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Calculation of Q decay amplitudes and partial widths

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All the Lorentz- and SU(3)-invariant coupling constants involved in the decays of axial-vector nonents into a ρ nonet and a π nonet are calculated. The S- and the D-wave amplitudes for Q decays are then expressed only in terms of the mixing angle α of Q mesons. The partial widths, the D/S ratios, and the F_1/F_0 ratios for Q decays are then plotted against α. We find that if $Q_2 \rightarrow K^*\pi$ is dominated by the S wave, then the processes $Q_1 \rightarrow \rho K$ and $Q_1 \rightarrow \omega K$ are dominated by the S wave, while the processes $Q_1 \rightarrow K^*\pi$, $Q_2 \rightarrow \rho K$, and $Q_2 \rightarrow \omega K$ are dominated by the D wave. We also find that $F_1 = F_0$ for the processes $Q_1 \rightarrow \rho K$ and $Q_1 \rightarrow \omega K$. The predicted decay amplitudes and partial widths for Q decays are given for $\alpha = 45°$.

I. INTRODUCTION

Recently, there has been convincing experimental evidence[1] that there are two Q mesons as predicted by the quark model. However, the measured partial widths are still rather uncertain. In the present paper, we calculate the Q decay amplitudes and partial widths based upon the D/S ratio[2] for $B \rightarrow \omega\pi$, SU(3) symmetry, the quark-model phase constraint,[3,4] and the Okubo[5]-Zweig[6]-Iizuka[7] (OZI) rule. In order to perform the above calculations, we should first calculate the SU(3)-invariant coupling constants involved in the decay of an axial-vector meson into a vector meson and a pseudoscalar meson. Since the above decay process involves two partial waves, there are two types of coupling constants, denoted by F and G. We thus have two D-type coupling constants, denoted by F_D and G_D, and two F-type coupling constants, denoted by F_F and G_F, and similarly for singlet coupling constants. The two D-type coupling constants are calculated by using the known D/S ratio for $B \rightarrow \omega\pi$ and the OZI rule, while the two F-type coupling constants are calculated through the quark-model phase constraint which related $(D/S)_{B\rightarrow\omega\pi}$ to $(D/S)_{A\rightarrow\rho\pi}$. All other singlet coupling constants are then calculated based upon the OZI rule. In this way, we have calculated all SU(3)-invariant coupling constants involved in the decays of axial-vector nonets into a ρ nonet and a π nonet.

In our analysis, the SU(3)-invariant coupling constants are also Lorentz invariant, these Lorentz- and SU(3)-invariant coupling constants are linear combinations of the partial-wave amplitudes. This is different from the conventional SU(3)-invariant coupling constants with definite orbital angular momentum, which are not Lorentz invariant when two or more partial waves are involved in the decay.

Let Q_A and Q_B belong to the A_1 and B octets, respectively, then the physical Q mesons, denoted by Q_1 and Q_2, are assumed to be the linear combinations of Q_A and Q_B with mixing angle α. This mixture of Q_A and Q_B is due to (SU(3) breaking. Based upon this mixing scheme and the above calculated Lorentz- and SU(3)-invariant coupling constants, the S- and D-wave amplitudes for Q decays are expressed only in terms of α. The partial widths, the D/S ratios, and the F_1/F_0 (F_λ is the helicity amplitude) ratios for Q decays are then plotted against α. We find that if $Q_2 \rightarrow K^*\pi$ is dominated by the S wave, then the processes $Q_1 \rightarrow \rho K$ and $Q_1 \rightarrow \omega K$ are dominated by the S wave, while the processes $Q_1 \rightarrow K^*\pi$, $Q_2 \rightarrow \rho K$, and $Q_2 \rightarrow \omega K$ are dominated by the D wave. We also find that $F_1 = F_0$ for the processes $Q_1 \rightarrow \rho K$ and $Q_1 \rightarrow \omega K$. We also tabulate the predicted amplitudes and partial widths for Q decays at $\alpha = 45°$, which are consistent with experiments. We note that the conclusion of the S-wave dominance of the processes $Q_1 \rightarrow \rho K$ is in agreement with the experiments,[8] while the conclusion of D-wave dominance for some Q decays is contrary to the usual thinking that all vector-pseudoscalar decays of Q mesons are dominated by the S wave.

In Sec. II, we calculate all the Lorentz- and SU(3)-invariant coupling constants. In Sec. III, we express the Q partial-wave amplitudes in terms of the mixing angle α and draw the figures for D/S ratios and partial widths. The helicity amplitudes are defined and the F_1/F_0 ratios are obtained in Sec. IV. The conclusions and a discussion are given in the last section.

II. LORENTZ- AND SU(3)-INVARIANT COUPLING CONSTANTS

The two Lorentz-invariant coupling constants, denoted by F and G, for an axial-vector meson decay into a vector meson and a pseudoscalar meson are defined by

$$(4p_0q_0)^{1/2}\langle 1^-(q,\lambda')|j_\pi(0)|1^+(p,\lambda)\rangle = \bar{\epsilon}_\mu(q,\lambda')\left(\delta_{\mu\nu}F + \frac{P_\mu q_\nu}{m^2}G\right)\epsilon_\nu(p,\lambda), \quad (1)$$

where λ's are helicities and m is the mass of the final vector meson. The above coupling constants can be shown to be real because of time-reversal invariance. The Lorentz- and SU(3)-invariant coupling constants are then obtained by dividing F and G by the appropriate SU(3) Clebsch-Gordan coefficients.

It is well known that there are two axial-vector octets: the A_1 octet and the B octet. The Lorentz- and SU(3)-invariant coupling constants for the A_1 octet decays into a vector octet and a pseudoscalar octet are of F type and are denoted by F_F and G_F. For the decays of the B octet, the Lorentz- and SU(3)-invariant coupling constants are of D type and are denoted by F_D and G_D. Therefore we have four octet coupling constants for the decays of axial-vector mesons into a vector meson and a pseudoscalar meson.

In order to calculate the coupling constants, we first express these coupling constants in terms of partial-wave amplitudes. The S- and D-wave amplitudes can be shown[9] to be

$$S=(72\pi)^{-1/2}\left[\left(2+\frac{q_0}{m}\right)F-\frac{q^2M}{m^3}G\right],$$
$$D=(36\pi)^{-1/2}\frac{q^2}{m^2}\left[-\frac{m}{q_0+m}F+\frac{M}{m}G\right], \tag{2}$$

where M is the mass of the decaying axial-vector meson, and q is the magnitude of the c.m. momentum of the final particles. The factors outside the brackets in expression (2) guarantee the following normalization:

$$\Gamma(1^+\to 1^-0^+)=\frac{q}{M^2}(|S|^2+|D|^2), \tag{3}$$

for the S- and D-wave amplitudes. From expression (2), we can express the coupling constants in terms of the partial-wave amplitudes. Since we want to calculate the Lorentz- and SU(3)-invariant coupling constants from the known width and D/S ratio of B meson decay, it is convenient to express the coupling constants in terms of the decay width and the D/S ratio. The expressions are

$$F=2M\left(\frac{3\pi\Gamma}{q}\right)^{1/2}\sin(\theta+\beta),$$
$$G=\frac{2m^3}{q^2}\left(\frac{6\pi\Gamma}{q}\right)^{1/2}\frac{\sin(\theta+\beta')}{\sin(\beta-\beta')}, \tag{4}$$

where we define $\tan\theta=D/S$, $\cos\beta=1/\sqrt{3}$, $\cos\beta'=(1+K^2)^{-1/2}$ with $K=\sqrt{2}(q_0-m)/(q_0+2m)$, and β's are in the first quadrant.

Now we consider the decay process $B\to\omega\pi$. The coupling constants are of D type because of charge-conjugation invariance. By means of the SU(3) Clebsch-Gordan coefficients and the ideal ω-ϕ mixing, we have

$$F_{\omega\pi B}=(\tfrac{2}{3})^{1/2}F_{(1)}+\frac{1}{(15)^{1/2}}F_D,$$
$$G_{\omega\pi B}=(\tfrac{2}{3})^{1/2}G_{(1)}+\frac{1}{(15)^{1/2}}G_D, \tag{5}$$

where $F_{(1)}$ and $G_{(1)}$ are the Lorentz- and SU(3)-invariant singlet-octet-octet coupling constants. Since the Okubo-Zweig-Iizuka rule implies[10] $F_{(1)}=(2/5)^{1/2}F_D$ and $G_{(1)}=(2/5)^{1/2}G_D$, expression (5) reduces to

$$F_{\omega\pi B}=\left(\frac{3}{5}\right)^{1/2}F_D,$$
$$G_{\omega\pi B}=\left(\frac{3}{5}\right)^{1/2}G_D. \tag{6}$$

By substituting expression (6) into expression (4), we can express F_D and G_D in terms of the decay width and the D/S ratio of the decay $B\to\omega\pi$. The values of F_D and G_D can then be calculated directly. We use $\Gamma(B\to\omega\pi)=150$ MeV and $D/S^2=0.3$, and obtain $F_D=\pm 6093.3$ MeV and $G_D=\pm 12\,730.9$ MeV with F_D and G_D having the same sign.

In order to calculate the F-type coupling constants, we consider the process $A_1\to\rho\pi$, in which only the F-type coupling is allowed by charge-conjugation invariance. By means of SU(3) Clebsch-Gordan coefficients we have

$$F_{\rho^+\pi^0A_1^+}=\frac{1}{\sqrt{3}}F_F,$$
$$G_{\rho^+\pi^0A_1^+}=\frac{1}{\sqrt{3}}G_F. \tag{7}$$

By substituting expression (7) into expression (4), we can express F_F and G_F in terms of the decay width and the D/S ratio of the decay, $A_1\to\rho\pi$. The D/S ratio of the process $A_1\to\rho\pi$ can be related to the D/S ratio for the process $B\to\omega\pi$ by means of the quark-model phase constraint[3,4]

$$2\left(\frac{F_1}{F_0}\right)_{A_1\to\rho\pi}=\left(\frac{F_0}{F_1}\right)_{B\to\omega\pi}+1, \tag{8}$$

where F_λ is the helicity amplitude. From Ref. 9, we know that F_1 is proportional to $S+D/\sqrt{2}$, while F_2 is proportional to $S-\sqrt{2}D$. Then expression (8) implies

$$\left(\frac{D}{S}\right)_{A_1\to\rho\pi}=-\frac{1}{2}\left(\frac{D}{S}\right)_{B\to\omega\pi}. \tag{9}$$

This gives the value -0.15 to the D/S ratio for the process $A_1\to\rho\pi$. With $\Gamma(A_1\to\rho\pi)=300$ MeV, the F-type Lorentz- and SU(3)-invariant coupling constants are calculated to be $F_F=\pm 6526.8$ MeV and $G_F=\mp 5405.8$ MeV.

Now we have four sets of values for the Lorentz- and SU(3)-invariant coupling constants. Since only the relative signs among them is important, the above four sets of values reduce to two sets of values with different relative signs among the coupling constants. In the following calculations of the D/S ratios, the helicity amplitudes, and the partial widths for Q decays, the Lorentz-invariant octet coupling constants given by

$$\begin{aligned} &F_F=-6526.8 \text{ MeV}, \quad G_F=5405.8 \text{ MeV}, \\ &F_D=6093.3 \text{ MeV}, \quad G_D=12\,730.1 \text{ MeV} \end{aligned} \tag{10}$$

are used. The other set of values with F-type coupling constants changing sign will be discussed in the last section.

In order to calculate the singlet coupling constants, we use the Okubo-Zweig-Iizuka (OZI) rule to relate the singlet coupling constants to the octet couplings. The complete relations are shown[10] to be

$$g_{111}=g_{188}=g_{818}=-\frac{1}{2(2)^{1/2}}g_{881}=(\tfrac{2}{5})^{1/2}g_D\,, \tag{11}$$

where $g_{\nu_2\nu_3\nu_1}$ is the SU(3)-invariant coupling constant in the process $\nu_1\to\nu_2+\nu_3$ with ν_i representing the SU(3) multiplet, and g_D is the D-type coupling. The normalization is such that $g_{\nu_2\nu_3\nu_1}{}^2(g_D)$ is the sum of the squares of the couplings of all pairs of particles in ν_2 and ν_3 to any one particle in ν_1. Since the process in which an A_1 nonet decays into a ρ nonet and a π nonet has zero D-type coupling constants, all the singlet coupling constants for this process vanish according to expression (11). For the process in which a B nonet decays into a ρ nonet and a pseudoscalar nonet, we have, by means of Eqs. (10) and (11), the following values for singlet coupling constants:

$$\begin{aligned} &F_{(1)}\equiv F_{188}=F_{818}=F_{111}=3853.7 \text{ MeV}, \\ &G_{(1)}\equiv G_{188}=G_{818}=G_{111}=805.2 \text{ MeV}, \\ &F_{881}=-10\,899.9 \text{ MeV}, \quad G_{881}=-2277.4 \text{ MeV}\,, \end{aligned} \tag{12}$$

where the notations $F_{(1)}$ and $G_{(1)}$ are used in expression (5). Therefore, we have calculated all the nonvanishing Lorentz- and SU(3)-invariant coupling constants involved in the decays of A_1 and B nonets into a ρ nonet and a π nonet.

III. D/S RATIOS FOR Q WIDTHS

Let Q_A and Q_B belong to the A_1 and B octets, respectively. The physical Q mesons, denoted by Q_1 and Q_2, are assumed to be mixing states of Q_A and Q_B, i.e.,

$$\begin{aligned} &Q_1=\cos\alpha\, Q_A+\sin\alpha\, Q_B\,, \\ &Q_2=-\sin\alpha\, Q_A+\cos\alpha\, Q_B\,, \end{aligned} \tag{13}$$

where α is the mixing angle. Then the above expression (13) and the isoscalar factors immediately give the following expressions for various coupling constants defined by expression (1):

$$\begin{aligned} &F_{K^*\pi Q_1}=\frac{1}{2}\left(\cos\alpha\, F_F+\frac{3}{(5)^{1/2}}\sin\alpha\, F_D\right), \\ &F_{\rho K Q_1}=\frac{1}{2}\left(\cos\alpha\, F_F-\frac{3}{(5)^{1/2}}\sin\alpha\, F_D\right), \\ &F_{\omega K Q_1}=-\frac{1}{2(3)^{1/2}}\left(\cos\alpha\, F_F-\frac{3}{(5)^{1/2}}\sin\alpha\, F_D\right), \\ &F_{K^*\pi Q_2}=\frac{1}{2}\left(-\sin\alpha\, F_F+\frac{3}{(5)^{1/2}}\cos\alpha\, F_D\right), \\ &F_{\rho K Q_2}=-\frac{1}{2}\left(\sin\alpha\, F_F+\frac{3}{(5)^{1/2}}\cos\alpha\, F_D\right), \\ &F_{\omega K Q_2}=\frac{1}{2(3)^{1/2}}\left(\sin\alpha\, F_F+\frac{3}{(5)^{1/2}}\cos\alpha\, F_D\right). \end{aligned} \tag{14}$$

In the above expressions for $F_{\omega KQ_1}$ and $F_{\omega KQ_2}$, the ideal ω-ϕ mixing and the Okubo-Zweig-Iizuka rule are used. A similar expression also holds for the coupling constants G. From expression (14), we obtain

$$\begin{aligned} &F_{\rho KQ_1}=-\sqrt{3}F_{\omega KQ_1}\,, \\ &F_{\rho KQ_2}=-\sqrt{3}F_{\omega KQ_2}\,, \end{aligned} \tag{15}$$

and similar equalities for G. These equalities then imply that $\Gamma(Q_1\to\rho K)=3\Gamma(Q_1\to\omega K)$ and $\Gamma(Q_2\to\rho K)=3\Gamma(Q_2\to\omega K)$ if ω and ρ mesons have the same mass.

We define the following quantities

$$\begin{aligned} &S^F_{VPQ}=(72\pi)^{-1/2}\left[\left(2+\frac{q_0}{m}\right)F_F-\frac{q^2M}{m^3}G_F\right], \\ &S^D_{VPQ}=(72\pi)^{-1/2}\left[\left(2+\frac{q_0}{m}\right)F_D-\frac{q^2M}{m^3}G_D\right], \\ &D^F_{VPQ}=(36\pi)^{-1/2}\frac{q^2}{m^2}\left(-\frac{m}{q_0+m}F_F+\frac{M}{m}G_F\right), \\ &D^D_{VPQ}=(36\pi)^{-1/2}\frac{q^2}{m^2}\left(-\frac{m}{q_0+m}F_D+\frac{M}{m}G_D\right), \end{aligned} \tag{16}$$

where M and m are the masses of the Q meson and the vector meson, respectively, V denotes vector meson, ρ denotes pseudoscalar meson, and q is the c.m. momentum of the final particles. Then from expressions (2) and (14), we obtain the following expressions for various S-wave amplitudes:

$$\begin{aligned} &S_{K^*\pi Q_1}=\frac{1}{2}\left(\cos\alpha\, S^F_{K^*\pi Q_1}+\frac{3}{(5)^{1/2}}\sin\alpha\, S^D_{K^*\pi Q_1}\right), \\ &S_{\rho KQ_1}=\frac{1}{2}\left(\cos\alpha\, S^F_{\rho KQ_1}-\frac{3}{(5)^{1/2}}\sin\alpha\, S^D_{\rho KQ_1}\right), \\ &S_{\omega KQ_1}=-\frac{1}{2(3)^{1/2}}\left(\cos\alpha\, S^F_{\omega KQ_1}-\frac{3}{(5)^{1/2}}\sin\alpha\, S^D_{\omega KQ_1}\right), \\ &S_{K^*\pi Q_2}=\frac{1}{2}\left(-\sin\alpha\, S^F_{K^*\pi Q_2}+\frac{3}{(5)^{1/2}}\cos\alpha\, S^D_{K^*\rho Q_2}\right), \\ &S_{\rho KQ_2}=-\frac{1}{2}\left(\sin\alpha\, S^F_{\rho KQ_2}+\frac{3}{(5)^{1/2}}\cos\alpha\, S^D_{\rho KQ_2}\right), \\ &S_{\omega KQ_2}=\frac{1}{2(3)^{1/2}}\left(\sin\alpha\, S^F_{\omega KQ_2}+\frac{3}{(5)^{1/2}}\cos\alpha\, S^D_{\omega KQ_2}\right). \end{aligned} \tag{17}$$

The corresponding D-wave amplitudes are obtained by replacing S^F_{VPQ} and S^D_{VPQ} by D^F_{VPQ} and D^D_{VPQ}, respectively. We note that the above S- and D-wave amplitudes satisfy the decay-width expression (3). From expression (17), we see that $S_{\rho K Q_1} = -\sqrt{3} S_{\omega K Q_1}$, $S_{\rho K Q_2} = -\sqrt{3} S_{\omega K Q_2}$, and similar relations for D-wave amplitudes if the ω and ρ mesons have the same mass.

Since the values of Lorentz- and SU(3)-invariant coupling constants are already given in Eq. (10), the quantities S^F_{VPQ}, S^D_{VPQ}, D^F_{VPQ}, and D^D_{VPQ} can then be calculated directly. After calculations, we obtain expressions which express the partial-wave amplitudes in terms of the mixing angle only. The final expressions for the S- and D-wave amplitudes of Q_1 decays are

$$\begin{aligned}
S_{K^*\pi Q_1} &= -696.3\cos\alpha + 729.7\sin\alpha\,,\\
D_{K^*\pi Q_1} &= 64.1\cos\alpha + 121.0\sin\alpha\,,\\
S_{\rho K Q_1} &= -660.6\cos\alpha - 796.0\sin\alpha\,,\\
D_{\rho K Q_1} &= 13.7\cos\alpha - 20.3\sin\alpha\,,\\
S_{\omega K Q_1} &= 379.0\cos\alpha + 464.5\sin\alpha\,,\\
D_{\omega K Q_1} &= -4.5\cos\alpha + 8.8\sin\alpha\,.
\end{aligned} \tag{18}$$

The corresponding expressions for Q_2 decays are

$$\begin{aligned}
S_{K^*\pi Q_2} &= 663.1\cos\alpha + 728.1\sin\alpha\,,\\
D_{K^*\pi Q_2} &= -215.3\cos\alpha + 109.0\sin\alpha\,,\\
S_{\rho K Q_2} &= -687.2\cos\alpha + 712.4\sin\alpha\,,\\
D_{\rho K Q_2} &= -181.1\cos\alpha - 86.9\sin\alpha\,,\\
S_{\omega K Q_2} &= 404.8\cos\alpha - 407.7\sin\alpha\,,\\
D_{\omega K Q_2} &= 93.3\cos\alpha + 45.0\sin\alpha\,.
\end{aligned} \tag{19}$$

In the above calculations, we use $m_{Q_1} = 1290$ MeV, $m_{Q_2} = 1400$ MeV, $m_K = 493.7$ MeV, $m_\omega = 782.7$ MeV, $m_\rho = 773$ MeV, $m_{K^*} = 892.2$ MeV, and $m_\pi = 139.6$ MeV. We note that the partial-wave amplitudes given in expressions (18) and (19) have the dimension of mass expressed in units of MeV. Through these relations, we can easily express the Q partial widths in terms of the mixing angle α.

If the process $Q_2 \to K^*\pi$ is dominated by the S wave as reported,[8] then both $\cos\alpha$ and $\sin\alpha$ terms in expression (19) should have substantial contributions to $S_{K^*\pi Q_2}$ and shouldlargely cancel in $D_{K^*\pi Q_2}$. This implies that the mixing angle should be positive and around 45°. A similar conclusion was also obtained by us in a previous analysis[11] based upon the S-wave dominance. We therefore plot, against the mixing angle α, the D/S ratios and the partial widths of Q mesons decaying into ρK, $K^*\pi$, and ωK in Figs. 1–6.

We also calculate the values of the S-wave width Γ_S, the D-wave width Γ_D, the total partial width

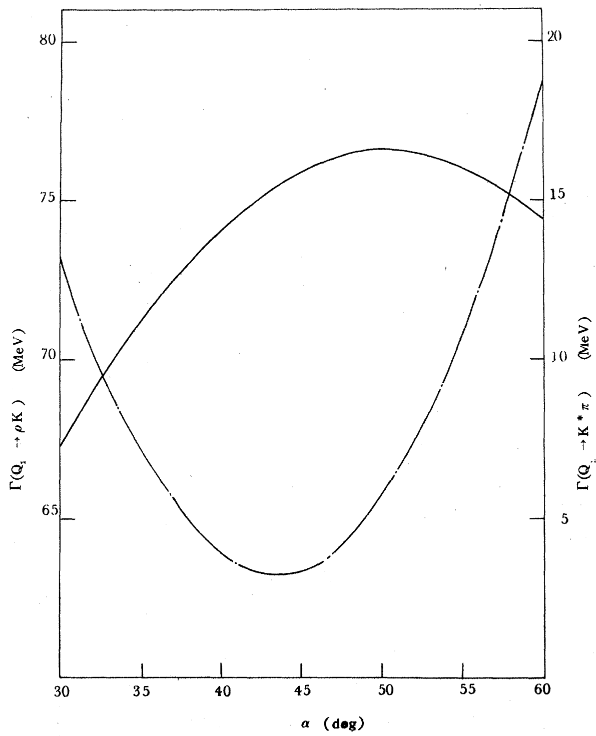

FIG. 1. α is the mixing angle in degrees. $\Gamma(Q_1 \to \rho K)$ and $\Gamma(Q_1 \to K^*\pi)$ are represented by the solid and the dash-dot curves, respectively.

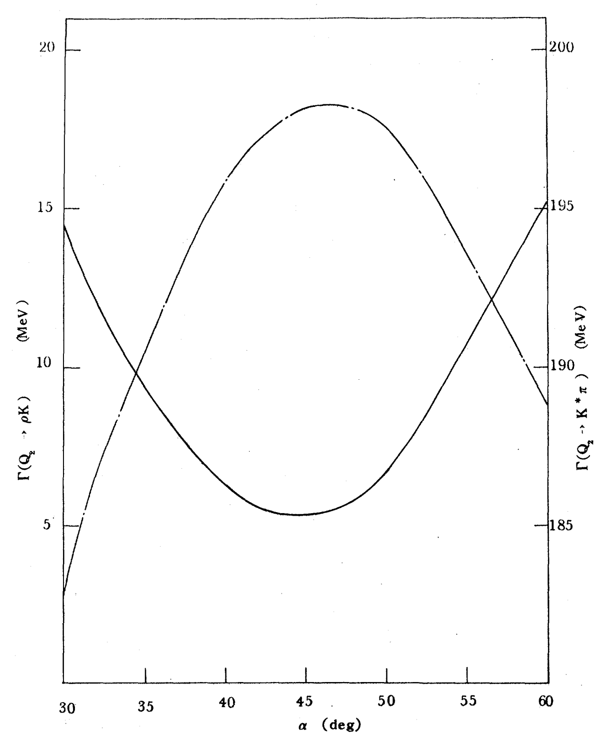

FIG. 2. $\Gamma(Q_2 \to \rho K)$ and $\Gamma(Q_2 \to K^*\pi)$ are represented by the solid and the dash-dot curves, respectively.

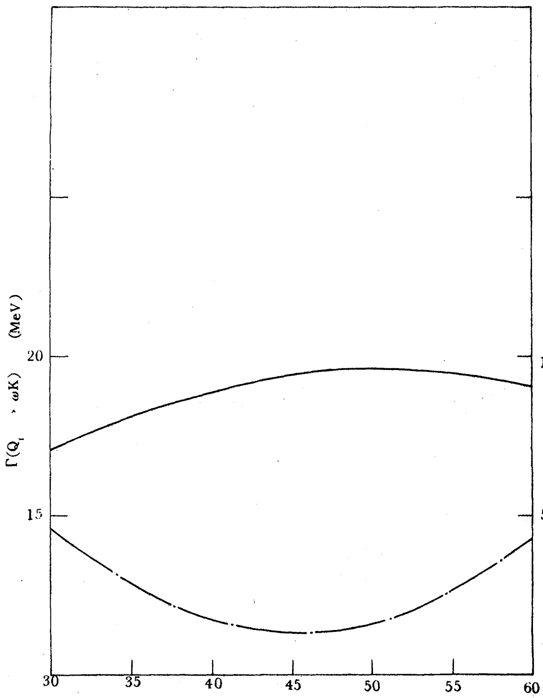

FIG. 3. $\Gamma(Q_1 \to \omega K)$ and $\Gamma(Q_2 \to \omega K)$ are represented by the solid and the dash-dot curves, respectively.

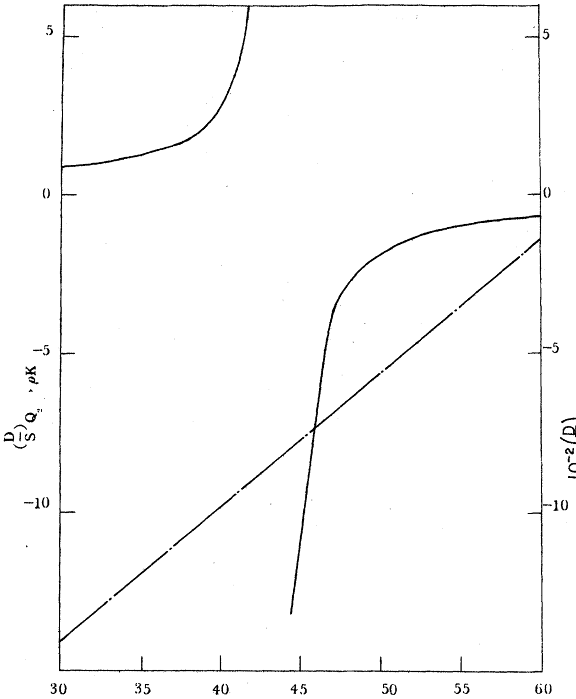

FIG. 5. $(D/S)_{Q_2 \to \rho K}$ and $(D/S)_{Q_2 \to K^* \pi}$ are represented by the solid and the dash-dot curves, respectively. $S_{\rho K Q_2} = 0$ at $\alpha = 44°$.

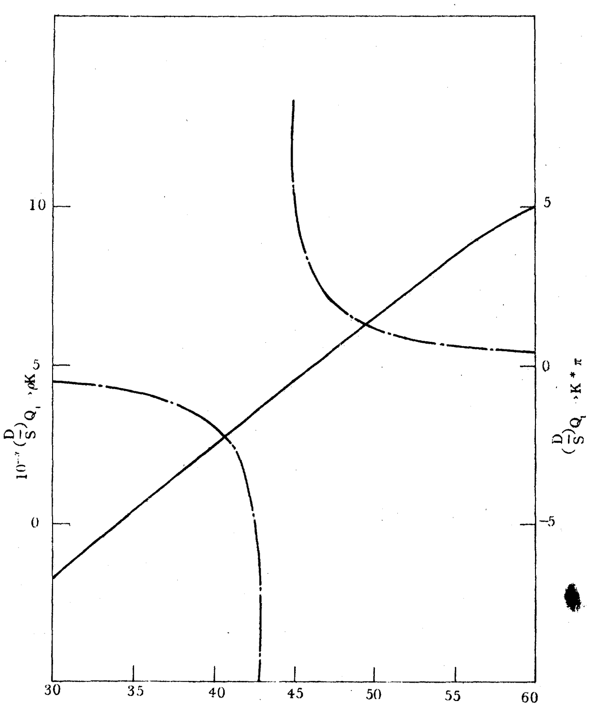

FIG. 4 $(D/S)_{Q_1 \to \rho K}$ and $(D/S)_{Q_1 \to K^* \pi}$ are represented by the solid and the dash-dot curves, respectively. $S_{K^* \pi Q_1} = 0$ at $\alpha = 43.7°$.

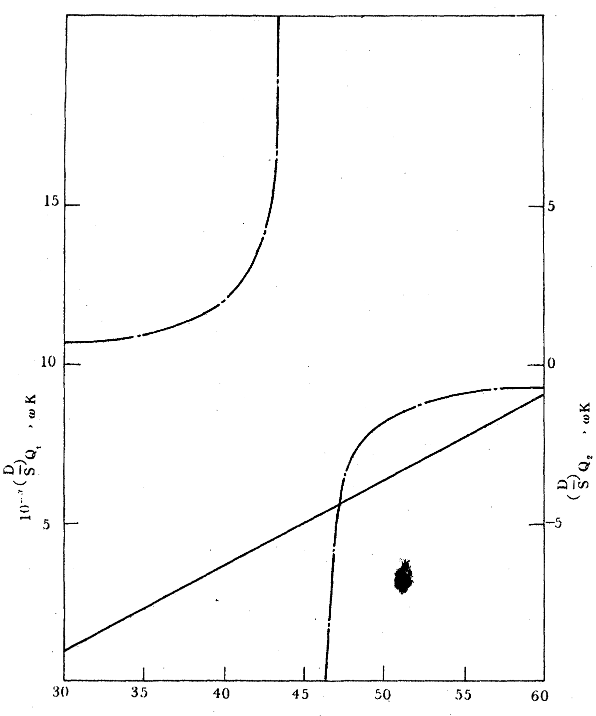

FIG. 6. $(D/S)_{Q_1 \to \omega K}$ and $(D/S)_{Q_2 \to \omega K}$ are represented by the solid and the dash-dot curves, respectively. $S_{\omega K Q_2} = 0$ at $\alpha = 44.8°$.

TABLE I. Q decay amplitudes and partial widths calculated at $\alpha = 45°$. Γ_{exp} is taken from Ref. 12.

Decay Modes	S (MeV)	D/S	F_0 (MeV)	F_1/F_0	Γ_S (MeV)	Γ_D (MeV)	Γ (MeV)	Γ_{exp} (MeV)
$Q_1(1290) \to \rho K$	−1030.0	4.6×10^{-3}	−5.7	1.0	75.9	1.6×10^{-3}	75.9	100 ± 35
$\to K^*\pi$	23.6	5.5	−7.0	−0.7	0.1	3.2	3.3	12 ± 13
$\to \omega K$	596.4	5.2×10^{-3}	5.7	1.0	19.5	5.2×10^{-4}	19.5	$\leqslant 32 \pm 11$
$Q_2(1400) \to \rho K$	17.8	−10.7	8.7	−0.4	4.7×10^{-2}	5.4	5.4	2 ± 1
$\to K^*\pi$	983.7	-7.6×10^{-2}	6.4	0.9	197.0	1.2	198.2	154 ± 52
$\to \omega K$	−2.0	−48.0	−8.3	−0.5	6.0×10^{-4}	1.4	1.4	~ 0

Γ, the D/S ratio, and the S-wave amplitude S by setting $\alpha = 45°$. These values are given in Table I.

IV. HELICITY AMPLITUDES FOR Q DECAYS

For the decays of an axial-vector meson into a vector meson and a pseudoscalar meson, there are three helicity amplitudes, denoted by F_{+1}, F_0, and F_{-1}, with +1, 0, and −1 representing the helicities. Since the amplitude with helicity +1 is equal to the amplitude with helicity −1 because of rotational invariance, we have only two independent helicity amplitudes in accordance with the two partial waves involved in the above process. It is well known that the helicity amplitudes can be expressed in terms of the partial-wave amplitudes as follows:

$$F_\lambda = N \sum_L \left(\frac{2L+1}{3}\right)^{1/2} C(L11;0\lambda\lambda) F^{(L)}, \qquad (20)$$

where λ is the helicity, and $F^{(L)}$ are the partial-wave amplitudes with $F^{(0)} = S$ and $F^{(2)} = D$. In the above expression, N is chosen such that the conventional normalization,

$$2|F_1|^2 + |F_0|^2 = 1 \qquad (21)$$

is satisfied. Then we have from expression (20) the following expressions for the helicity amplitudes:

$$F_0 = (S - \sqrt{2}D)[3(S^2+D^2)]^{-1/2},$$
$$F_1 = (\sqrt{2}S + D)[6(S^2+D^2)]^{-1/2}. \qquad (22)$$

From expression (22), we immediately obtain

$$\frac{F_1}{F_0} = \frac{\sqrt{2}S + D}{\sqrt{2}S - 2D}. \qquad (23)$$

Since the S-wave and the D-wave amplitudes for Q decays are explicitly given in expressions (16) and (17) as functions of the mixing angle α, the helicity amplitude can also be expressed in terms of the mixing angle by simple substitutions. We then plot the ratio F_1/F_0 against the mixing angle in Figs. 7–9. We also give the F_1/F_0 ratios and the values of F_0 for $\alpha = 45°$ in Table I.

V. CONCLUSIONS AND DISCUSSION

The values of all octet and singlet coupling constants involved in the decays of axial-vector nonets into a vector nonet and a pseudoscalar nonet are calculated and given in Eqs. (10) and (12). In the above calculations, we use the decay widths of $B \to \omega\pi$ and $A_1 \to \rho\pi$, the D/S ratio for $B \to \omega\pi$, the quark-model phase constraint (8), and the OZI relations (11). We then use the above-calculated coupling constants to express the S- and D-wave amplitudes of Q decays in terms of the mixing angle α. These expressions are given in (18) and (19). Through these relations we can easily express the Q widths, the D/S ratios, and the F_1/F_0 ratios in terms of α. These results are plotted in Figs. 1–9.

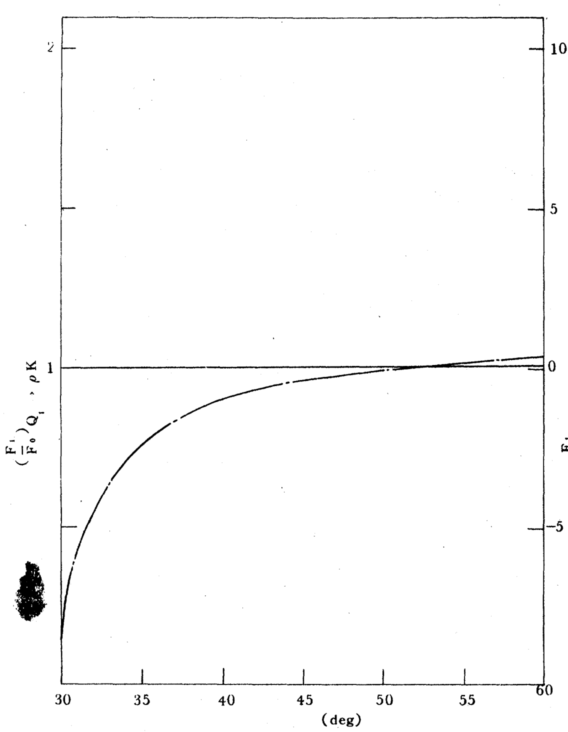

FIG. 7. $(F_1/F_0)_{Q_1\to\rho K}$ and $(F_1/F_0)_{Q_2\to\rho K}$ are represented by the solid and the dash-dot curves, respectively. $F_{1\rho K Q_2} = 0$ at $\alpha = 51.4°$.

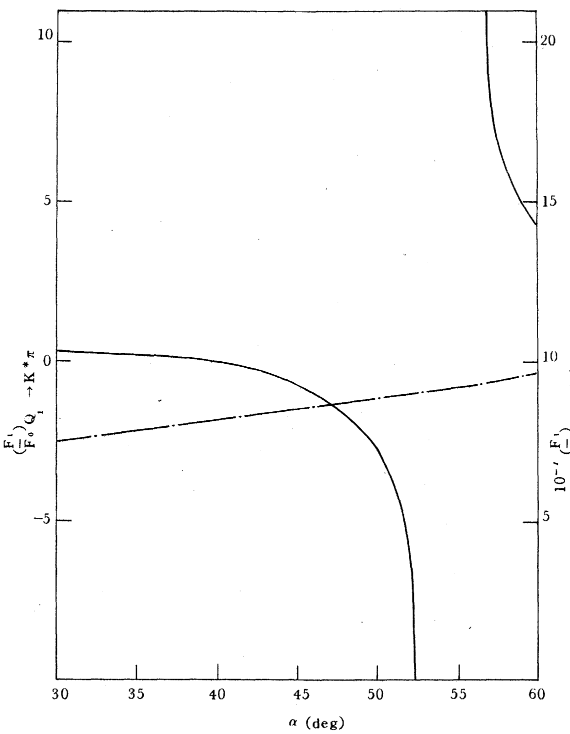

FIG. 8. $(F_1/F_0)_{Q_1\to K^*\pi}$ and $(F_1/F_0)_{Q_2\to K^*\pi}$ are represented by the solid and the dash-dot curves, respectively. $F_{0K^*\pi Q_1}=0$ at $\alpha=54.6°$.

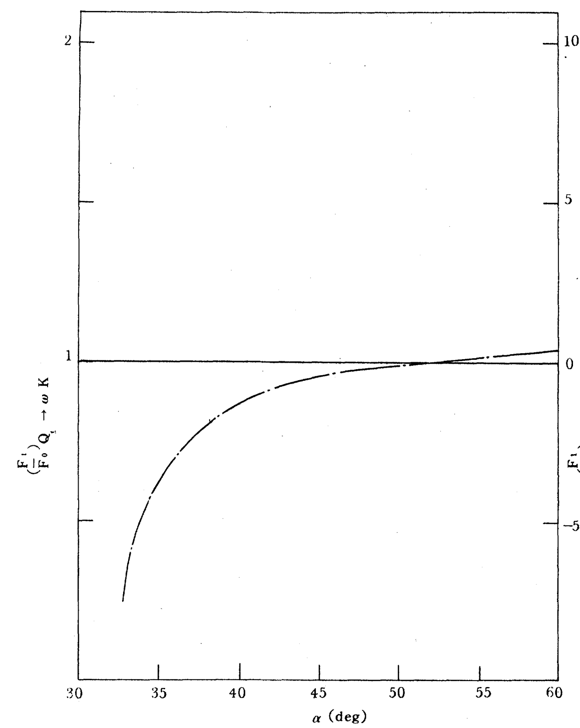

FIG. 9. $(F_1/F_0)_{Q_1\to\omega K}$ and $(F_1/F_0)_{Q_2\to\omega K}$ are represented by the solid and the dash-dot curves, respectively. $F_{1\omega KQ_2}=0$ at $\alpha=52.4°$ and $F_{0\omega KQ_2}=0$ at $\alpha=30°$.

From Figs. 1–3, the following can be seen clearly: (1) The partial widths of $Q_1\to\rho K$ and $Q_2\to K^*$ are dominant for α around 45° in accordance with the conventional belief. (2) $\Gamma(Q_1\to\rho K\geq 3\Gamma(Q_1\to\omega K)$ and $\Gamma(Q_2\to\rho K)\geq 3\Gamma(Q_2\to\omega K)$. (3) The maximum or minimum values of all partial widths occur at $\alpha\sim 45°$. Therefore all partial widths change smoothly with α around $\alpha=45°$.

From Figs. 4–6, we draw the following conclusions: (1) The processes $Q_1\to\rho K$ and $Q_1\to\omega K$ are dominated by S waves, while the processes $Q_1\to K^*\pi$, $Q_2\to\rho K$, and $Q_2\to\omega K$ are dominated by D waves, if $Q_2\to K^*\pi$ is dominated by the S wave as reported. (2) The S- and D-wave amplitudes have the same sign for the processes $Q_1\to\rho K$ and $Q_1\to\omega K$, and have the opposite sign for the process $Q_2\to K^*\pi$. (3) For the processes $Q_1\to K^*\pi$, $Q_2\to\rho K$, and $Q_2\to\omega K$, the S-wave amplitudes vanish at $\alpha\sim 44°$, and the relative sign between the S- and the D-wave amplitudes changes across this angle.

From Figs. 7–9, we obtain the following conclusions: (1) $F_1=F_0$ for the processes $Q_1\to\rho K$ and $Q_1\to\omega K$. (2) For the processes $Q_1\to K^*\pi$, $Q_2\to\rho K$, and $Q_2\to\omega K$, F_1 and F_0 are of approximately the same order of magnitude and have opposite sign for α near 45°. (3) For the process $Q_2\to K^*\pi$, F_1 and F_0 are of the same order of magnitude and have the same sign for all α.

We note that if $m_\omega=m_\rho$, then we have $\Gamma(Q_i\to\rho K)=3\Gamma(Q_i\to\omega K)$ for $i=1$ and 2. The conclusion $\Gamma(Q_i\to\rho K)\geq 3\Gamma(Q_i\to\omega K)$, is due to $m_\omega\geq m_\rho$ and is consistent with the estimation given by Carnegie *et al.*[12] It is interesting to point out that in our analysis we find that the processes $Q_1\to K^*\pi$, $Q_2\to\rho K$, and $Q_2\to\omega K$ are dominated by the D-wave, contrary to the belief[12] that all the partial widths are dominated by the S wave. The actual sign of the D/S ratio for the above processes may help to pin down the precise value of the mixing angle.

In Table I, $\Gamma_{\rm exp}$ is taken from Ref. 12, in which the authors claim that all D-wave contributions are negligible. Therefore $\Gamma_{\rm exp}$ should compare with our total partial width Γ. We suspect that the S-wave widths for $Q_1\to K^*\pi$ and $Q_2\to\rho K$ given in Ref. 12 are actually the D-wave widths. From Table I, we see that the predicted total partial widths are consistent with $\Gamma_{\rm exp}$.

In Sec. II, we mention that there is another set of octet coupling constants with F-type coupling constants changing sign. The transformation $F_F\to -F_F$ and $G_F\to -G_F$ implies the changes $S^F_{VPQ}\to -S^F_{VPQ}$ and $D^F_{VPQ}\to -D^F_{VPQ}$, as can be seen from expression (16). These changes are equivalent to the replacement $\alpha \to \pi - \alpha$, as can be seen from expression (17). Therefore this set of coupling constants gives exactly the same partial widths, the same D/S and F_1/F_0 ratios as those given by set (10) except for the mixing angle, which now takes a value around 135°.

ACKNOWLEDGMENT

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