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# Field-theoretic model for diffraction scattering of hadrons

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We discuss here diffraction scattering of hadrons in a field-theoretic model with quark field operators. We first note that a nonrelativistic model qualitatively describes correct baryon-baryon and baryon-meson slopes in terms of harmonic-oscillator wave functions. Next, for a relativistic description, we apply a Lorentz-boosting scheme proposed earlier. It then becomes apparent that a current-current type of interaction in quark space can possibly reproduce baryon-baryon diffraction scattering. When we consider baryon-meson diffraction scattering, a change in the usual form of the current-current interaction becomes necessary to reproduce the quark additivity rule for hadron-hadron scattering; but even with this there is some mismatch with the pion charge radius. An alternative scheme of Lorentz boosting is discussed which has some similarity with the quasipotential approach. This field-theoretic analysis indicates the necessity of assumptions which are at present difficult to understand.

## I. INTRODUCTION

Diffraction scattering of hadrons[1] is usually understood in a phenomenological manner with a Pomeron exchange.[2] Alternative descriptions in terms of the impact parameter[3] or with a geometrical model[4] are also successful. An understanding of the latter type of models is also possible in terms of a group contraction[5] of O(3) to E(2) as the relevant little group for the Lorentz group at high energies. All these methods depend on general principles, and *do not* recognize that hadrons are composite in nature with quarks and/or antiquarks as constituents and thus have no explicit dynamical basis. In view of this, we try here to describe the diffractive elastic scattering of hadrons taking into account the composite nature of hadrons with a relevant form of quark-quark interactions. Thus in a way we expect that the hadronic matter distribution in the Chou-Yang model[3] or the geometrical model[4] may be related to the quark distribution content in hadrons. We discuss this problem here with a simple four-fermion interaction in quark space,[6] with the harmonic-oscillator wave functions for the hadrons providing an average good description for them.

In Sec. II, we first give some calculations of the nonrelativistic quark model in a field-theoretic language.[7] This reproduces not only the expected familiar results of the interrelationships of baryon-baryon and baryon-meson scattering at high energies, but also yields the magnitudes of the slope parameters for diffraction scattering in both the cases in terms of hadronic wave functions *without* any other fresh parameter. The motivation in this section is to see in a simple manner the possibilities of such a picture with only nonrelativistic quark field operators,[7,8] and, although the results are good, the limitation of such an approach is obvious.

In Sec. III, we utilize a field-theoretic description of hadrons in terms of quark field operators with a theory of relativistic boosting developed by one of the authors[9] to describe hadrons in motion. In this model, we have quark field operators $\psi_Q(x)$ which describe the quark $Q$ as a constituent of a hadron at rest, the operators $Q^{L(p)}(x)$ which describe $Q$ as a constituent of a hadron with four-momentum $p$, and quark field operators $\psi_\alpha(x)$ which describe the quark annihilation or antiquark creation of the $Q$ quark in *any* hadron with any four-momentum.[9] The last-mentioned field operators allow us to take a relativistic version of the four-fermion interaction proposed in Sec. II and to deal with relativistic hadrons.[9] This model, proposed earlier, was interesting, since with a field-theoretic description[8] it is possible to obtain $\pi^0$ and $\eta$ decay into $2\gamma$ *without* the assumption of partially conserved axial-vector current (PCAC) and with minimal electromagnetic coupling, and since it was possible to generate almost quantitatively strong interaction couplings[8,9] from quark-model parameters only, which is consistent with the Okubo-Zweig-Iizuka (OZI) rule.[10] As was seen earlier, the model proposed seems to describe also in a reasonable manner the radiative decays of mesons[11] automatically through a photon–vector-meson coupling.[9] In another application of these ideas,[12] if we further assume a point-interaction Hamiltonian generating the known universal hadron-hadron diffraction scattering, then we are able to understand in a qualitative manner the vector-dominance hypothesis at high energies for electromagnetic interaction with a modification which yields an energy-dependent cross section for $\gamma p \rightarrow Vp$ consistent with experimental results. This, in addition to the motivation of Sec. II prompts us to *explicitly* try a four-fermion point interaction to obtain diffraction scattering of hadrons in the field-theoretic quark-model picture. We find in the present paper that a vector-vector four-fermion point interaction yields a finite $(d\sigma/dt)_0$ as the energy goes to infinity. In the context of the present discussions, the form of the quark-antiquark interaction has to be taken as the same as that of the quark-quark interaction to maintain the additivity rules of the quark model. This is a necessary feature of the vector-vector interaction model which at present we are unable to understand. The results appear to be qualitatively satisfactory for baryon-baryon diffraction scattering, but not so for meson-baryon diffraction scattering. We further notice that a Lorentz-boosting factor present in this model is responsible for this mismatch. Hence in Sec. IV we propose an *alternative* scheme of Lorentz boosting, which is similar to the quasipotential approach or to superheavy quarks in the sense that certain three-vectors are *not* Lorentz transformed. This removes the experimental anomalies of Sec. III. The input effectively is a universal vector-vector four-fermion interaction in quark space, involving a single parameter for the strength of this interaction which gets determined from known $\sigma_t(pp)$. Without any other parameter it becomes possible to get reasonable predictions for the charge radius of the proton and pion, and for the diffraction slopes of baryon-baryon and baryon-meson scattering, along with the respective total scattering cross sections for baryons and mesons, as is familiar in the quark model. The universality of diffraction scattering results from quark-quark interactions and the composite nature of the hadrons. Further possibilities of the present model are discussed in Sec. V. Although we aim at constructing a successful model for diffraction scattering, difficulties are encountered in all the approaches we have considered. This can be attributed to the harmonic-oscillator wave functions not being adequate for high-momentum probes, and, to an incomplete understanding of color-gauge dynamics. Our objective here is a *phenomenological* description of this problem in the context of an effective field-theoretic quark-quark interaction, and we consider different assumptions which become necessary to achieve this.

## II. NONRELATIVISTIC QUARK MODEL

We shall consider diffraction scattering of hadrons in the context of the nonrelativistic quark model in a field-theoretic language. In the colored-quark model, we shall take, e.g., the proton as[8]:

$$
\begin{aligned}
|p_{1/2}(\vec{p})\rangle = \frac{\epsilon_{ijk}}{3\sqrt{2}} \int \delta_3(\vec{k}_1+\vec{k}_2+\vec{k}_3)\, u(\vec{k}_1,\vec{k}_2,\vec{k}_3)\, d^3k_1 d^3k_2 d^3k_3 \\
\times[\mathcal{P}^i_{I1/2}(\vec{k}_1+\tfrac{1}{3}\vec{p})^\dagger \mathcal{P}^j_{I1/2}(\vec{k}_2+\tfrac{1}{3}\vec{p})^\dagger \mathfrak{N}^k_{I\,-1/2}(\vec{k}_3+\tfrac{1}{3}\vec{p})^\dagger \\
-\mathcal{P}^i_{I1/2}(\vec{k}_1+\tfrac{1}{3}\vec{p})^\dagger \mathcal{P}^j_{I\,-1/2}(\vec{k}_2+\tfrac{1}{3}\vec{p})^\dagger \mathfrak{N}^k_{I1/2}(\vec{k}_3+\tfrac{1}{3}\vec{p})^\dagger]\,|\mathrm{vac}\rangle\,,
\end{aligned} \tag{2.1}
$$

and the pion as

$$
|\pi_\alpha(\vec{p})\rangle = \frac{1}{\sqrt{6}} \int \delta_3(\vec{k}_1+\vec{k}_2)\, u_\pi(\vec{k}_1)\, d^3k_1 d^3k_2\, q^j_I(\vec{k}_1+\tfrac{1}{2}\vec{p})^\dagger \tau_\alpha \tilde{q}^j_I(\vec{k}_2+\tfrac{1}{2}\vec{p})\,|\mathrm{vac}\rangle\,. \tag{2.2}
$$

In the above, $i,j,k$ stand for the color indices $q_I=(\mathcal{P}_I,\mathfrak{N}_I)$, and we are taking the large components of the quark field operators.[8] Summation over repeated indices is understood. We take also,[13] explicitly,

$$
u(\vec{k}_1,\vec{k}_2,\vec{k}_3) = \left(\frac{3R^4}{\pi^2}\right)^{3/4} \exp\left[-\frac{R^2}{6}\sum_{i<j}(\vec{k}_i-\vec{k}_j)^2\right] \tag{2.3}
$$

and

$$
u_\pi(\vec{k}_1) = \left(\frac{R_\pi{}^2}{\pi}\right)^{3/4} \exp\left[-\frac{R_\pi{}^2}{2}\vec{k}_1{}^2\right] \tag{2.4}
$$

We now take a nonrelativistic form of interaction with a scalar isospin-singlet meson given by[7]

$$\mathcal{H}_I(x) = g q_I^i(x)^\dagger q_I^i(x)\,\Phi(x)\,. \tag{2.5}$$

We can define $|p_{-1/2}(\vec{p})\rangle$ in a manner similar to (2.1).

We shall now consider proton-proton elastic scattering. Let the momenta and spins of the incident protons be $(p_1, r_1)$, $(p_2, r_2)$, and that of the outgoing protons be $(p_1', r_1')$, $(p_2', r_2')$. We then obtain

$$\langle f|S|i\rangle = \delta_4(p_f - p_i)\mathfrak{M}_{fi}\,, \tag{2.6}$$

where, in the second order,[14]

$$\mathfrak{M}_{fi} = (-i)(2\pi)^7 \sum_n \langle p_1' r_1', p_2' r_2'|\mathcal{H}_I(0)|n\rangle\langle n|\mathcal{H}_I(0)|p_1 r_1, p_2 r_2\rangle(p_1^0 + p_2^0 - E_n)^{-1}\,. \tag{2.7}$$

In (2.7), the states $|n\rangle$ will be only hadronic states,[9,12] and momentum conservation has already been taken into account. We *do not* use Feynman graphs, since in our theory we do not have quark propagators although we have quark operators[9,12]; however, the results for the process in (2.7) are identical if we take the Feynman theory, as is easily seen from below. From (2.7), we obtain the matrix elements, with $M$ as the mass of the exchanged meson:

$$\mathfrak{M}_{fi}^{(d)} = ig^2(2\pi)^4\langle p_1' r_1'|q_I^i(0)^\dagger q_I^i(0)|p_1 r_1\rangle\langle p_2' r_2'|q_I^i(0)^\dagger q_I^i(0)|p_2 r_2\rangle(M^2 - t)^{-1}\,, \tag{2.8}$$

$$\mathfrak{M}_{fi}^{(e)} = \mathfrak{M}_{fi}^{(d)}(p_1 r_1 \rightleftarrows p_2 r_2) \tag{2.9}$$

and

$$\mathfrak{M}_{fi} = \mathfrak{M}_{fi}^{(d)} - \mathfrak{M}_{fi}^{(e)}\,. \tag{2.10}$$

We now have,[15] from (2.1) and (2.3):

$$\langle p_1' r_1'|q_I^i(0)^\dagger q_I^i(0)|p_1 r_1\rangle = \delta_{r_1' r_1}(2\pi)^{-3} F_B(t)\,3\,, \tag{2.11}$$

where

$$F_B(t) = \exp(\tfrac{1}{6}R^2 t)\,, \tag{2.12}$$

with

$$-t = (\vec{p}_1 - \vec{p}_1')^2\,. \tag{2.13}$$

Clearly, in the above results, we have made nonrelativistic approximations. We thus finally get

$$\mathfrak{M}_{fi}^{(d)} = \delta_{r_1' r_1}\delta_{r_2' r_2}\frac{9ig^2}{(2\pi)^2(M^2 - t)}[F_B(t)]^2\,, \tag{2.14}$$

such that, using the c.m. frame of reference,

$$\frac{d\sigma}{dt} = \frac{4\pi^3}{\vec{p}^2 s}(p_1^0 p_2^0 p_1'^0 p_2'^0)|\mathfrak{M}_{fi}|^2 \tag{2.15}$$

yields, for the $p$-$p$ diffraction scattering,[7] *ignoring the fact that we have made nonrelativistic approximations*

$$\left(\frac{d\sigma}{dt}\right)_{pp} = \frac{81}{16\pi}\left(\frac{g^2}{M^2 - t}\right)^2[F_B(t)]^4\,. \tag{2.16}$$

We note that $F_B(t)$ is merely the form factor of the proton[6,9] in the nonrelativistic quark model. Thus, if $|t| \ll M^2$, we trivially have the result that $(d\sigma/dt) \propto [F_B(t)]^4$, a result recognized earlier in the context of the Chou-Yang model.[3] Also, in such a case we can approximate (2.5) by the point interaction

$$\mathcal{H}_I(x) = f[q_I^i(x)^\dagger q_I^i(x)][q_I^i(x)^\dagger q_I^i(x)]\,, \tag{2.17}$$

where

$$f = g^2/(2M^2)\,. \tag{2.18}$$

Also, we introduce in (2.5) the antiquark-antiquark term to consider meson-baryon scattering. We then obtain, e.g., for $\pi^+p$ scattering, with obvious notations, corresponding to (2.14):

$$\mathfrak{M}_{fi}(p_1 r, p_2 r \to p_1' r', p_2' r') = \delta_{rr'}\frac{6ig^2}{(2\pi)^2(M^2 - t)}F_B(t)F_M(t)\,, \tag{2.19}$$

where,[8] corresponding to (2.11) and (2.12),

$$\langle\pi^+(\vec{p}')|[q_I^i(0)^\dagger q_I^i(0) - \bar{q}_I^i(0)^\dagger \bar{q}_I^i(0)]|\pi^+(\vec{p})\rangle = (2\pi)^{-3}F_M(t)\,2\,, \tag{2.20}$$

with[9]

$$F_M(t) = \exp\left(\frac{R_\pi^{\,2}}{16}t\right). \tag{2.21}$$

We thus obtain[7]

$$\left(\frac{d\sigma}{dt}\right)_{\pi p} = \frac{36}{16\pi}\left(\frac{g^2}{M^2 - t}\right)^2[F_B(t)F_M(t)]^2\,. \tag{2.22}$$

Clearly, with the usual assumption that $(d\sigma/dt)_0 = \sigma_t^2/(16\pi)$, we get

$$\sigma_t(pp) = \tfrac{3}{2}\sigma_t(p\pi)\,, \tag{2.23}$$

a well-known result of the quark model. We further get for the slope parameters, with large $M^2$

$$b_{pp} = \tfrac{2}{3}R^2 \simeq 10 \text{ GeV}^{-2} \tag{2.24}$$

and

$$b_{p\pi} = \tfrac{1}{3}R^2 + \tfrac{1}{8}R_\pi^{\,2} = 7.8 \text{ GeV}^{-2}\,. \tag{2.25}$$

We have taken in the above,[9,11] $R^2 = 15$ GeV$^{-2}$ and $R_\pi^{\,2} = 22.5$ GeV$^{-2}$. Thus, with the quark-model parameters obtained earlier *from the static properties of the hadrons*, the slope parameters for baryon-baryon and baryon-meson diffraction scattering get fixed in reasonable quantitative agreement with experiments. Further, if we substitute $\sigma_t(pp) = 39$ mb, such that $\sigma_t(\pi p) = 26$ mb, we then obtain that in (2.18)

$$f = g^2/(2M^2) = 5.7 \text{ GeV}^{-2}\,. \tag{2.26}$$

If we further utilize the fact that[5] for diffraction scattering

$$\sigma_{\rm el} = \sigma_t^{\,2}/(16\pi b)\,, \tag{2.27}$$

we then obtain

$$\sigma_{\rm el}(pp) = 7.8 \text{ mb} \tag{2.28}$$

and

$$\sigma_{\rm el}(\pi p) = 4.4 \text{ mb}\,, \tag{2.29}$$

which are in reasonable agreement with experiments.[1]

The universality of baryon-baryon and baryon-meson diffraction scattering becomes now a consequence of (2.18) (or its equivalent) and the composite nature of hadrons. Experimentally, this has been observed not only for scattering of stable particles,[16] but also through vector-meson dominance,[17] it can be regarded as verified for vector-meson–nucleon scattering[12] through photoproduction of vector mesons.[18] We note that we attempted the above problem[12] by introducing in an *ad hoc* manner a local invariant interaction Hamiltonian $\mathcal{V}(x)$ in quark space, the matrix elements of which for meson-nucleon interaction are determined from the universal nature of diffraction scattering, e.g., from photoproduction of vector mesons, we have extrapolated that[12] for $\rho$ and $\omega$ mesons

$$\sigma_t(\rho N) = 26.5 \text{ mb} \tag{2.30}$$

and

$$\sigma_t(\omega N) = 26 \text{ mb}\,, \tag{2.31}$$

which agree with what we take above for $\sigma_t(\pi p)$. The success of such a model for the above problem leads us to believe that $\mathcal{V}(x)$ taken in an *ad hoc* manner in Ref. 12 could be some interaction similar to (2.17). Obviously we are investigating this problem in the present paper, and the above nonrelativistic considerations show the possible richness of such an approach.

We note that pointlike four-fermion interactions for *strong interactions* is not a new concept.[19] With free use of infinite constants, the possible equivalence of interactions such as (2.5) and (2.17) has also been recognized for some time.[20] It has further been demonstrated that by such techniques, local gauge symmetries may even get generated from only global gauge invariance along with other interesting results,[21] which indicates that interactions such as (2.5) and (2.17) probably cannot really be distinguished.[22]

Next, we shall further examine the possibility of generalizations of interactions like (2.17) yielding high-energy diffraction scattering in a manner envisaged in the present section, and taking relativistic boosting, proposed earlier[9] for high energies.

## III. RELATIVISTIC MODELS

The simplest thing to do now is to generalize the Hamiltonian (2.17) in quark space to[9]

$$\mathcal{H}_I^s(x) = f_s\,[\bar{q}^{\,g}(x)\,q^g(x)]^2\,, \tag{3.1}$$

where

$$q^g(x) = \begin{pmatrix} \mathcal{P}^g(x) \\ \mathcal{N}^g(x) \end{pmatrix}. \tag{3.2}$$

We shall specifically consider proton-proton elastic scattering, where (2.1) is replaced by[9]

$$\begin{aligned} |p_{1/2}(\vec{p})\rangle = \frac{\epsilon_{ijk}}{3\sqrt{2}}\left(\frac{p^0}{m}\right) \int \delta_3(\vec{k}_1+\vec{k}_2+\vec{k}_3)\,u(\vec{k}_1,\vec{k}_2,\vec{k}_3)\,d^3k_1\,d^3k_2\,d^3k_3 \\ \times [\mathcal{P}^i_{I1/2}(L(p)k_1)^\dagger\,\mathcal{P}^j_{I1/2}(L(p)k_2)^\dagger\,\mathcal{N}^k_{I\,-1/2}(L(p)k_3)^\dagger \\ -\mathcal{P}^i_{I1/2}(L(p)k_1)^\dagger\,\mathcal{P}^j_{I\,-1/2}(L(p)k_2)^\dagger\,\mathcal{N}^k_{I1/2}(L(p)k_3)^\dagger\,]\,|\text{vac}\rangle\,, \end{aligned} \tag{3.3}$$

with similar changes in other equations defining the states. As before, Eqs. (3.1) and (3.3) yield

$$\langle f|S|i\rangle = \delta_4(p_f - p_i)(\mathfrak{M}_{fi}^{(d)} - \mathfrak{M}_{fi}^{(e)}), \tag{3.4}$$

where

$$\mathfrak{M}_{fi}^{(d)} = (-i)(2\pi)^4\, 2f_s \langle p_1' r_1' | \bar{q}^g(0) q^g(0) | p_1 r_1\rangle \langle p_2' r_2' | \bar{q}^g(0) q^g(0) | p_2 r_2\rangle \,. \tag{3.5}$$

*Suppressing the spin dependence* of both hadrons and quarks, and concentrating on Lorentz-boosting factors for the evaluation of

$$\langle p_1' | \bar{q}^g(0)\, q^g(0) | p_1\rangle \,,$$

we shall need the expression[9]

$$\left(\frac{m}{p^0}\right)\left(\frac{3R^2}{2\pi}\right)^{3/2} \int d^3k_1 \exp\left[-\frac{3R^2}{4}(\vec{k}_1^{\,2} + \vec{k}_1'^{\,2})\right] \bar{u}(\vec{k}_1')\, S^{-1}(L')\, S(L)\, u(\vec{k}_1)\,, \tag{3.6}$$

where

$$\vec{k}_1' = \overleftrightarrow{L}'^{-1} \cdot \overleftrightarrow{L} \cdot \vec{k}_1 - \tfrac{2}{3} \overleftrightarrow{L}'^{-1} \cdot (\vec{p} - \vec{p}')\,, \tag{3.7}$$

with $\overleftrightarrow{L}'$ and $\overleftrightarrow{L}$ being the space parts of $L(p')$ and $L(p)$, respectively.[23] We use the c.m. frame of reference. Considering the forward scattering matrix amplitude and Eq. (2.5) for $(d\sigma/dt)_0$, we easily see that this will go to zero like $s^{-1}$, and hence *cannot* yield the known form of diffraction scattering.

We next consider the vector-vector interaction Hamiltonian in quark space:

$$\mathcal{H}_I^V(x) = f_V [\bar{q}^g(x)\gamma^\mu q^g(x)][\bar{q}^g(x)\gamma_\mu q^g(x)] \,. \tag{3.8}$$

This yields Eq. (3.4) with

$$\mathfrak{M}_{fi}^{(d)} = (-i)(2\pi)^4\, 2f_V \langle p_1' r_1' | \bar{q}^g(0)\gamma^\mu q^g(0) | p_1 r_1\rangle \times \langle p_2' r_2' | \bar{q}^g(0)\gamma_\mu q^g(0) | p_2 r_2\rangle \,, \tag{3.9}$$

and with $\mathfrak{M}_{fi}^{(e)}$ given by (2.9). In order to evaluate (3.9), we shall need expressions like (3.6). As before, we choose a c.m. frame of reference, and explicitly take in the high-energy limit

$$\vec{p} = \vec{p}_1 = (0, 0, p) \tag{3.10}$$

and

$$p_1' = p' = (\sqrt{-t}\cos\phi, \sqrt{-t}\sin\phi, p(1 + 2t/s)). \tag{3.11}$$

We then note that, in the high-energy limit,[23]

$$S^{-1}(L')\gamma^0 S(L) \simeq \frac{p}{m}\beta(1 + \alpha_z)\,, \tag{3.12}$$

$$S^{-1}(L')\gamma^i S(L) \simeq \delta_{i3}\frac{p}{m}\beta(1 + \alpha_z)\,, \tag{3.13}$$

$$S(L')\gamma^0 S^{-1}(L) \simeq \frac{p}{m}\beta(1 - \alpha_z)\,, \tag{3.14}$$

and

$$S(L')\gamma^i S^{-1}(L) \simeq -\delta_{i3}\frac{p}{m}\beta(1 - \alpha_z)\,. \tag{3.15}$$

Now, while evaluating

$$\langle p_1' r_1' | \bar{q}^g(0)\gamma^\mu q^g(0) | p_1 r_1\rangle \,, \tag{3.16}$$

the spin dependence in quark space need not be the same as the spin dependence of the proton. In fact, to evaluate (3.16), we shall need the expressions,[9] in the high-energy limit,

$$A^\mu \equiv \frac{m}{p^0}\left(\frac{3R^2}{2\pi}\right)^{3/2} \int d^3k_1 \exp\left[-\frac{3R^2}{4}(\vec{k}_1^{\,2} + \vec{k}_1'^{\,2})\right] \times \bar{u}(\vec{k}_1') S^{-1}(L')\gamma^\mu S(L) u(\vec{k}_1) = (\delta_{\mu 0} + \delta_{\mu 3})A\,, \tag{3.17}$$

where

$$A = \left(\frac{3R^2}{2\pi}\right)^{3/2} \int d^3k_1 \exp\left[-\frac{3R^2}{4}(\vec{k}_1^{\,2} + \vec{k}_1'^{\,2})\right] \times [f_1' f_1 + g^2(\vec{\sigma}\cdot\vec{k}_1')(\vec{\sigma}\cdot\vec{k}_1) + g f_1(\vec{\sigma}\cdot\vec{k}_1')\sigma_z + g f_1'\sigma_z(\vec{\sigma}\cdot\vec{k}_1)] \tag{3.18}$$

Clearly, $A$ is a matrix in the spin space of quarks. In writing (3.17) and (3.18) we have utilized the approximations (3.12) and (3, 13) of the high-energy limit, and $f_1 = f(\vec{k}_1^{\,2})$, $f_1' = f(\vec{k}_1'^{\,2})$, with $\vec{k}_1'$ as in (3.7). We now use the fact that for fixed momentum transfers, in the high-energy limit,

$$\overleftrightarrow{L}'^{-1} \cdot \overleftrightarrow{L} = \begin{pmatrix} 1 & 0 & -(\sqrt{-t}/m)\cos\phi \\ 0 & 1 & -(\sqrt{-t}/m)\sin\phi \\ 0 & 0 & 1 \end{pmatrix}, \tag{3.19}$$

such that (3.7) yields

$$\vec{k}_1' = \vec{k}_1 - \frac{k_{1z}}{m}\vec{p}_\perp' + \tfrac{2}{3}\vec{p}_\perp'\,, \tag{3.20}$$

where

$$\vec{p}_\perp' = (\sqrt{-t}\,\cos\phi, \sqrt{-t}\,\sin\phi, 0)\,. \tag{3.21}$$

In order to evaluate this integral (3.18), we now substitute

$$\vec{k}_1 = \vec{k} + \left(\frac{k_z}{2m} - \frac{1}{3}\right)\vec{p}_\perp'\,, \tag{3.22}$$

$$k_z' = k_z + \frac{\tau}{6m}\,, \tag{3.23}$$

$$\tau = t/\gamma, \tag{3.24}$$

and

$$\gamma = \left(1 - \frac{t}{4m^2}\right). \tag{3.25}$$

We then obtain

$$\vec{k}_1^{\,2} + \vec{k}_1'^{\,2} = 2(\vec{k}_\perp^{\,2} + \gamma k_z'^2 - \tfrac{1}{9}\tau)\,, \tag{3.26}$$

where $\vec{k}_\perp = (k_x, k_y)$. We shall integrate (3.18) first with respect to $\vec{k}_\perp$ and then $k_z'$. For this purpose we note that up to second order in small quark components[9]

$$f_1 f_1' \simeq 1 - \tfrac{1}{2}g^2(\vec{k}_1^{\,2} + \vec{k}_1'^{\,2}) = 1 - g^2(\vec{k}_\perp^{\,2} + \gamma k_z'^2 - \tfrac{1}{9}\tau)\,, \tag{3.27}$$

$$\begin{aligned} g^2(\vec{\sigma}\cdot\vec{k}_1')(\vec{\sigma}\cdot\vec{k}_1) &\equiv g^2[\vec{k}^2 - (k_z/2m - \tfrac{1}{3})^2\vec{p}_\perp'^2 + 2k_z(k_z/2m - \tfrac{1}{3})\sigma_z(\vec{\sigma}\cdot\vec{p}_\perp')] \\ &\equiv g^2\left[k_\perp^2 + \gamma k_z'^2 + \frac{\tau^2}{36m^2} + \frac{\tau}{9\gamma} + 2\sigma_z(\vec{\sigma}\cdot\vec{p}_\perp')\left(\frac{k_z'^2}{2m} + \frac{\tau}{18m\gamma}\right)\right], \end{aligned} \tag{3.28}$$

and

$$g f_1'\sigma_z(\vec{\sigma}\cdot\vec{k}_1) + g f_1(\vec{\sigma}\cdot\vec{k}_1')\sigma_z \equiv -\frac{g\tau}{3m} - \frac{2g}{3\gamma}\sigma_z(\vec{\sigma}\cdot\vec{p}_\perp')\,. \tag{3.29}$$

In writing (3.28) and (3.29) we have ignored terms which will not contribute on subsequent integration. Evaluation of (3.18) is now straightforward, and we obtain, in (3.17),

$$A = (\gamma)^{-1/2}\exp(R^2\tau/6)\left[\left(1 + \frac{2g^2\tau}{9\gamma} - \frac{g\tau}{3m} + \frac{g^2\tau}{6m^2R^2}\right) + 2\sigma_z(\vec{\sigma}\cdot\vec{p}_\perp')\left(\frac{g^2}{6mR^2\gamma} + \frac{\tau}{18m\gamma} - \frac{g}{3\gamma}\right)\right]. \tag{3.30}$$

Using (3.3), (3.17), and (3.30), we can evaluate (3.16) for arbitrary proton spins. Clearly, the second term in (3.30) permits a spinflip of the proton for nonzero momentum transfers.

We now also need to evaluate

$$\langle p_2' r_2' | \bar{q}^g(0)\gamma^\mu q^g(0) | p_2 r_2\rangle\,, \tag{3.31}$$

which requires in quark space the expression

$$\begin{aligned} B^\mu &= \left(\frac{m}{p^0}\right)\left(\frac{3R^2}{2\pi}\right)^{3/2}\int d^3k_2 \exp\left[-\frac{3R^2}{4}(\vec{k}_2^{\,2} + \vec{k}_2'^{\,2})\right]\bar{u}(\vec{k}_2')\,S(L')\gamma^\mu S^{-1}(L)u(\vec{k}_2) \\ &= (\delta_{\mu 0} - \delta_{\mu 3})B\,, \end{aligned} \tag{3.32}$$

where

$$B = \left(\frac{3R^2}{2\pi}\right)^{3/2}\int d^3k_2 \exp\left[-\frac{3R^2}{4}(\vec{k}_2^{\,2} + \vec{k}_2'^{\,2})\right][f_2 f_2' + g^2(\vec{\sigma}\cdot\vec{k}_2')(\vec{\sigma}\cdot\vec{k}_2) - g f_2(\vec{\sigma}\cdot\vec{k}_2')\sigma_z - g f_2'\sigma_z(\vec{\sigma}\cdot\vec{k}_2)]\,. \tag{3.33}$$

In (3.32) and (3.33), (3.14) and (3.15) have been used, and we also have

$$\begin{aligned} \vec{k}_2' &= \vec{L}'^{-1}\cdot\vec{L}\cdot\vec{k}_2 + \tfrac{2}{3}\vec{L}'^{-1}\cdot(\vec{p} - \vec{p}') \\ &\simeq \vec{k}_2 - (k_{2z}/m + \tfrac{2}{3})\vec{p}_\perp'\,. \end{aligned} \tag{3.34}$$

Evaluation of (3.33) is similar to that of (3.18). We may evaluate this directly or we may note that (3.33) becomes the same as (3.18) when we change the signs of the parameters $m$, $g$, and $\vec{p}_\perp'$. Hence, from (3.30) we easily obtain

$$B = A\,. \tag{3.35}$$

Using (3.3), we now note that[9] the matrix $A$ in quark spin space remains the same in proton spin space also. We thus get, in proton space

$$\langle p_1' r_1' | \bar{q}^g(0)\gamma^\mu q^g(0) | p_1 r_1\rangle = (2\pi)^{-3}\, 3(\delta_{\mu 0}+\delta_{\mu 3})\langle r_1' |A| r_1\rangle \quad (3.36)$$

and

$$\langle p_2' r_2' | \bar{q}^g(0)\gamma^\mu q^g(0) | p_2 r_2\rangle = (2\pi)^{-3}\, 3(\delta_{\mu 0}-\delta_{\mu 3})\langle r_2' |A| r_2\rangle\,. \quad (3.37)$$

Hence, (3.9) now yields,

$$\mathfrak{M}_{fi}^{(d)} = (-i)(2\pi)^{-2} f_V \times 4\times 9\times \langle r_1'|A|r_1\rangle\langle r_2'|A|r_2\rangle\,. \quad (3.38)$$

Obviously, the above contribution will give rise to $p$-$p$ diffractive scattering. Using (2.15) and making appropriate spin summations, we then get

$$\frac{d\sigma}{dt} = \frac{f_V{}^2}{\pi} 81[F_B(t)]^4\,, \quad (3.39)$$

where

$$[F_B(t)]^2 = \tfrac{1}{4}(\mathrm{Tr}AA^\dagger) = \gamma^{-1}\exp(\tfrac{1}{3}R^2\tau)\left[\left(1+\frac{2g^2\tau}{9\gamma}-\frac{g\tau}{3m}+\frac{g^2\tau}{6m^2R^2}\right)^2 - \frac{4\tau}{\gamma}\left(\frac{g^2}{6mR^2}+\frac{\tau}{18m}-\frac{g}{3}\right)^2\right]. \quad (3.40)$$

If we substitute $\sigma_t{}^2/(16\pi) = (d\sigma/dt)_0$, we then obtain, with $\sigma_t(pp) = 39$ mb.

$$f_V = \sigma_t/36 \simeq 2.8\ \mathrm{GeV}^{-2}\,. \quad (3.41)$$

We shall next consider $\pi^+p$ elastic scattering. For this purpose, we note that, to start with, quarks and antiquarks in $\pi^+$ should give the same contribution, i.e., for zero momentum transfers, the contribution for them should be additive, giving broadly a factor 2 instead of the factor 3 as in (3.36). Hence, we now take the strong current in quark and antiquark space as, with $i$ as the color index,

$$J^\mu_{\mathrm{str}}(x) = [\bar{q}_i^g(x)\gamma^\mu q_i^g(x) - \bar{\tilde{q}}_i^g(x)\gamma^\mu \tilde{q}_i^g(x)]\,, \quad (3.42)$$

and generalize (3.8) to

$$\mathcal{H}_V(x) = f_V J^\mu_{\mathrm{str}}(x) J_{\mathrm{str}\,\mu}(x)\,. \quad (3.43)$$

We take (3.42) so that, effectively, it corresponds to quarks and antiquarks having the *same* "charge" and not charges of opposite sign. As mentioned earlier, we take (3.42) and (3.43) so that, effectively, quark-quark and quark-antiquark interactions become identical, *in spite of* the current-current form of effective interaction. We may recognize that (3.43) is an *effective* interaction[24] expected to be valid for small momentum transfers with $f_V$ being a constant, which may be the result of gluon interactions we cannot solve.

Now, parallel to Eq. (3.36), we obtain, *for the pion*,[9]

$$\langle \pi^+(\vec{p}\,')|J^\mu_{\mathrm{str}}(0)|\pi^+(\vec{p})\rangle = (2\pi)^{-3}\, 2(\delta_{\mu 0}+\delta_{\mu 3})F_\pi(t)\,, \quad (3.44)$$

where

$$F_\pi(t) = \gamma'^{-1/2}\exp(R_\pi{}^2\tau'/16) \times\left(1+\frac{g^2\tau'}{8\gamma'}-\frac{g\tau'}{4m_\pi}+\frac{g^2\tau'}{6m_\pi{}^2R_\pi{}^2}\right)\,. \quad (3.45)$$

In (3.45) we have taken

$$\gamma' = 1+\frac{\vec{p}_\perp'^2}{4m_\pi{}^2} \quad (3.46)$$

and

$$\tau' = t/\gamma'\,. \quad (3.47)$$

Also, with (3.42) Eq. (3.37) remains unaltered, and we get

$$\langle -\vec{p}\,', r'|J^\mu_{\mathrm{str}}(0)|-\vec{p}, r\rangle = (2\pi)^{-3}\, 3(\delta_{\mu 0}-\delta_{\mu 3})\langle r'|A|r\rangle\,. \quad (3.48)$$

Equations (3.45) and (3.48) show that in the high-energy limit, parallel to (3.39),

$$\left(\frac{d\sigma}{dt}\right)_{\pi p} = \frac{f_V{}^2}{\pi} 36[F_B(t)]^2[F_\pi(t)]^2\,, \quad (3.49)$$

which becomes similar to (2.22) of the nonrelativistic model and with usual assumptions we get, with (3.41),

$$\sigma_t(\pi p) = \tfrac{2}{3}\sigma_t(pp) = 26\ \mathrm{mb}\,. \quad (3.50)$$

We note that the present section is based on the Lorentz-boosting scheme proposed in Ref. 9. We shall now mention some disagreements of the results with this scheme with experimental observations. Firstly, we may immediately note that $F_\pi(t)$ in (3.45) is essentially the pion form factor,[25] and for $-t \gg m_\pi^2$ we get $\tau' = -4m_\pi^2$, such that, essentially, the slope characteristic for diffractive scattering through the pion form factor gets lost. Further, in Ref. 9 we had, for $R^2 = 15\ \text{GeV}^{-2}$, $\langle r_\pi^2 \rangle = 0.31\ \text{fm}^2$ and for $R^2 = 22.5\ \text{GeV}^{-2}$, $\langle r_\pi^2 \rangle = 0.4\ \text{fm}^2$. These are quite agreeable results in the context of recent measurements of the pion charge radius.[26] However, in contrast to earlier measurements[27] which had $|t| \ll m_\pi^2$, in Ref. 26, $|t| \gg m_\pi^2$, and there seems to be no effective change of pion charge radius. In view of this and in view of the mismatch with the slope parameter for $\pi p$ diffractive scattering with moderate momentum transfers, we are led to believe that the replacement of $t$ by $\tau'$ in the exponential between Eqs. (2.21) and (3.45) may be wrong, in view of the fact that the nonrelativistic model discussed in Sec. II falls so neatly into the pattern of experimental results along with the value of pion charge radius. Also, Licht and Pagnamenta,[28] in the context of Lorentz boosting through the wave function, had a similar replacement of $t$ by $\tau'$ and thus had to effectively reject this picture for consideration of pion form factors. In view of all this, we now suggest an alternative scheme for Lorentz boosting to what we had in Ref. 9, with the purpose of retaining the nice features of Sec. II.

## IV. AN ALTERNATIVE LORENTZ-BOOSTING SCHEME

In order to develop an alternative theory of Lorentz boosting, we first note the basic assumptions in the Lorentz-boosting scheme developed earlier.[9] We first define, e.g., the quark field operator[8,9]

$$Q(x) = (2\pi)^{-3/2} \int u(\vec{k})\, Q_I(\vec{k}) \exp(i\vec{k}\cdot\vec{x})\, d^3k\ . \quad (4.1)$$

We then consider that, when $Q$ is the constituent of a hadron at rest, the time dependence in (4.1) is given by[9]

$$Q(\vec{x}, t) = Q(\vec{x})\, e^{-i\omega t}\,, \quad (4.2)$$

where the value of $\omega$ depends on the quark and the hadron. We then define

$$Q^{L(p)}(x) \equiv U(L(p))\, S(L(p))\, Q(L(p)^{-1}x)\, U^{-1}(L(p))\ . \quad (4.3)$$

Equation (4.3) is given a meaning in operator space through the assumption that[9]

$$U(L(p))\, Q_I(\vec{k})\, U^{-1}(L(p)) = \left(\frac{p^0}{m}\right)^{1/2} Q_I([L(p)k]) \quad (4.4)$$

where $k = (\vec{k}, \omega)$. One then assumes the normal anticommutation rule for the two-component quark operators $Q_I(L(p)k)$, even when the quark $Q$ may belong to different hadrons with arbitrary momenta. Regarding the comments at the end of Sec. III, basically, the equation

$$[L(p)k]_i = (\overleftrightarrow{L}\cdot k)_i + \lambda p_i\,, \quad (4.5)$$

with $\lambda = (\omega/m)$ and $\overleftrightarrow{L}$ being the space part of $L(p)$, yields the replacement of $t$ by $\tau'$ through[9,28] $\overleftrightarrow{L}$. We shall now replace the scheme of Lorentz boosting of Ref. 9 with this in mind.

We obviously keep (4.1) and (4.2) as before and now replace (4.3) by the alternative definition[29]

$$Q^{(p)}(x) = U(L(p))\, S(L(p))\, Q(\vec{x})\, U^{-1}(L(p)) \times \exp\{-i[L(p)^{-1}x]^0\}, \quad (4.6)$$

with the identification in quark operator space given as

$$U(L(p))\, Q_I(\vec{k})\, U^{-1}(L(p)) = Q_I(\vec{k} + \lambda\vec{p})\,, \quad (4.7)$$

where, as before, $\lambda = (\omega/m)$, $m$ being the mass of the hadron. Equation (4.7) avoids the Lorentz boosting of "quark momentum" $\vec{k}$ in (4.5). Also, the Lorentz transformation in (4.7) behaves like a translation. In fact, $\vec{k}$ appears to be an *invariant* three-vector of the quark momentum in the rest frame of the hadrons, and the Lorentz transformation has the effect of *addition* of the hadronic fractional momentum as a translation.[30] From (4.1) and (4.2), (4.6) now becomes equivalent to

$$Q^{(p)}(x) = (2\pi)^{-3/2} \int d^3k\, S(L(p))\, u(\vec{k})\, Q_I(\vec{k} + \lambda\vec{p}) \times \exp[i(\vec{k} + \lambda\vec{p})\cdot\vec{x} - i\lambda p^0 t]\ . \quad (4.8)$$

In the same manner we also obtain for the antiquark creation component

$$\tilde{Q}^{(p)}(x) = (2\pi)^{-3/2} \int d^3k\, S(L(p))\, v(\vec{k})\, \tilde{Q}_I(\vec{k} + \lambda\vec{p}) \times \exp[-i(\vec{k} + \lambda\vec{p})\cdot\vec{x} + i\lambda p^0 t]\ . \quad (4.9)$$

$u(\vec{k})$, $v(\vec{k})$ and the quantization conditions of the two-component quark field operators remain unaltered.[9] Also, with

$$|p_{1/2}(\vec{p})\rangle = U(L(p))\, |p_{1/2}(\vec{0})\rangle\,, \quad (4.10)$$

the state $|p_{1/2}(\vec{p})\rangle$ is now defined by (2.1) instead of (3.3), along with the normalization condition

$$\langle p_r(\vec{p}')|p_s(\vec{p})\rangle = \delta_{rs}\,\delta(\vec{p}-\vec{p}')\,. \tag{4.11}$$

We shall next consider the generalized quark field operator $Q^g(x)$ which can describe[9] the $Q$ quark as a constituent of any hadron with any momentum, along with the identification,[9]

$$Q^g(x) = \alpha Q^{(p)}(x)\,. \tag{4.12}$$

For this purpose, as earlier, we consider the electromagnetic current[9]

$$J^\mu(x) = \sum_{i,Q} e_Q \bar{\psi}^{ig}_Q(x)\gamma^\mu \psi^{ig}_Q(x) \tag{4.13}$$

and evaluate that, with (2.1),

$$\langle p_{1/2}(\vec{p})|J^\mu(0)|p_{1/2}(\vec{p})\rangle = (2\pi)^{-3}|\alpha|^2 e\frac{p^\mu}{m}\,. \tag{4.14}$$

This shows that the prescription[9] for the identification (4.12) should now be

$$Q^g(x) = \left(\frac{m}{p^0}\right)^{1/2} Q^{(p)}(x)\,, \tag{4.15}$$

when the quark operator in $Q^g(x)$ gets contracted with the $Q$ quark of a hadron of four-momentum $p$.

We now proceed as in the last section with the changed pattern of Lorentz boosting given by (4.8) and the interaction described by (3.43) with (3.42). We then obtain, in the high-energy limit

$$\langle p_{r'}(\vec{p}')|J^\mu_{\rm str}(0)|p_r(\vec{p})\rangle = (2\pi)^{-3}\times 3\times(\delta_{\mu 0}+\delta_{\mu 3})\langle r'|A'|r\rangle\,, \tag{4.16}$$

where

$$\begin{aligned} A' &= \left(\frac{3R^2}{2\pi}\right)^{3/2}\int d^3k_1\,\exp[-\tfrac{3}{4}R^2(\vec{k}_1^{\,2}+\vec{k}_1'^{\,2})] \\ &\quad\times[f_1f_1' + g^2(\vec{\sigma}\cdot\vec{k}_1')(\vec{\sigma}\cdot\vec{k}_1) \\ &\qquad + f_1 g(\vec{\sigma}\cdot\vec{k}_1')\sigma_z + f_1' g\sigma_z(\vec{\sigma}\cdot\vec{k}_1)] \\ &= \exp(R^2t/6)\left[1+\frac{2g^2}{9}t - \frac{2g}{3}\sigma_z(\vec{\sigma}\cdot\vec{p}_\perp')\right]. \end{aligned} \tag{4.17}$$

We note that in (4.17), we have taken, appropriately with spectator quarks, momentum conservation[9]

$$\begin{aligned} \vec{k}_1' &= \vec{k}_1 - \tfrac{2}{3}(\vec{p}-\vec{p}') \\ &= \vec{k}_1 + \tfrac{2}{3}\vec{p}_\perp'\,. \end{aligned} \tag{4.18}$$

We also note that the matrix $A$ in the spin space[31] of the proton evaluated in (3.30) now gets replaced by (4.17), and as earlier is the same as the matrix in the spin space of quarks. As earlier we now obtain

$$\frac{d\sigma}{dt} = \frac{81}{\pi}\times f_V{}^2\times[F_B'(t)]^4\,, \tag{4.19}$$

where, with (4.17),

$$\begin{aligned} [F_B'(t)]^2 &= \tfrac{1}{4}(\mathrm{Tr}A'^\dagger A') \\ &= \exp\left(\frac{R^2t}{3}\right)\left(1+\frac{4g^4}{81}t^2\right). \end{aligned} \tag{4.20}$$

$F_B'(t)$ clearly corresponds to the form factor of the proton.

For the pion, we again obtain (3.44) with (3.45) replaced by

$$F_\pi'(t) = \exp(R_\pi{}^2t/16)\times(1+\tfrac{1}{8}g^2t)\,. \tag{4.21}$$

Hence we obtain

$$\left(\frac{d\sigma}{dt}\right)_{\pi p} = \frac{36}{\pi}f_V{}^2[F_B'(t)]^2[F_\pi'(t)]^2\,. \tag{4.22}$$

We note that with this alternative scheme of Lorentz boosting, the nonrelativistic results of Sec. II get reproduced. However, the three vectors $\vec{x}$ or $\vec{k}$ in quark field operators attain an invariant meaning, the significance of which we do not understand except in the context of a quasipotential approach.[30] We are not tempted to give it a geometrical meaning[32] since we add the fractional hadronic momentum to quark momentum for spectator quarks.

## V. DISCUSSIONS

Let us examine the broad assumptions which have gone into the description of hadrons here. It is assumed that hadrons at rest can be described by four-component quark field operators satisfying equal-time Dirac anticommutators, for which we take a simple but an unconventional form.[8] Hadrons in motion are described by Lorentz boosting of these field operators with some specific assumptions.[9] The description of hadronic states for nonzero momenta depends on the equal-time wave functions of hadrons[33] and on the scheme of Lorentz boosting.[9] In Sec. IV of the present paper we have suggested an alternative scheme of Lorentz boosting to what we had taken earlier.[9]

In this picture, hadronic interactions are extremely conplicated. (i) We may have exchange of constituents; (ii) we may have reactions with virtual exchange of hadrons arising from conventional hadron couplings from quark-pair creation terms[8,9]; or (iii) we may have Hamiltonians with quark field operators which give rise to hadronic interactions. The second type of interactions above are in many ways conventional interactions. We have investigated here the possibility of the third type of interaction mentioned above for the purpose of understanding diffraction scattering of hadrons.

We recognize that we do not know the primary quark-quark interactions which will yield hadrons as bound states of quarks, although many beautiful attempts have been made in this direction.[34] Therefore, *a fortiori*, the residual effective quark-quark interaction which will operate for hadron-hadron scattering is also unknown, and thus we are to proceed by trial and error, assuming that such an idea may be successful. The motivation for this is twofold. Firstly, the success of the quark-counting rule has been impressive, which has been derived along with some new results in Sec. II. Secondly, the assumption that hadron-hadron elastic diffraction scattering is described by a pointlike Hamiltonian $\mathcal{V}(x)$ in the context of our ideas seems adequate to generate a vector-dominance model for $\gamma p \to Vp$ with corrections in an agreeable manner,[12] so that we may now look for an explicit form for $\mathcal{V}(x)$. Naturally a simple form to try in this context is a current-current interaction.[19,20,21]

A major objective of the present paper has been to *illustrate* how an effective Hamiltonian in quark space can generate hadron-hadron scattering using Lorentz-boosted states, and *not* using Feynman theory.[35] We choose diffraction scattering because of its universal nature, so that we can hope to have a simple description.

With this approach, we achieve the following: With the alternative scheme of Lorentz boosting suggested in Sec. IV, and with quark parameters and radii of the harmonic-oscillator wave functions determined earlier,[9] we obtain the appropriate form of diffraction-scattering results along with the appropriate value of the slope parameters. These agreements are reached in addition to a quantitative prediction for the charge radius of the proton and the pion. It thus appears that there may be some truth in the phenomenological interaction we have introduced. However, assumptions are needed in both the models of Secs. III and IV which are at present not at all clear. If the alternative scheme of Lorentz boosting is relevant, the exponential decrease of form factors in (4.20) is not experimentally true, and we may probably attribute this to the fact that harmonic-oscillator wave functions need not be good for large momentum transfers and a power-law behavior may become necessary.[36] However, it is difficult to understand the form of quark-antiquark interactions; in any case, we are aiming at a phenomenological description which is related to specific assumptions of the quark model.

Some further generalizations may be worthwhile. Let us for a moment take

$$\chi_q(x) = \begin{pmatrix} q(x) \\ \tilde{q}(x) \end{pmatrix}, \tag{5.1}$$

and define

$$J^{j\mu}(x) = \bar{\chi}_q^i(x)\gamma^\mu \tau^j \chi_q^i(x). \tag{5.2}$$

We can then consider

$$\begin{aligned}\mathcal{H}_I(x) &= f_V J^{j\mu}(x) J^j_\mu(x) \\ &= f_V [J^\mu_{\text{str}}(x) J_{\text{str}\,\mu}(x) \\ &\quad + 2\bar{q}^{ig}(x)\gamma^\mu \tilde{q}^{ig}(x)\bar{\tilde{q}}^{ig}(x)\gamma_\mu q^{ig}(x)].\end{aligned} \tag{5.3}$$

The Hamiltonian (5.3) is "artificial,"[37] but it has the following possibilities. With multiple-gluon exchange or in a manner parallel to (2.18), $f_V$ may have a dependence on momentum transfers, in which case it will correspond to "hard collisions" of quarks and antiquarks.[38] It may yield results for elastic backward scattering, charge-exchange scattering, or, with the second term in (5.3), scattering of hadrons with creation of hadron pairs with fresh quantum numbers. However, it may also on the same measure yield couplings in violation of the OZI rule; at the present stage, it is not clear that such a contribution will be small. These processes will be much more complicated in nature, and will need further assumptions regarding the dynamical form of the naive interaction Hamiltonian (5.3), regarding the dependence of $f_V$ on $s$, $t$, or $u$ corresponding to the different processes.

Considering all this, it appears that calculating explicitly two-body exclusive strong interactions with some strong field-theoretic effective interaction of quarks will be useful in this field-theoretic language, which may be worth investigating. For example, it leads directly to quark-quark scattering amplitudes[39] or may indirectly lead to the assumptions of the constituent-interchange model[40] with meson-quark scattering, or even to effective quark fusion[41] brought in an *ad hoc* manner for many hadronic processes, particularly for large momentum-transfer phenomena. We note that in spite of the artificial assumptions which become necessary in the present case, bringing in here the language of quark field operators can be extemely rewarding. We believe that these may not only give explanations to specific problems, but may let us understand how the quarks really behave inside the hadrons, both in their rest frame and in arbitrary frames of reference, and possibly how "free" quarks behave under impulse approximation.

We would finally like to remark that our main interest has been to employ field-theoretic techniques with Lorentz boosting to high-energy scattering, with *phenomenological* quark field operator interactions taken as needed for this purpose. It is hoped that such a description may ultimately arise from, maybe, color gauge theory; our present phenomenological theory *is to find out what to expect* from any exact theory regarding wave functions and residual phenomenological interaction among quarks, assuming such an approach to be meaningful.

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[7]The basic ideas regarding this were contained in S. P. Misra in *Proceedings of the III High Energy Physics Symposium, Bhubaneswar,* 1976, edited by K. V. L. Sharma (Tata Institute of Fundamental Research, Bombay, 1976) Vol. II, p. 423.

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[14]We use:

$$S_n = (-i)^n \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{t_1} dt_2 \cdots \int_{-\infty}^{t_{n-1}} dt_n V(t_1) \cdots V(t_n),$$

where $V(t) = \int \mathcal{H}_I(\vec{x}t)d^3x$, and use space-time translational invariance. See Ref. 12. We do not have any quark propagators; but hadronic intermediate states may simulate hadronic propagators.

[15]See Ref. 8 and 9 for such calculations. These results have also been derived in Ref. 7 in a slightly different manner.

[16]See, e.g., C. W. Akerlof *et al.*, Phys. Rev. D 14, 2864 (1976), or Ref. 1.

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[22]We do not understand this aspect fully as long as we make use of the infinite constants.

[23]S. Gasiorowicz, *Elementary Particle Physics* (Wiley, New York, 1966). See also Ref. 9.

[24]It may be recognized that we are not introducing a basic theory here, but a phenomenological model which may be the result of interactions we do not at present understand. Effective interaction (3.43) is expected to result from a color Yang-Mills type gauge group or in some alternative manner. To be able to get the quark additivity property for such interactions for mesons, we need the form (3.43), which we have introduced in a phenomenological manner.

[25]We may compare this expression with that of Ref. 9.

[26]E. D. Dally *et al.*, Phys. Rev. Lett. 39, 1176 (1977).

[27]See, e.g., S. Dubnicka and O. V. Dumbraja, Phys. Lett. 53B, 285 (1974).

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[29]We distinguish the two schemes of Lorentz boosting by using the notations $Q^{L(p)}(x)$ and $Q^{(p)}(x)$ for the respective cases.

[30]We note that the present approach strongly reminds one of the quasipotential approach of A. A. Longunov and A. N. Tavkhelidze, Nuovo Cimento 29, 380 (1963). See also V. G. Kadyshevsky, R. M. Mirkasinov, and N. B. Skachkov, Nuovo Cimento 55A, 233 (1968), where the role of Lorentz transformation as a translation is noted in this context, although the corresponding transformations are different. The present authors have been *unable* to establish a close link between the present *ad hoc* rules and the corresponding theory of representations of the Lorentz group. This is mainly because of the complications of the Shapiro functions which replace Fourier transforms, and the authors want to continue to use Fourier transforms because of their simplicity. It may also be noted that if we have extremely heavy quarks with large binding energies,

but describe the wave functions of hadrons as we have, the matrix $\overleftrightarrow{L}$ will naturally become identity since the quark field operators will still have the same form as in Sec. II on Lorentz boosting.

[31]J. R. O'Fallon *et al.*, [Phys. Rev. Lett. 39, 733 (1977)] have observed this; however, further calculations will be needed to know whether what we propose is relavant here. Our objective here is to obtain the gross features of diffractive scattering only.

[32]G. Preparata, in *Proceedings of the 1974 Erice School*, edited by A. Zichichi (Academic, New York, 1975) p. 54; G. Preparata and N. Craigie, Nucl. Phys. B102, 478 (1976). See also F. C. Hoh, Phys. Rev. D 14, 2790 (1976); 14, 2801 (1976).

[33]We do not use the Bethe-Salpeter equations since the time degree of freedom creates difficulties. See, e.g., R. P. Feynman, M. Kislinger and F. Ravndal, Phys. Rev. D 3, 2706 (1971), discussions in Ref. 8.

[34]K. G. Wilson, Phys. Rev. D 10, 2445 (1975); J. Kogut and L. Susskind, Phys. Rev. D 9, 3501 (1974); A. De Rújula, H. Georgi, and S. L. Glashow, Phys. Rev. D 12, 147 (1975); E. Eichten *et al.*, Phys. Rev. Lett. 34, 369 (1975); A. M. Polyakov, Nucl. Phys. B120, 429 (1977). For a review see H. Joos, *Current Induced Reactions*, (Springer Berlin, 1975), p. 428, or H. Fritzsch, Lectures, International School of Physics "Enrico Fermi" Varenna, 1977, CERN Report No. Ref. TH-2359-CERN, 1977 (unpublished).

[35]We avoid using the Feynman theory because we regard the hadronic space as the physical vector space and the quark space as an underlying unphysical vector space, and so far, quarks have being meaningless except as constituents of hadrons. See Refs. 9,12, and 14.

[36]E.g., J. F. Gunion, S. J. Brodsky, and R. Blankenbecler, Phys. Rev. D 8, 287 (1973) take this in the infinite-momentum frame.

[37]See, e.g., N. Kemmer, Proc. Cambridge Philos. Soc. 34, 354 (1938) and W. Heitler, Proc. R. Irish Acad. Soc. A51, 33 (1946) who introduce conventional isospin.

[38]S. M. Berman, J. D. Bjorken, and J. B. Kogut, Phys. Rev. D 4, 3388 (1971).

[39]R. D. Field and R. P. Feynman, Phys. Rev. D 15, 2590 (1977).

[40]D. Sivers, S. J. Brodsky, and R. Blankenbecler, Phys. Rep. 23C, 1 (1976), and references therein.

[41]P. V. Landshoff and J. C. Polkinghorne, Phys. Rev. D 8, 4157 (1973).