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Monte Carlo approach to multiparticle production in a quark-parton model. III. e^+e^- annihilation and quark fragmentation functions

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A Monte Carlo quark-parton model of multiparticle production in e^+e^- annihilation is presented. The model is based on the assumption that the polarization cloud in e^+e^- annihilation behaves in a similar way as the parton system near $y^* \approx 0$ in hadronic collisions. The model is constructed in a close analogy with our previous work on multiple production in hadron collisions. With all the parameters fixed by data on hadron collisions the model gives reasonable results for e^+e^- annihilation. Quark fragmentation functions are extracted from our results on particle production in e^+e^- annihilation. Our results for fragmentation of quarks to mesons are similar to those obtained reecently by Field and Feynman in a different model. Scale breaking of quark fragmentation functions following from our model is discussed. Predictions for fragmentation of quarks to baryons are presented.

I. INTRODUCTION

In the two preceding papers[1,2] of this series (referred to as I and II in what follows) we have proposed and described in detail a Monte Carlo quark-parton model of multiparticle production in hadron-hadron collisions. Taking into account the constraints imposed by space-time evolution[3,4] of the collision, the model gave a reasonable description[5,6] of the low-mass[7] dimuon production in hadronic collisions.

In the present paper we shall pursue further the approach of I and II and we shall construct a Monte Carlo quark-parton model for multiparticle production in e^+e^- annihilation. The philosophy of the present series of papers is rather simple. At this stage we want to compare one simple model with various sets of data, hoping that in this way we shall be able to locate those points in the model which require modifications and that at the same time we shall find those pieces of data which are most sensitive to such modifications. Because of that we made no attempts at quantitative fits of the data by introducing additional parameters, but we rather kept one set of parameters fixed and looked at the qualitative agreement (or disagreement) of the model with the data. We think that a more detailed specification of the model will make sense only after having compared it on such a qualitative level with a broad and representative sample of the data on different reactions.

To start with we have to show why we think that it is possible to apply the model of I and II, originally designed for hadronic collisions, to e^+e^- annihilation. We shall first briefly describe the assumed pictures of both processes.

The hadron-hadron collision is assumed to be initiated[3,4,8] by the interaction of wee partons at $y \approx 0$ [in the present paper all quantities refer to the center-of-mass system (c.m.s.)]. The excited region at $y \approx 0$ then cools down by emitting hadrons and by exciting neighboring regions in rapidity. The emission of hadrons is assumed to proceed through the intermediate stage in which the gluons are converted to $Q\bar{Q}$ pairs and hadrons are subsequently formed by the recombination of $Q\bar{Q}$ pairs to mesons and of QQQ and $\bar{Q}\bar{Q}\bar{Q}$ triplets to baryons and antibaryons. In fact there is some sea consisting of Q's and $\bar{Q}$'s present in the colliding hadrons already before the collision, but the magnitude of the sea created during the collision (by the conversion of gluons) is about 5 times larger than the original one.[1,9]

In the model of I and II we start with generating configurations of quarks (Q's) and antiquarks ($\bar{Q}$'s) before their recombination to hadrons. The Q's and $\bar{Q}$'s are generated according to the cylindrical phase space, with an additional factor which forces valence quarks to keep large momentum fractions. The Q's and $\bar{Q}$'s which are nearby in rapidity then recombine to resonances from the SU(6) 35-plet of mesons and 56-plets of baryons and antibaryons. Resonances then decay to stable particles. The short-range nature of the recombination makes the model consistent with the assumed picture of the space-time evolution of the collision.[10]

According to Feynman,[8] Casher, Kogut, and Susskind,[11] and Bjorken[3] the process $e^+e^- \to$ hadrons starts with creation of a Q and an $\bar{Q}$ moving in opposite directions. Subsequently, a polarization cloud[3,11] is formed at $y \approx 0$. The cloud expands, its central part cools down by emitting hadrons, and the front parts finally reach the originally produced Q and $\overline{Q}$.

It seems that at presently available energies the general features of $e^+e^- \rightarrow$ hadrons and of hadronic collisions are rather similar.[12,13] We shall therefore assume that (in what concerns the multiparticle production) the "polarization cloud" in e^+e^- annihilation behaves similarly to the "central region" in multiple production in hadron collisions. In particular, we shall assume that the energy of the polarization cloud is first converted to Q's and $\overline{Q}$'s and the hadrons are formed by the recombination process, which is similar to that occuring in hadronic collisions.

Following I and II we shall again make no attempt at decribing the whole process but we shall instead generate the distribution of Q's and $\overline{Q}$'s before the recombination. We hope the short-range character of the recombination makes such a simple and naive picture consistent with the assumed space-time evolution of the process.

In what concerns the role ascribed to gluons, our model is admittedly oversimplified. (In principle, gluons produce mesons also "directly", i.e., without converting first to $Q\overline{Q}$ pairs.) This is because we do not know how to calculate such "direct" contributions. In this respect we accept the attitude expressed by Feynman in discussion following his recent talk.[14] According to this view, the best way how to see the role played by the gluons is to ascribe to them the most modest role possible and to see what happens.[15]

Various features of multiparticle production in hadronic collisions have been recently successfully described by models[16-18,20] based on the idea[21,22] that hadrons in the final state are originated by the recombination of Q's and $\overline{Q}$'s.

In the present paper we advocate the extension of this idea to multiple production in e^+e^- annihilation. It is sometimes claimed[17,19] that multiparticle production in e^+e^- annihilation and in hadronic collisions is caused by two basically different mechanisms. In the former case the multiple production is assumed to be given by the fragmentation of the originally created Q and $\overline{Q}$, whereas in the latter the particle production proceeds via the recombination. In our opinion, making deep differences between the two mechanisms is misleading. The term "quark fragmentation" in fact denotes the result (and not the dynamics) of the process starting with a Q (or an $\overline{Q}$) separated by a large rapidity gap from the rest of the system.

We shall show below that a "recombination mechanism" for e^+e^- annihilation leads to "fragmentation functions" which agree (at least qualitatively) with the available information as summarized by Field and Feynman.[23]

The paper is organized as follows. In the next section we describe in some detail our model. In Sec. III we compare our results with the data on multiparticle production in e^+e^- annihilation. The calculations are performed separately for the three initial configurations.

$$e^+e^- \rightarrow u\overline{u}\,, \tag{1a}$$

$$e^+e^- \rightarrow d\overline{d}\,, \tag{1b}$$

$$e^+e^- \rightarrow s\overline{s}\,, \tag{1c}$$

and the results are weighted by squares of the quark charges and summed up to give the multiparticle production in e^+e^- annihilation. It is to be stressed that in this way we obtain only that part of e^+e^- annihilation which proceeds via the three configurations (1). Because of that we can make a comparison with the data only in the region where the ratio $R = (e^+e^- \rightarrow \text{hadrons})/(e^+e^- \rightarrow \mu^+\mu^-)$ is close to 2 (for instance, at $W = \sqrt{s} = 3$ GeV, where R is about 2.5).

At higher energies one would need also the initial configurations $e^+e^- \rightarrow c\overline{c}$ and $e^+e^- \rightarrow \tau^+\tau^-$. The latter could be estimated, but the former one presents serious problems due to threshold effects and rather complicated many-particle decays of charmed hadrons.

The results obtained for multiparticle production in reactions (1) give directly quark fragmentation functions. For instance, if in the reaction (1a) the up quark moves (at the very beginning) to the right, then

$$D_u^{\pi^+}(z) \equiv N_u^{\pi^+}(z) \equiv \frac{1}{\sigma}\,\frac{d\sigma_{\pi^+}}{dz} \tag{2}$$

is the function describing the fragmentation of the up quark to π^+ (z is the ratio of the longitudinal c.m.s. momenta of the π^+ and the originally produced up quark). Fragmentation functions obtained in this way are presented in Sec. IV and compared with those found recently by Field and Feynman.[23,24]

In our model we take explicitly into account transverse momenta and effective masses of quarks and we perform all the calculations at finite energies. Because of that our fragmentation functions show scale-breaking effects for small z and differ here from Field-Feynman fragmentation functions.

In Sec. V we discuss the question[25] of quark-quantum-number retention (QQNR). It is pointed out that our fragmentation functions lead at high energies to full QQNR. The reason for this is simply that our model includes also the production of baryons and antibaryons, differing in this respect from numerous models[26] discussed in the literature.

Comments and conclusions are presented in Sec. VI.

II. THE MODEL OF MULTIPARTICLE PRODUCTION IN e^+e^- ANNIHILATION

The model is constructed in close analogy to our model for multiparticle production in hadronic collisions. We shall therefore start with discussing briefly the latter and then we shall specify the modifications necessary for the e^+e^- case.

In our model (for details see I and II) we make no attempt to describe the whole space-time evolution of the hadronic collision. Instead we make a model which in a sense should roughly represent the whole evolution by a single picture of the distribution of Q's and $\bar{Q}$'s prior to the recombination (sort of a stroboscopic picture of the whole process). The Q's and $\bar{Q}$'s contained in a system formed by the two colliding protons consist of the six valence quarks (three for each proton), from Q's and $\bar{Q}$'s from the two seas presented in colliding protons before the collision and from $Q\bar{Q}$ pairs coming from the conversion of gluons during the space-time evolution of the process. These Q's and $\bar{Q}$'s are distributed according to the cylindrical phase space and valence quarks are pushed by a simple prescription to the corresponding ends of the available rapidity region. Such a model thus takes into account the energy-momentum conservation (at the level of Q's and $\bar{Q}$'s), the cutoff on transverse momenta, and the tendency of valence quarks to keep their momenta during the collision.

The probability of finding six valence quarks with rapidities and transverse momenta $y_1, p_{T1}, y_2, p_{T2}, \ldots, y_6, p_{T6}$, n quarks with y_i, p_{Ti} $(i=7,\ldots,n+6)$, and n antiquarks with y_j, p_{Tj} $(j=n+7,\ldots,2n+6)$ is given by the expression

$$dP_N(y_1,p_{T1},\ldots,y_N,p_{TN}) = KW_{id}G^n\left(\prod_1^6 \sqrt{|x_i|}\right) \times \exp\left(-\sum_1^N p_{Ti}^{\,2}/R_T^{\,2}\right)\delta\left(\sum_1^N \vec{p}_{Ti}\right)\delta\left(\sum_1^N p_{Li}\right)\delta\left(E-\sum_1^N E_i\right)\prod_1^N (dy_i d^2p_{Ti}) . \tag{3}$$

Here K is the normalization factor, independent of energy, W_{id} is the factor for identical particles (see I), G is "the coupling constant" regulating the average multiplicity of additional $Q\bar{Q}$ pairs and thereby also the multiplicity of final-state particles, and $R_T^{\,2}$ specifies the cutoff on transverse momenta of Q's and $\bar{Q}$'s prior to recombination (note that these transverse momenta are not necessarily identical to those of Q's and $\bar{Q}$'s in a free proton[5,27]). The factor $\sqrt{|x_i|}$ for each valence quark[28] gives a larger probability for configurations in which valence quarks have rather large momentum fractions.

In assigning the quantum numbers to nonvalence Q's and $\bar{Q}$'s we have suppressed the s and $\bar{s}$ quarks by a phenomenological factor $\lambda = P(s)/P(u) = P(s)/P(d)$, where, for instance, $P(u)$ is a probability that a given nonvalence quark is of the u type.

After having generated the "initial" configuration of Q's and $\bar{Q}$'s, the program recombines the neighbors in rapidity to mesons and baryons. The prescription for recombination is contrived so as to avoid large rapidity gaps between the recombining partons (details are given in I). The momentum of the product of the recombination is set equal to the vector sum of momenta of the recombining partons.

In the next step the program specifies the hadron which is being formed for a particular $Q\bar{Q}$, QQQ, or $\bar{Q}\bar{Q}\bar{Q}$ combination. Probabilities are given by squares of the coefficients in wave functions of hadrons within the SU(6) scheme, averaged over spins of initial partons and summed over spins of hadrons being formed. In the final step, resonances decay to stable particles. Branching ratios are taken from Particle Data Group tables.

In the present paper we apply this model to the multiparticle production in e^+e^- annihilation. The crucial assumption here is that the "polarization cloud" in the e^+e^- annihilation behaves similarly as the central region in hadronic collisions.

Using the same model for both e^+e^- annihilation and for hadronic collisions is motivated by the desire to learn whether there are some basic differences between multiple production in the two cases. By using the same model and studying in detail its consequences we can see what pieces of data, and in what respect, indicate differences between underlying dynamics. In the model of multiple production in e^+e^- annihilation we thus start with the following "stroboscopic" distribution of Q's and $\bar{Q}$'s prior to the recombination:

$$dP_N(y_1,\vec{p}_{T1},\ldots,y_N,\vec{p}_{TN})=K'W_{id}G^n\left(\prod_1^2\sqrt{|x_i|}\right)\exp\left(-\sum_1^N p_{Ti}^2/R_T^2\right)$$
$$\times\delta\left(\sum_1^N\vec{p}_{Ti}\right)\delta\left(\sum_1^N p_{Li}\right)\delta\left(E-\sum_1^N E_i\right)\prod_1^N(dy_i d^2p_{Ti}),\quad N=2+2n. \tag{4}$$

The originally produced Q and $\overline{Q}$ are in the present model treated in the same way as the valence quarks in a hadronic collision.

The factor for identical particles W_{id}, the coupling constant G (specifying the average multiplicity), the p_T-cutoff parameter R_T^2, and the factor λ suppressing the production of $s\overline{s}$ pairs are assigned values obtained in the analysis of multiparticle production in hadronic collisions.

The momentum distribution of the originally produced (leading) Q and $\overline{Q}$ just prior to the recombination depends on details of the dynamics of e^+e^- annihilation and in particular on what happends when the "polarization cloud" is catching the leading Q and $\overline{Q}$. Supposing that these Q and $\overline{Q}$ behave similarly to valence quarks in hadronic collisions is a hypothesis which, within the present framework, corresponds to the closest possible similarity between the two processes. Following the philosophy mentioned in the introduction we shall consider here only this simplest possibility.

III. COMPARISON OF THE MODEL WITH DATA ON e^+e^- ANNIHILATION

We shall start with comparing the results of our model with data on $e^+e^-\to$ hadrons at the c.m.s. energy $W=3$ GeV. At this energy the ratio $R=(e^+e^-\to\text{hadrons})/(e^+e^-\to\mu^+\mu^-)$ is equal[29] to 2.6 ± 0.2 and one can therefore hope that most of the events proceed via (1a), (1b), or (1c).

The parameters of our model [see Eq. (3)] were fixed at the following values: $G=1.47$, $\lambda=0.38$, $R_T^2=0.20$ GeV$^2/c^2$, which were obtained in our analysis of multiparticle production in hadronic collisions, and the constant K' is fixed by the requirement that the sum of probabilities $P_N(y_i,\vec{p}_{Ti})$ in Eq. (4) is equal to one. Our model gives in this way the expressions for $(1/\sigma)d\sigma/da\,db\cdots$, where $a,b,\cdots$ are variables of interest. In calculating the cross sections we have used the cross section σ corresponding to $R=2$ of the simple quark-parton model. The recombination of $Q\overline{Q}$, QQQ, and $\overline{Q}\overline{Q}\overline{Q}$ to hadrons and decays of resonances to final-state particles was treated exactly as in our model of multiple production in hadronic collisions.[1,2] In this way our model contains no free parameters.

In Fig. 1 we show the comparison of our results with the inclusive production of charged hadrons in e^+e^- annihilation at $W=3$ GeV. It is not surprising that we are somewhat below the data at $x_p=0.2-0.3$, where most of the cross section comes from. In fact, by multiplying our results by $2.6/2=1.3$ (the ratio of experimental to naive quark-model value of R) we could reach better agreement with the data, but this would be somewhat unfair since our model can describe only that part of the e^+e^- annihilation which corresponds to $R=2$.

Our results are also significantly below the data in the region of large x_p. This could be remedied by introducing into Eq. (4) matrix elements pressing the additional $Q\overline{Q}$ pairs to lower values of x. In our opinion such attempts are at present premature since there is a well known discrepancy between the SLAC and DESY data at large x_p. In fact, the DESY data[30] are slightly below our results at large values of x_p.

Figure 2 shows that our model reproduces reasonably the multiplicity distribution of charged particles at $W=3$ GeV.

In Fig. 3 we compare our results on $s d\sigma/dx_p$ with the SPEAR data[31] at $W=4.8$ GeV. The comparison is rather inconclusive since the SPEAR data are normalized to the experimental value $R=5.5$,

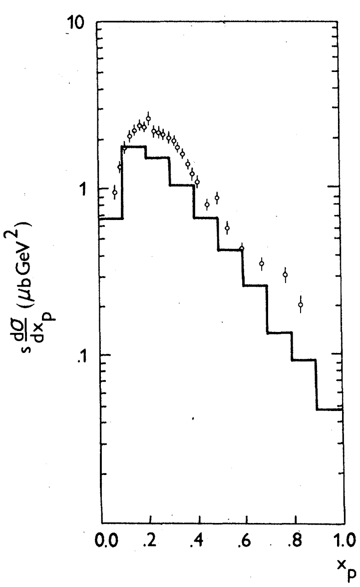


FIG. 1. The comparison of our calculations (histogram) with the SPEAR data (Ref. 29) on $sd\sigma/dx_p$ for charged hadrons at $W=3$ GeV. Note the difference in normalization; our results correspond to $R=2$, the data to the experimental value (Ref. 29) $R\approx2.6$. Here $x_p=2p/W$.

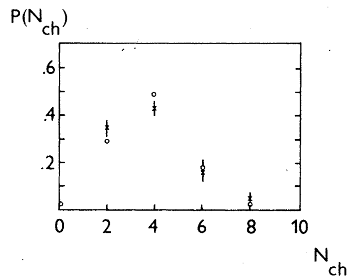


FIG. 2. Multiplicity distribution of charged particles in e^+e^- annihilation at $W=3$ GeV. The data (Ref. 32) are denoted by crosses, our results by open circles.

whereas our results correspond to $R=2$. If we multiplied our results by 5.5/2 we could reach much better agreement with the data, but taking that seriously would mean to assume that e^+e^- annihilations proceeding via (1) behave similarly to that due to the sum of other mechanisms ($c\bar{c}$ or $\tau^+\tau^-$ production) and this can perhaps be trusted only on a very rough qualitative level.

The same remarks apply also to the comparison of our results and the data on $sd\sigma/dx_p$ and $(1/\sigma)d\sigma/dp_T$ at $W=7.4$ GeV shown in Figs. 4 and 5.

It is to be noted that the data[32] on $(1/\sigma)d\sigma/dp_T$ correspond only to a selected class of events, namely, those in which one of the hadrons in the final state has $x_p>0.5$. Such a selection together with possible inaccuracies in the determination of the jet axis can modify to some extent the ex-

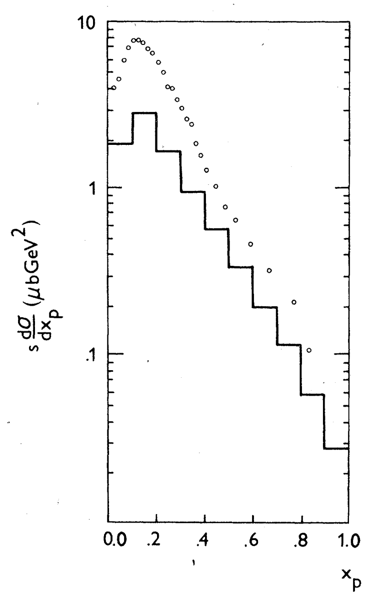


FIG. 3. The data (Ref. 32) (open circles) on $sd\sigma/dx_p$ at $W=4.8$ GeV and results of our model (histogram). Note the difference in normalization [data (Ref. 29): $R\approx 5.5$, our model: $R=2$].

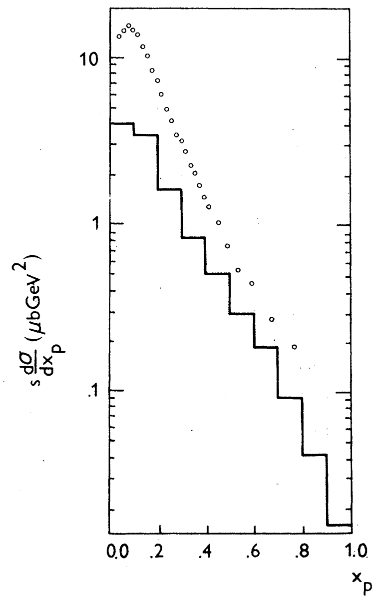


FIG. 4. The data (Ref. 31) (open circles) on $sd\sigma/dx_p$ at $W=7.4$ GeV and results of our model (histogram). The same difference in normalization as in Fig. 3.

perimental results on $(1/\sigma)d\sigma/dp_T$.

Still, one of the simplest ways to see the similarity between e^+e^- annihilation and the hadronic collisions is to look at the p_T distribution in both cases. According to our model, the p_T distributions in both cases should be rather similar except perhaps for the sea-gull effect present in hadronic collisions.[33]

More information about the mechanism of e^+e^-

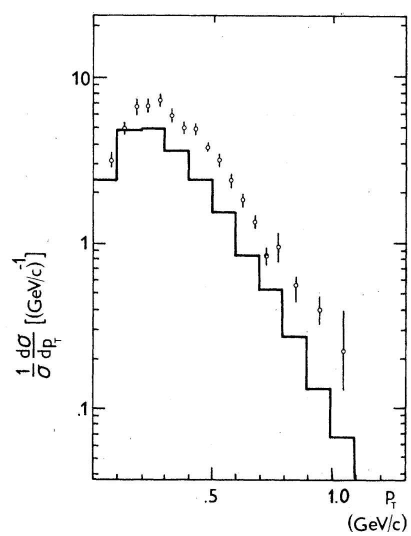


FIG. 5. The data (Ref. 32) (open circles) on $(1/\sigma)(d\sigma/dp_T)$ at $W=7.4$ GeV and results of our model (histogram).

annihilation can be obtained from the inclusive spectra of identified final-state hadrons. The data are, however, rather scarce at $W=3$ GeV and at higher energies the data on strange particle production are seriously modified by the $e^+e^-\to c\bar{c}$ channel which is not included in the present version of our model. We shall therefore make no attempts in this direction and we shall rather compare our results with the available information on fragmentation functions. This will be done in the following section.

IV. FRAGMENTATION FUNCTIONS OF QUARKS FOLLOWING FROM OUR MODEL OF e^+e^- ANNIHILATION

The information on quark fragmentation functions comes from various sources including e^+e^- annihilation, deep-inelastic lepton-nucleon scattering, and large-p_T phenomena. Instead of trying to compare our results with various pieces of data we shall take here as a standard of reference the recent work by Field and Feynman[23] (FF).

Our Monte Carlo model gives quark fragmentation functions in a direct way, indicated in Eq. (2). The model takes into account the energy-momentum conservation laws, transverse momenta of Q's and $\bar{Q}$'s, and finite effective masses of quarks.[34] All these factors lead to some scale breaking.

In contradiction to most of the previous work on the subject,[23,35] we obtain also results for fragmentation of quarks to baryons.

Our results for fragmentation of the up quark to π^+ and π^- are compared with the results of FF[23] in Fig. 6. Taking into account that our model contains no free parameters, the agreement is surprisingly good. The scale-breaking effects

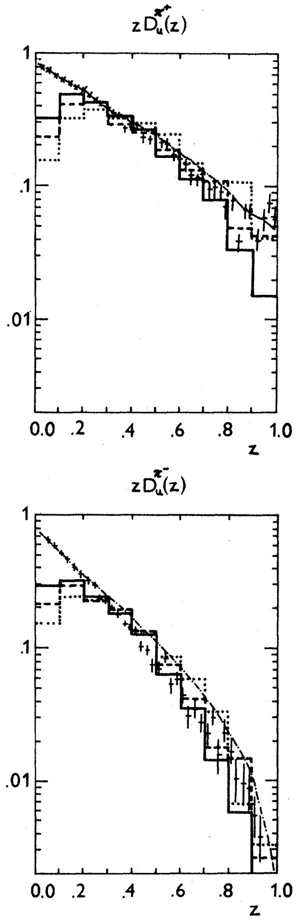


FIG. 6. Fragmentation functions $zD_u^{\pi^+}(z)$ and $zD_u^{\pi^-}(z)$ compared with results of Ref. 23 ($z\equiv x_F=p_L/p_{L,\max}$). Our results are represented by histograms (dotted line $W=3.0$ GeV, dashed line $W=4.8$ GeV, solid line $W=7.4$ GeV). The results of Monte Carlo calculations by Field and Feynman (Ref. 23) are denoted by crosses and their analytic approximation by dash-dot line.

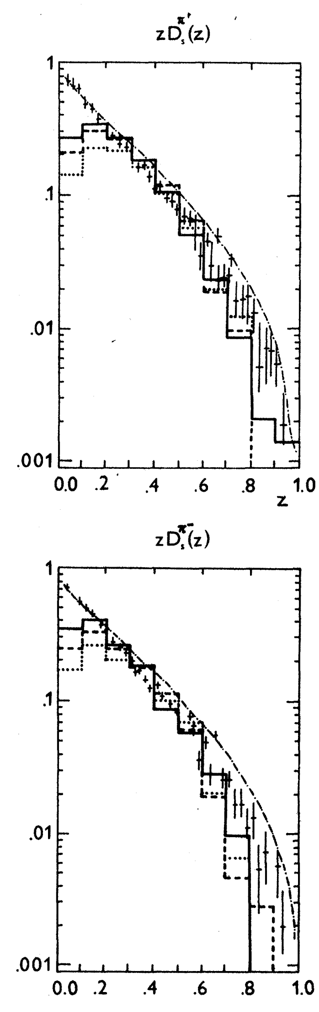
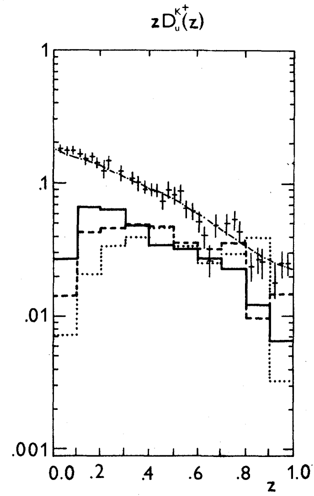


FIG. 7. Fragmentation functions $zD_s^{\pi^+}(z)$ and $zD_s^{\pi^-}(z)$. The notation is the same as in Fig. 6.

are clearly seen in our results. At low values of $z = x_F = p_L/p_{L,\max}$, our results on $zD_u^{\pi^+}(z)$ and $zD_u^{\pi^-}(z)$ increase rather strongly with W. This is quite natural. Our model for e^+e^- annihilation is built in a close analogy with multiparticle production in hadronic collisions. The inclusive distributions therefore develop a plateau at $y \approx 0$. Since $dy \approx dz/(z^2 + m_T^2/p_{L,\max}^2)^{1/2}$, the $zD_u^{\pi}(z)$ behave at low z as $z(z^2 + m_T^2/p_{L,\max}^2)^{-1/2}$. Furthermore, at large z our fragmentation functions decrease with increasing W. Still, within the energy range considered they are quite close to those of Ref. 23.

In Fig. 7 we give the results for fragmentation of the s quark to π^+ and π^-. The agreement with Ref. 23 is again reasonably good except for z near 1, where our results are considerably below the FF ones. The scale breaking is similar to



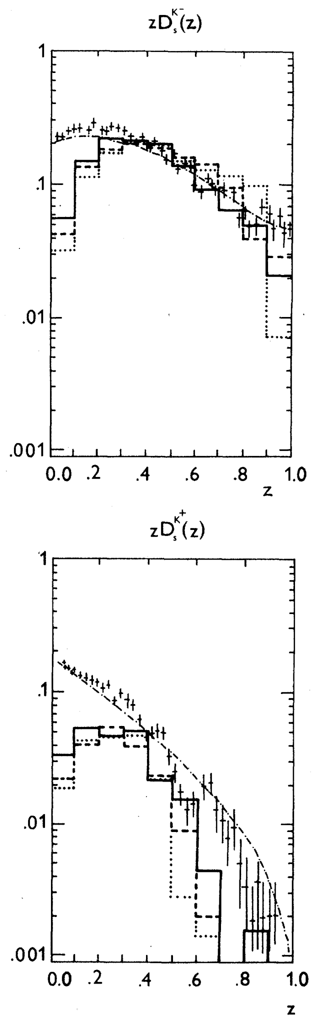


FIG. 8. Fragmentation functions $zD_s^{K^-}(z)$ and $zD_s^{K^+}(z)$. The notation is the same as in Fig. 6.

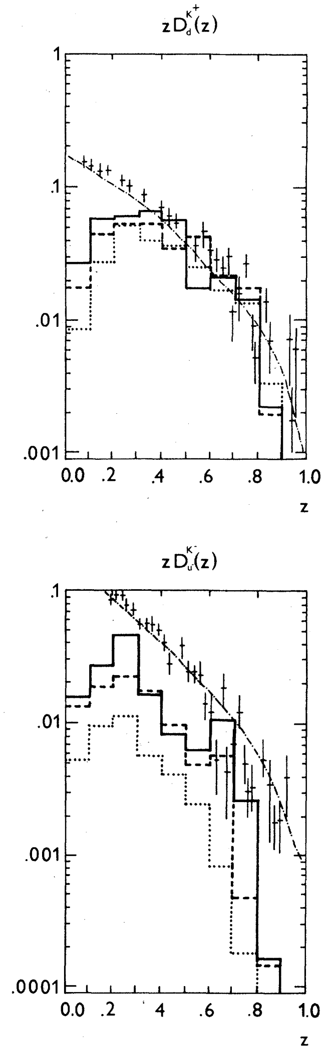


FIG. 9. Results for $zD_d^{K^+}(z)$, $zD_u^{K^-}(z)$ following from our model. The notation is the same as in Fig. 6.

that in Fig. 6.

Fragmentation of the s quark to K^- shown in Fig. 8(a) is similar to that of the up quark to π^+. In both cases the mechanism is rather similar. For fragmentation of the s quark to K^+ [Fig. 8(b)] we have rather large fluctuations at large values of z and our results are (within the W range studied) systematically below those of FF.[23]

In Figs. 9(a)–9(c) we present our results for $zD_u^{K^+}$, $zD_d^{K^+}$, and $zD_u^{K^-}$. The comparison of the three cases is instructive. In $u\to K^+$ fragmentation the K^+ with a large z can be formed either by the recombination $u\bar{s}=K^+$ or by $u\bar{s}=K^{*+}\to K^+\pi^0$; for $d\to K^+$, only the production via $d\bar{s}\to K^0\to K^+\pi^-$ is possible and in the $u\to K^-$ fragmentation both $\bar{u}$ and s contained in K^- have to come from nonleading partons. This is apparently connected with a strong W dependence of $D_u^{K^-}(z)$ clearly visible in Fig. 9(c).

Our results for the fragmentation of quarks to proton are presented in Fig. 10. The W dependence, as expected, is rather large. The similarity between $s\to p$, $d\to p$, and $s\to p$ is due to copious production of baryon resonances and hyperons which finally decay to proton or neutron.

A remarkable result is the large probability of $u\to p$ fragmentation for large z, where $D_u^p(z) \sim D_u^{\pi^+}(z)$. The explanation is rather simple. Suppose that the up quark moves with a large z. Then there are four possible configurations of partons near $z=1$, namely QQu, $Q\bar{Q}u$, $\bar{Q}Qu$, and $\bar{Q}\bar{Q}u$. According to our recombination rules[1] we obtain in the first case a fast baryon or a baryonic resonance. After the resonance decay the resulting baryon still keeps a large momentum fraction. In the other three cases the recombination leads to pseudoscalar- or vector-meson production. However, decays of vector mesons like $\rho\to\pi\pi$ lead to two stable mesons with smaller momentum fractions. Since according to SU(6) weights, which we are using in the present version of the program, vector mesons are three times more frequent than direct pseudoscalar mesons, the fragmentation of the quark for a large z to a stable meson is about as frequent as that to a baryon.

Finally, in Fig. 11 we present our predictions for the fragmentation of the up quark to antiproton (fragmentations $d\to\bar{p}$ and $s\to\bar{p}$ are quite similar to $u\to\bar{p}$).

V. THE RETENTION OF QUARK QUANTUM NUMBERS

Since the original Feynman's formulation,[25] the hypothesis of the quark-quantum-number retention (QQNR) has received well deserved attention.[26] It is well known that in models where quarks fragment only to mesons the full QQNR is not possible. The situation may be changed

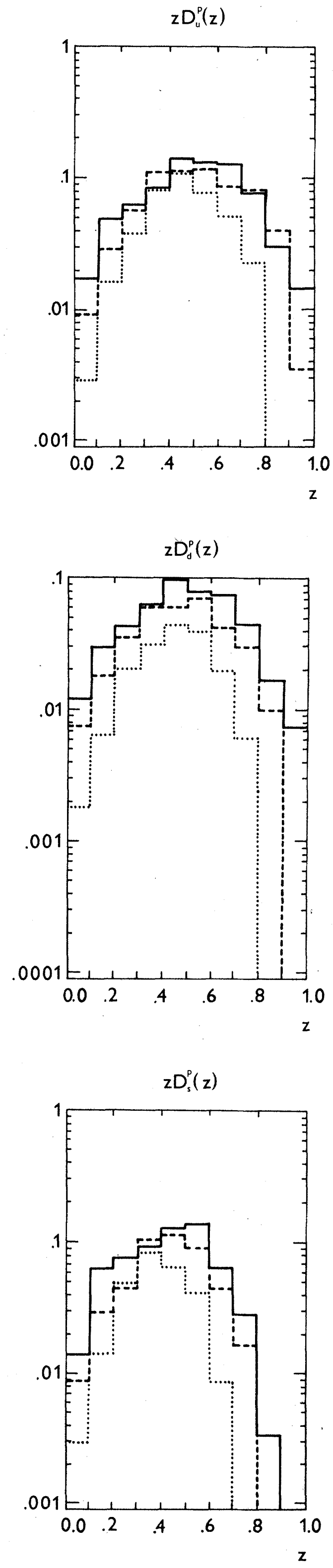


FIG. 10. Predictions for fragmentation of quarks to proton. The notation is the same as in Fig. 6.

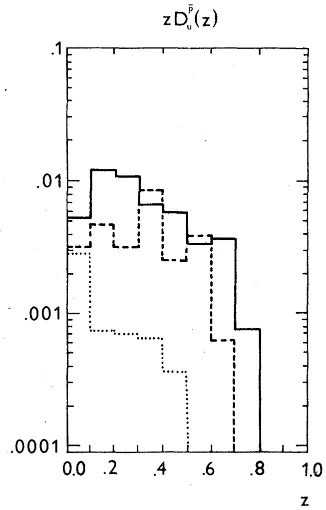


FIG. 11. Prediction for the fragmentation of the up quark to proton. Notation as in Fig. 6.

if baryons are also produced (Cahn and Colglazier[26]).

In this section we shall show that our model gives asymptotically full QQNR. This is essentially due to the statistical (random) distribution of quantum numbers of all Q's and $\bar{Q}$'s except for the "leading" Q and $\bar{Q}$ (the two created at the very beginning of the e^+e^- annihilation) and to the fact that baryons can appear in the final state.

In order to simplify the discussion we shall consider only the question of baryonic-charge retention and for a while we shall forget about other quantum numbers. Let us suppose that the e^+e^- annihilation starts with $e^+e^- \to Q\bar{Q}$ with the Q moving left (negative rapidity) and $\bar{Q}$ moving right. Later on there appear n additional Q's and n $\bar{Q}$'s. Rapidities of these $N=(2n+2)$ Q's and $\bar{Q}$'s are denoted as $y_1 < y_2 < \cdots < y_N$ and the originally created Q and $\bar{Q}$ are supposed to have rapidities y_1 and y_N, respectively. The quantum numbers of the additional Q's and $\bar{Q}$'s are supposed to be randomly distributed. We shall now look at Q's and $\bar{Q}$'s with negative rapidities. Their baryonic charge averaged over many events is certainly equal to the baryonic charge of the "leading" quark. Suppose further that all Q's and $\bar{Q}$'s with $y<0$ recombine to hadrons according to the rules given in I. As a result we obtain a set of hadrons with negative rapidities and possibly also remaining Q's and/or $\bar{Q}$'s which have finally to recombine with the Q's or $\bar{Q}$'s with (small) positive rapidities. The average baryonic charge of hadrons with $y<0$ is equal to $\frac{1}{3}$ (the baryonic charge of the originally produced Q) minus the charge of those Q's and $\bar{Q}$'s which remained in the left hemisphere.

In order to show that the baryonic charge is retained we have to show that the leftover after the recombinations has, in the average, vanishing baryonic charge.

Suppose that we are starting with the Q with the lowest rapidity. After having added his closest neighbors in rapidity we can have one of the following four triplets (QQQ), $(Q\bar{Q}Q)$, $(QQ\bar{Q})$, $(Q\bar{Q}\bar{Q})$, each of them occurring with probability $\frac{1}{4}$. Using our rules[1] for the recombination, we obtain in the first case a baryon and no leftover, in the second and the third case a meson and a Q as a leftover, and in the fourth case a meson and an $\bar{Q}$. Looking only at leftovers we have the following "transition probabilities"

$$P(Q\to 0)=\tfrac{1}{4}\,,\quad P(Q\to \bar{Q})=\tfrac{1}{2}\,,\quad P(Q\to \bar{Q})=\tfrac{1}{4}\,.$$

In the symbol $P(a\to b)$, a denotes the quark we have started with and b the leftover. If there is no leftover we put $b=0$. The leftover is again completed to the triplet by rapidity neighbors and the game is played again. We obtain

$$P(\bar{Q}\to Q)=\tfrac{1}{4}\,,\quad P(\bar{Q}\to \bar{Q})=\tfrac{1}{2}\,,\quad P(\bar{Q}\to 0)=\tfrac{1}{4}\,,$$
$$P(0\to Q)=\tfrac{3}{8}\,,\quad P(0\to 0)=\tfrac{2}{8}\,,\quad P(0\to\bar{Q})=\tfrac{3}{8}\,.$$

If $[p(b)]_K$ denotes the probability that after k recombinations we have b as a leftover, we can rewrite the preceding results by using the transition matrix (used by Cahn and Colglazier[26] in a slightly different context)

$$\begin{bmatrix} p(Q) \\ p(0) \\ p(\bar{Q}) \end{bmatrix}_{K+1} = \begin{bmatrix} \frac{1}{2} & \frac{3}{8} & \frac{1}{4} \\ \frac{1}{4} & \frac{2}{8} & \frac{1}{4} \\ \frac{1}{4} & \frac{3}{8} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} p(Q) \\ p(0) \\ p(\bar{Q}) \end{bmatrix}_K$$

The transition matrix has eignevalues $\lambda_1=1$, $\lambda_2=0$, and $\lambda_3=\frac{1}{4}$. The corresponding eigenvectors are

$$u_1=\tfrac{1}{8}\begin{bmatrix} 3 \\ 2 \\ 3 \end{bmatrix},\quad u_2=\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix},\quad u_3=\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}.$$

The vector of probabilities with which we start can be written as a linear superposition of u_1, u_2, and u_3. However, only the component proportional to u_1 survives very many transitions (after n transitions the magnitudes of components along u_1, u_2, and u_3 are multiplied by λ_1^n, λ_2^n, and λ_3^n). However, in u_1 there are equal probabilities of having a Q and an $\bar{Q}$ and consequently the leftovers after many "transitions" has in the average the vanishing baryonic charge.

This together with the argument sketched above leads asymptotically to the full retention of the baryonic charge of the originally produced quark in the corresponding hemisphere. The same holds true for the retention of other quantum numbers.

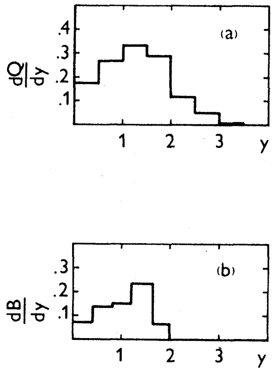


FIG. 12. The distribution of charge (a) and baryonic charge (b) on the side of the up quark in the e^+e^- annihilation proceeding via $e^+e^- \to u\bar{u}$ at $W=7.4$ GeV.

In order to show how the QQNR works at nonasymptotic energies, we present in Fig. 12 the average distribution of charge and of baryonic charge in jets initiated by the u quark following from our model at $W=7.4$ GeV. The average charge in this jet is $\langle Q\rangle = 0.63$ and the baryonic charge $\langle B\rangle = 0.26$. The corresponding fluctuations between the two hemispheres in $e^+e^- \to u\bar{u} \to$ hadrons are $\langle(\Delta Q)^2\rangle = 1.05$ and $\langle(\Delta B)^2\rangle = 0.27$.

VI. CONCLUDING REMARKS

In this paper we have studied a model of e^+e^- annihilation constructed in a close analogy with the model[1,2] of multiparticle production in hadronic collisions. In what concerns the inclusive spectra and fragmentation functions of quarks our results agree on a rough qualitative level with general features of Field and Feynman results. Notable differences are in (i) scale-breaking effects, which are present in our model at low z and to a lesser extent at large z, and (ii) the production of baryons which is disregarded in Ref. 23. The rough qualitative agreement is rather surprising since both models in fact assume different underlying dynamics. In our language the essential difference is in the prescription for the ordering of quarks and antiquarks before the recombination. The Field and Feynman model can be viewed as a recombination of a chain with strict ordering $Q\bar{Q}Q\bar{Q}\cdots Q\bar{Q}$. The Q and $\bar{Q}$ at both ends represent the originally produced Q and $\bar{Q}$ and the rest of Q's and $\bar{Q}$'s come from the "polarization cloud". This strict ordering corresponds to the "adiabatic" or "cold" polarization cloud. In our model the distribution of Q's and $\bar{Q}$'s in the cloud is random, what may be thought of as representing the polarization cloud with an "infinite temperature". Because of that we can have chains like $QQQ\bar{Q}Q\bar{Q}\cdots\bar{Q}Q$ which by a short-range recombination lead to the production of both mesons and baryons. The truth probably lies somewhere in between, which means that one can expect that our model predicts too large baryon production. Anyway, the baryon production is a particularly important piece of data since it measures deviations from the strict $Q\bar{Q}Q\bar{Q}\cdots$ ordering of parton chains and in a vague sense the "temperature" of the polarization cloud.[36,37]

The intermediate picture (probably closer to reality) would respect the space-time evolution[3,4,11] literally and the excited (hot) region of the polarization cloud would extend only over a few (~2) units in rapidity. Such a picture leads to a reasonable description of the production of low-mass dimuons.[5,6] In order to see these differences clearly one needs in general the data on e^+e^- annihilation at higher energies, perhaps 8–10 rapidity units, and in particular the data on quantum-number distribution and fluctuations (the latter are expected[38] to be smaller in models with strict ordering than in models with random distribution[3] of Q's and $\bar{Q}$'s). This data and the data on baryon production can, in our opinion, contribute largely to the understanding of the dynamics of e^+e^- annihilation.

The above-mentioned differences between Ref. 23 (taken here as a standard of reference) and our model are basic since they have a direct relation to the dynamics of the process. There are also some other more technical and less important differences; for instance, in our calculations we have taken the ratio of vector- to pseudoscalar-meson production as 3:1 and decuplet to baryon octet as 2:1 although the former is probably too high (1:1 would be more reasonable). In fact, changes in this ratio do not affect results presented above in a qualitative way (they would of course influence the production of ρ^0 by a factor of 3) and we wanted to keep all parameters at values used in our earlier work on multiparticle production in hadronic collisions.

Furthermore, our prescription for the distribution of Q's and $\bar{Q}$'s in the cylindrical phase space, Eq. (4) is admittedly oversimplified. One probably needs also an explicit introduction of matrix elements which would specify the distribution of Q's and $\bar{Q}$'s in more detail. This will be probably possible only after rather accurate data on inclusive particle production in e^+e^- annihilation become available at higher energies.

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