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Unitarity corrections to short-range order: Long-range rapidity correlations

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Although the effective hadronic forces have short range in rapidity space, one nevertheless expects long-range dynamical correlations induced by unitarity constraints. This paper contains a thorough discussion of long-range rapidity correlations in high-multiplicity events. In particular, we analyze in detail the forward-backward multiplicity correlations, measured recently in the whole CERN ISR energy range. We find from these data that the normalized variance of the number n of exchanged cut Pomerons, $\langle (n/\langle n \rangle - 1)^2 \rangle$, is most probably in the range 0.32 to 0.36. We show that such a number is obtained from Reggeon theory in the eikonal approximation. We also predict a very specific violation of local compensation of charge in multiparticle events: The violation should appear in the fourth-order zone correlation function and is absent in the second-order correlation function, the only one measured until now.

I. INTRODUCTION

The effective hadronic forces have a short range in rapidity space and are soft (in the sense of suppression of large invariant momentum transfers—in particular, the transverse momenta of secondaries are, on the average, small). Thus, the ensemble of particles produced in a high-energy collision has some resemblance to a noncritial liquid enclosed in a long (but thin) container. Consequently, it has been suggested that multiparticle states exhibit a short-range order (SRO), in the sense that there are only short-range dynamical rapidity correlations (cf. Ref. 1 and references therein). The introduction into the context of multiparticle physics of the model-independent concept of SRO turned out to be very fruitful. It has been rapidly realized, however, that SRO is expected to hold only in the approximation where effects induced by unitarity constraints are neglected.[2,3] The purpose of this paper is to discuss long-range rapidity correlations in multiparticle production.[4] In order to put the content of this work in the proper perspective, let us briefly recall the chain of arguments which lead one to expect that SRO is broken.

In the context of high-energy physics the concept of SRO is intimately related to the idea of representing diffractive scattering, in first approximation, as the exchange of a specific Regge pole, the Pomeron.[5] Now, the Pomeron is a quasiparticle strongly coupled to hadron sources. Therefore, in the complete theory, the forward elastic-scattering amplitude is expected to be a sum of single- and multi-Pomeron-exchange amplitudes.[9] However, the existence of the multi-Pomeron-exchange contributions to the elastic amplitude has far reaching consequences for multiparticle production. Cross sections for physical processes can be obtained by isolating the appropriate discontinuities of the forward elastic-scattering amplitude or, using a more pictorial language, by cutting in a suitable manner the diagrams representing this amplitude.[10] The operation of cutting through a single Pomeron defines an "elementary" rapidity density of secondaries (a random function of the rapidity variable with the property of SRO). Cutting through n Pomerons defines a rapidity density which is a sum of n elementary ones. Hence, the rapidity density of secondaries produced in a highly inelastic hadron-hadron collision is a superposition of a fluctuating number of elementary densities. It is easy to see that this picture implies long-range rapidity correlations.

Let δN denote the number of particles found in a single event within a given rapidity subinterval. Multiplicity fluctuations within elementary densities are of the order $O(\langle \delta N \rangle^{1/2})$, while the multiplicity fluctuations due to the variation of the number of superposed elementary densities are of the order $O(\langle \delta N \rangle)$. Therefore, when one observes $\delta N \gg \langle \delta N \rangle$ and provided $\langle \delta N \rangle$ is large enough, one can be almost sure that one has picked up an event where several elementary densities are superposed. Consequently, one can bet, with a considerable chance of success, that, in the event in question, the multiplicity within another, distant rapidity subinterval is also well above the average. A more rigorous version of this argument will be given in the next section.

Summarizing: although the effective hadronic forces have short range in rapidity space, one expects SRO to be broken, even if one considers exclusively those events which are, so to say, "dense" in rapidity (no large rapidity gaps). Nevertheless, as we shall see, the pattern of SRO breaking is predictable and very specific.

As already mentioned, the above chain or arguments is not new. There are new data, however, providing a new challenge to theorists: CERN ISR collaboration has measured the correlation between multiplicities in the two center-of-mass hemispheres (the forward and the backward one), in the whole ISR energy range.[11] Such a correlation has already been observed at Fermilab,[12] but its persistence at the highest ISR energies (where it is the strongest) is a rather clear evidence for a nontrivial long-range rapidity correlation.

A natural tool for studying long-range rapidity correlations is the Reggeon theory. Unfortunately, this is an asymptotic theory where finite-energy effects, and in particular the energy-momentum-conservation constraints, are practically beyond control. We do not mean that Reggeon theory is useless for phenomenology. Simply, at present accelerator energies, the applicability of this theory to different sectors of phenomenology is not equally legitimate. The theory seems sound when applied to elastic scattering, but it has to be handled with care in the context of multiparticle production. This is the reason why we formulate, in Sec. II, a probabilistic analog model, which is asymptotically isomorphic to Reggeon theory (without Reggeon-Reggeon interaction), but which has a natural extension towards lower energies. This latter extension and a technique of estimating finite energy corrections is presented in Sec. III. In Sec. IV we set forth the formalism relevant for the analysis of the forward-backward multiplicity correlation. In Sec. V we come back to the Reggeon theory *sensu stricto*. We conjecture that the probability distribution of n, the number of elementary densities, calculated from Reggeon theory can be trusted at present energies. This conjecture, together with the probabilistic analog model, provides an interesting connection between the phenomenology of multiparticle production and the phenomenology of two-body scattering. The most significant part of our numerical results and a discussion of the data are given in Sec. V B. In Sec. VI we show that local compensation of charge is broken in a very subtle manner, when unitary corrections to SRO are taken into account. We suggest measuring the fourth-order zone correlation function. Section VII contains the summary and the conclusions of the paper. Technical details concerning inelastic diffraction are put in the Appendix, to make the paper more readable.

We should mention that the phenomenological implications of Reggeon theory for multiparticle production have already been studied by other authors.[13] In the absence of better data, these studies were usually limited to the discussion of multiplicity distributions. For this reason, these earlier studies seem to us less conclusive than it might appear. In principle, the energy variation of multiplicity distributions reveals the pattern of correlations between secondary particles. In practice, this variation is very slow and one can hardly distinguish a transitory phenomenon from a truly asymptotic trend.

II. A PROBABILISTIC ANALOG MODEL

The model formulated in this section gives identical results to the Reggeon theory (with Reggeon-Reggeon interactions neglected), provided the collision energy is high enough and as long as one is in the so-called "central region" (see Sec. V A and note added in proof). However, the probabilistic analog model has a natural extension to lower energies, where Reggeon theory has no predictive power. We employed an almost identical model in the context of hadron-nucleus collisions.[14] The remarkable success of this earlier study encourages us to use similar arguments again. Furthermore, the probabilistic analog model offers an attractive intuitive picture of the mechanism of long-range rapidity correlations.

Let $N(y)$ denote the random function representing the rapidity density of secondary particles (or, just of a given species of secondaries). For definiteness, we work in the center-of-mass reference frame. The total energy in this frame is denoted by W.

We claim that $N(y)$ is a sum of a fluctuating number of elementary densities:

$$N(y)=\sum_{j=1}^{n} N_0^{(j)}(y)\,, \tag{1}$$

where n is itself a random variable. Furthermore:

(a) Distinct elementary densities are statistically independent. For example,

$$\langle N_0^{(j)}(y)N_0^{(k)}(y')\rangle_n=\langle N_0^{(j)}(y)\rangle_n\langle N_0^{(k)}(y')\rangle_n \tag{2}$$

for $j\neq k$. The subscript n indicates that the average has been taken keeping n fixed.

(b) All elementary densities have identical average properties. Thus, the subscript (j) is usually superfluous,

$$\langle N_0^{(j)}(y)\rangle_n\equiv\langle N_0(y)\rangle_n\,, \tag{3a}$$

$$\langle N_0^{(j)}(y)N_0^{(j)}(y')\rangle_n\equiv\langle N_0(y)N_0(y')\rangle_n\,,\quad \text{etc}. \tag{3b}$$

(c) The moment functions $\langle N_0(y_1)\cdots N_0(y_k)\rangle_n$ exhibit short-range rapidity correlations only.

At asymptotic energies the elementary densities should have all the conventional SRO properties. However, this asymptotic regime is not reached yet. The n dependence of the moment functions $\langle N_0(y_1)\cdots N_0(y_k)\rangle_n$ comes through their energy dependence. Indeed, W is partitioned among n elementary densities. Thus, the energy relevant for an elementary density is W/n rather than W (see

Sec. III).

From Eq. (1) one finds

$$\langle N(y)\rangle = \langle n\langle N_0(y)\rangle_n\rangle . \tag{4}$$

Likewise,

$$\langle N(y)N(y')\rangle = \langle n\langle N_0(y)N_0(y')\rangle_n\rangle + \langle n(n-1)\langle N_0(y)\rangle_n\langle N_0(y')\rangle_n\rangle . \tag{5}$$

Let us introduce the (second-order) *density* correlation function (cf. Ref. 15)

$$B(y,y') = \langle N(y)N(y')\rangle - \langle N(y)\rangle\langle N(y')\rangle . \tag{6}$$

This function is simply related to the standard *inclusive* correlation function $C(y,y')$

$$B(y,y') = C(y,y') + \langle N(y)\rangle\delta(y-y') . \tag{7}$$

From Eqs. (4)–(6) one further obtains

$$B(y,y') = B_{SR}(y,y') + \langle n^2\langle N_0(y)\rangle_n\langle N_0(y')\rangle_n\rangle - \langle n\langle N_0(y)\rangle_n\rangle\langle n\langle N_0(y')\rangle_n\rangle , \tag{8}$$

where

$$B_{SR}(y,y') = \langle n[\langle N_0(y)N_0(y')\rangle_n - \langle N_0(y)\rangle_n\langle N_0(y')\rangle_n]\rangle \tag{9}$$

tends rapidly to zero when $|y-y'|\to\infty$ (the subscript SR stands for short range). Equation (8) can also be rewritten in the form

$$C(y,y') = C_{SR}(y,y') + \langle n^2\langle N_0(y)\rangle_n\langle N_0(y')\rangle_n\rangle - \langle n\langle N_0(y)\rangle_n\rangle\langle n\langle N_0(y')\rangle_n\rangle . \tag{10}$$

In the *asymptotic regime*, where $\langle N_0(y)\rangle_n$ becomes independent of n, one finds

$$\langle N(y)\rangle = \langle n\rangle\langle N_0(y)\rangle \tag{11}$$

and

$$C(y,y') = C_{SR}(y,y') + x(0)\langle N(y)\rangle\langle N(y')\rangle . \tag{12}$$

We introduce here the specific symbol

$$x(\alpha) = \langle n^{2(1-\alpha)}\rangle/\langle n^{1-\alpha}\rangle^2 - 1 , \tag{13}$$

which will be often used in the rest of this paper. Obviously, $x(0)$ is the normalized variance of the number of elementary densities. Furthermore, in the asymptotic regime, $\langle N_0(y)\rangle$ becomes independent of y, provided y is far enough from kinematic boundaries and, according to Eq. (11), so does $\langle N(y)\rangle$. Hence there is a constant and positive contribution to $C(y,y')$, equal to $(\langle n^2\rangle/\langle n\rangle^2-1)\langle N(0)\rangle^2$. Notice that this long-range correlation is present if and only if $\langle n^2\rangle\neq\langle n\rangle^2$: *SRO is broken because n fluctuates*. Superposing a fixed number of elementary densities does not produce any long-range effect. A corollary: the long-range rapidity correlations should be significantly reduced in a sample of events where the multiplicity within some rapidity subinterval is kept at a (nearly) fixed value.[16] The last few remarks should also be obvious from the intuitive argument presented in the Introduction.

III. FINITE-ENERGY EFFECTS

We have argued in the Introduction that the observation of a large multiplicity in a given rapidity subinterval is, most likely, an indication that several elementary densities are superposed in the event under consideration. This, in turn, means that multiplicity is also enhanced beyond the originally observed subinterval. Question: how far in rapidity is a local density fluctuation felt, at a finite collision energy? The answer to this question necessarily involves a discussion of the energy-momentum-conservation constraints.

The total available energy is partitioned among n ($n=1,2,\ldots,$) elementary densities. Furthermore, the elementary densities are not necessarily at rest with respect to each other. Consequently, the elementary densities do not fully overlap and, in any case, the rapidity extension of a single elementary density is roughly $2\ln(W/nW_0)$, with $W_0\simeq 1$ GeV or so. Thus, the range of the long-range correlations is reduced due to kinematic constraints. In particular, there should be no correlation, or very little, between the two fragmentation regions (neglecting diffraction dissociation). Such a correlation is indeed not observed.[11,17]

Finite-energy effects relevant for the following discussion can be estimated using a rough, but very simple technique which we are going to explain now. We illustrate this technique with a few examples, which are sufficient to convey the idea.

Integrating Eq. (4) with respect to y, we find the average multiplicity

$$\langle N\rangle = \langle n\langle N_0\rangle_n\rangle . \tag{14}$$

Let us also integrate Eq. (8), with respect to y and y', to obtain

$$D^2 = \int\int dy\,dy' B_{SR}(y,y') + \langle n^2\langle N_0\rangle_n{}^2\rangle - \langle n\langle N_0\rangle_n\rangle^2 . \tag{15}$$

Here, D is the dispersion of the multiplicity distribution: $D^2 = \langle N^2\rangle - \langle N\rangle^2$.

Consider now the elementary multiplicity $\langle N_0\rangle_{n=1}$. It is a function of W: $\langle N_0\rangle_{n=1} = F(W)$. We assume that, in first approximation, one can neglect the collective motion of elementary densities with respect to the center-of-mass frame. We also assume that $F(W)$ is sufficiently smooth to write

$$\langle N_0\rangle_n \simeq F(W/n) . \tag{16}$$

In hadron-hadron collisions, the probability that

n takes a large value is small: typically, the values of n which really matter are $n=1$ to 4 or so (this becomes evident when one works with specific models). Within a finite energy interval, say for $W/4 < W' < W$, the functional dependence of $F(W')$ on W' can be fairly well approximated by a power law:

$$F(W') = \text{const} \times (W')^{\gamma}. \qquad (17)$$

Of course, the exponent γ depends on the energy interval under consideration. Now, Eqs. (16) and (17) imply that

$$\langle N_0 \rangle_n = n^{-\gamma} \langle N_0 \rangle_{n=1}. \qquad (18)$$

Inserting (18) into (14) we find

$$\langle N \rangle = \langle n^{1-\gamma} \rangle \langle N_0 \rangle_{n=1}. \qquad (19)$$

From (15), (18), and (19) we get after straightforward algebra

$$D^2 = \int \int dy\, dy' B_{\text{SR}}(y, y') + x(\gamma) \langle N \rangle^2. \qquad (20)$$

This equation is formally identical to the one which holds in the asymptotic regime, except that $x(0)$ has been replaced by $x(\gamma)$. The long-range correlation is now controlled by the parameter $x(\gamma)$.

In first approximation,

$$\langle n^{1-\alpha} \rangle \simeq \langle n \rangle^{1-\alpha} [1 - \alpha(1-\alpha) x(0)/2 + \cdots]. \qquad (21)$$

Thus, roughly

$$x(\alpha) \simeq (1-\alpha)^2 x(0). \qquad (22)$$

We shall see in Sec. V that both $\langle n \rangle$ and $x(0)$ are expected to be very weakly dependent on W. Furthermore, for charged secondaries and in the energy interval $19 < W' < 63$ GeV, the data on the average multiplicity[18] are well described by the formula

$$\langle N \rangle = 2.10 (W')^{0.435}, \qquad (23a)$$

where W' is in GeV. Hence, for $W = 63$ GeV, $\gamma = 0.435$, and Eq. (22) indicates that the long-range contribution to D^2 is reduced (roughly) by a factor of 4. Qualitatively, this reduction is easy to understand: the long-range correlation is the stronger the larger is the multiplicity. However, the energy conservation reduces the probability of producing large multiplicity events.

For $5 < W' < 30$ GeV the data are well described by the equation

$$\langle N \rangle = 1.67 (W')^{0.506}. \qquad (23b)$$

Hence, at the lowest ISR energies it is more appropriate to use $\gamma = 0.506$. The data on multiplicities are quite precise and one can see the decrease of the effective exponent γ.

Consider now the central region, near $y = 0$. The data on the average rapidity density of charged secondaries[19] give

$$\langle N(0) \rangle \simeq 0.777 (W')^{0.256} \qquad (24)$$

for $W' < 63$ GeV. Repeating the arguments developed above,[21] we obtain

$$\langle N_0(0) \rangle_n = n^{-\beta} \langle N_0(0) \rangle_{n=1} \qquad (25)$$

with

$$\beta = 0.256. \qquad (26)$$

Equation (25), together with Eqs. (4) and (10), yields

$$\langle N(0) \rangle = \langle n^{1-\beta} \rangle \langle N_0(0) \rangle_{n=1} \qquad (27)$$

and

$$C(0,0) = C_{\text{SR}}(0,0) + x(\beta) \langle N(0) \rangle^2. \qquad (28)$$

IV. FORWARD-BACKWARD MULTIPLICITY CORRELATIONS

In the following, N_F denotes the number of secondaries with positive center-of-mass rapidities

$$N_F = \int_{y>0} dy\, N(y). \qquad (29a)$$

Similarly, N_B is defined as the number of secondaries with negative rapidities

$$N_B = \int_{y<0} dy\, N(y). \qquad (29b)$$

We are interested in the correlation between N_B and N_F. More precisely, we consider the regression of N_B versus N_F, i.e., the dependence of the average backward multiplicity $\langle N_B(N_F) \rangle$ on the forward multiplicity N_F.[24] We first derive a few equations which hold quite generally, i.e., independently of the range of correlations.

As is well known, $\langle N_B(N_F) \rangle$ would be a linear function of N_F if $N_{B,F}$ were normally distributed.[28] This, in turn, would be (approximately) true if all density correlations of order >2 were negligible. Although this is not really the case, the Gaussian approximation is presumably not a bad one, as evidenced by the behavior of the cumulants of multiplicity distributions (the normalized cumulants decrease rapidly with increasing order[29]). Hence, a rough linearity of $\langle N_B(N_F) \rangle$ with respect to N_F is actually expected. This expectation is borne out by the data. Let us find now the slope of $\langle N_B(N_F) \rangle$ versus N_F.

The linear regression is a standard problem in probability theory. We set $\langle N_B(N_F) \rangle = a + bN_F$, and we determine the coefficients a and b by requiring that $\langle [N_B - (a + bN_F)]^2 \rangle$ is as small as possible. The result is

$$\langle N_B(N_F) \rangle = \langle N_B \rangle + b(N_F - \langle N_F \rangle), \qquad (30)$$

where

$$b=(\langle N_BN_F\rangle-\langle N_B\rangle\langle N_F\rangle)/(\langle N_F{}^2\rangle-\langle N_F\rangle^2)\,. \qquad (31)$$

It is obvious that

$$\langle N_BN_F\rangle-\langle N_B\rangle\langle N_F\rangle=\int_{y<0}dy\int_{y'>0}dy'[\langle N(y)N(y')\rangle-\langle N(y)\rangle\langle N(y')\rangle]\,. \qquad (32)$$

Since $y\neq y'$, the integrand in (32) can be identified with the *inclusive* correlation function $C(y,y')$. On the other hand,

$$\langle N_F{}^2\rangle-\langle N_F\rangle^2=\int_{y>0}dy\int_{y'>0}dy'[\langle N(y)N(y')\rangle-\langle N(y)\rangle\langle N(y')\rangle]\,. \qquad (33)$$

Here, the integrand must be identified with the *density* correlation function $B(y,y')$.
Finally,

$$b=\int_{y>0}dy\int_{y'<0}dy'\,C(y,y')\Big/\left[\int_{y>0}dy\int_{y'>0}dy'\,C(y,y')+\langle N_F\rangle\right]. \qquad (34)$$

We stress that this result is exact and independent of any model.

From now on we shall assume forward-backward symmetry. With this symmetry one further has

$$\int_{y>0}dy\int_{y'>0}dy'\,C(y,y')=f_2/2-\int_{y>0}dy\int_{y'<0}dy'\,C(y,y')\,, \qquad (35)$$

where f_2 is Mueller's correlation parameter[30] (the integral of the inclusive correlation function)

$$f_2=D^2-\langle N\rangle\,. \qquad (36)$$

With exact SRO, b tends to zero when $\langle N\rangle\to\infty$. This is obvious from Eq. (34); the numerator is of order 0(1), while the denominator increases indefinitely. The situation changes radically in the presence of the long-range correlation. Using Eqs. (12) and (34) one easily convinces oneself that asymptotically, when $\langle N\rangle\to\infty$, one expects $b\to1$, provided $x(0)\neq0$. This limit should be approached from below if the short-range correlation is positive (this is the case experimentally). In the ISR experiment, the slope b is about 0.3 at the highest ISR energy, $W=62.8$ GeV. This value is very far from the asymptotic one and indicates that the asymptotic regime is far from being reached.

The short-range rapidity correlations have been extensively studied. The shape of $C_{SR}(y,y')$ is often fitted with a Gaussian

$$C_{SR}(y,y')\propto\exp[-(y-y')^2/4\delta^2]\,, \qquad (37)$$

and $\delta\simeq0.6$.[31] The normalization for $y\simeq y'\simeq0$ can be obtained from Eq. (28)

$$C_{SR}(0,0)=\langle N(0)\rangle^2[R(0)-x(\beta)]\,, \qquad (38)$$

where R is the normalized inclusive correlation function

$$R(0)=C(0,0)/\langle N(0)\rangle^2\,, \qquad (39)$$

which is almost constant through the Fermilab and ISR energy ranges, $R(0)\simeq0.6$.[32]

Let us evaluate the double integral which appears in the numerator of Eq. (34) [the denominator is easily found using (35)]. The long-range contribution to $C(y,y')$ is easily integrated and yields $x(\gamma)\langle N\rangle^2/4$. Only the neighborhood of $y=y'=0$ is relevant for the integration of the short-range contribution to $C(y,y')$. Near $y=y'=0$ one can use Eqs. (37) and (38). After a straightforward calculation one obtains

$$b=Q/[(D/\langle N\rangle)^2-Q]\,, \qquad (40a)$$

where

$$Q=x(\gamma)/2+4\langle N(0)\rangle^2\delta^2[R(0)-x(\beta)]/\langle N\rangle^2\,. \qquad (40b)$$

It is obvious that with $x(0)=x(\beta)=x(\gamma)=0$, the above equation gives the value of the slope b expected when there are only short-range rapidity correlations.

Equation (40), together with the approximate formula (22), give the slope b as a function of a few well-known observable parameters and of $x(0)$. At the highest ISR energies, where the theory is the better founded, the observed value of b is close to 0.3. From this value we get (roughly) $x(0)=0.3$ to 0.4. The dynamical significance of this result can only be appreciated in a theoretical framework where the averaging with respect to n, involved in the definition of $x(0)$, is precisely defined. Such a framework is provided by the Reggeon theory which gives a relation between $x(0)$ and parameters describing the elastic hadron-hadron scattering. Anticipating slightly, let us mention that a correct value of $x(0)$ is obtained in the eikonal model, using as the only input the observed values of the total and the total inelastic proton-proton cross sections. Our analysis of the data on the forward-backward multiplicity correlation is presented in Sec. V B. The reader who is not interested in theoretical details can skip Sec. V A and go directly to Sec. V B.

V. BACK TO REGGEON THEORY

A. Generalities

We first recall some of the basic equations of the Reggeon theory. For simplicity, we neglect the triple-Pomeron coupling.

Let f_k be the contribution to the forward elastic-scattering amplitude represented by the Reggeon diagram where k Pomerons are exchanged between the projectile and the target. The contribution to σ_{tot} obtained by cutting f_k through n Pomerons is denoted by $\sigma_{k,n}$. The cutting rules yield[3]

$$\sigma_{k,n}=(-1)^{k-n}\binom{k}{n}\sigma_{k,k}, \quad n>0 \tag{41a}$$

and, neglecting the real part of the single-Pomeron-exchange amplitude,

$$\sigma_{k,0}=(-1)^{k}(1-2^{1-k})\sigma_{k,k}. \tag{41b}$$

By definition

$$\sigma_n=\sum_{k=n}^{\infty}\sigma_{k,n}. \tag{42}$$

Thus, σ_n is the cross section for all processes which are represented by diagrams with n cut Pomerons. The cross section for all "genuine" inelastic processes (excluding diffraction dissociation) is obtained by summing σ_n from $n=1$ to ∞

$$\sigma_{in}=\sum_{k=1}^{\infty}(-1)^{k-1}\sigma_{k,k}. \tag{43}$$

The total cross section is obtained by summing σ_n from $n=0$ to ∞

$$\sigma_{tot}=\sum_{k=1}^{\infty}(-1/2)^{k-1}\sigma_{k,k}. \tag{44}$$

The calculation of inclusive cross sections rests on Mueller's optical theorem[6] combined with the cutting rules. Neglecting, for simplicity of writing, the exchanges of Reggeons other than the Pomeron (in the three-to-three and in the four-to-four forward scattering amplitudes), one obtains

$$d\sigma/dy=\rho\sum_{n=1}^{\infty}\sum_{k=n}^{\infty}n\sigma_{k,n}\equiv\rho\sum_{n=1}^{\infty}n\sigma_n \tag{45a}$$

and, provided y and y' are separated enough,

$$d^2\sigma/dy\,dy'=\rho^2\sum_{n=1}^{\infty}\sum_{k=n}^{\infty}n^2\sigma_{k,n}\equiv\rho^2\sum_{n=1}^{\infty}n^2\sigma_n. \tag{45b}$$

In the above equations ρ denotes the two-Pomerons-two-hadrons coupling, integrated with respect to transverse momentum. Defining the averaging with respect to n

$$\langle\langle\cdots\rangle\rangle=\sum_{n=1}^{\infty}(\cdots)\sigma_n\Big/\sum_{n=1}^{\infty}\sigma_n, \tag{46}$$

one easily obtains from (45) the asymptotic equations (11) and (12) [with $C_{SR}(y,y')$ missing, because of our neglect of secondary exchanges—the generalization is straightforward]. Using the identity

$$\sum_{n=1}^{\infty}\binom{n}{m}\sigma_{k,n}\equiv\sigma_{m,m}\delta_{k,m}, \tag{47}$$

one further gets

$$\langle n\rangle=\sigma_{1,1}/\sigma_{in} \tag{48}$$

and

$$\langle n(n-1)\rangle=2\sigma_{2,2}/\sigma_{in}. \tag{49}$$

The physical content of the above equations, together with Eqs. (45), is the celebrated Abramovskii-Kancheli-Gribov (AKG) cancellation[3] of absorptive corrections to inclusive spectra.

In the ISR energy range, σ_{in} increases by about 10%.[33] Hence, in this energy range

$$\sigma_{in}\propto W^{0.1}. \tag{50}$$

On the other hand, $\sigma_{1,1}$ is the discontinuity of the single-Pomeron-exchange amplitude. Therefore,

$$\sigma_{1,1}\propto(W^2)^{\alpha(0)-1}, \tag{51}$$

so we obtain from Eq. (48)

$$\langle n\rangle\propto W^{2[\alpha(0)-1]-0.1}, \tag{52}$$

where $\alpha(0)$ is the intercept of the (bare) Pomeron. The absence of any energy dependence of $\langle n\rangle$ corresponds to $\alpha(0)=1.05$, which is a reasonable value for the bare intercept (cf. Ref. 34 and footnote 35). For such a value of $\alpha(0)$, the energy variation of $\langle n^2\rangle$ obtained from Eqs. (48) and (49) is also mild.

Only the lowest-order moments of n are relevant for our calculations and these moments depend weakly on energy. Therefore, we conjecture that the probability distribution of n, calculated from Reggeon theory, can be trusted even at present energies. More precisely, we shall use the averaging prescription (46) with σ_n computed from the standard perturbative Reggeon calculus.

B. The eikonal model and an exploratory data analysis

The unknown parameters entering Eq. (40), which yields the slope b, are $x(\beta)$ and $x(\gamma)$. Both $x(\beta)$ and $x(\gamma)$ are readily found once a model for Regge cuts is adopted. In this section we calculate the slope parameter b within the framework of the most popular model of Regge cuts, the eikonal model. For simplicity, we neglect the real part of the single-Pomeron-exchange amplitude (we checked that including this real part one gets essentially identical results). We also assume that the single-Pomeron-exchange amplitude falls exponentially with the invariant momentum transfer $|t|$ (in fact our results are insensitive to the large-$|t|$ behavior of Regge residues). With these provisos, the eikonal model is defined by the equation

$$\sigma_{k,k}=AB^k/(kk!)\,. \tag{53}$$

Of course, B is dimensionless and A has the dimension of a cross section. [Using the identity

$$A/k=\int d_2x\,\exp(-x^2k\pi/A)\,, \tag{54}$$

and (53) one easily rewrites Eqs. (43) and (44) in the familiar form, $\vec{x}$ playing the role of the impact vector.]

A straightforward calculation yields

$$\langle n\rangle=AB/\sigma_{\text{in}} \tag{55}$$

and

$$x(0)=\tfrac{1}{2}(\sigma_{\text{in}}/A)(1+2/B)-1\,. \tag{56}$$

An easy numerical calculation enables one to find A and B using Eqs. (43), (44), and (53) and the observed values of σ_{in} and σ_{tot}.[35] In the ISR energy range the value of the parameter $x(0)$ is stable, $x(0)\simeq 0.33$. The parameters $x(\beta)$ and $x(\gamma)$ are found numerically using A and B and Eqs. (42), (46), and (53)

$$x(\beta=0.256)=0.166\,, \tag{57a}$$

$$x(\gamma=0.435)=0.088\,, \tag{57b}$$

$$x(\gamma=0.506)=0.063\,. \tag{57c}$$

The value of $x(\gamma)$ given by (57b) [(57c)] is appropriate for the highest [lowest] ISR energy, as explained in Sec. III. Notice that the approximate formula (22) would give numbers larger by 15 to 35% than those given in (57).

We will not drown the reader in the details of the numerical games we have played. We have given him all the elements necessary to check our assertions. Our conclusions are summarized below. The set of parameters qualified as the "standard" one is the following: $\delta=0.6$, $R(0)=0.6$, $\langle N(0)\rangle$ and $\langle N\rangle$ are calculated using the fits (23) and (24), the ratio $D/\langle N\rangle$ is taken from the paper by Thomé *et al.*[18]

(1) In the lowest part of the ISR energy range the formula (40) for the slope b becomes very sensitive to the choice of input parameters. At $W=23.6$ GeV one can get for b any value between 0.2 and 0.5, just playing with the uncertainties associated with the values of these parameters. With the standard choice we find $b=0.31$, to be compared with the experimental value at $W=23.6$ GeV, which is $b=0.217\pm0.018$. In fact, at this energy, the data are compatible with the absence of any long-range correlation: setting $x(0)=0$ and using the standard parameters one finds $b=0.18$. Nothing firm can be said at this point of the discussion.

(2) At $W=62.3$ GeV the typical values of b calculated from (40) fall around 0.3. With standard parameters one finds $b=0.27$. The experimental value is $b=0.312\pm0.014$. The results depend weakly on the choice of the Gaussian shape for the short-range component of the inclusive correlation function. It is excluded that the forward-backward multiplicity correlation results from the short-range correlation alone: with $x(0)=0$ and using standard parameters one finds $b=0.10$. Now, we claim that the model is successfull. We could not expect more precision. And we observe that there are no free parameters in the game.

(3) In the neighborhood of $y=y'=0$ one has [cf. Eqs. (37) and (38)]

$$C(y,y')=\langle N(0)\rangle^2[R(0)-x(\beta)]\times\exp[-(y-y')^2/4\delta^2]+x(\beta)\langle N(0)\rangle^2\,, \tag{58}$$

with $x(\beta)$ given by (57a). It is not unreasonable to assume that, at $W=62.8$ GeV, say, Eq. (58) is a good approximation within the whole "central" region $y,y'\in(-1,1)$. Using Eq. (34) one can calculate the slope b corresponding to the forward-backward correlation in the central region alone. Of course, now

$$\langle N_F\rangle\simeq\langle N(0)\rangle\,. \tag{59}$$

The slope b is, in the present case, mostly determined by the short-range component of $C(y,y')$ and does depend on the choice of the shape of $C_{\text{SR}}(y,y')$. With the Gaussian shape and using the standard parameters one finds $b=0.39$ at $W=62.8$ GeV. With the exponential shape $C_{\text{SR}}(y,y')\propto\exp(-|y-y'|/\lambda)$ and choosing $\lambda=2\delta/\sqrt{\pi}$ (to have the same average $|y-y'|$), one obtains $b=0.32$. The experimental number is $b=0.354\pm0.009$. Again the model seems reasonable.

(4) Consider now the slope b for the "outer" rapidity region: $|y|>1$. The calculation is analogous to that employed to derive Eq. (40). The

contribution of $C_{SR}(y,y')$ to the numerator in (34) is negligible. The term, in the denominator, which involves $C_{SR}(y,y')$ is written as the difference of two integrals: the first is over the region where $y,y'>0$ and has been already calculated. The second is over the region where one argument is within $(0,1)$ and the other is within $(0, \ln(W/W_0))$. In computing this second integral we use (58), since the contribution of the region where (58) fails is small. Another piece of information necessary to carry out the calculation is the energy variation of the outer multiplicity

$$\langle N_{outer}\rangle = \int_{|y|>1} dy \langle N(y)\rangle . \qquad (60)$$

In the ISR energy range one has $\langle N_{outer}\rangle \sim W^{\epsilon}$, with ϵ being (roughly) 0.52 to 0.59. The corresponding $x(\epsilon)=0.043$ to 0.061. With the standard parameters, the calculated slope, at $W=62.8$ GeV, is $b=0.08$ to 0.11. The experimental number, at this energy, is $b=0.156\pm0.013$.

Below the ISR energy range $\langle N_{outer}\rangle$ falls rapidly with decreasing W. Thus, $x(\epsilon)$ decreases dramatically when one moves from the ISR to the Fermilab energies. Consequently, the multiplicity correlation between the outer forward and backward regions is expected to disappear. This expectation is borne out by the data: at $W=23.6$ GeV one already finds $b=0.032\pm0.015$ for the outer region. This fits nicely with the assertion we made in point (1), that the full-rapidity-range forward-backward correlation comes mainly from the short-range component of $C(y,y')$.

(5) One can wonder what are the implications of the long-range rapidity correlation, discussed in this paper, for the earlier analyses of multiparticle data. In particular, in papers using the independent-cluster-emission model,[36] it has been postulated that dynamical long-range correlations are absent, except for low-multiplicity events. A thorough discussion of this problem is outside the scope of this work and we limit ourselves to a few remarks only.

The most elaborate application of the cluster model concerns the semi-inclusive data. However, when the total (charged) multiplicity is kept fixed, the long-range correlation is suppressed, since n is no longer allowed to fluctuate freely. Our tentative conclusion is that the study of semi-inclusive data based on the independent-cluster-emission model would not be significantly altered by the long-range rapidity correlation we are discussing.

Let us consider now the theoretical arguments which have been put forward to determine the average rapidity density of clusters.

Consider the tail of the rapidity-gap distribution in the compound density $N(y)$. It is not likely that there is a large rapidity gap in all the n elementary densities at exactly the same place. When a very large gap is observed, then most probably $n=1$. Hence, if Δy denotes the length of the rapidity gap, one predicts[37] at large Δy

$$\text{Prob}(\Delta y) \sim \exp(-\rho_0 \Delta y) , \qquad (61)$$

where ρ_0 is the average rapidity density of clusters in $N_0(y)$. Regge theory also predicts an exponential fall of Prob(Δy) (apart from a slowly varying corrective factor associated with the nonvanishing slope of the Regge trajectory, cf. the review Ref. 38), but with the exponent equal to $1+\alpha(0)-2\alpha_R(0)$ instead of ρ_0. We denote by $\alpha(0)$ and $\alpha_R(0)$ the intercepts of the Pomeron and of the leading secondary Regge trajectory. Thus, with $\alpha(0)=1$ and $\alpha_R(0)=\frac{1}{2}$, one expects $\rho_0=1$. A typical eikonal model value of $\langle n\rangle$, at least in the ISR energy range, is $\langle n\rangle = 1.56$. Using (21), (26), and (27), one finds

$$\langle N(0)\rangle / \langle N_0(0)\rangle \simeq \langle n\rangle^{1-\beta} \simeq 1.4 . \qquad (62)$$

Hence, the average rapidity density of clusters in real events is expected to be roughly 40% larger than $\rho_0=1$. This, in turn, implies that the average number of the cluster decay products is 40% smaller than is usually believed. We do not insist on this point, since the relevance of the rapidity-gap argument for the cluster model has been questioned.

Another argument is due to Stodolsky,[39] who identifies the distribution of the energy "left over" in the process of cluster production with the energy distribution of the leading particle. The approximate flatness of the leading-particle spectrum $(d\sigma/dx)_{leading}$ (remember that we are neglecting inelastic diffraction) is used to determine the average density of clusters. In our model one writes, following Stodolsky,

$$(d\sigma/dx)_{leading} = \rho_0 \sum_{n=1}^{\infty} n\sigma_n (1-x)^{n\rho_0 -1} . \qquad (63)$$

The leading-particle spectrum is now flat for $\rho_0 \simeq 0.7$. Applying the correction implied by (62), one finds that the (compound) cluster density is close to unity, as conventionally assumed. Notice that the two arguments sketched above do not give the same ρ_0, as was the case when long-range correlations were neglected.

C. What about inelastic diffraction?

Until now the inelastic diffraction has been neglected. This may appear as a poor approximation, since the cross section σ_{SD} for single diffraction dissociation in a proton-proton collision is $\sigma_{SD} \simeq 8$

mb at highest ISR energies.[40] Therefore, we feel obliged to discuss the consequences of taking into account the triple-Pomeron interaction (to the first order, however). As we shall see, the "improved" theory is not better than the most naive one (a rather common phenomenon in high-energy physics). For this reason the following discussion is not very elaborate: we just summarize the important points, leaving aside technicalities (for more details see the Appendix).

Compared to the preceding sections the theory involves now a new building block, the so-called Y diagram. We mean the simplest Reggeon diagram, with a single Pomeron emitted at the bottom (top) and two Pomerons absorbed at the top (bottom). The amplitude corresponding to the Y diagram (the Y amplitude) involves the triple-Pomeron vertex function which, in turn, depends on three momentum-transfer variables. The study of the inclusive spectra in the triple-Regge limit provides (some) information on the dependence of the triple-Pomeron vertex on two momentum-transfer variables.[41] The dependence of this vertex function on its third argument is unknown. Consequently, the behavior of the Y amplitude as a function of the overall momentum transfer is also unknown.

Define

$$r=\frac{(\text{slope of the one-Pomeron-exchange ampl})}{(\text{slope of the } Y \text{ ampl})}\,.$$

Since the behavior of both amplitudes is governed by the same soft-hadronic dynamics, we do not expect r to be dramatically different from unity. Nevertheless, nothing prevents r from being close to 0.5, to give an example.

Having nothing better at our disposal we stick to the eikonal model. Compared to the preceding section, we have now one more observable quantity to fit: σ_{SD}. Unfortunately, there are two new parameters: the triple-Pomeron coupling and r. Thus, one parameter, say r, is left free.

The implications of the nonvanishing triple-Pomeron coupling for the long-range rapidity correlations are twofold:

(i) Taking into account the AKG cancellation, one has four new diagrams contributing to $d^2\sigma/dy\,dy'$. The relevance of these diagrams for the forward-backward correlation can be estimated by calculating the ratio

$$\langle N_F N_B\rangle/(\langle N_F\rangle\langle N_B\rangle)=1+x(0)+\Delta x(0)+O(1/\langle N_{F,B}\rangle^2)\,. \tag{64}$$

In the above equation, $\Delta x(0)$ is proportional to the triple-Pomeron coupling. It turns out that for all reasonable values of r the ratio $|\Delta x(0)|/x(0)\lesssim 15\%$. Hence, the long-range rapidity correlation associated with inelastic diffraction is much less important than the long-range correlation resulting from the polyperipheral production of multiparticle states. This result is asymptotic, in the sense that it rests on the AKG cancellation. Strictly speaking, we do not know whether this result holds at a finite energy, but we suspect that it does. Consequently, we neglect $\Delta x(0)$ also at finite energy.

(ii) There is also an indirect implication of $\sigma_{SD}\neq 0$. For a given value of r, the eikonal model parameters A and B depend on the value taken by σ_{SD}. In other words, the relative weight of multi-Pomeron-exchange diagrams (which do not involve the triple-Pomeron coupling) becomes modified when $\sigma_{SD}\neq 0$: even if one neglects $\Delta x(0)$, the value of $x(0)$ one finds depends on the magnitude of σ_{SD}.[43] This effect is, in general, much more important than the one discussed in (i) above.

Without further ado let us mention a few typical numerical results: for $r=1$, one finds $\langle n\rangle\propto W^{0.1}$ and, at $W=62.8$ GeV, $x(0)=0.44$ and $b=0.44$ (with the standard parameters). For $r=0.5$, one gets $\langle n\rangle\propto W^{0.03}$, $x(0)=0.35$, and $b=0.29$, in agreement with the data.

In summary: because of the uncertainty on r the theory loses a part of its predictive power. Values of r close to unity are unlikely. Nevertheless, with other, equally acceptable values of r, like $r=0.5$, one obtains results which are close to those found in the last section with the most naive eikonal model. It is worth mentioning that the results are weakly sensitive to the choice of r, provided $r\lesssim 0.5$ or so.

An aficionado of the eikonal model would presumably turn this last conclusion into a more positive statement: the analysis of the long-range rapidity correlations provides precious information about the (otherwise) unaccessible slope of the Y amplitude. This, in turn, enables one to control better the absorptive corrections to the triple-Regge formula and to reduce the uncertainty on the value of the "true" triple-Pomeron coupling (for the relation between the value of the triple-Pomeron coupling and the choice of r, consult Ref. 42).

VI. VIOLATION OF THE LOCAL COMPENSATION OF CHARGE

Following Ref. 44, we denote by $Z(y)$ the random function representing the transfer of the electric charge across rapidity y. The charge is said to be locally compensated when the moment functions $\langle Z(y_1)\cdots Z(y_k)\rangle$ have the short-range order properties: cluster decomposition and translational in-variance.

In the model discussed heretofore one expects

$$Z(y)=\sum_{j=1}^{n} Z_0^{(j)}(y)\,, \tag{65}$$

where $Z_0^{(j)}(y)$, $j=1,2,\ldots,n$, satisfy the constraints of local compensation. Notice, that, for symmetry reasons, the moment functions of odd order must vanish, provided all rapidities are far from kinematic boundaries:

$$\langle Z_0(y_1)\cdots Z_0(y_{2k+1})\rangle=0\,. \tag{66}$$

In analogy to Eq. (5) and using (66) we find (in the central region)

$$\langle Z(y)Z(y')\rangle=\langle n\langle Z_0(y)Z_0(y')\rangle_n\rangle\,. \tag{67}$$

Thus, at the level of two-point functions there is no violation of the local compensation of charge. The (observable) function $\langle Z(y)Z(y')\rangle$ is expected to fall rapidly towards zero with increasing distance between y and y'.

Consider, however, the fourth order zone correlation function

$$D_4(y_1,y_2,y_3,y_4)=\langle Z(y_1)Z(y_2)Z(y_3)Z(y_4)\rangle - [\langle Z(y_1)Z(y_2)\rangle\langle Z(y_3)Z(y_4)\rangle + (2\leftrightarrow 3)+(2\leftrightarrow 4)]\,. \tag{68}$$

Limiting, for simplicity, our attention to the asymptotic regime, we derive from (65)

$$D_4(y_1,y_2,y_3,y_4)=\langle n\rangle D_4^{(0)}(y_1,y_2,y_3,y_4) + x(0)[\langle Z(y_1)Z(y_2)\rangle\langle Z(y_3)Z(y_4)\rangle + (2\leftrightarrow 3)+(2\leftrightarrow 4)]\,. \tag{69}$$

It is obvious from the above equation that for $x(0)\neq 0$ there is a long-range contribution to the correlation function D_4. An experimental study of a four-point function requires enormous statistics and is therefore very difficult. It is, however, sufficient to set $y_1=y_2=y$ and $y_3=y_4=y'$:

$$D_4(y,y,y',y')=\langle n\rangle D_4^{(0)}(y,y,y',y') + x(0)[\langle Z(0)^2\rangle^2+2\langle Z(y)Z(y')\rangle^2]\,, \tag{70}$$

where we have used the fact that in the asymptotic regime $\langle Z(y)^2\rangle=\langle Z(0)^2\rangle$. Hence, we predict that there is a constant, positive contribution to $D_4(y,y,y',y')$, equal to $x(0)\langle Z(0)^2\rangle^2$. We observe that the same parameter $x(0)$ controls the forward-backward multiplicity correlation and the violation of the local compensation of charge.

At finite energy one should apply the appropriate corrections. A very rough estimate indicates that the long-range contribution to $D_4(y,y,y',y')$ might be as large as 0.2. The correlation function D_4 has never been measured. Its value at $y_j=0$ can be estimated from the observed distribution of the charge transfer between forward and backward hemispheres. Using the data at 200 GeV (read) from Fig. 100 in Ref. 45, one obtains a rough estimate $D_4(0,0,0,0)\simeq 0.7\pm 0.2$.

Let us observe that even at very high energies the central region is not exactly neutral. For example,[46] in pp collisions, $\langle Z(0)\rangle=0.07\pm 0.02$ and 0.03 ± 0.03 at $p_{1ab}=100$ and 400 GeV/c, respectively: A small fraction of the leading charge is leaking to the central region. In testing the predictions of this section it is, therefore, recommended, to measure instead of $D_4(y,y,y',y')$ the correlation function defined below:

$$\tilde{D}_4(y,y,y',y')=\langle Z(y)^2Z(y')^2\rangle-\langle Z(y)^2\rangle\langle Z(y')^2\rangle-2\langle Z(y)\rangle[\langle Z(y)Z(y')^2\rangle-\langle Z(y)\rangle\langle Z(y')^2\rangle] - 2\langle Z(y')\rangle[\langle Z(y')Z(y)^2\rangle-\langle Z(y')\rangle\langle Z(y)^2\rangle]-2[\langle Z(y)Z(y')\rangle-\langle Z(y)\rangle\langle Z(y')\rangle]^2 + 4\langle Z(y)\rangle\langle Z(y')\rangle[\langle Z(y)Z(y')\rangle-\langle Z(y)\rangle\langle Z(y')\rangle]\,. \tag{71}$$

The functions $D_4(y,y,y',y')$ and $\tilde{D}_4(y,y,y',y')$ become identical when $\langle Z(y)\rangle=\langle Z(y')\rangle=0$. Equation (71) can be checked by comparing it with the expression for the full fourth-order correlation function, to be found in textbooks.[47]

The rapid fall towards zero of the second-order correlation function

$$\langle Z(y)Z(y')\rangle-\langle Z(y)\rangle\langle Z(y')\rangle$$

has been observed.[46] This behavior is a beautiful evidence for the short range in rapidity of the effective hadronic forces. A measure of $\tilde{D}_4(y,y,y',y')$ would be a very interesting test of the ideas discussed and developed in this paper.

VII. SUMMARY AND CONCLUSIONS

As argued in the Introduction, the short-range-order dynamics implies, via unitarity, a well-defined pattern of long-range rapidity correlations in high-multiplicity events. In this paper, these long-range correlations have been studied in detail. In particular, we paid much attention to the forward-backward multiplicity correlation, which has been recently measured in the whole ISR energy range.[11] We summarize below what we have learned from this investigation.

We find that, independently of any specific model, the strength of the forward-backward multiplicity correlation at the highest ISR energies is incompatible with short-range order alone.

We observe that in order to understand data one must carefully take into account finite-energy effects and, in particular, the energy-momentum-conservation constraints. The use of an asymptotic formalism at present accelerator energies is usually misleading.

We suggest that the long-range rapidity correlations associated with inelastic diffraction are a secondary effect (at least at the inclusive level) compared to the long-range correlations resulting from the (roughly speaking) polyperipheral production of multiparticle states.

We extract from the data the normalized dispersion (fluctuation, in the language of statistical mechanics) of the number n of exchanged *cut* Pomerons. A very conservative estimate is

$$\langle\!\langle n/\langle n\rangle - 1)^2\rangle \simeq 0.3 \text{ to } 0.4\,,$$

but there is an indication from the work with models that the correct value is rather 0.32 to 0.36.

The value of $\langle\!\langle n/\langle n\rangle - 1)^2\rangle$ being (roughly) known from multiparticle data, we have a nontrivial constraint on any model of Regge cuts. It is remarkable that the good old eikonal model (with inelastic diffraction neglected) gives $\langle\!\langle n/\langle n\rangle - 1)^2\rangle = 0.33$ in the ISR energy range. With this model we obtain a very reasonable overall picture of the forward-backward multiplicity correlations (except for the energy dependence of the effect: the theoretical uncertainties, inherent of our approach, increase considerably when one enters into the lower half of the ISR energy range).

The *a priori* more realistic theory, taking into account the triple-Pomeron interaction (to first order), does not bring any improvement from the phenomenological point of view. The lack of knowledge of the triple-Pomeron vertex, as a function of all the three independent momentum-transfer variables, reduces the predictive power of the theory. Nevertheless, the theory is still compatible with the data.

We predict a very specific violation of the local compensation of charge: the violation should appear in the fourth-order zone correlation function, being absent in the second-order correlation function. We recall that only the second-order function has been measured. The observed behavior of the second-order zone correlation function is a nice evidence for the short range in rapidity of the effective hadronic forces. Of course, this evidence would not be invalidated if the predicted violation of the local compensation of charge were observed. On the contrary, such a very specific violation of the local compensation is a consequence of the basic short-range nature of the effective forces together with unitarity.

Note added in proof. In order to avoid any misunderstanding, it is worth mentioning that our probabilistic analog model does not properly incorporate the effects of Bose-Einstein (or Fermi-Dirac) statistics. However, in this paper, we are interested in the long-range rapidity correlations, whereas quantum-statistics effects are only relevant for the fine structure of the short-range correlations. Taking into account quantum-statistics effects one would obtain, in Reggeon theory, an extra (short-range) contribution to the right-hand side of Eq. (9). In our analysis of the data we only use the fact that $C_{SR}(y,y')$ tends rapidly to zero as $|y-y'|\to\infty$. We never use Eq. (9).

Dr. N. N. Nikolaev recently called our attention to a paper by E. M. Levin and M. G. Ryskin, Yad. Fiz. 21, 396 (1975) [Sov. J. Nucl. Phys. 21, 206 (1975)]. Like ours, this work is inspired by the Reggeon theory. It attempts to evaluate the long-range contribution to two-body correlations. There is little overlap between the phenomenology presented in their paper and in ours.

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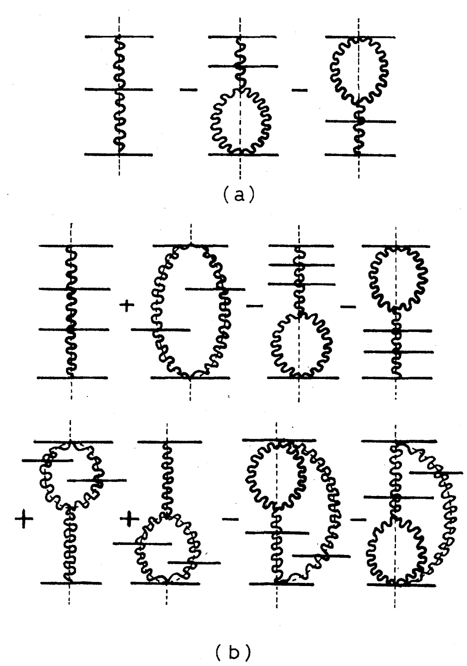

FIG. 1. (a) Diagrammatic representation of $d\sigma/dv$. (b) Diagramatic representation of $d^2\sigma/dydy'$.

grateful to I. Derado and to T. Ferbel for information on the data. The assistance of C. Panda is also acknowledged.

APPENDIX

The contribution of single diffraction dissociation to the inclusive peak near (Feynman) $x=1$ is well represented by the formula

$$d\sigma_{SD}/dx = \text{const}/(1-x). \tag{A1}$$

The right-hand side of (A1) is identical to the triple-Regge PPP term with $\alpha(0)=1$ and $\alpha'(0)=0$. In the following we shall neglect the corrections to the Y amplitude due to the fact that $\alpha(0)$ is not exactly at 1 and that $\alpha'(0)$ is nonvanishing. This approximation leads to much more transparent formulas without altering seriously the results. In the spirit of the inclusive-exclusive connection[48] we also assume that (A1) gives a good (average) description of diffraction dissociation even at x very close to 1.

In the eikonal model and to the first order in the triple-Pomeron coupling one has

$$\sigma_{tot} = A\sum_{k=1}^{\infty}(-1/2)^{k-1}B^k/(kk!) - G\sum_{k=0}^{\infty}(-1/2)^k B^k/[(k+r)k!]\,, \tag{A2a}$$

$$\sigma_{in} = A\sum_{k=1}^{\infty}(-1)^{k-1}B^k/(kk!) - G\sum_{k=0}^{\infty}(-1)^k B^k/[(k+r)k!]\,, \tag{A2b}$$

$$\sigma_{SD} = G\sum_{k=0}^{\infty}(-1)^k B^k/[(k+r)k!]\,. \tag{A2c}$$

In the above equations G/r is the contribution of the unabsorbed Y amplitude to σ_{SD} (r has been defined in Sec. VC).

We use the Mueller-Regge model, taking into account the AKG cancellation. Exchanges of Regge trajectories other than the Pomeron are neglected. The one-particle (two-particle) inclusive spectrum is represented diagramatically in Fig. 1(a) (Fig. 1(b)). Each diagram represents a particular discontinuity of the corresponding amplitude. With a vanishing triple-Pomeron coupling, $d\sigma/dy$ and $d^2\sigma/dy\,dy'$, corresponding to the diagrams in Figs. 1(a) and 1(b), respectively, are, of course, identical to those obtained from Eqs. (45) and (46). Notice that with the approximation (A1) all positions of the rapidity y_0 of the triple-Pomeron vertex are equally likely.

A straightforward calculation yields asymptotically

$$\langle N_{F,B}\rangle/\ln(W/W_0) = (A\rho/\sigma_{in})[B - G/(2rA)] \tag{A3}$$

and

$$\langle N_F N_B\rangle/\ln^2(W/W_0) = (A\rho^2/\sigma_{in})\{B + B^2/2 + 3G/(4rA) - GB/[A(1+r)]\}. \tag{A4}$$

Setting $r=1$ and with $\sigma_{tot}=43$ mb, $\sigma_{in}=35.2$ mb, and $\sigma_{SD}\simeq 8$ mb (approximate values of cross-sections at $W=63$ GeV), one finds from (A2) that $A=23.8$ mb, $B=3.43$, and $G=28.4$ mb. With these values of the parameters A, B, and G, Eqs. (64) and (A3) and (A4) give $x(0)+\Delta x(0)=0.51$. Setting $G=0$ (also in σ_{in}) but keeping the above values of A and B, one obtains $x(0)=0.44$. With $r=0.5$, but otherwise with the same input as before, one finds $A=30.1$ mb, $B=2.29$, and $G=7.05$ mb. This corresponds to $x(0)+\Delta x(0)=0.36$ and $x(0)=0.34$.

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[4]We should perhaps remark that the observed near constancy of diffractive cross sections is *by itself* a manifestation of the breaking of SRO. Indeed, as shown by Le Bellac (Ref. 2), exact SRO implies that all $\sigma_{exclusive}/\sigma_{tot}$ fall with energy faster than any inverse power of log$E$. Let us emphasize, therefore, that in this paper we focus our attention on *large multiplicity* production processes.

[5]In the Mueller-Regge framework (Ref. 6) one derives SRO from the assumption that diffraction can be represented by Pomeron exchange. Conversely, the existence of the Pomeron pole has been explicitly verified in models exhibiting SRO (Ref. 7). A model-independent demonstration that SRO implies the existence of the Pomeron pole does not exist, to our knowledge. However, one can show, using unitarity equations and assuming SRO (more generally, local compensation of

transverse momentum), that the forward diffraction peak should shrink indefinitely with increasing energy (Ref. 8), a behavior strongly reminiscent of the Regge one.

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