P_\mu, \mathrm{SU}(4)_E] = 0\,, \text{ where } \hat P_\mu = P_\mu M^{-1}\,. \tag{1.3}$$

Under assumption (1.3) the natural choice for basis vectors in the hadron space is

$$|\hat p s s_3; \alpha\rangle = |\hat p s s_3\rangle \otimes |\alpha\rangle\,. \tag{1.4}$$

This is the basis in the direct-product space $\mathcal{H}^{\hat{\mathcal{P}}} \otimes \mathcal{H}^{\mathrm{SU}(4)}$ where $\mathcal{H}^{\hat{\mathcal{P}}}$ is the representation space of the velocity-Poincaré group $\mathcal{P}_{L_{\mu\nu}, \hat p_\mu}$ characterized by the invariants $\hat p_\mu \hat p^\mu = 1$ and s, and $\mathcal{H}^{\mathrm{SU}(4)}$ is the representation space of $\mathrm{SU}(4)_E$. Thus the generators of $\mathrm{SU}(4)_E$ are represented by $1 \otimes E_\alpha$, and the generators of $\mathcal{P}_{L_{\mu\nu}, \hat p_\mu}$ are represented by $L_{\mu\nu} \otimes 1$ and $\hat P_\mu \otimes 1$. However, the mass operator and consequently the momentum operator, i.e., the representative of the generator of the physical Poincaré group in the hadron space $\mathcal{H}^{\hat{\mathcal{P}}} \otimes \mathcal{H}^{\mathrm{SU}(4)}$, cannot be given as a direct product in which one of the factors is the unit operator.

In our quantum-mechanical description of the one-hadron system the algebra of observables contains in addition to the operators of the extended Poincaré group and the generators E_α of the spectrum-generating $\mathrm{SU}(4)_E$ other observables which are defined by their algebraic relations with these operators. Such observables are the transition operators or "nonlocal currents" V_μ^α; μ indicates their transformation property as a Lorentz-vector operator and the label α, which can take all the particle labels π, K, η, D, F, χ, and σ,[3] is connected with their $\mathrm{SU}(4)_E$ transformation property.[4] However, as discussed in Ref. 1, there is no reason to assume that the transition operators V_μ^α themselves are regular tensor operators. Instead, experimental data give preference to the assumption that functions of V_μ^α and the mass operator, e.g., $\hat V_\mu^\alpha = \{V_\mu^\alpha, M^{-1}\}$ transform irreducibly under $\mathrm{SU}(4)_E$. In the direct-product space for the one-hadron system $\mathcal{H}^{\hat{\mathcal{P}}} \otimes \mathcal{H}^{\mathrm{SU}(4)}$ these irreducible tensor operators are then without any further assumption written in their most general form as a linear combination of direct products of operators V_μ^I in $\mathcal{H}^{\hat{\mathcal{P}}}$ and irreducible tensor operators $\hat V_I^\alpha$ in $\mathcal{H}^{\mathrm{SU}(4)}$:

$$\hat V_\mu^\alpha = \sum_I V_\mu^I \otimes \hat V_I^\alpha\,, \tag{1.5}$$

where I are some additional labels.

Instead of (1.2) one uses in the spectrum-generating-group approach the Wigner-Eckart theorem for the matrix elements of the irreducible tensor operator $\hat{V}_I^\alpha$ between the generalized eigenvectors of velocity (1.4):

$$\langle\alpha'\hat{p}'|T^\beta|\hat{p}\alpha\rangle=\sum_\gamma C(\alpha\beta\alpha')_\gamma\langle\hat{p}'\|T\|\hat{p}\rangle_\gamma. \qquad (1.6)$$

Now the $\langle\hat{p}'\|T\|\hat{p}\rangle_\gamma$ are $SU(4)_E$-*invariant* reduced matrix elements which are functions only of $SU(4)_E$-invariant quantities like the four-velocity $\hat{p}$ and do *not* depend upon α.

In Sec. II we apply the above ideas of a relativistic quantum mechanics, using the spectrum-generating-group assumption (1.3) and a well specified transformation property of the V_μ^α, to the calculation of the radiative decays $V\to P\gamma$. The result (2.20) differs from the conventional expression for $\Gamma(V\to P\gamma)$ by a suppression factor $\phi(m_p, m_V)$ (Ref. 5) whose detailed form depends upon the precise assumption of the $SU(4)_E$ properties of the electromagnetic transition operator. The SU(4) matrix elements can be expressed in terms of four reduced matrix elements which can be reduced to two after the charge-conjugation property and the group-theoretical substitute for the Okubo-Zweig-Iizuka (OZI) rule has been used. The number of parameters may be further reduced to one by the requirement that for the old mesons in SU(3) the electromagnetic current operator is given by the Gell-Mann–Nishijima formula. However, this assumption cannot reproduce the experimental data for $\Gamma(\rho\to\pi\gamma)$, so we have also tried dropping it. Both cases are capable of describing the general features of the experimental situation, in particular the suppression of the radiative decays of the new mesons $\psi\to\chi(2.8)\gamma$ and $D^*\to D\gamma$. The recently measured value of $\Gamma(\eta'\to\rho\gamma)/\Gamma(\eta'\to\omega\gamma)$ is also compatible with both cases. In Sec. III the same assumptions that were used in Sec. II for the radiative decays are applied to the leptonic decays of vector mesons, $V\to e\bar{e}$, and agreement with the experimental values is obtained. Therewith all electromagnetic decay rates of vector mesons can be derived from one precise theoretical assumption and given in terms of three reduced matrix elements. Although this particular form of the suppression factor is phenomenologically acceptable within this context, it is not yet firmly established. Nevertheless, the general idea of the spectrum-generating-group approach appears to be the only tenable interpretation of SU(4).[5a]

II. RADIATIVE DECAYS

In this section we will discuss in detail the decay of a vector meson V into a pseudoscalar meson P and a photon γ. Owing to the possible nonlocality one can already not use the conventional expression for the decay rate; we start with its derivation. From the general principles of quantum mechanics it follows that the rate for the decay of a system V into a system $P\gamma$ is given by[6]

$$\Gamma(V\to P\gamma)=2\pi\sum_b\delta(E_{P\gamma}-E_V)\,|\langle\gamma Pb|T|V)|^2, \qquad (2.1)$$

where $|V)$ denotes the state vector describing the state of the decaying system V and $|bP\gamma\rangle$ denotes (not a state vector but) an element of a basis system of (generalized) eigenvectors which span the space of the decay products P,γ. $\sum_b$ means summation (or integration) over all values of b which are detected. T is the transition operator (interaction Hamiltonian).

In relativistic processes one usually chooses these generalized basis vectors to be generalized eigenvectors of the momentum operators. If these generalized momentum operators $|k,\lambda\rangle$ are "normalized" according to

$$\langle\lambda'k'|k\lambda\rangle=\delta_{\lambda'\lambda}2E(k)\delta(k-k'), \qquad (2.2)$$

then

$$\sum_b=\sum_\lambda\int\frac{d^3k}{2E(k)}. \qquad (2.3)$$

However, one could also choose any other basis system. As we have remarked above, under assumption (1.3) it is very advantageous to choose for the hadron spaces not the generalized momentum eigenvectors but generalized velocity eigenvectors $|\hat{p}\lambda\rangle$ which are normalized according to

$$\langle\lambda'\hat{p}'|\hat{p}\lambda\rangle=\delta_{\lambda'\lambda}2\hat{E}(\hat{p})\delta^3(\hat{p}-\hat{p}'), \qquad (2.4)$$

where $\hat{p}=p/m$ and $\hat{E}=E/m=(1+\vec{p}^2/m^2)^{1/2}$. Then

$$\sum_b=\sum_\lambda\int\frac{d^3\hat{p}}{2\hat{E}}. \qquad (2.5)$$

Using the generalized momentum eigenvectors (2.2) for the photon and the generalized velocity eigenvectors (2.4) for the pseudoscalar mesons, one has the basis system

$$|bP\gamma\rangle=|\hat{p}P\rangle\otimes|k\lambda\gamma\rangle=|\hat{p}k\lambda P\gamma\rangle; \qquad (2.6)$$

so (2.1) is written as

$$\Gamma(V\to P\gamma)=2\pi\int\frac{d^3\hat{p}}{2\hat{E}_p}\sum_\lambda\int\frac{d^3p}{2E_\gamma}\delta(E_P+E_\gamma-E_V)\times|\langle P\gamma\lambda k\hat{p}|T|V)|^2. \qquad (2.7)$$

The decaying state vector $|V)$ may be expanded in terms of the generalized velocity eigenvectors

$$|V) = \sum_{\lambda'} \int \frac{d^3\hat{p}'}{2\hat{E}} |\hat{p}'\lambda' V\rangle \phi_{\lambda'}(\hat{p}'), \qquad (2.8)$$

where the "velocity wave function" $\phi_\lambda(\hat{p}_V)$ is normalized such that

$$\sum_{\lambda'} \int \frac{d^3\hat{p}'}{2\hat{E}'} |\phi_{\lambda'}(\hat{p}')|^2 = 1 .$$

The label V in $|\hat{p}\lambda V\rangle$ stands for the internal quantum numbers α of the vector mesons, and $|\phi_\lambda(\hat{p})|^2$ describes the probability density that the momentum of the decaying system is $p = m_V \hat{p}$ and the polarization is λ. For the case that the decaying particles have the momentum $\vec{p}_V(=0)$ and completely undetermined polarization

$$\phi_{\lambda''}(\hat{p}')\phi^*_{\lambda'}(\hat{p}') = \tfrac{1}{3}\delta_{\lambda''\lambda'} 2\hat{E}\delta(\hat{p}' - \hat{p}_V) = \tfrac{1}{3}\delta_{\lambda''\lambda'} m_V{}^2 2E\delta(p' - p_V) \qquad (2.9)$$

as there are three polarizations λ.

As a consequence of momentum conservation (or the Wigner-Eckart theorem for the scalar operator T of the translation group) one can write the T matrix

$$\langle P\gamma\lambda k\hat{p}|T|\hat{p}'\lambda_V V\rangle = \delta^3(k + p - p')\langle\!\langle \gamma P|T|V\rangle\!\rangle , \qquad (2.10)$$

where the reduced matrix element depends only upon two of the momenta k, p, p'.

Using (2.8) with (2.9) and (2.10) in (2.7) one obtains after some calculations:

$$\Gamma(V \to P\gamma) = 2\pi \int \frac{d^3k}{2E_\gamma} \frac{d^3p}{2E_p} \overline{\sum_{\lambda_\gamma \lambda_V}} \delta(E_P + E_\gamma - E_V)\delta^3(p + k - p_V) \frac{1}{m_P{}^2 m_V{}^2} \frac{1}{2E_V} |\langle\!\langle \lambda_\gamma P\hat{p}|T|\hat{p}_V V\lambda_V\rangle\!\rangle|^2 \qquad (2.11)$$

where $\overline{\sum}_{\lambda_\gamma \lambda_V}$ means summing over the photon polarizations and averaging over the vector-meson polarization.

Equation (2.11) is still completely general. For the T matrix of the radiative decays we assume, in analogy to (3.15) of Ref. 1 and in concord with the lowest-order perturbation theory expression, that the T matrix can be written as the product of a photonic part $\epsilon^\mu(k, \lambda)$ and a hadronic part:

$$\langle\!\langle \lambda\gamma P\hat{p}|T|\hat{p}_V V\lambda_V\rangle\!\rangle = \epsilon^\mu(k, \lambda)\langle\!\langle P\hat{p}|H^{\text{el}}_\mu|\hat{p}_V V\lambda_V\rangle\!\rangle , \qquad (2.12)$$

where H^{el}_μ is the transition operator (nonlocal[4] current) in the electromagnetic transitions of hadrons.

The hadronic matrix element $\langle\!\langle P\hat{p}|H^{\text{el}}_\mu|\hat{p}_V V\rangle\!\rangle$ is obtained from the theory that describes the hadron structure, which is assumed to be a relativistic quantum mechanics. For the model which describes the vector mesons and pseudoscalar mesons it is given by an algebra of operators, which in addition to the generators of the extended Poincare group and the group $SU(4)_E$ [or the suitably defined group SU(8)] contains a sixteenplet of Lorentz-vector operators V^α_μ, $\alpha = \sigma, \pi, \eta, K, D, F, \chi$. (In addition to the Lorentz-vector operators there are also Lorentz-axial-vector operators A^α_μ and perhaps others which, however, do not concern us for electromagnetic transitions.) These vector operators are very reminiscent of, but not identical with, the local currents. Their properties are just specified by their relations to the other operators of the algebra (algebraic relations), e.g., that they are Lorentz-vector operators with a certain charge-conjugation and time-inversion transformation property and have a particular transformation property under $SU(4)_E$. In analogy to the property of the local current operators, one would want to assume that the V^α_μ are scalar (for $\alpha = \sigma$) and regular tensor operators. However, as SU(4) is not considered to be a symmetry group, one may want to admit a more general transformation property of the V^α_μ which in the symmetry limit goes over into the old assumption. Various possibilities for these new assumptions have been discussed in Ref. 1 for the weak leptonic decays.

We choose here in analogy

$$H^{\text{el}}_\mu = G\{M, \{M^q, V^{\text{el}}_\mu\}\}, \quad q = \tfrac{1}{2}, 1, \tfrac{3}{2} \qquad (2.13)$$

where M is the mass operator and where V^{el}_μ for SU(4) according to (23) of I is given by

$$V^{\text{el}}_\mu = V^{\pi^0}_\mu + \frac{1}{\sqrt{3}} V^\eta_\mu - (\tfrac{2}{3})^{1/2} V^\chi_\mu + V^\sigma_\mu . \qquad (2.14)$$

As in Ref. 1:

$$\{V^\alpha_\mu, M^{-1}\} = SU(4)_E\text{-regular-tensor operator} \qquad (2.15)$$

for $\alpha = \pi, K, \ldots,$ and an $SU(4)_E$-scalar operator for $\alpha = \sigma$.

The strength constant G is dimensionless for $q = 1$. However, as there is no theoretical derivation of basic assumptions like (2.13) (2.15) and their only justification can be given by agreement with experiment, we include in the fits of Table II also the values $q = \frac{1}{2}$ and $\frac{3}{2}$.

From this assumption (for $q = 1$), from the transformation properties with respect to the homogeneous Lorentz group, space reflection and time inversion, and from CVC (conservation of vector current) in the form

$$[P^\mu, H_\mu^{\rm el}]=0, \quad [P^\mu, V_\mu^{\rm el}]=0, \tag{2.16}$$

where P_μ is the momentum operator in hadron space, it follows that the matrix element describing the hadronic structure can be written as

$$\langle\!\langle P\hat{p}|H_\mu^{\rm el}|\hat{q}\,V\lambda\rangle\!\rangle = G(m_P+m_V)\langle P|V^{\rm mag}|V\rangle \times \epsilon_{\mu\nu\rho\sigma}\,p^\nu q^\rho e^\sigma(\hat{q},\lambda)\,. \tag{2.17}$$

Here $e^\sigma(\hat{q},\lambda)$ is the polarization vector of the vector meson, $q^\nu = m_V\hat{q}^\nu$ is its momentum, p^ν is the momentum of the pseudoscalar meson and $\langle P|V^{\rm mag}|V\rangle$ is the SU(4) matrix element. $V^{\rm mag}$ is one or a linear combination of the operators $V_I^{\rm el}$ in $\mathcal{H}^{\rm SU(4)}$ occurring in (1.5), mag standing for the index I.

Inserting (2.17) into (2.12) gives

$$\langle\!\langle \lambda\gamma P\hat{p}|T|\hat{q}\,V\lambda_V\rangle\!\rangle = G(m_P+m_V)\langle P|V^{\rm mag}|V\rangle \times \epsilon_{\mu\gamma\rho\sigma}\,p^\nu q^\rho e^\sigma(\hat{q}\,\lambda)\,\epsilon^\mu(k,\lambda)\,, \tag{2.18}$$

which is identical with the usual matrix element for magnetic dipole transitions, except that here, due to the use of generalized velocity eigenvectors, the magnetic moments depend not upon the internal quantum numbers through $\langle P|V^{\rm mag}|V\rangle$ or the Clebsch-Gordan coefficients, but also upon the masses (m_P+m_V).

To obtain the decay rate one inserts the T matrix (2.18) into (2.11)

$$\Gamma(V\to P\gamma) = 2\pi\left|\frac{m_P+m_V}{m_P m_V}G\langle P|V^{\rm mag}|V\rangle\right|^2 \times \overline{\sum_{\rm pol}}\int \frac{d^3k}{2E_\gamma}\frac{d^3p}{2E_p}\delta^4(q-p-k)\frac{1}{2E_V} \times |\epsilon_{\mu\nu\rho\sigma}\,p^\nu q^\rho e^\sigma(q\lambda_V)\,\epsilon^\mu(k,\lambda)|^2\,, \tag{2.19}$$

which after integration becomes

$$\Gamma(V\to P\gamma) = 2\pi\left|\frac{m_P+m_V}{m_P m_V}G\langle P|V^{\rm mag}|V\rangle\right|^2 \times \frac{\pi}{24}m_V{}^3\left[1-\left(\frac{m_P}{m_V}\right)^2\right]^3\,. \tag{2.20}$$

This expression differs from the usual expression for the $V\to P\gamma$ decay rate, by the appearance of a mass-dependent suppression factor

$$\frac{m_P+m_V}{m_P m_V} = \phi(m_P, m_V)\,. \tag{2.21}$$

As mentioned above, different assumptions than (2.13) with $q=1$ and (2.15) will give different suppression factors. From the general form of (2.13) suppression factors of the form

$$\phi^{(q)}(m_P, m_V) = \frac{m_P{}^q + m_V{}^q}{m_P m_V}\,, \quad q=\tfrac{1}{2}, 1, \tfrac{3}{2} \tag{2.22}$$

would follow. However, basic assumptions like (2.13) or (2.15) have only an empirical justification. Other forms of the suppression factors are also possible and should be tested.

The SU(4) matrix element is expressed in terms of reduced matrix elements $a_{(\gamma)}$ (Ref. 7) and the SU(4) Clebsch-Gordan coefficients $C(V, {\rm el}, P)$:

$$\langle P|V^{\rm mag}|V\rangle = \sum_\gamma C(V, {\rm el}, P)_\gamma a_{(\gamma)}\,. \tag{2.23}$$

As a consequence of (2.14) these electromagnetic components of the Clebsch-Gordan coefficients are given by

$$C(V, {\rm el}, P)_\gamma = C(V, \pi^0, P)_\gamma + \frac{1}{\sqrt{3}}C(V, \eta, P)_\gamma - (\tfrac{2}{3})^{1/2}C(V, \chi, P)_\gamma + C(V, \sigma, P)\,, \tag{2.24}$$

where γ stands for the additional label F (antisymmetric) or D (symmetric) and V and P for the internal quantum numbers of the mesons.

For the four reduced matrix elements $a_{(\gamma)}$ we use the following notation:

$$\begin{aligned} a_{(F)} &= \langle 0^-\{15\}\|V^{(15)}\|\{15\}1^-\rangle_F = F\,,\\ a_{(D)} &= \langle 0^-\{15\}\|V^{(15)}\|\{15\}1^-\rangle_D = D = \sqrt{3}\,d\,,\\ a_{(A)} &= \langle 0^-\{15\}\|V^{(15)}\|\{1\}1^-\rangle \\ &= -\frac{1}{\sqrt{15}}\langle 1^-\{1\}\|V^{(15)}\|\{15\}0^-\rangle = A\,,\\ a_{(S)} &= \langle 0^-\{15\}|V^\sigma|\{15\}1^-\rangle = S\,. \end{aligned} \tag{2.25}$$

From Eq. (36) of I it follows that $F=0$. In order to obtain the SU(4) matrix elements $\langle P|V^{\rm mag}|V\rangle$ the vector mesons $|V\rangle$ and pseudoscalar mesons $|P\rangle$ have to be assigned to vectors with definite SU(4) properties. As described in I (Sec. IV) we shall choose for the pseudoscalar mesons the basis vectors of the reduction chain (37) of I and for the vector mesons the basis vectors of the reduction chain (38) of I. The $I=0$ vector mesons are then described by (42) of I and the $I=0$ pseudoscalar mesons are described by the unmixed SU(4) basis vectors. We shall call this particle assignment the ideal mixing limit. This limit ignores not only deviation from ideal mixing for vector mesons, but also η-η'-χ mixing and isospin mixing. Deviation from ideal mixing should not be considered separately without considering isospin mixing (ρ^0-ω, π^0-η) because they are of the same magnitude and perhaps of the same origin.[8]

The Clebsch-Gordan coefficients $C(V_i, \text{el}, P)$, i.e., the coefficients appearing in the matrix elements $\langle P|V^{\text{mag}}|V_i\rangle$ where $|V_i\rangle$ stands for the basis vector in the SU(4) basis [Eq. (41) of I] are given in Table VI of I, and the coefficient appearing in the matrix element $\langle P|V^{\text{mag}}|\sigma\rangle$ in front of A is given by $C(\sigma, \beta, P)=\delta_{\beta P}$.[9] Using for the ideally mixed vector mesons the expressions (42) of I one then obtains immediately the SU(4) matrix elements $\langle P|V^{\text{mag}}|V\rangle$ in terms of the three parameters d, A, and S. These are listed in Table I, except for:

$$\begin{aligned}\langle\pi^0|V^{\text{mag}}|\psi\rangle &= -\langle\pi^0|V^{\text{mag}}|\phi\rangle\\ &= \sqrt{3}\,\langle\eta|V^{\text{mag}}|\psi\rangle\\ &= \frac{1}{2\sqrt{2}}d+\tfrac{1}{2}A\,. \end{aligned}\tag{2.26}$$

The reduced matrix elements d, A, and S are free parameters which can be determined by a fit to the experimentally known decay rates of the old vector mesons (see Table II). The experimental rate $\Gamma(\phi\to\pi^0\gamma)=4.9\pm1.6$ keV is orders of magnitude smaller than expected from its phase-space value, so one may conclude that $|\langle\pi^0|V^{\text{mag}}|\phi\rangle|\approx 0$. From (2.26) it then follows that the decay rates of $\psi\to\pi^0\gamma$ and $\psi\to\eta\gamma$ must also be small. The requirement

$$A=-d/\sqrt{2}\tag{2.27}$$

TABLE I. SU(4) matrix elements of the transition operator between pseudoscalar and vector mesons.

	General	With (2.27)
$\langle\pi\|V^{\text{mag}}\|\omega\rangle$	$\frac{1}{2}d-\frac{1}{\sqrt{2}}A$	d
$\langle\eta\|V^{\text{mag}}\|\phi\rangle$	$(\frac{3}{8})^{1/2}d-\frac{1}{2\sqrt{3}}A+(\frac{2}{3})^{1/2}S$	$(\frac{2}{3})^{1/2}(d+S)$
$\langle\pi\|V^{\text{mag}}\|\rho\rangle$	S	S
$\langle K^0\|V^{\text{mag}}\|K^{0*}\rangle$	$d+S$	$d+S$
$\langle K^+\|V^{\text{mag}}\|K^{+*}\rangle$	S	S
$\langle\eta\|V^{\text{mag}}\|\omega\rangle$	$-\frac{1}{2\sqrt{3}}d-\frac{1}{\sqrt{6}}A-\frac{1}{\sqrt{3}}S$	$-\frac{1}{\sqrt{3}}S$
$\langle\eta\|V^{\text{mag}}\|\rho\rangle$	$-\frac{1}{\sqrt{3}}d$	$-\frac{1}{\sqrt{3}}d$
$\langle\chi\|V^{\text{mag}}\|\psi\rangle$	$-\frac{1}{\sqrt{3}}d+\frac{1}{\sqrt{6}}A+\frac{\sqrt{3}}{2}S$	$\frac{\sqrt{3}}{2}(-d+S)$
$\langle D^0\|V^{\text{mag}}\|D^{0*}\rangle$	$-d+S$	$-d+S$
$\langle D^+\|V^{\text{mag}}\|D^{+*}\rangle$	S	S
$\langle F^+\|V^{\text{mag}}\|F^{+*}\rangle$	S	S

will make the amplitude for these transitions zero if one uses, as done here, the ideal mixing limit. We postulate relation (2.27) (and corresponding relations for other decays), which then explains the smallness of the decay rates of ψ into the old mesons as well as the smallness of $\phi\to\pi\gamma$ as slight deviations from ideal mixing. In the quark model the consequences of (2.27) are explained by the OZI rule.[10,11]

There is another possible theoretical requirement that would further reduce the number of parameters and relate d to S. This requirement would be that the SU(3)-scalar part of V^{mag} would not contribute to the matrix elements of V^{mag} between the old mesons. That is,

$$\langle 0^-, P'|[-(\tfrac{2}{3})^{1/2}V_\mu^\chi+V_\mu^\sigma]|V', 1^-\rangle=0\,,\tag{2.28}$$

where P' and V' are any of the *old* pseudoscalar and vector mesons, respectively. This leads to the following relation between the reduced matrix elements:

$$S=-\tfrac{1}{3}d\,.\tag{2.29}$$

In the symmetry limit, (2.29) leads to the same predictions as the naive quark model.[12] It is well known that the prediction $\Gamma(\rho\to\pi\gamma)/\Gamma(\omega\to\pi\gamma)\approx\frac{1}{9}$ of that model is in disagreement with the experimental value $\Gamma^{\text{exp}}(\rho\to\pi\gamma)/\Gamma^{\text{exp}}(\omega\to\pi\gamma)\approx\frac{1}{25}$. Furthermore, as the ρ and ω masses are so close to each other, any suppression factor will fulfill $\phi(m_\pi, m_\rho)\approx\phi(m_\pi, m_\omega)$. Thus it is highly unlikely that this disagreement could be explained by a symmetry-breaking effect. Therefore, if $\Gamma^{\text{exp}}(\rho\to\pi\gamma)$, which was obtained in only one experiment,[13] is correct, (2.28) cannot hold and the Gell-Mann–Nishijima formula for the current must be augmented by an SU(3) scalar term V_μ^S, so that in SU(3) the electromagnetic current is given by

$$V_\mu^{\text{el}}=V_\mu^{\pi^0}+\frac{1}{\sqrt{3}}V_\mu^\eta+V_\mu^S\,.\tag{2.30}$$

This still leads to the old Gell-Mann–Nishijima formula for the meson charges, since

$$\langle M|V_\mu^S|M\rangle=0\tag{2.31}$$

according to (29) of I. It should be remarked that V_μ^S cannot contribute to the baryon charges but might contribute to the baryon magnetic moments.[14,15]

From the experimental value of $\Gamma(\rho\to\pi\gamma)/\Gamma(\omega\to\pi\gamma)$ one obtains $|S/d|\approx\frac{1}{5}$. The experimental value for $\Gamma(\phi\to\eta\gamma)/\Gamma(\omega\to\pi\gamma)$ favors a positive sign, so

$$S=\tfrac{1}{5}d\tag{2.32}$$

is the empirically determined relation between S and d. A detailed comparison of the predictions of

TABLE II. Calculated values of the decay rates, in keV. In the first three fits [these fits have already been published in A. Bohm and R. B. Teese, Phys. Rev. Lett. 38, 629 (1977)] the ratio d/S is fixed empirically [Eq. (2.32)], using the suppression factor (2.22) depending on q. In the last two fits, the relation (2.29) is used with the suppression factor (2.33) depending on p. The parameter d is fixed by the $\omega\to\pi\gamma$ rate.

Decay	Experiment	$q=\frac{1}{2}$	$q=1$	$q=\frac{3}{2}$	$p=1$	$p=\frac{3}{2}$
$\omega\to\pi\gamma$	870 ± 61 [a]	870	870	870	870	870
$\phi\to\eta\gamma$	66 ± 9 [b]	51	76	98	77	60
$\rho\to\pi\gamma$	35 ± 10 [c]	35	35	35	95	96
$K^{0*}\to K^0\gamma$	75 ± 35 [a]	66	87	98	89	79
$K^{+*}\to K^+\gamma$	<80 [a]	1.9	2.6	3.0	23	20
$\omega\to\eta\gamma$	$3.0^{+2.5}_{-1.8}$ [d]	0.18	0.22	0.23	2.2	2.1
$\rho\to\eta\gamma$	50 ± 13 [d]	3.9	4.8	5.0	18	17
$\psi\to\chi\gamma$	<3.5 [e]	0.30	1.6	7.3	18	4.0
$D^{0*}\to D^0\gamma$	...	0.10	0.35	1.0	3.9	1.3
$D^{+*}\to D^+\gamma$	...	0.006	0.022	0.07	0.24	0.08
$F^{+*}\to F^+\gamma$	...	0.006	0.022	0.07	0.24	0.08

[a] Particle Data Group, Rev. Mod. Phys. 48, S1 (1976).

[b] See Ref. 16. The Orsay results are $\Gamma(\phi\to\eta\gamma)=107\pm29$ keV [D. Benaksas *et al.*, Phys. Lett. 42B, 511 (1972)] and $\Gamma(\phi\to\eta\gamma)=62\pm16$ keV [G. Cosme *et al.*, Phys. Lett. 63B, 352 (1976)].

[c] See Ref. 13.

[d] See Ref. 17.

[e] B. H. Wiik and G. Wolf, DESY Report No. 77/01 (unpublished).

(2.32) using (2.22) with the experimental data[16] is given in columns 3, 4, and 5 of Table II; except for the 3σ deviation from the value for $\Gamma(\rho\to\eta\gamma)$ (Ref. 17) the agreement is good.

If we assume the absence of any SU(3)-scalar contribution to the old decays, as expressed by (2.28), then the suppression factor is chosen to be

$$\phi(m_P, m_V)=\frac{1}{m_V{}^p+m_P{}^p}\,. \tag{2.33}$$

Fits of this hypothesis are shown in columns 7 and 8 of Table II. Except for the value of $\Gamma(\rho\to\pi\gamma)$ these fits are good. One may think that the value for $\Gamma(\rho\to\eta\gamma)$ is still off but, taking $\eta-\eta'$ mixing into account, this can be brought into perfect agreement with the experimental data.

To conclude this section we want to discuss whether the recently obtained experimental value[18]

$$\Gamma(\eta'\to\rho\gamma)/\Gamma(\eta'\to\omega\gamma)=9.9\pm2.0 \tag{2.34}$$

can discriminate between (2.29) and (2.32). Taking for the physical η':

$$|\eta'_{\rm ph}\rangle=\sin\phi\,|\eta\rangle+\cos\phi\,|\sigma_p\rangle\,,$$

one calculates for this ratio

$$\frac{\Gamma(\eta'_{\rm ph}\to\rho\gamma)}{\Gamma(\eta'_{\rm ph}\to\omega\gamma)}=\left|\frac{\sqrt2\,\tilde d-d\tan\phi}{\sqrt2\,\tilde S-S\tan\phi}\right|^2\,. \tag{2.35}$$

Here d and S are the reduced matrix elements given in (2.25) which can be expressed in terms of the SU(3) reduced matrix elements

$$s=\langle 0^-\{8\}\|V^s\|\{8\}1^-\rangle\,,$$
$$a=(\tfrac13)^{1/2}\langle 0^-\{8\}\|V^{\{8\}}\|\{1\}1^-\rangle \tag{2.36}$$

by

$$d=\frac{3}{\sqrt2}a\,,\quad S=s-\frac{1}{\sqrt2}a\,. \tag{2.37}$$

And $\tilde d$ and $\tilde S$ are given by

$$\tilde d=\frac{3}{\sqrt2}\tilde a\,,\quad \tilde S=\tilde s-\tfrac13\tilde d=\tilde s-\frac{1}{\sqrt2}\tilde a\,,$$

where

$$\tilde a=(\tfrac13)^{1/2}\langle 1^-\{8\}\|V^{\{8\}}\|\{1\}0^-\rangle^*\,,$$
$$\tilde s=\langle 0^-\{1\}\|V^s\|\{1\}1^-\rangle\,.$$

If $d=\tilde d$ and $S=\tilde S$, i.e., if $a=\tilde a$ and $s=\tilde s$, then one obtains from (2.35)

$$\frac{\Gamma(\eta'_{\rm ph}\to\rho\gamma)}{\Gamma(\eta'_{\rm ph}\to\omega\gamma)}=\left|\frac{d}{S}\right|^2\,, \tag{2.38}$$

and the experimental value (2.34) clearly excludes (2.32) and is in agreement with (2.29). But for this assumption there exists no more justification than for the assumption $V_\mu^S=0$. And for the general case (2.35), (2.34) does not determine $|d/S|$.

Concluding, we have found that if the experimental value for $\Gamma(\rho\to\pi\gamma)$ (Ref. 13) is correct then there must exist an SU(3)-scalar term V_μ^S (or higher multiplet terms) in the electromagnetic current operator. The experimental value (2.34) for the radiative decay of η' does not disagree with this assumption but the experimental value[17] for $\rho \to \eta\gamma$ and $\omega \to \eta\chi$ are then hard to explain with the symmetry breaking suggested here. If the experimental value[13] for $\Gamma(\rho \to \pi\gamma)$ is ignored, then all other experimental data can be fitted without an additional term in the Gell-Mann–Nishijima formula for the electromagnetic current.

III. LEPTONIC DECAY OF VECTOR MESONS

It may be instructive to compare our calculations for the radiative decays with the calculations for the leptonic decays $V \to e\bar{e}$ of vector mesons.[19]

The decay rate for $V \to e\bar{e}$ is derived by the same arguments given in the derivation of (2.11)

$$\Gamma(V \to e\bar{e}) = 2\pi \sum_{\text{pol}} \int \frac{d^3p_+}{2E_+} \frac{d^3p_-}{2E_-} \delta^4(p_V - p_+ - p_-) \times \frac{1}{2E_V m_V{}^2} |\langle\langle e\bar{e}|T|V\rangle\rangle|^2 . \tag{3.1}$$

The T matrix is written in analogy to (2.12)

$$\langle\langle e\bar{e}|T|V\rangle\rangle = \bar{u}(p_+)\gamma^\mu v(p_-) \frac{1}{q^2} \langle\langle \sigma | H_\mu^{\text{el}} | V\rangle\rangle , \tag{3.2}$$

where the leptonic part is chosen in complete analogy to the usual perturbation-theory expression with the one-photon exchange term (q being the momentum transfer) and where the hadronic part $\langle\langle \sigma | H^{\text{el}} | V\rangle\rangle$ is to come from our relativistic quantum mechanics of the one-hadron system. $|\sigma\rangle$ here denotes the vector with the hadron quantum numbers of the vacuum, i.e., it is according to (1.4) the direct product of the trivial representation of the velocity-Poincaré group $\mathcal{P}_{L_{\mu\nu}\hat{p}_\mu}$ and of the trivial representation of the spectrum-generating $SU(4)_E$. With (2.15) and (2.13), i.e., the same assumptions that were used for the radiative decays (and also the leptonic and semileptonic decays of pseudoscalar mesons[1]) one obtains

$$\langle\langle \sigma | H_\mu^{\text{el}} | V\rangle\rangle = G m_V m_V{}^q \langle\langle \sigma | V_\mu^{\text{el}} | V\rangle\rangle$$

and

$$\langle\langle \sigma | \{V_\mu^{\text{el}}, M^{-1}\} | V\rangle\rangle = \langle \sigma | V^{\text{el}} | V\rangle \langle\langle \sigma | V_\mu | V, \hat{p}_V, 1^-\rangle\rangle .$$

Consequently,

$$\langle\langle \sigma | V_\mu^{\text{el}} | V\rangle\rangle = m_V \langle \sigma | V^{\text{el}} | V\rangle e_\mu(p_V, \lambda_V) ,$$

and therewith

$$\langle\langle \sigma | H_\mu^{\text{el}} | V\rangle\rangle = G m_V{}^{2+q} \langle \sigma | V^{\text{el}} | V\rangle e_\mu(p_V \lambda_V) . \tag{3.3}$$

Here $\langle \sigma | V^{\text{el}} | V\rangle$ is the $SU(4)_E$ matrix element which in analogy to (2.23) is given by

$$\langle \sigma | V^{\text{el}} | V\rangle = \sum_\gamma C(V, \text{el}, \sigma) \tilde{a}_{(\gamma)} . \tag{3.4}$$

However, here the reduced matrix elements $\tilde{a}_{(\gamma)}$ are different from those given in (2.25), and the ones different from zero are denoted

$$\begin{aligned} \tilde{a}_{(A)} &= \langle 0^+\{1\} \| V^{(15)} \| \{15\} 1^-\rangle = a , \\ \tilde{a}_{(S)} &= \langle 0^+\{1\} \| V^{\sigma} \| \{1\} 1^-\rangle = s . \end{aligned} \tag{3.5}$$

Whereas the transitions in $V \to P\gamma$ were magnetic-type transitions, the transitions here are electric-type transitions. We therefore use the notation V^{el} for this V_I^{el} of the direct-product decomposition (1.5).

Using for the vector mesons the vectors (41) and (42) of I (ideal mixing limit) and inserting the Clebsch-Gordan coefficients into (3.4), one obtains for these matrix elements

$$\begin{aligned} \langle \sigma | V^{\text{el}} | \rho\rangle &= -\frac{1}{\sqrt{15}} a , \\ \langle \sigma | V^{\text{el}} | \omega\rangle &= -\frac{1}{\sqrt{2}} s , \\ \langle \sigma | V^{\text{el}} | \phi\rangle &= -\frac{1}{\sqrt{30}} a - \tfrac{1}{2} s , \\ \langle \sigma | V^{\text{el}} | \psi\rangle &= -\frac{1}{\sqrt{30}} a + \tfrac{1}{2} s . \end{aligned} \tag{3.6}$$

The decay rate is calculated by inserting (3.3) into (3.2) and (3.2) into (3.1). The result is[20]

$$\Gamma(V \to e\bar{e}) = 2\pi |G m_V{}^{q-1} \langle \sigma | V^{\text{el}} | V\rangle|^2 \tfrac{1}{12} \pi m_V . \tag{3.7}$$

In Ref. 19 a detailed comparison of (3.7) with the experimental decay rates of the old vector mesons was made, and it was found that for $q = 0, 1$ reasonable fits were obtained and that $q = \frac{1}{2}$ gave an excellent fit. The case $q = 0$ will not be considered further as it does not work for the radiative decays. For the case of $q = 1$ and $q = \frac{1}{2}$ one can calculate the two parameters Gs and Ga from the decay rates of the old vector mesons and then calculate the rate for $\psi \to e\bar{e}$. The prediction for $q = 1$ is then $\Gamma(\psi \to e\bar{e}) = 23.3$ keV $\pm 30\%$, which is certainly too large compared to the experimental value $\Gamma^{\exp}(\psi \to e\bar{e}) = 4.85 \pm 0.55$ keV. For $q = \frac{1}{2}$ the prediction is $\Gamma(\psi \to e\bar{e}) = 5.63 \pm 1.08$ keV [or $\Gamma(\psi \to e\bar{e}) = 5.48 \pm 0.38$ keV if one uses only the Orsay data for the decay rates of the old vector mesons], which is in excellent agreement with the experimental value.

The values of the reduced matrix elements that one obtains for the case $q = \frac{1}{2}$ from the experimental decay rates of the old vector mesons are

$$|Gs| = 0.96 \text{ keV}^{1/2} \pm 11\%, \quad \text{for } q = \tfrac{1}{2}, \tag{3.8}$$

$$|Ga| = 7.46 \text{ keV}^{1/2} \pm 6\%, \quad \text{for } q = \tfrac{1}{2}, \tag{3.9}$$

and

$$\mathrm{sign}\, a = -\,\mathrm{sign}\, s\,. \tag{3.10}$$

The ratio of a/s that one obtains from these empirical values is

$$a/s = -7.76 \pm 14\%\,. \tag{3.11}$$

The empirical value (3.11) is interesting because there is a theoretical argument that will allow one to calculate this ratio: Unlike for the operator V^{mag} in the SU(4) space describing the magnetic-type transitions $V \to P\gamma$, the operator V^{el} in the SU(4) space describing the electric-type transitions should obey the same restrictions that lead to the Gell-Mann–Nishijima formula when SU(4) is restricted to SU(3). Therefore, it is tempting to postulate

$$\langle 0^+, \sigma | [-(\tfrac{2}{3})^{1/2} V^{\chi} + V^{\sigma}] | V', 1^- \rangle = 0 \tag{3.12}$$

for any of the old vector mesons V', because then the electric-type transitions for the old vector mesons in SU(3) are given by $\langle \sigma | [V^{\pi^0} + (1/\sqrt{3}) V^{\eta}] | V' \rangle$. Inserting the values of the Clebsch-Gordan coefficients into (4.12) leads to

$$a/s = -8.22\,. \tag{3.13}$$

This is in perfect agreement with the empirical value (3.11).

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[1](a) A. Bohm, Phys. Rev. D 13, 2110 (1976); (b) A. Bohm and J. Werle, Nucl. Phys. B106, 165 (1976); (c) A. Bohm, M. Igarashi, and J. Werle, Phys. Rev. D 15, 2461 (1977); (d) A. Bohm, R. B. Teese, A. Garcia, and J. S. Nilsson, *ibid.* 15, 689 (1977).

[2]As "dynamical stability group of P/M" the same general idea has been arrived at by H. van Dam and L. C. Biedenharn, Phys. Rev. D 14, 405 (1976). A related nonperturbative approach to SU(3) had for years been advocated by S. Matsuda and S. Oneda, Phys. Rev. 174, 1992 (1968). The present spectrum-generating-group approach to SU(3) originates from the following series of papers: J. Werle, ICTP report, Trieste, 1965 (unpublished); A. Bohm, Phys. Rev. 158, 1408 (1967); Phys. Rev. D 7, 2701 (1973); A. Bohm and E. C. G. Sudarshan, Phys. Rev. 178, 2264 (1969).

[3]The properties of SU(4), the labeling of basis vectors, the particle assignments, and the group-theoretical properties of the charge and electromagnetic current operator are discussed in A. Bohm, M. Hossain, and R. B. Teese, Phys. Rev. D 18, 248 (1978), hereafter referred to as I.

[4]To avoid further misunderstanding we wish to emphasize that we work here under very general assumptions. Neither quarks nor local fields are used; therefore, some of our later assumptions may contradict conventional conceptions. Ours is a purely quantum-mechanical framework in which the properties of the operators are specified by their relations with each other [see Ref. 1(a)]. We specify as many relations as we need, so that more mathematical structure can still later be imposed upon the set of observables. We could certainly impose as many requirements as are valid in quark model and local current algebra; only then we will not get any result different from the well known predictions of these models. The V_μ^α are just defined by their relations to the algebra of $\mathrm{SU}(4)_E$ and the extended Poincaré group. One can, of course, define fields $V_\mu^\alpha(x) = e^{-iP^\mu x_\mu} V_\mu^\alpha e^{iP^\mu x_\mu}$, only these fields are in general nonlocal unless SU(4) is a symmetry group and the masses are degenerate. The hadrons are just defined by the assignment to their subspace in the representation space of our algebraic structure. One can, of course, assume that they are quark bound states; only then one may lose some of the freedom for the choice of the electromagnetic current operator which we want to use below.

[5]That, in addition to the purely group-theoretical predictions, a correction factor depending upon the masses may be needed and is theoretically permissible has been noted several times before: M. Gourdin, in *Symmetries and Quark Models*, edited by R. Chand (Gordon and Breach, New York, 1970); R. P. Feynman, *Photon-Hadron Interaction* (Benjamin, New York, 1972); D. R. Yennie, Phys. Rev. Lett. 34, 239 (1975); G. J. Aubrecht II and M. S. K. Razmi, Phys. Rev. D 12, 2120 (1975). An octet-broken SU(3) model incorporating current mixing has been used in L. M. Brown and P. Singer, Phys. Rev. D 15, 3438 (1977), where further references are given. Mass-dependent correction factors have also been derived from the Duffin-Kemmer-Petiau (DKP) formalism: B. G. Kenney, D. C. Peaslee, and M. M. Nieto, Phys. Rev. D 13, 757 (1976), and references therein. The DKP formalism has also other results in common with our spectrum-generating-group approach, cf. footnote 16 of Ref. 1(a). Another way of describing symmetry-breaking effects by mass-de-

pendent correction factors in the effective coupling constants has been suggested by J. Schwinger [in *Proceedings of the Seventh Hawaii Topical Conference on Particle Physics, Honolulu, 1977,* edited by R. J. Cence *et al.* (University Press of Hawaii, Honolulu, 1977)]; and L. F. Urrutia [Phys. Rev. D 18, 819 (1978), where further references to the description of symmetry breaking effects are given]. Their suppression factor, obtained there by a transformation from the fields in a U(3) non-Abelian gauge-invariant Lagrangian to the observable particle fields, can be obtained in our spectrum-generating-group approach by an appropriate choice for H^{el}_{μ} in place of (3.17) below. Equation (3) in this reference is the direct analog of Eq. (7) in Ref. 1(b).

[5a]This paper is an abridged version; the full-length version can be obtained from the authors as CPT Report No. ORO 317 (unpublished).

[6]A. Bohm, *Quantum Mechanics* (Springer, New York, 1979), Chap. XXI, Sec. 5.

[7]The effect of a strong dependence of $a_{(\gamma)}$ upon $\hat{p}_{\mu}\hat{q}^{\mu}$ will be the same as a change of the assumption (3.16)

[8]H. Hallock, S. Oneda, and Milton D. Slaughter, University of Maryland Report No. 76-294, 1976 (unpublished), and references therein.

[9]Tables of SU(4) Clebsch-Gordan coefficients are also given by V. Rabl, G. Campbell, Jr., and K. C. Wali, J. Math. Phys. 16, 2494 (1975); Y. Miyata, S. Iwai, and K. Kudoh, Tokyo Institute of Technology Report No. TIT/HEP-21, 1975 (unpublished).

[10]J. Iizuka, Prog. Theor. Phys. 37–38, 21 (1966); S. Okubo, Phys. Lett. 5, 165 (1963); G. Zweig, CERN Report No. TH412, 1964 (unpublished).

[11]A theoretical explanation of (2.27) can be given in terms of the transformation property of the operator V^{mag} with respect to the $SU(6)\otimes SU_{S_X}(2)$ and $SU_W(4)\otimes SU_{S_Y}(2)$ occurring in the subgroup chain (38) of I: So far we have only specified the transformation property of $\hat{V}^{el}_{\mu}$ and V^{mag} with respect to $SU(4)_E$. If we assume in addition that V^{mag} is the restriction to the $SU(4)_E$ representation space $\mathcal{H}^{SU(4)}$ of an operator which commutes with $SU(6)\otimes SU(2)_{S_X}$ and $SU(4)_W\otimes SU(2)_{S_Y}$, then it follows that

$\langle\pi^0|V^{mag}|\psi\rangle=\langle\pi^0|V^{mag}|\phi\rangle=\langle\eta|V^{mag}|\psi\rangle=0$, as one can immediately see from Table 5 of I. With (2.26) this then leads to (2.27). The precise algebraic formulation of this $SU(6)\otimes SU(2)_{S_X}$ and $SU(4)_W\otimes SU(2)_{S_Y}$ probably can be given only after the precise meaning of the relativistic SU(8), SU(6), and $SU(4)_W$ is known.

[12]R. Van Royen and V. F. Weisskopf, Nuovo Cimento 50A, 617 (1967).

[13]B. Gobbi *et al.*, Phys. Rev. Lett. 33, 1450 (1974); 37, 1439 (1976).

[14]A. Bohm, Phys. Rev. D 18, 2547 (1978).

[15]A. Bohm and R. B. Teese, Phys. Rev. D 18, 330 (1978).

[16]E. H. Thorndike, in *Experimental Meson Spectroscopy, 1977,* proceedings of the Fifth International Conference, Boston, edited by E. von Goeler and R. Weinstein (Northeastern University Press, Boston, 1978), p. 354.

[17]Recently the following experimental data have been published by D. E. Andrews *et al.*, Phys. Rev. Lett. 38, 198 (1977); $\Gamma(\omega\rightarrow\eta\gamma)=3.0^{+2.5}_{-1.8}$ keV or $\Gamma(\omega\rightarrow\eta\gamma)=29\pm7$ keV, $\Gamma(\rho\rightarrow\eta\gamma)=50\pm13$ keV or $\Gamma(\rho\rightarrow\eta\gamma)=76\pm15$ keV and $\Gamma(\phi\rightarrow\eta\gamma)=55\pm12$ keV.

[18]C. J. Zanfino *et al.*, Phys. Rev. Lett. 38, 930 (1977).

[19]A. Bohm, Phys. Rev. D 13, 2656 (1977).

[20]Here the mutually consistent normalization of the v, $\sum v\bar{v}=p-m$, and the measure $\int d^3p/2E$ for the electrons has been used in order to facilitate comparison with the radiative decays, where the leptonic part is different.