Analysis of hadronic decays of ψ/J particles in generalized Veneziano models. III. The $\psi \to K\bar{K}\pi$ decay

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Constructing the amplitude for the $\psi \to K\bar{K}\pi$ decay from the five-point Veneziano function for $K\bar{K} \to K\bar{K}\pi$, we calculate the $K\bar{K}\pi$ Dalitz-plot density. This amplitude explains the features of the experimental data well, especially the fact that the quasi-two-body final state KK^* is dominant.

I. INTRODUCTION

In our previous papers[1,2] (referred to hereafter as I and II) we presented a speculation on the mechanism which governs the hadronic decays of the ψ/J particle into ordinary hadrons, and studied the $\psi \to 3\pi$ decay channel as a first application. We assumed that the ψ decays into ordinary hadrons through its mixing with the $J=1$ daughters of the ω and/or ϕ recurrences. Hence, the amplitude for $\psi \to$ ordinary hadrons may be written as

$$A(\psi \to \text{hadrons}) = \sum_i \sum_\beta g_{\psi\omega_{i,\beta}} \frac{1}{m_\psi^2 - \alpha_\omega^{-1}(i)} A(\omega_{i,\beta} \to \text{hadrons}) + \sum_i \sum_\beta g_{\psi\phi_{i,\beta}} \frac{1}{m_\psi^2 - \alpha_\phi^{-1}(i)} A(\phi_{i,\beta} \to \text{hadrons}), \quad (1)$$

where α_ω (α_ϕ) is the Regge trajectory of ω (ϕ). The $\omega_{i,\beta}$ ($\phi_{i,\beta}$) are daughters of the $\omega(\phi)$ recurrences, satisfying the equation

$$\alpha_\omega(m_{\omega_{i,\beta}}^2) = i \quad (\alpha_\phi(m_{\phi_{i,\beta}}^2) = i),$$

and having spin 1. In general, daughters are degenerate and the subscript β distinguishes one state from another in the degenerate level. The constants $g_{\psi\omega_{i,\beta}}$ ($g_{\psi\phi_{i,\beta}}$) express the strength of the ψ mixing with $\omega_{i,\beta}$ ($\phi_{i,\beta}$).

In the case of the $\psi \to 3\pi$ decay, only daughters of the ω recurrences contribute. We constructed the amplitude $A(\psi \to 3\pi)$ for the process $\psi \to 3\pi$ from the five-point Veneziano function for $K\bar{K} \to 3\pi$. Then we found that our amplitude $A(\psi \to 3\pi)$ well reproduces the characteristic features of the experimental data.[1,2]

In this paper, we apply our idea further to the study of the $\psi \to K\bar{K}\pi$ decay channel. The Dalitz plot[3] in Fig. 1 for the decay process $\psi \to K_S^0 K^\pm \pi^\mp \to (\pi^+\pi^-)K^\pm\pi^\mp$ shows the following features: (i) Just as the $\pi^+\pi^-\pi^0$ channel is dominated by $\rho\pi$, the $K_S K\pi$ channel is dominated by the quasi-two-body final state KK^* (Ref. 4); (ii) the decay modes KK^{**} are not seen.

Unlike the case of the $\psi \to 3\pi$ decay, daughters of both the ω and ϕ recurrences contribute to the

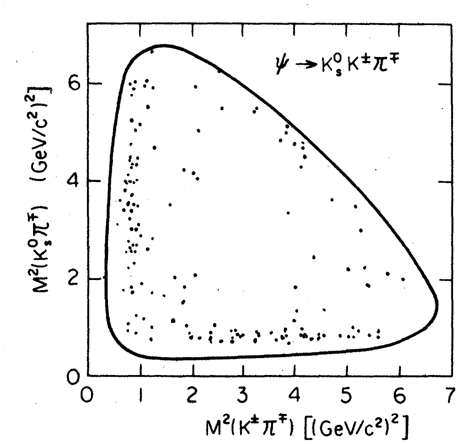

FIG. 1. Experimental $K_S^0 K^\pm \pi^\mp$ Dalitz plot (Ref. 3).

channel $\psi \to K\bar{K}\pi$. This forces us to introduce one parameter, which indicates the relative weight of the two contributions. Estimating the parameter under a plausible assumption that the daughters $\omega_{i,\beta}$ ($\phi_{i,\beta}$) have very large widths, we can show that the amplitude constructed from the five-point Veneziano function for $K\bar{K} \to K\bar{K}\pi$ well reproduces the features of the experimental plot for $\psi \to K_S^0 K^{\pm}\pi^{\mp}$, in particular the strong suppression of the central region in the Dalitz plot.

Some time ago Cohen-Tannoudji *et al.*[5] proposed that the Dalitz plot for $\psi \to K\bar{K}\pi$ (as well as for $\psi \to 3\pi$) exhibits characteristic structures described by a Virasoro amplitude. This Virasoro amplitude does not have poles at even-integer points of the K^* trajectory. So the signals of K^{**} do not appear, which is consistent with the experimental data. However, this amplitude gives rise to an enhancement in the central region of the $K\bar{K}\pi$ Dalitz plot just like the $\psi \to 3\pi$ case.[2] This result disagrees with the features of the experimental plot.

This paper is organized as follows. In Sec. II we describe the way to construct the amplitude $A(\psi \to K\bar{K}\pi)$. In Sec. III we introduce the imaginary parts into the ρ and K^* trajectories, and calculate the Dalitz-plot density for $\psi \to K\bar{K}\pi$. Comparison with the experimental plot is made. In Sec. IV we present concluding remarks.

II. CONSTRUCTION OF THE AMPLITUDE FOR $\psi \to K\bar{K}\pi$

As stated in Sec. I, daughters of both the ω and ϕ recurrences contribute to the process $\psi \to K\bar{K}\pi$. If we fix the ω trajectory by assuming the exact exchange degeneracy of the ω and ρ trajectories, just as we did in papers I and II, we obtain[6]

$$\alpha_\omega(s) = 0.48 + 0.89s\,. \tag{2}$$

Further if we determine the ϕ trajectory by requiring the universal slope and by the condition $\alpha_\phi(m_\phi^2) = 1$ we obtain[6]

$$\alpha_\phi(s) = 0.08 + 0.89s\,. \tag{3}$$

Evaluating Eqs. (2) and (3) at $s = m_\psi^2$, we find $\alpha_\omega(m_\psi^2) = 9.0$ and $\alpha_\phi(m_\psi^2) = 8.6$. Then the contribution of the daughters $\omega_{i=9,\beta}$ would seem to dominate the amplitude for $\psi \to K\bar{K}\pi$.

But here we should notice the following point: Because the ψ mass is very heavy, a little change in the slope parameter brings about a slight variation to the values of $\alpha_\omega(m_\psi^2)$ and $\alpha_\phi(m_\psi^2)$. For example, when we determine the slope and intercept of the ω trajectory from the masses of $\omega(783)$ with $J^P = 1^-$ and $\omega(1675)$ with $J^P = 3^-$, we find

$$\alpha_\omega(s) = 0.44 + 0.91s$$

and $\alpha_\omega(m_\psi^2) = 9.2$. And fixing the ϕ trajectory just in the same way as before, we obtain

$$\alpha_\phi(s) = 0.05 + 0.91s$$

and $\alpha_\phi(m_\psi^2) = 8.8$. Then the contribution of the daughters $\phi_{i=9,\beta}$ would become as important as that of $\omega_{i=9,\beta}$. Moreover, if we consider the propagators in Eq. (1) as in the resonance form such as $1/(m_\psi^2 - m_R^2 + i m_R \Gamma_R)$ with decay width Γ_R and $m_R = \alpha^{-1}(i)$, the relative weight of the two contributions depends also on the unknown decay widths of the daughters $\omega_{i,\beta}$ and $\phi_{i,\beta}$.

From these arguments, we assume in the following that only the daughters $\omega_{i=9,\beta}$ (Ref. 7) and $\phi_{i=9,\beta}$ contribute to the decay amplitude $A(\psi \to K\bar{K}\pi)$, and leave the relative strength of the two contributions as a free parameter (in general, a complex number). For the actual calculation later on of the pole residues and the projection of the $J = 1$ states, we shall use the values in Eq. (2) and (3) for the (universal) slope and intercepts of the ω and ϕ trajectories.

Now we construct the $A(\psi \to K\bar{K}\pi)$ from a five-point Veneziano function for the process $K\bar{K} \to K\bar{K}\pi$. The amplitude $A(K\bar{K} \to K\bar{K}\pi)$ is given by Bardakci-Ruegg as follows,[8]

$$A(K\bar{K} \to K\bar{K}\pi) \propto \sum_{P[(1,3),(2,4)]} (K_1^\dagger \tau_i \tau_5 K_4)(K_3^\dagger K_2) K_F B_5(\alpha_{12}^\rho - 1, \alpha_{23}^\phi - 1, \alpha_{34}^\omega - 1, \alpha_{45}^{K^*} - 1, \alpha_{51}^{K^*} - 1)\,, \tag{4}$$

where K_F is a kinematical factor written as

$$K_F = \epsilon_{\mu_1\mu_2\mu_3\mu_4} P_1^{\mu_1} P_2^{\mu_2} P_3^{\mu_3} P_4^{\mu_4}\,. \tag{5}$$

The indices $1, \ldots, 5$ label the particles in Fig. 2. The sum is over the permutations $1 \leftrightarrow 3$, $2 \leftrightarrow 4$, and $(1 \leftrightarrow 3) \otimes (2 \leftrightarrow 4)$. The function B_5 can be written in many particular forms,[8,9] one of which being

$$B_5(\alpha_{12} - 1,\ \alpha_{23} - 1, \alpha_{34} - 1, \alpha_{45} - 1, \alpha_{51} - 1)$$
$$= \int_0^1 du_1 du_4 u_1^{-\alpha_{12}} (1 - u_1)^{-\alpha_{23}} u_4^{-\alpha_{45}} (1 - u_4)^{-\alpha_{34}} (1 - u_1 u_4)^{-\alpha_{51} + \alpha_{23} + \alpha_{34} - 1}$$

$$\equiv B_5(1,2,3,4,5)\,. \tag{6}$$

This B_5 function has the following cyclic and reflection symmetries:

$$B_5(1,2,3,4,5)=B_5(2,3,4,5,1)=\ldots \tag{7}$$

and

$$B_5(1,2,3,4,5)=B_5(5,4,3,2,1)\,. \tag{8}$$

We shall make full use of these symmetry properties. For other notations and symbols in Eq. (4), refer to paper I. From the Fierz identity such as

$$(K_1^\dagger \tau_a K_4)(K_3^\dagger K_2)=\tfrac{1}{2}(K_1^\dagger \tau_a K_4)(K_3^\dagger K_2)+\tfrac{1}{2}(K_1^\dagger K_2)(K_3^\dagger \tau_a K_4)-\tfrac{1}{2}i\epsilon_{abc}(K_1^\dagger \tau_b K_2)(K_3^\dagger \tau_c K_4)\,, \tag{9}$$

Eq. (4) requires the exact exchange degeneracy between the ρ and ω trajectories and also between the K^* and K^{**} trajectories.

In the amplitude of Eq. (4), the $I=0$ part for the initial two kaons labeled as 1 and 2 can be found by using the above identity Eq. (9) as

$$\begin{aligned}A^{I=0}(K\bar{K}\to K\bar{K}\pi)\propto (K_1^\dagger K_2)(K_3^\dagger \tau_{i_5} K_4)K_F[&B_5(\alpha_{12}^{\omega}-1,\alpha_{23}^{\phi}-1,\alpha_{34}^{\rho}-1,\alpha_{45}^{K^*}-1,\alpha_{51}^{K^*}-1)\\ &+B_5(\alpha_{12}^{\omega}-1,\alpha_{25}^{K^*}-1,\alpha_{53}^{K^*}-1,\alpha_{34}^{\rho}-1,\alpha_{41}^{\phi}-1)\\ &-2B_5(\alpha_{21}^{\phi}-1,\alpha_{14}^{\rho}-1,\alpha_{45}^{K^*}-1,\alpha_{53}^{K^*}-1,\alpha_{32}^{\rho}-1)]\,.\end{aligned} \tag{10}$$

The first and second terms in Eq. (10) contain the process $K\bar{K}\to\omega_{i=9}\to K\bar{K}\pi$, corresponding to Figs. 3(a) and 3(b), respectively. On the other hand the process $K\bar{K}\to\phi_{i=9}\to K\bar{K}\pi$ corresponding to Fig. 3(c) is contained in the third term.

Hence the contribution of the daughters $\omega_{i=9,\beta}$ to the amplitude $A(\psi\to K\bar{K}\pi)$ may be obtained by evaluating the pole residues of the first and second terms in Eq. (10) at $\alpha_{12}^{\omega}=9$, projecting out the $J=1$ state, factorizing the $K\bar{K}\omega_{i=9,\beta}$ vertex and finally multiplying it by the coupling constant $g_{\psi\omega_{i=9,\beta}}$. For the details of the evaluation of pole residues and the spin projection, we refer the reader to paper I.

However, daughters may be degenerate and the factorization does not hold in general. Here we shall take either of the following two assumptions:

(i) The $g_{\psi\omega_{i=9,\beta}}$ is proportional to the coupling strength of the $K\bar{K}\omega_{i=9,\beta}$ vertex $f_{K\bar{K}\omega_{i=9,\beta}}$, with the same proportional constant for all β, i.e.,

$$g_{\psi\omega_{i=9,\beta}}=\lambda_\omega f_{K\bar{K}\omega_{i=9,\beta}}\quad\text{for all }\beta\,. \tag{11}$$

(ii) Only one state among the degenerate states $\omega_{i=9,\beta}$ dominantly couples to ψ and also to $K\bar{K}$ channel. In this case the same relation as Eq. (11) holds for the dominant state.[10]

Then in either case the factorization of the $K\bar{K}\omega_{i=9,\beta}$ vertex and the succeeding multiplication by $g_{\psi\omega_{i=9,\beta}}$ together amount to a single procedure of multiplication by a constant λ_ω.

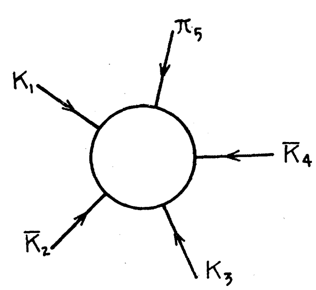

FIG. 2. Variables for the $K\bar{K}K\bar{K}\pi$ reaction.

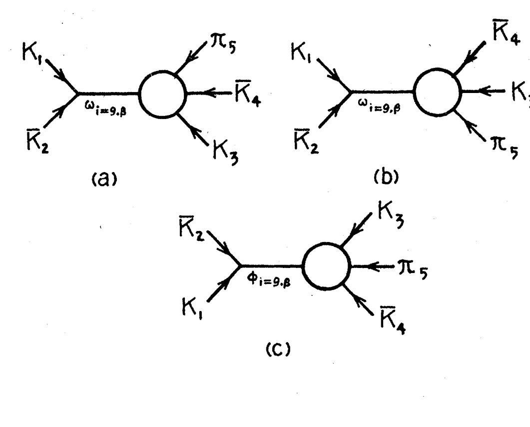

FIG. 3. (a) The process $K\bar{K}\to\omega_{i=9,\beta}\to K\bar{K}\pi$ (b) the process $K\bar{K}\to\omega_{i=9,\beta}\to K\bar{K}\pi$ (c) the process $K\bar{K}\to\phi_{i=9,\beta}\to K\bar{K}\pi$.

Just in the same way we can obtain the contribution of the daughters $\phi_{i=9,\beta}$ to the amplitude $A(\psi\to K\bar{K}\pi)$. We make either of the similar assumptions on the $g_{\psi\phi_{i=9,\beta}}$ and the coupling strength of the $K\bar{K}\phi_{i=9,\beta}$ vertices $f_{K\bar{K}\phi_{i=9,\beta}}$. In either case, we can write

$$g_{\psi\phi_{i=9,\beta}}=\lambda_{\phi}f_{K\bar{K}\phi_{i=9,\beta}}. \tag{12}$$

Then successive procedures of factorization of the $K\bar{K}\phi_{i=9,\beta}$ vertex and multiplication by $g_{\psi\phi_{i=9,\beta}}$ amount to multiplication by a constant λ_{ϕ}.

After tedious but straightforward calculations, we finally obtain the amplitude for $\psi\to K\bar{K}\pi$

$$\begin{aligned}A(\psi\to K\bar{K}\pi)=\epsilon_{\mu\nu\sigma\lambda}P_3^{\mu}P_4^{\nu}P_5^{\sigma}e^{\lambda}\\ \times\{\gamma_1[D^1(t,u,s)+D^1(s,u,t)]\\ +\gamma_2D^2(s,t,u)\},\end{aligned} \tag{13}$$

where e is the polarization vector of ψ, and

$$\begin{aligned}s&=(P_3+P_5)^2=(P_K+P_{\pi})^2,\\ t&=(P_4+P_5)^2=(P_{\bar{K}}+P_{\pi})^2,\\ u&=(P_3+P_4)^2=(P_K+P_{\bar{K}})^2.\end{aligned} \tag{14}$$

The parameters γ_1 and γ_2 express the weight of contributions of the daughters $\omega_{i=9,\beta}$ and $\phi_{i=9,\beta}$, respectively, and their explicit forms are

$$\begin{aligned}\gamma_1&=\lambda_{\omega}\times(\text{propagator of }\omega_{i=9,\beta}),\\ \gamma_2&=-2\lambda_{\phi}\times(\text{propagator of }\phi_{i=9,\beta}).\end{aligned} \tag{15}$$

The scalar amplitudes $D^1(t,u,s)$ and $D^2(s,t,u)$ have, respectively, the following forms:

$$D^1(t,u,s)=\sum_{n=1}^{9}C_n^1(t,u,s)B(n-\alpha_{K^*}(t),1-\alpha_{\rho}(u)), \tag{16}$$

$$D^2(s,t,u)=\sum_{n=1}^{9}C_n^2(s,t,u)B(n-\alpha_{K^*}(s),1-\alpha_{K^*}(t)). \tag{17}$$

The remaining amplitude $D^1(s,u,t)$ is obtained from Eq. (16) by interchanging s and t. The coefficients C_n^1 and C_n^2 are polynomials in s, t, and u. Their expressions are essentially the same as C_n in paper II, except for some modifications which are due to the appearance of different trajectories. We show in the Appendix the necessary modifications. The pole structures arise from the Euler beta functions B.

Using the symmetry properties of the B_5 function in Eqs. (7) and (8), more specifically, the fact that

$$\begin{aligned}B_5(\alpha_{21}^{\phi}-1,\alpha_{14}^{\rho}-1,\alpha_{45}^{K^*}-1,\alpha_{53}^{K^*}-1,\alpha_{32}^{\rho}-1)\\ =B_5(\alpha_{12}^{\phi}-1,\alpha_{23}^{\rho}-1,\alpha_{35}^{K^*}-1,\alpha_{54}^{K^*}-1,\alpha_{41}^{\rho}-1),\end{aligned}$$

we can show

$$D^2(s,t,u)=D^2(t,s,u).$$

Therefore, the amplitude $A(\psi\to K\bar{K}\pi)$ in Eq. (13) is indeed symmetric in the variables s and t.

III. DALITZ-PLOT DENSITY FOR $K\bar{K}\pi$

Now we calculate the Dalitz-plot density for $\psi\to K\bar{K}\pi$. First we must introduce the imaginary parts into the ρ and K^* trajectories to smear out the infinities arising from the beta functions. We have already determined the imaginary part of the ρ trajectory in paper II,

$$\alpha_{\rho}(s)=0.48+0.89s+i0.14(s-4m_{\pi}^2)^{1/2}. \tag{18}$$

Also we choose the imaginary part of the K^* trajectory as follows,[11]

$$\alpha_{K^*}(s)=0.28+0.89s+i0.064[s-(m_K+m_{\pi})^2]^{1/2}. \tag{19}$$

[The real part of $\alpha_{K^*}(s)$ was already determined in paper I by requiring the universal slope and the Adler partially conserved axial-vector current (PCAC) condition.]

Now the trajectories have imaginary parts, hence $D^2(s,t,u)$ in Eqs. (13) and (17), which corresponds to the contribution of the daughters $\phi_{i=9,\beta}$, becomes asymmetric in the variables s and t. So we rewrite Eq. (13) such that the amplitude is manifestly symmetric in s and t,

$$\begin{aligned}\tilde{A}(\psi\to K\bar{K}\pi)=\epsilon_{\mu\nu\sigma\lambda}P_3^{\mu}P_4^{\nu}P_5^{\sigma}e^{\lambda}\\ \times\{\gamma_1[D^1(t,u,s)+D^1(s,u,t)]\\ +\tfrac{1}{2}\gamma_2[D^2(s,t,u)+D^2(t,s,u)]\}.\end{aligned} \tag{20}$$

It is easy to see that the Dalitz-plot density for the final states $K_S^0K^{\pm}\pi^{\mp}$ is proportional to

$$|\tilde{A}(\psi\to K\bar{K}\pi)|^2. \tag{21}$$

If we want to obtain numerical results, we must fix the parameters γ_1 and γ_2 in Eq. (20). For our present purpose we require only the ratio $\kappa\equiv\gamma_2/\gamma_1$ as the overall normalization is not necessary for the calculation of the Dalitz-plot density. From Eq. (15) we have

$$\kappa=\frac{-2\lambda_{\phi}}{\lambda_{\omega}}\frac{(\text{propagator of }\phi_{i=9,\beta})}{(\text{propagator of }\omega_{i=9,\beta})}. \tag{22}$$

We can obtain the ratio of λ_{ω} to λ_{ϕ}, if we assume the SU(3) symmetry both for the $K\bar{K}\omega_{i=9,\beta}$ and $K\bar{K}\phi_{i=9,\beta}$ vertices and for the $\psi-\omega_{i=9,\beta}$ and $\psi-\phi_{i=9,\beta}$ mixing. With the assumption of ψ being an SU(3) singlet, we find

$$f_{K\bar{K}\omega_{i=9,\beta}}/f_{K\bar{K}\phi_{i=9,\beta}}=-1/\sqrt{2},$$

$$g_{\psi\omega_{i=9,\beta}}/g_{\psi\phi_{i=9,\beta}}=\sqrt{2}\,,$$

and (23)

$$\lambda_\omega = -2\lambda_\phi\,.$$

Furthermore, if the SU(3) symmetry is exact also in mass, the result is that $\alpha_\omega(s)=\alpha_\phi(s)$. Then in this case we have $\kappa=1$.

However, as stated in Sec. II, in order to estimate κ in the real world, we have to know also the decay widths of $\omega_{i=9,\beta}$ and $\phi_{i=9,\beta}$. Now the $\omega_{i=9,\beta}$ ($\phi_{i=9,\beta}$) are daughters of the much higher $\omega(\phi)$ recurrences. Then it may be plausible that they have very large widths. In such a case, Eq. (22) will be approximated to be

$$\kappa=\frac{-2\lambda_\phi}{\lambda_\omega}\frac{m_\psi{}^2-m_{\omega_{i=9,\beta}}{}^2+im_{\omega_{i=9,\beta}}\Gamma_{\omega_{i=9,\beta}}}{m_\psi{}^2-m_{\phi_{i=9,\beta}}{}^2+im_{\phi_{i=9,\beta}}\Gamma_{\phi_{i=9,\beta}}}$$

$$\sim\frac{-2\lambda_\phi}{\lambda_\omega}\frac{m_{\omega_{i=9,\beta}}\Gamma_{\omega_{i=9,\beta}}}{m_{\phi_{i=9,\beta}}\Gamma_{\phi_{i=9,\beta}}}\,.$$

Note that the differences between the mass of ψ and $\omega_{i=9,\beta}$ ($\phi_{i=9,\beta}$) are very small. Therefore with a good approximation that the SU(3) symmetry for the vertices is nearly exact, κ is a real positive value. However, the precise estimation of κ is beyond the scope of the present paper. Here we shall adopt, as a rough approximation, the value $\kappa=1$ under a supposition that the widths $\Gamma_{\omega_{i=9,\beta}}$ and $\Gamma_{\phi_{i=9,\beta}}$ may be comparable in magnitude.

With this choice of κ, we calculate the Dalitz-plot density for $\psi\to K_S^0K^\pm\pi^\mp$. Our result is shown in Fig. 4. Comparing with the experimental plot in Fig. 1, we find a very good agreement. Especially it can be seen that the quasi-two-body final state KK^* is dominant.

The kinematical factor in Eq. (20) has the same effect as that in the case of $\psi\to3\pi$, i.e., enhancing the central region of the Dalitz plot, because

$$\sum_{\text{spin}}|\epsilon_{\mu\nu\sigma\lambda}P_3^\mu P_4^\nu P_5^\sigma e^\lambda|^2$$
$$=\tfrac{1}{4}[stu+(m_\psi{}^2-m_K{}^2)(m_K{}^2-m_\pi{}^2)u-m_K{}^2(m_\psi{}^2-m_\pi{}^2)^2]\,. \quad (24)$$

But our result shows that the effect of this kinematical factor is surpassed by the sum of the scalar amplitudes D^1 and D^2, and the suppression in the central region of the Dalitz plot is brought about.

In order to see how much our result depends on the choice of the parameter κ, we have calculated the Dalitz-plot density in the following four cases: (i) $\kappa=\frac{1}{2}$, (ii) $\kappa=2$, (iii) $\kappa=0$ (i.e., $\gamma_2=0$), and (iv) $\kappa=\infty$ (i.e., $\gamma_1=0$). Then we find that in the first two cases, the results are not so different from

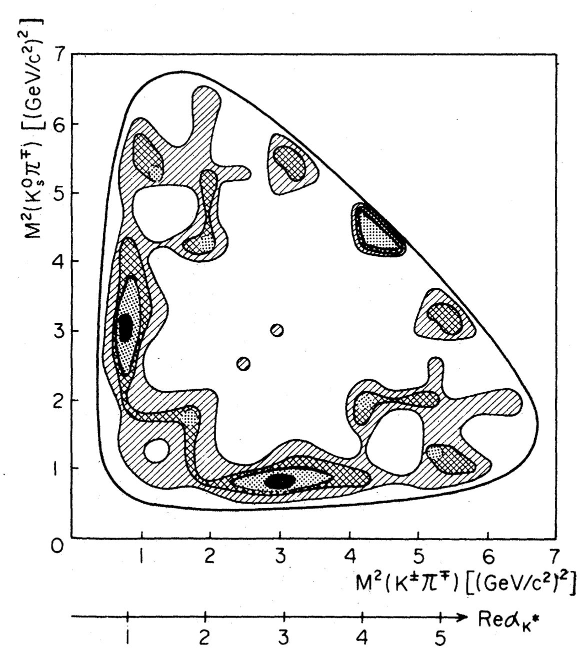

FIG. 4. Predictions of our Veneziano model for $K_S^0K^\pm\pi^\mp$ Dalitz plot in the case of $\kappa=1$. The diagram is divided into five parts according to the density of events (maximum = 10) as follows: ☐, 0–1; ▨, 1–2; ▩, 2–3; ▦, 3–6; ■, 6–10.

the case of $\kappa=1$. However, in the last two extreme cases, agreement with the experimental plot is lost. In particular, the K^{**} signals appear much more strongly than those of the K^* mesons. Furthermore we find that in the case of κ = negative value, the central region in the Dalitz plot is not suppressed. These arguments mean that contributions of both the daughters $\omega_{i=9,\beta}$ and $\phi_{i=9,\beta}$ are important at about equal strength.

Finally, it may be interesting to consider the exact-SU(3)-symmetry limit. In this limit we have

$$m_K{}^2=m_\pi{}^2\,,$$
$$\alpha_\rho(s)=\alpha_{K^*}(s)=\alpha_\omega(s)=\alpha_\phi(s)\,, \quad (25)$$
$$\gamma_1=\gamma_2\,,$$

and the coefficients C_n^1 and C_n^2 in Eqs. (16) and (17) are reduced to C_n's in papers I and II. Thus the amplitude for $\psi\to K\bar{K}\pi$ turns out to be identical with that for $\psi\to3\pi$ and also symmetric in all variables s, t, and u. Consequently the signals of K^{**} disappear from the final $K\bar{K}\pi$ states. This is our version of the statement that the ψ cannot decay into KK^{**} meson pairs in the SU(3) limit.[12] In the real world, the SU(3) symmetry is not exactly realized, then we expect the appearance of K^{**} signals in the $K\bar{K}\pi$ Dalitz plot. But from Fig. 4 we find that K^{**} signals are much suppressed as compared with those of the K^* mesons.

IV. CONCLUDING REMARKS

In the present series of papers, we have shown that the *aged* Veneziano model is still applicable to the hadronic decay phenomena of the *new* particle ψ. In fact, the characteristic features of the final-state distributions for both the $\psi \to 3\pi$ and $\psi \to K\bar{K}\pi$ decays can be well described by the amplitudes which are constructed from five-point Veneziano functions.

The essential point is our assumption that ψ decays into 3π and $K\bar{K}\pi$ through mixing with *heavy* daughters of the ω and/or ϕ recurrences and that the final-state distributions are determined only by the decay amplitudes of those daughters into 3π and $K\bar{K}\pi$. In consequence, the $\rho\pi$ channel is dominant in the $\psi \to 3\pi$ decays, and so is the KK^* channel in the $\psi \to K\bar{K}\pi$ decays. Also deep suppression is brought about in the central region of the Dalitz plots for both decays.

Finally it may be possible to apply our idea to the study of other hadronic decays of ψ into more than three final particles. Application might be also possible to the future study of hadronic decays of heavier quarkonium states such as $\Upsilon(9.5)$. From the results of our analysis of the ψ decays into 3π and $K\bar{K}\pi$, we do expect that there also appears deep suppression in the central region of the Dalitz plots for the final three-body hadronic decays of heavier quarkonium states.

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APPENDIX

In this appendix we refer to the expressions of the coefficients C_n^1 and C_n^2 in Eqs. (16) and (17). Because the structures of C_n^1 and C_n^2 are essentially the same as C_n in the Appendix of paper II, we present here the necessary modifications only. We can obtain the $C_n^1(s,t,u)$ and $C_n^2(s,t,u)$ by modifying the expressions of a_s, b_u, g_s, g_u, and $h_{s,u}$ appearing in the $C_n(s,t,u)$ in paper II as follows:

(i) For $C_n^1(s,t,u)$,

$$a_s = \tfrac{1}{2}[\alpha' s + \alpha_\rho(0) - 3\alpha_{K^*}(0) + 2\alpha_\phi(0) - \tfrac{15}{2}],$$

$$b_u = \tfrac{1}{2}[\alpha' u - \alpha_\rho(0) + 5\alpha_{K^*}(0) - 2\alpha_\phi(0) - \tfrac{17}{2}],$$

$$g_s = \alpha'^2 K[m_\psi{}^4 - 2(s + m_K{}^2)m_\psi{}^2 + (s - m_K{}^2)^2],$$

$$g_u = \alpha'^2 K[m_\psi{}^4 - 2(u + m_K{}^2)m_\psi{}^2 + (u - m_K{}^2)^2],$$

$$h_{s,u} = \alpha'^2 K[su + (m_\psi{}^2 - m_K{}^2)(s+u) + m_K{}^4 - m_\psi{}^4 + 2m_\psi{}^2(m_K{}^2 - m_\pi{}^2)].$$

(ii) For $C_n^2(s,t,u)$,

$$a_s = \tfrac{1}{2}[\alpha' s + 3\alpha_\rho(0) - 3\alpha_{K^*}(0) - \tfrac{15}{2}],$$

$$b_u = \tfrac{1}{2}[\alpha' u + 2\alpha_\rho(0) - \tfrac{17}{2}],$$

$$g_s = \alpha'^2 K[m_\psi{}^4 - 2(s + m_K{}^2)m_\psi{}^2 + (s - m_K{}^2)^2],$$

$$g_u = \alpha'^2 K[m_\psi{}^4 - 2(u + m_\pi{}^2)m_\psi{}^2 + (u - m_\pi{}^2)^2],$$

$$h_{s,u} = \alpha'^2 K[(s - m_\psi{}^2 - m_K{}^2)(u - m_\psi{}^2 - m_\pi{}^2) + 2m_\psi{}^2(s + u - m_\psi{}^2 - m_K{}^2)].$$

In the above expressions,

$$K \equiv \frac{m_\psi{}^2 - 4m_K{}^2}{4m_\psi{}^2} = \frac{9 - \alpha_\rho(0) - 4[\frac{1}{2} - \alpha_{K^*}(0)]}{4[9 - \alpha_\rho(0)]}$$

and α' is the universal trajectory slope ($=0.89$ GeV^{-2}).

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[4]According to Ref. 3, we use the following notation conventions: $K^* \equiv K^*(892)$, $K^{**} \equiv K^*(1420)$, $KK^* \equiv K\bar{K}^* + \bar{K}K^*$, $KK^{**} \equiv K\bar{K}^{**} + \bar{K}K^{**}$.

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[7]See also the arguments in paper I.

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[10]In papers I and II, we made the second assumption (ii). Even if we take the first one (i), all the results of I and II are not altered.

[11]We have determined the imaginary part of the K^* trajectory so as to reproduce the K^* width which is 0.049 GeV, since its signals are clearly seen in the experimental data in Fig. 1.

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