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Production of orbitally excited charmed hadrons in e^+e^- annihilation

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New spin-counting predictions are derived for relative production rates of orbitally excited singly charmed mesons and baryons in e^+e^- annihilation. $c\bar{c}$ dominance is assumed and a covariant free-quark model based on U(6,6) is used with an l-breaking given by the generalized $^3P_0$ model. The predictions of this model for ground-state charmed hadrons are compared with existing results.

I. INTRODUCTION

The recent discoveries of charmed mesons in e^+e^- annihilation[1] lead to the hope that excited states of charmed mesons and charmed baryons may soon be found in such experiments. This makes it important to explore the predictions of simple quark models for such processes. An important simplification in this task is the $c\bar{c}$-dominance hypothesis[2] used by De Rújula, Georgi, and Glashow[3,4] (hereafter referred to as DGG). In this picture the virtual photon is assumed to produce a $c\bar{c}$ pair which then picks up light quarks and becomes a pair of charmed hadrons. There are two possible justifications for this picture.[4,5] One is based on the vector-dominance model: The energy region under consideration is much closer to the $c\bar{c}$ vector-meson poles than it is to the light-quark vector-meson poles. The other is based on asymptotic freedom: If the $c\bar{c}$ pair materializes first, then only a small amount of momentum needs to be transferred across to the light quarks and this can be accomplished by the long-range confinement forces; if on the other hand the light quarks are produced first, a large momentum must be transferred to the charmed quarks, and for such processes the effective strong coupling constant is small.

We shall assume the $c\bar{c}$-dominance picture throughout this paper and use a covariant form of the free-quark model which we shall call U(2,2), based on the U(6,6) scheme of Delbourgo *et al.*[6] The covariance property is obviously very important for discussing a pair production process but the internal symmetry aspects are less useful because of the huge mass splitting between the charmed and the other quarks. Hence, we will only consider *singly charmed*, *nonstrange* hadrons and use only spin-isospin symmetry. The results, however, can be trivially modified to deal with charmed hadrons with one strange quark by replacing isospin by U spin or V spin.[7] To treat the breaking of the symmetry by orbital excitations we shall use the generalized $^3P_0$ (GTPZ) model.[8] For decay processes this model is closely related to Melosh-based schemes[9] and lends itself to application in the cross channel. Calculations in this model, using the diagram method,[8] are very simple, as we hope to demonstrate.

In Sec. II we briefly describe the GTPZ model and the diagram method in the context of creation-channel processes. In Sec. III the production of ground-state mesons is used as a simple example of the method and we proceed to discuss the production of an $l=1$ excited meson with a ground-state meson. In Sec. IV we turn to the production of baryons, comparing our results with those of Matsuda[10] before going on to discuss excited-baryon production.

II. THE U(2,2)-GTPZ MODEL

A. U(6,6) and the generalized $^3P_0$ model

The original formulation of U(6,6) uses quark spinors $u_{a\alpha}$ labeled by an SU(3) index a ($a=1,2,3$) and a Dirac-spinor index α ($\alpha=1,2,3,4$). Meson states are constructed from a quark and an antiquark moving in tandem with equal momenta. Meson wave functions are, therefore, formed from two spinors combined together with Clebsch-Gordan (CG) coefficients to make states of definite spin. For the ground-state mesons the wave function is written as[11]

$$\Phi_{a\alpha}{}^{b\beta}(p)=\frac{1}{2\sqrt{2M}}\{(\not{p}+M)[\gamma_5 P^b_a-\gamma^\sigma(V^b_a)_\sigma]\}^\beta{}_\alpha\,, \quad (1)$$

where P^b_a represents the pseudoscalar nonet and $(V^b_a)_\sigma$ the vector-meson nonet, where the Lorentz index σ is attached to the spin-1 polarization vector. This is an elegant expression of the schematic form

$$\Phi(p;J,m)=\begin{pmatrix}\text{CG coefficient}\\ J=\tfrac{1}{2}\oplus\tfrac{1}{2}:m=\lambda_1+\lambda_2\end{pmatrix}\begin{pmatrix}\text{SU(3) CG}\\ \text{coefficient}\end{pmatrix}\times u^{(\lambda_1)}_{a\alpha}(p)\,v^{(\lambda_2)b\beta}(p) \quad (2)$$

for the spin-J, helicity-m member of the $\underline{35}$ multiplet.

Baryon wave functions are constructed similarly from three u spinors (see Ref. 8, Appendix 1.). Wave functions for orbitally excited hadrons are built up by adding in "spurions", which are spin-1 objects which symbolize the orbital angular momentum between the quarks. Thus [dropping the SU(3) labels] the wave function for the $l=1$ excited mesons is written

$$\Phi^{\beta}_{\alpha,\mu}=\frac{1}{2\sqrt{2}M}\left[(\not{p}+M)(\gamma_5 P_{,\mu}-\gamma^{\sigma}V_{\sigma,\mu})\right]^{\beta}_{\alpha}, \qquad (3)$$

where $P_{,\mu}$ and $V_{\sigma,\mu}$ are reduced to the usual polarization vectors for spin 0, 1, and 2 as follows:

$$P_{,\mu}=P_{\mu} \quad ({}^1P_1 \text{ states}),$$

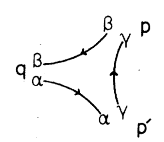
$$V_{\sigma,\mu}=V^{(2)}_{(\sigma\mu)}+\frac{i}{\sqrt{2}}\frac{p^{\alpha}}{M}\epsilon^{\beta}_{\alpha\sigma\mu}V^{(1)}_{\beta}+\frac{1}{\sqrt{3}}\left(g_{\sigma\mu}-\frac{p_{\sigma}p_{\mu}}{M^2}\right)V^{(0)} \qquad (4)$$

$({}^3P_2,\ {}^3P_1,$ and ${}^3P_0$ states$)$.

This wave function corresponds to a polarization vector $\xi^{\mu}(p)$, representing the orbital excitation, and two spinors $u(p)$ and $v(p)$ linked together with Clebsch-Gordan coefficients appropriate to an $(S\oplus\bar{S})\oplus l$ coupling scheme.

Hadronic vertices are obtained by tracing over the internal and Dirac indices of the wave functions: Each way of performing the trace corresponds to a quark diagram (see Fig. 1). This trace procedure is equivalent to calculating the diagram as a trivial Feynman diagram, associating the appropriate spinor with each external line and combining these spinors together with Clebsch-Gordan coefficients to make hadrons in the initial and final states.

When orbitally excited hadrons are involved the U(6,6) symmetry can only be maintained by contracting the spurion index with the momentum. In the generalized ${}^3P_0$ (GTPZ) model[8] the symmetry is broken by spurion interactions with the quarks in which the orbital-excitation index is contracted



$$=\bar{\Phi}_{\beta}^{\ \gamma}(p)\bar{\Phi}_{\gamma}^{\ \alpha}(p')\Phi_{\alpha}^{\ \beta}(q)$$

$$=(\text{C-G's})\times[\bar{V}^{\beta}(q)V_{\beta}(p)][\bar{U}^{\alpha}(p')U_{\alpha}(q)][\bar{U}^{\gamma}(p)V_{\gamma}(p')]$$

FIG. 1. Hadronic vertices in U(2,2) are obtained by traces over the wave functions involved. This is equivalent to contracting the spinors together in pairs in the way suggested by the diagram.

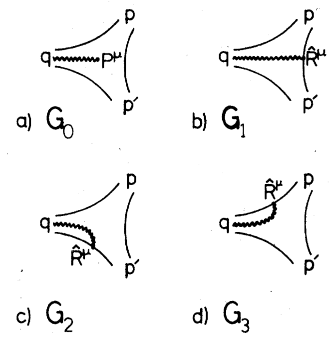


FIG. 2. The four amplitudes which, in the GTPZ model (see Ref. 8.), contribute to the decay of an $l=1$ excited meson into two ground-state mesons. The wavy line represents the spurion, i.e., the orbital excitation, $P^{\mu}=\frac{1}{2}(p^{\mu}-p'^{\mu})$, $\hat{R}^{\mu}=(2\sqrt{2}M)^{-1}i\gamma_5\gamma^{\alpha}\epsilon_{\mu\alpha\beta\gamma}p^{\beta}p'^{\gamma}$.

with $R_{\mu}\equiv i\gamma_5\gamma^{\alpha}\epsilon_{\mu\alpha\beta\gamma}p^{\beta}p'^{\gamma}$ inserted into any of the quark lines. Thus the decay of an $l=1$ excited meson into two $l=0$ mesons is parametrized in terms of four amplitudes (symmetry requirements reduce this to three, see later) according to which the quark interacts with the spurion (see Fig. 2).

The relationship of the GTPZ model to other models is discussed in Ref. 8. The ${}^3P_0$ model[12] corresponds to using only G_0 and G_1, amplitudes, while the results of Melosh-based approaches[9] for pion and photon transitions correspond to using G_0, G_1, and G_2 amplitudes (where the pion or photon couple to the p' quark lines in Fig. 2). This equivalence arises because in the Melosh scheme the pure γ^{μ} photon coupling is modified by $\gamma_{\perp}p_{\perp}$ factors arising from the transformation between constituent and current quarks. The $Y_{lm}(\Omega_p)$ dependence of the $l=1$ meson wave function and the identity

$$\left(\frac{3}{4\pi}\right)^{1/2}\frac{1}{|\vec{p}|}\int d\Omega_p Y_{1m}(\Omega_p)p_{\mu}=\xi^{(m)}_{\mu}(\hat{z}) \quad (m=\pm 1) \qquad (5)$$

turn these $\gamma_{\perp}p_{\perp}$ factors into spurion couplings to γ^{μ}, i.e., a mixture of P^{μ} and R^{μ} couplings to the active quark. [Equation (5) is the sense in which the spurion represents the orbital angular momentum, allowing one to ignore relative motions of the quarks even for excited mesons.]

The G_3 amplitude, involving the spectator quark, does not appear in the Melosh scheme, but is important in the GTPZ model, e.g., in pion transitions where the other final-state meson is also a member of the ground-state $\underline{35}$ multiplet. In this case symmetry requires the G_2 and G_3 amplitudes to have equal strength, thus reducing the number of independent amplitudes to three, as remarked earlier. For the present purposes the G_3 amplitude is an important feature of GTPZ since it gives the model a kind of crossing symmetry.

We shall now fix our attention on the creation channel $\gamma_v \to HH'$ and describe the diagram method[8] for GTPZ calculations.

B. The diagram method for $\gamma_v \to HH'$

First we describe the notation and conventions we adopt for the process $\gamma_v \to HH'$.[13] The anticharmed hadron (which carries the orbital excitation, if any) is produced with four-momentum $p^\mu = (E, 0, 0, k)$, with $M^2 = E^2 - k^2$, and is always depicted at the top of the diagram. The other hadron has four-momentum $p'^\mu = (E', 0, 0, -k)$, with $M'^2 = E'^2 - k^2$. We also define $q^\mu = p^\mu + p'^\mu$, $s = q^2$, and $P^\mu = \frac{1}{2}(p^\mu - p'^\mu)$. The z axis makes an angle θ with the beam axis along which the virtual photon is assumed to be purely transverse.[14]

To calculate a particular helicity amplitude one can perform the trace over two U(2,2) wave functions, such as (1) or (3), with an appropriate insertion for the photon coupling. Equivalently, one could use the spinor form, such as (2), and calculate the diagram by contracting spinors in pairs. This requires repeated use of results such as

$$\bar{u}^{(\lambda)}(p) v^{(\lambda')}(p') = (-1)^{1/2-\lambda} \frac{k}{M} \delta_{\lambda\lambda'}, \tag{6}$$

[This is the case $M = M'$. We remark later on the perennial problem of mass splittings in U(2,2).] Such results involve three factors (i) a sign, dependent on the quark helicities, (ii) a kinematic factor, and (iii) a Kronecker δ describing the spin structure. In any particular helicity amplitude the signs multiply up to produce an overall sign, dependent on the total helicity of one of the final-state hadrons, but not on the internal quark-spin configuration. The kinematic factors similarly multiply up to produce an overall kinematic factor K which is the same for a whole set of amplitudes. The heart of the amplitude lies in the spin structure and this can be calculated very simply, using wave functions constructed from Pauli spinors, written as $\uparrow$ or $\downarrow$ according to the J_z value (N.B. J_z and helicity values are the negatives of each other for quarks in the p' hadron).

Amplitudes involving excited hadrons require the use of results such as

$$\bar{u}^{(\lambda)}(p) R_\mu v^{(\lambda')}(p') = (-1)^{1/2+\lambda} \frac{2\sqrt{2} E k^2}{M} \times \xi_\mu^{(2\lambda')}(p) \delta_{\lambda,-\lambda'} \tag{7}$$

(assuming $M = M'$). This shows the convenience of R_μ as a symmetry-breaking term: It corresponds to a definite spin structure. If this R^μ is coupled to a spurion in the hadron p, i.e., to $\xi_\mu^{*(m)}(p)$, the only nonzero contributions come from spurions with $m = \pm 1 = 2\lambda'$. (Spurions coupling to P^μ only contribute for $m = 0$.) These considerations enable one to draw diagrams of allowed spin configurations which have local conservation of J_z. We shall say that the spins "match up" in these circumstances. Some examples are shown in Fig. 3.

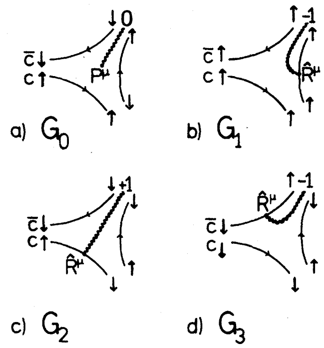


FIG. 3. Examples of allowed spin configurations for each of the four GTPZ amplitudes for $c\bar{c} \to$ excited meson + ground-state meson.

Because of the difference in the kinematic factors involved in $P \cdot \xi^*(p)$ and $R \cdot \xi^*(p)$ it is convenient to define the G_1, G_2, G_3 amplitudes in terms of $\hat{R}_\mu \equiv R_\mu / 2\sqrt{2}M$, so as to retain the result that each amplitude has the same overall kinematic factor. For the photon coupling to the $c\bar{c}$ pair we shall use a mixture of $\bar{P}_\mu$ and $\bar{R}_\mu$ couplings (because of the constituent/current quark distinction), where

$$\bar{P}_\mu = \frac{(M + M')}{(p \cdot p' - MM')} P_\mu, \tag{8}$$

$$\bar{R}_\mu = \frac{1}{(2s)^{1/2}} \frac{(M + M')}{(p \cdot p' - MM')} R_\mu, \tag{9}$$

so that the overall kinematic factor K is the same as that obtained from a γ_μ coupling. This is to ensure that the G's are free of kinematic singularities. We shall return to the question of the kinematic factor at the end of the next section in which we use ground-state meson production as an example of the calculational procedure.

III. MESON PRODUCTION

A. $\gamma_v \to$ ground-state mesons

$c\bar{c}$ dominance implies that the photon coupling can be written as an effective $c\bar{c}$ wave function[10]

$$\langle i | = \langle a D^{1*}_{\pm 1,0}(\theta)(\uparrow\downarrow + \downarrow\uparrow) + b(D^{1*}_{\pm 1,1}(\theta)\uparrow\uparrow + D^{1*}_{\pm 1,-1}(\theta)\downarrow\downarrow) | . \tag{10}$$

The parameters a and b (where $|a|^2 + |b|^2 = 1$) represent the strength of $\bar{P}_\mu$ and $\bar{R}_\mu$ couplings, respectively. Matching this wave function to those of the final-state mesons produces the helicity amplitudes $\Gamma^{\lambda\lambda'}$. For example, for $\gamma_v \to \overline{D}D$,

$$(\Gamma^{00}/G_0K)D^{1*}_{\pm1,0}(\theta)=\left\langle i\left|\frac{1}{\sqrt{2}}(\uparrow\downarrow-\downarrow\uparrow);\frac{1}{\sqrt{2}}(\uparrow\downarrow-\downarrow\uparrow)\right.\right\rangle$$

$$=\tfrac{1}{2}(1+1)aD^{1*}_{\pm1,0}(\theta)=aD^{1*}_{\pm1,0}(\theta)\,. \quad (11)$$

The two $\uparrow\downarrow$ terms match up with the $\uparrow\downarrow$ term in $|i\rangle$. The two $\downarrow\uparrow$ terms similarly match up with the $\downarrow\uparrow$ term in $|i\rangle$. [This spin configuration is that of Fig. 3(a), if one ignores the spurion line. The ordering convention is such that the spins read from top to bottom of the diagrams, i.e., $\langle\langle \overline{c}c|\overline{c}q;\overline{q}c\rangle\rangle$.] This procedure is repeated for all the helicity amplitudes to determine the coefficients α and β defined by

$$\alpha\equiv\frac{1}{2}\sum_\lambda|\Gamma^{\lambda\lambda}|^2/(|a|^2|G_0K|^2)\,, \quad (12)$$

$$\beta\equiv\frac{1}{2}\sum_\lambda(|\Gamma^{\lambda+1,\lambda}|^2+|\Gamma^{\lambda-1,\lambda}|^2)/(|b|^2|G_0K|^2)\,. \quad (13)$$

The general formula for the differential cross section is then given by[14]

$$\frac{d\sigma}{d(\cos\theta)}=\frac{4\pi\overline{\alpha}^2}{3s^2\sqrt{s}}k|G_0K|^2[\alpha|a|^2g_a(\theta)+\beta|b|^2g_b(\theta)]\,, \quad (14)$$

where

$$g_a(\theta)\equiv\tfrac{3}{4}\sin^2\theta\,,\quad g_b\equiv\tfrac{3}{8}(1+\cos^2\theta) \quad (15)$$

($\overline{\alpha}$ is the fine-structure constant). A slightly more convenient form is obtained using Matsuda's[10] $f(\theta)$ (also normalized to unity) defined by

$$f(\theta)\equiv|a|^2g_a(\theta)+|b|^2g_b(\theta)\,. \quad (16)$$

The differential cross section then reads

$$\frac{d\sigma}{d(\cos\theta)}=\frac{4\pi\overline{\alpha}^2}{3s^2\sqrt{s}}k|G_0K|^2[\beta f(\theta)+(\alpha-\beta)|a|^2g_a(\theta)]\,. \quad (17)$$

The results for $\gamma_v\to$ ground-state mesons are in Table I and these reproduce Matsuda's[10] results. An s-wave $c\overline{c}$ pair corresponds to $b/a=\sqrt{2}$, i.e., $|a|^2=\frac{1}{3}$, and in this case integrated cross sections become proportional to $(\alpha+2\beta)$ leading to the famous $\overline{D}D{:}\overline{D}D^*+\overline{D}^*D{:}\overline{D}^*D^*=1{:}4{:}7$.[3,4,15]

The overall kinematic factor K is found by multiplying the kinematic factors from the photon coupling, $\overline{P}\cdot\xi(q)$, and the two quark lines, $\overline{u}(p)v(p')$ and $\overline{u}(p')v(p)$, giving, for $M=M'$,[16]

$$K_{\text{mesons}}=\left(\frac{M}{k^2}k\right)\left(\frac{k}{M}\right)\left(\frac{k}{M}\right)=\frac{k}{M}\,. \quad (18)$$

TABLE I. Results for $\gamma_v\to$ ground-state mesons. α and β inserted into Eq. (14) or (17) give the differential cross section.

	α	β
$\overline{D}D$	$\frac{1}{2}$	0
$\overline{D}D^*$	0	$\frac{1}{2}$
$\overline{D}^*D^*$	$\frac{3}{2}$	1

For baryons, since there are three $\overline{u}v$ pairs, $K_{\text{baryons}}=(k/M)^2$. The mass splitting between members of the same U(2,2) multiplet, e.g., between D and D^*, causes the interpretation of K to be somewhat ambiguous, because one could use either some average multiplet mass or the actual, physical masses in evaluating K. Calculating K with nondegenerate masses one obtains

$$K_{\text{mesons}}=\frac{(M+M')}{2MM'}k\,, \quad (19)$$

$$K_{\text{baryons}}=\frac{(M+M')}{2MM'}\left(\frac{p\cdot p'-MM'}{2MM'}\right)^{1/2}k\,. \quad (20)$$

In the meson case the factor k [which comes from $P\cdot\xi(q)$] is necessary to satisfy the threshold condition for a p-wave final state,[17] in the baryon case it is required to evade the threshold constraints on the helicity amplitudes.[18] The ambiguity in the baryon case is whether $[\frac{1}{2}(pp'-MM')]^{1/2}$ behaves like a factor k (the physical three-momentum) or not, depending on the interpretation of M and M'.

Hence the ratios such as the 1:4:7 quoted above are to be understood in terms of reduced cross sections defined by removing n powers of k from the raw cross section, where $n=3$ for mesons (2 from $|K|^2$ and 1 from phase space) and $n=3$ or 5 for baryons. This is the conventional statement for mesons but differs from Matsuda[10] (where $n=1$) for baryons because, although there is no general consideration which forces the helicity amplitudes to be $O(k)$ as for mesons, there are threshold constraints which in U(2,2) are satisfied only by evasion.[19]

B. $\gamma_v\to$ excited meson + ground-state meson

In this section we consider the production of a meson pair where the anticharmed meson has an $l=1$ orbital excitation. The heavy mass of the charmed quark makes the usual $(\overline{S}_c\oplus S_q)\oplus l$ coupling scheme inappropriate, and a better approximation is to use wave functions constructed by $(S_q\oplus l)\oplus\overline{S}_c$ coupling.[3,4] The states are then labeled by J_{2j} where $j=S_q\oplus l$.

In the GTPZ model there are four amplitudes (illustrated in Fig. 3.), but if we neglect the spin-orbit couplings of heavy quarks, following DGG, we can ignore G_2 and G_3 amplitudes. The remaining symmetry-breaking term G_1 involves the "spectator" quark, i.e., it is the crossed version of the *non-Melosh* coupling. Of course in this channel the term "spectator" is less appropriate; there is an "audience participation" effect.

To illustrate the calculation of G_1 amplitudes consider the following helicity amplitude:

$$
\begin{aligned}
(\Gamma^{0,-1}/G_1K)D^{1*}_{\pm1,1}(\theta) &= \langle i\,|\,D^{**}(1_1{:}\lambda=0);D^*(\lambda'=-1(J_z=1))\rangle \\
&= \langle i\,|-\frac{1}{\sqrt{6}}[\sqrt{2}(\downarrow\downarrow,1)-\sqrt{2}(\uparrow\uparrow,-1)-(\downarrow\uparrow,0)+(\uparrow\downarrow,0)];\uparrow\uparrow\rangle \\
&= -\frac{1}{\sqrt{6}}(-\sqrt{2})bD^{1*}_{\pm1,1}(\theta)=+\frac{1}{\sqrt{3}}bD^{1*}_{\pm1,1}(\theta)\,.
\end{aligned}
\tag{21}
$$

The $(\uparrow\uparrow,-1);\uparrow\uparrow$ term matches up with the $\uparrow\uparrow$ term in $|i\rangle$. [This spin configuration is shown in Fig. 3(b)]. Notice that only spurion $J_z=\pm1$ terms contribute to G_1 amplitudes, and only spurion $J_z=0$ terms contribute to G_0 amplitudes.

The results of the calculation are shown in Table II, with g equal to the relative strength (G_1/G_0) of the G_1 amplitude. The processes divide into two classes according to the value of $j=S_q\oplus l$. For $j=\frac{1}{2}$ mesons $\alpha=\beta$ so that these processes all have an $f(\theta)$ dependence and we obtain a generalization of DGG's[3,4] result,

$$\bar{D}^{**}[1_1]D^*:\bar{D}^{**}[1_1]D:\bar{D}^{**}[0_1]D^*=2:1:1 \quad (\text{for all } \theta, |a|^2, g)\,. \tag{22}$$

The other processes, involving $j=\frac{3}{2}$ mesons, will have different angular dependences and the general result [obtainable from Eq. (17)] will depend on θ and $|a|^2$. However, if we assume s wave $c\bar{c}$ ($|a|^2=\frac{1}{3}$) and integrate over $\cos\theta$ we obtain

$$\bar{D}^{**}[2_3]D^*:\bar{D}^{**}[2_3]D:\bar{D}^{**}[1_3]D^*:\bar{D}^{**}[1_3]D = 4:1:2:1 \quad (\text{for } |a|^2=\tfrac{1}{3},\ \text{all } g)\,. \tag{23}$$

To relate the two sets of processes requires an assumption as to the value of g, the relative strength of the G_1 amplitude. DGG's result is that all the $j=\frac{3}{2}$ processes are forbidden, which corresponds to $g=\sqrt{2}$. This stems from their assumption that $S_q\oplus\bar{S}_q\oplus l=0$, where S_q and $\bar{S}_q$ are the spins of the light quark and antiquark and l is, as before, the orbital angular momentum between $\bar{c}$ and q in the excited meson. (It is easy to see that to satisfy this equation requires the particular $P_\mu,\hat{R}_\mu$ mixture given by $g=\sqrt{2}$.) The basis of the DGG assumption $S_q\oplus\bar{S}_q\oplus l=0$ seems somewhat obscure; in general one can only say that $S_q\oplus\bar{S}_q\oplus L=0$, where L is the orbital angular momentum between q and $\bar{q}$, and is not obviously related to l.

TABLE II. Results for $\gamma_v\to$ excited meson + ground-state meson. α and β inserted into Eq. (14) or (17) give the differential cross section. The parameter g is the relative strength of the G_1 amplitude, G_1/G_0. See text and Fig. 3.

	α	β
$\bar{D}^{**}[0_1];D$	0	0
$\bar{D}^{**}[0_1];D^*$	$\frac{1}{6}\lvert 1+\sqrt{2}g\rvert^2$	$\frac{1}{6}\lvert 1+\sqrt{2}g\rvert^2$
$\bar{D}^{**}[1_1];D$	$\frac{1}{6}\lvert 1+\sqrt{2}g\rvert^2$	$\frac{1}{6}\lvert 1+\sqrt{2}g\rvert^2$
$\bar{D}^{**}[1_1];D^*$	$\frac{1}{3}\lvert 1+\sqrt{2}g\rvert^2$	$\frac{1}{3}\lvert 1+\sqrt{2}g\rvert^2$
$\bar{D}^{**}[1_3];D$	$\frac{1}{6}\lvert\sqrt{2}-g\rvert^2$	$\frac{1}{24}\lvert\sqrt{2}-g\rvert^2$
$\bar{D}^{**}[1_3];D^*$	$\frac{1}{12}\lvert\sqrt{2}-g\rvert^2$	$\frac{5}{24}\lvert\sqrt{2}-g\rvert^2$
$\bar{D}^{**}[2_3];D$	0	$\frac{1}{8}\lvert\sqrt{2}-g\rvert^2$
$\bar{D}^{**}[2_3];D^*$	$\frac{5}{12}\lvert\sqrt{2}-g\rvert^2$	$\frac{7}{24}\lvert\sqrt{2}-g\rvert^2$

IV. BARYON PRODUCTION

A. $\gamma_v\to$ ground-state baryons

We now consider the production of a baryon-antibaryon pair. The construction of spin-flavor wave functions for singly charmed baryons is simpler than in U(6, 6) since only the spin-isospin symmetry of the light quarks need be considered. An immediate result, since $c\bar{c}$ is isosinglet, is that Λ_c's are only produced with $\bar{\Lambda}_c$'s and Σ_c's only with $\bar{\Sigma}_c$'s or $\bar{\Sigma}_c^*$'s.[3,5] As emphasized by Matsuda[5] this is a strong prediction of $c\bar{c}$ dominance. Another strong prediction is that the three charge modes of $\Sigma_c\bar{\Sigma}_c$ production contribute equally.[3,20] (The tabulated results are for a *single* charge mode.)

The U(2, 2) results (see Table III, with $r=0$) correspond to the general results of Matsuda with his parameter $c/d=+\sqrt{2}$. (c and d multiply the

TABLE III. Results for $\gamma_v \to$ ground-state baryons. α and β inserted into Eq. (14) or (17) give the differential cross section. r is a measure of the s/d ratio of the light diquark state (see text). $\bar{\Sigma}_c \Sigma_c$ results are quoted for a single charge mode.

	α	β
$\bar{\Lambda}_c;\ \Lambda_c$	1	1
$\bar{\Sigma}_c;\ \Sigma_c$	$\frac{1}{9}\lvert 3+r\rvert^2$	$\frac{1}{9}\lvert 1+r\rvert^2$
$\bar{\Sigma}_c^*;\ \Sigma_c$	$\frac{2}{9}\lvert r\rvert^2$	$\frac{2}{9}(3+\lvert 1+r\rvert^2)$
$\bar{\Sigma}_c^*;\ \Sigma_c^*$	$\frac{1}{9}(9+\lvert 3+2r\rvert^2)$	$\frac{2}{9}(3+2\lvert 1+r\rvert^2)$

helicity ±1 and helicity 0 components of the $Q\bar{Q}$ wave function in his approach, where Q denotes a spin-1 diquark.[21]) Notice that this value of c/d does not correspond to an s-wave $Q\bar{Q}$ pair (this would be $c/d = -\sqrt{2}$) and is, in fact, mostly ($\frac{8}{9}$) d wave, as found by Körner and Kuroda.[18] The origin of this slightly surprising result can be traced back to the charge-conjugation matrix: A straight pair of quark lines is obviously a relative s wave between the incoming Q and the outgoing Q; if we "bend" these lines to represent a $Q\bar{Q}$ pair springing from the vacuum, charge-conjugation changes the sign of the helicity zero component of the wave function. Thus a pair of bent quark lines in U(2,2) quark diagrams naturally corresponds to a $Q\bar{Q}$ state with $c/d = +\sqrt{2}$, rather than $-\sqrt{2}$.

To produce the most general $Q\bar{Q}$ state, i.e., with arbitrary c/d, one must allow for the exchange of angular momentum between the quarks making up the diquark. This would mean modifying the GTPZ model to include an H_0 amplitude corresponding to "spurion exchange" between the light quarks (see Fig. 4) in spin-1 diquarks. This H_0 amplitude can be defined with a suitably normalized $\tilde{R}_\mu = R_\mu / \{2[(p\cdot p')^2 - M^2 M'^2]\}^{1/2}$ to keep the overall kinematic factor the same as for G_0 amplitudes. Defining r as the relative strength (H_0/G_0) of this amplitude we obtain the results in Table III. Noting that H_0 amplitudes are only nonvanishing for helicity-0

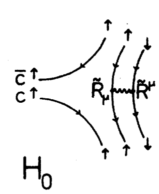


FIG. 4. Example of an allowed spin configuration for an 'H_0'' (spurion exchange) amplitude. (This amplitude is outside the pure GTPZ scheme of Ref. 8.)

diquarks, one easily sees that $r+1$ is to be identified with $\sqrt{2}\,d/c$. s-wave quarks correspond to $r=-2$ for which all processes have an $f(\theta)$ dependence. (It is amusing to note that in this case the total $G_0 + H_0$ amplitude corresponds to the exchange of an "object" with that mixture of $P_\mu, \tilde{R}_\mu$ couplings which, at threshold, corresponds to a γ_μ coupling.) The results for $r=-2$ give Matsuda's[10] generalization of DGG's[3,4] result:

$$\bar{\Lambda}_c\Lambda_c : \bar{\Sigma}_c\Sigma_c : \bar{\Sigma}_c^*\Sigma_c + \bar{\Sigma}_c\Sigma_c^* : \bar{\Sigma}_c^*\Sigma_c^* \text{ (all charge modes)}$$
$$= 3:1:16:10 \text{ (for all } \theta, |a|^2). \quad (24)$$

The corresponding pure U(2,2) results ($r=0$) will depend on θ and $|a|^2$, but integrating over $\cos\theta$ and assuming s-wave $c\bar{c}$ ($|a|^2 = \frac{1}{3}$) one finds the very different ratios[22]

$$9:11:32:38\,. \quad (25)$$

B. $\gamma_v \to$ excited baryon + ground-state baryon

It is now quite straightforward to treat excited baryon production. The heaviness of the charmed quark again makes the usual U(2,2) wave functions inappropriate and we have used wave functions constructed by $[(S_q' \oplus S_q)\oplus l]\oplus S_c$ coupling, so that l is the orbital angular momentum between the light diquark and the charmed quark. For Σ_c^{**} states $S_q \oplus S_q' \equiv S_Q = 1$ and the states may be labeled by $[J, J_Q]$ where $J_Q \equiv S_Q \oplus l$.

The results in Table IV are calculated using a spurion exchange, H_0, amplitude as well as a G_1 amplitude (see Fig. 5) though G_2 and G_3 amplitudes are neglected as before. Again the processes fall into classes according to the value of J_Q. We see immediately that

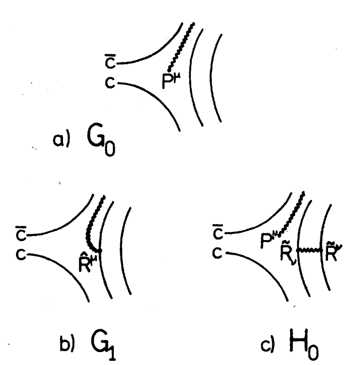


FIG. 5. The three amplitudes considered for $c\bar{c} \to$ excited baryon + ground-state baryon. The spurion-exchange amplitude H_0 is outside the pure GTPZ scheme. (Symmetric insertion of spurion exchange in the G_1 amplitude simply reproduces the existing structure, due to the property that $\{R_\mu, R_\nu\}$ is proportional to the identity Dirac matrix.)

TABLE IV. Results for $\gamma_v \to$ excited baryon + ground-state baryon. α and β inserted into Eq. (14) or (17) give the differential cross section. r and g are the relative strengths of the H_0 and G_1 amplitudes.

	α	β
$\bar{\Lambda}_c[\frac{1}{2}];\Lambda_c$	$\frac{1}{3}$	$\frac{1}{3}$
$\bar{\Lambda}_c[\frac{3}{2}];\Lambda_c$	$\frac{2}{3}$	$\frac{2}{3}$
$\bar{\Sigma}_c[\frac{1}{2},0];\Sigma_c$	$\frac{1}{9}\lvert 1+r+\sqrt{2}g\rvert^2$	$\frac{1}{9}\lvert 1+r+\sqrt{2}g\rvert^2$
$\bar{\Sigma}_c[\frac{1}{2},0];\Sigma_c^*$	$\frac{2}{9}\lvert 1+r+\sqrt{2}g\rvert^2$	$\frac{2}{9}\lvert 1+r+\sqrt{2}g\rvert^2$
$\bar{\Sigma}_c[\frac{1}{2},1];\Sigma_c$	$\frac{1}{9}\lvert\sqrt{2}+g\rvert^2$	0
$\bar{\Sigma}_c[\frac{1}{2},1];\Sigma_c^*$	$\frac{1}{18}\lvert\sqrt{2}+g\rvert^2$	$\frac{1}{6}\lvert\sqrt{2}+g\rvert^2$
$\bar{\Sigma}_c[\frac{3}{2},1];\Sigma_c$	$\frac{1}{18}\lvert\sqrt{2}+g\rvert^2$	$\frac{1}{6}\lvert\sqrt{2}+g\rvert^2$
$\bar{\Sigma}_c[\frac{3}{2},1];\Sigma_c^*$	$\frac{5}{18}\lvert\sqrt{2}+g\rvert^2$	$\frac{1}{6}\lvert\sqrt{2}+g\rvert^2$
$\bar{\Sigma}_c[\frac{3}{2},2];\Sigma_c$	$\frac{5}{18}\lvert\sqrt{2}-g+\frac{2\sqrt{2}r}{5}\rvert^2$	$\frac{1}{90}(3\lvert\sqrt{2}-g\rvert^2+4\lvert\sqrt{2}-g+\sqrt{2}r\rvert^2)$
$\bar{\Sigma}_c[\frac{3}{2},2];\Sigma_c^*$	$\frac{1}{20}\lvert\sqrt{2}-g\rvert^2+\frac{1}{180}\lvert\sqrt{2}-g+4\sqrt{2}r\rvert^2$	$\frac{1}{6}\lvert\sqrt{2}-g\rvert^2+\frac{4}{45}\lvert\sqrt{2}-g+\sqrt{2}r\rvert^2$
$\bar{\Sigma}_c[\frac{5}{2},2];\Sigma_c$	$\frac{2}{15}\lvert r\rvert^2$	$\frac{1}{15}(2\lvert\sqrt{2}-g\rvert^2+\lvert\sqrt{2}-g+\sqrt{2}r\rvert^2)$
$\bar{\Sigma}_c[\frac{5}{2},2];\Sigma_c^*$	$\frac{1}{5}\lvert\sqrt{2}-g\rvert^2+\frac{3}{10}\lvert\sqrt{2}-g+\frac{2\sqrt{2}r}{3}\rvert^2$	$\frac{1}{6}\lvert\sqrt{2}-g\rvert^2+\frac{2}{15}\lvert\sqrt{2}-g+\sqrt{2}r\rvert^2$

$$\left.\begin{aligned}\bar{\Lambda}_c[\tfrac{3}{2}]\Lambda_c:\bar{\Lambda}_c[\tfrac{1}{2}]\Lambda_c=2:1\\ \bar{\Sigma}_c[\tfrac{1}{2},0]\Sigma_c^*:\bar{\Sigma}_c[\tfrac{1}{2},0]\Sigma_c=2:1\end{aligned}\right\}\ (\text{for all } \theta, |a|^2, r, g)\,. \tag{26}$$

The other general results are more complicated and depend on θ. If we specialize to $|a|^2=\frac{1}{3}$ (s-wave $c\bar{c}$) and integrate over $\cos\theta$ we obtain

$$\begin{aligned}\bar{\Sigma}_c[\tfrac{3}{2},1]\Sigma_c^*:\bar{\Sigma}_c[\tfrac{3}{2},1]\Sigma_c:\bar{\Sigma}_c[\tfrac{1}{2},1]\Sigma_c^*:\bar{\Sigma}_c[\tfrac{1}{2},1]\Sigma_c\\ =11:7:7:2\ \ (\text{for } |a|^2=\tfrac{1}{3}, \text{all } r,g).\end{aligned} \tag{27}$$

The equality of the middle two processes in fact holds at all angles, independently of $|a|^2$.

In the pure GTPZ case ($r=0$) we can also obtain (for $|a|^2=\frac{1}{3}$ and integrated over $\cos\theta$)

$$\begin{aligned}\bar{\Sigma}_c[\tfrac{5}{2},2]\Sigma_c^*:\bar{\Sigma}_c[\tfrac{5}{2},2]\Sigma_c:\bar{\Sigma}_c[\tfrac{3}{2},2]\Sigma_c^*:\bar{\Sigma}_c[\tfrac{3}{2},2]\Sigma_c\\ =33:12:17:13\ \ (\text{for } |a|^2=\tfrac{1}{3},\ r=0, \text{ all } g).\end{aligned} \tag{28}$$

These are some of the simpler special cases of our results. One might ask what approximations would be "in the spirit of DGG"? We cannot follow the analogy with excited mesons and require $S_Q\oplus\bar{S}_Q\oplus l=0$, since parity forces $S_Q\oplus\bar{S}_Q$ to be even and $l=1$ for the excited baryons considered here. Presumably the "spirit of DGG" assumption would be $r=-2$, since this corresponds to s-wave diquarks in the ground-state baryon case. Of the results quoted above only (28) is changed: The ratios now have a complicated dependence on g. However, if we were to set $g=0$, for example, these ratios would become 5:4:5:1. Thus the $J_Q=2$ processes are very sensitive to the values of r and g. This is also true of relationships between processes of different J_Q.

V. DISCUSSION

We have compared U(2,2) results for the production of ground-state charmed hadrons in e^+e^- annihilation with existing results and derived new predictions for excited charmed hadrons, including their angular dependence. As emphasized by Close[23] the angular dependence is important when the detector has finite acceptance in $|\cos\theta|$. These spin-counting predictions are of course a continuum result and cannot be expected to apply close to $c\bar{c}$ resonances. In particular, the failure of the 1:4:7 ratios of DGG at 4.028 GeV (Ref. 24) cannot be regarded as a fair test of that prediction and can be understood in terms of the *spatial* structure of the 3s charmonium wave function.[25] Excited mesons and baryons should have thresholds far enough above ψ resonances for such considerations to be unimportant.

Other dynamical effects have been neglected here and a possible extension is to use form factors obtained from U(2,2) with an energy dependence suggested by the generalized vector-dominance model (cf. Ref. 26). In view of the difficulty mentioned earlier, of the dependence on k, the three-momentum of the charmed hadrons, it is interesting to note a suggestion of Humpert and Clark[27] that comparisons of cross sections should be made at equal k values rather than at equal s values.

Apart from presenting the results themselves, a second aim of this paper has been to emphasize the convenience of the U(2, 2)-GTPZ model as a framework for such calculations. For example, it would be straightforward to calculate amplitudes for the light-quark contribution if the $c\bar{c}$-dominance hypothesis were found not to be a good approximation: One could even include the effects of single-gluon exchange between the light- and heavy-quark pairs as suggested by the asymptotic-freedom argument.

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