

## Deep-inelastic scattering beyond the leading order in asymptotically free gauge theories

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We calculate the full order- $g^2$  corrections to the coefficient functions which determine the  $Q^2$  dependence of the moments of deep-inelastic structure functions. The calculation is performed in the minimal-subtraction scheme of 't Hooft. The results are combined with the recent two-loop calculations of anomalous dimensions by Floratos, Ross, and Sachrajda to give the full  $\bar{g}^2$  corrections to the leading order of asymptotic freedom. We present results for  $C_n(1, \bar{g}^2)$  relevant for electroproduction and neutrino reactions for both nonsinglet and singlet combinations of the structure functions. Phenomenological consequences of the full  $\bar{g}^2$  corrections to the nonsinglet structure function are discussed. The corrections to the Gross-Llewellyn Smith and Bjorken sum rules are estimated to be of the order of 15%.

### I. INTRODUCTION

In the last two years there has been considerable interest<sup>1</sup> in the comparison of asymptotically free gauge theories (ASF)<sup>2</sup> with the deep-inelastic data. Most analyses so far have concentrated on the leading asymptotic behavior of the moments of the structure functions, in which only the  $[\ln(Q^2/\Lambda^2)]^{d_n}$  terms are retained. The conclusion has been reached<sup>1</sup> that this leading behavior is consistent with the scaling violations observed in  $ep$ ,  $\mu p$ ,  $\nu N$ , and  $\bar{\nu} N$  deep-inelastic scattering<sup>3</sup> except for a large ratio  $\sigma_L/\sigma_T$  seen at SLAC.<sup>4</sup> However, at  $Q^2 \approx \text{few GeV}^2$ , the leading asymptotic behavior cannot be the whole effect, and it is of interest to ask whether mass corrections and higher-order corrections in the effective coupling constant  $\bar{g}^2(Q^2)$  modify these results. Mass corrections when treated in the manner of Nachtmann-Georgi-Politzer<sup>5-10</sup> improve the agreement of the theory with the data for  $\nu W_2^\phi$ ,  $\nu W_2^{\nu, \bar{\nu}}$  and give a small but nonvanishing  $\sigma_L/\sigma_T$  ratio.

As concerns higher-order corrections in  $\bar{g}^2$ , only the calculation of  $\sigma_L/\sigma_T \sim O(\bar{g}^2)$  is simple and the result is well known.<sup>11-14</sup> These corrections together with the mass corrections are unable<sup>6, 7, 14</sup> to explain the large value of  $\sigma_L/\sigma_T$  measured at SLAC at large  $x$ .

The analysis of the  $\bar{g}^2$  corrections as applied to  $\nu W_2^\phi$ ,  $\nu W_2^{\nu, \bar{\nu}}$ , and  $\nu W_3^{\nu, \bar{\nu}}$  is much more involved<sup>6, 7, 11</sup>. To see this, consider the moments of a nonsinglet structure function ( $\nu W_2^{ep} - \nu W_2^{en}$ ,  $\nu W_2^{\nu p} - \nu W_2^{\bar{\nu} p}$ , etc.), which in ASF are given<sup>2</sup> as follows:

$$M_n(Q^2) \equiv \int_0^1 dx x^{n-2} F_{NS}(x, Q^2) \\ = A_n C_n(1, \bar{g}^2) \exp \left[ - \int_{\bar{g}(Q_0^2)}^{\bar{g}(Q^2)} d\tau \frac{\gamma_n(\tau)}{\beta(\tau)} \right]. \quad (1.1)$$

Here,  $\bar{g}^2(Q^2)$ ,  $\gamma_n(\bar{g}^2)$ , and  $\beta(\bar{g}^2)$  are the effective

coupling constant, anomalous dimension of the spin- $n$  operator in the Wilson expansion of the product of two currents and the standard  $\beta$  function, respectively. Each of these quantities and  $C_n(1, \bar{g}^2)$  can be calculated in perturbation theory. The constants  $A_n$  are to be found from the data at some arbitrary value of  $Q^2 = Q_0^2$ .

By expanding  $C_n(1, \bar{g}^2)$ ,  $\gamma_n(\bar{g}^2)$ , and  $\beta(\bar{g}^2)$  in powers of  $\bar{g}^2$ , calculating  $\bar{g}^2$  in the two-loop approximation for  $\beta(\bar{g}^2)$ , and inserting everything into (1.1), one finds<sup>7, 11</sup>

$$M_n(Q^2) = A_n \left( 1 + \frac{f_n^{(1)}}{\ln(Q^2/\Lambda^2)} + \frac{f_n^{(2)}}{\ln(Q^2/\Lambda^2)} \ln \ln \frac{Q^2}{\Lambda^2} \right) \\ \times \left( \frac{\ln(Q^2/\Lambda^2)}{\ln(Q_0^2/\Lambda^2)} \right)^{d_n}, \quad (1.2)$$

with  $f_n^{(1)}$ ,  $f_n^{(2)}$ , and  $d_n$  depending on the parameters in the expansion of  $C_n$ ,  $\gamma_n$ , and  $\beta$ . We shall give explicit formulas for  $f_n^{(1)}$ ,  $f_n^{(2)}$ , and  $d_n$  in Sec. II. Here we only recall<sup>7, 11</sup> that in order to find  $f_n^{(1)}$  and  $f_n^{(2)}$  the knowledge of  $\gamma_n(\bar{g}^2)$  and  $\beta(\bar{g}^2)$  to  $\bar{g}^4$  order and of  $C_n(1, \bar{g}^2)$  to  $\bar{g}^2$  order is required. The  $\bar{g}^2$  corrections to  $C_n(1, \bar{g}^2)$  and the  $\bar{g}^4$  corrections to  $\gamma_n(\bar{g}^2)$  deserve particular attention. As pointed out in Ref. 15, these two corrections are renormalization-prescription dependent and only when both are calculated in the same renormalization scheme can a physically meaningful answer be obtained. So far only in Ref. 7 have both contributions in question been included in a phenomenological analysis. To this end the results for  $C_n(1, \bar{g}^2)$  of Ref. 6 together with those for  $\gamma_n(\bar{g}^2)$  of Ref. 15 have been used. Unfortunately, it is now clear<sup>16</sup> that the renormalization scheme used in Ref. 6 is not the same as that used in the two-loop calculation of  $\gamma_n(\bar{g}^2)$ .<sup>15</sup> Therefore the results of Ref. 6 and 15 should not be used together.

In order to calculate  $C_n(1, \bar{g}^2)$ , both the  $\bar{g}^2$  corrections to the virtual Compton amplitude and the

$\bar{g}^2$  corrections to the matrix elements of local operators are needed.<sup>17</sup> In a general renormalization scheme both of these quantities are sensitive to infrared structure of the theory and are gauge dependent. We show, however, that in the renormalization scheme of Ref. 15 the gauge dependences of the virtual Compton amplitude and of the matrix elements of local operators are the same and cancel when  $C_n(1, \bar{g}^2)$  is calculated. Consequently, in the renormalization scheme in question,  $C_n(1, \bar{g}^2)$  is gauge independent. Of course,  $C_n(1, \bar{g}^2)$  remains renormalization-prescription dependent. This renormalization-scheme dependence of  $C_n(1, \bar{g}^2)$  is then canceled by that of two-loop contributions to  $\gamma_n(\bar{g}^2)$ . Since  $\gamma_n(\bar{g}^2)$  in the minimal-subtraction scheme<sup>18</sup> used in Ref. 15 is gauge independent<sup>19,20</sup> and  $\beta(g)$  to two loops is gauge and renormalization-prescription independent,<sup>19-21</sup> when all corrections are combined in (1.2) the physical result is obtained.

In the present paper we shall calculate  $C_n(1, \bar{g}^2)$  to  $\bar{g}^2$  order in the renormalization scheme which has been used in Ref. 15 to calculate  $\gamma_n(\bar{g}^2)$  to  $\bar{g}^4$  order. This we do not only for the nonsinglet electromagnetic structure functions but also for singlet structure functions and neutrino process-

es.

Our paper is organized as follows. In Sec. II we formulate the problem in greater detail and state a general procedure for the calculation of  $C_n(1, \bar{g}^2)$ . We subsequently focus on the renormalization scheme used by Floratos, Ross, and Sachrajda.<sup>15</sup> In Sec. III, we calculate those quantities necessary to determine the coefficient function  $C_n(1, \bar{g}^2)$  for nonsinglet operators, and the significance of the scale parameter  $\Lambda$  is discussed.

In Sec. IV we calculate  $C_n(1, \bar{g}^2)$  for nonsinglet contributions to neutrino deep-inelastic scattering and discuss  $\bar{g}^2$  corrections to the various neutrino sum rules and parton-model relations.

We compare our results to the recently measured<sup>10</sup> moments of the nonsinglet structure function  $xF_3$  in Sec. V. We find that the order- $\bar{g}^2$  corrections do not change the conclusions of previous analyses<sup>1,10</sup> based on the leading order.

In Sec. VI we extend our calculations of  $C_n(1, \bar{g}^2)$  to singlet structure functions. We use again the renormalization scheme of the authors of Ref. 15. Section VII contains a brief summary of our paper. The contributions to the virtual Compton amplitude and to the matrix elements from the individual Feynman diagrams are collected in an Appendix.

## II. BASIC FORMALISM

### A. Preliminaries

In what follows we shall discuss the spin-averaged amplitude  $T_{\mu\nu}^{a\bar{a}}$  for the forward scattering of a current  $J_\mu^a$ . In our case  $J_\mu^a$  will stand either for the electromagnetic current ( $e\bar{p}$  scattering) or a weak current ( $\nu, \bar{\nu}$  scattering). The amplitude  $T_{\mu\nu}^{a\bar{a}}$  can be decomposed into invariant amplitudes as follows:

$$T_{\mu\nu}^{a\bar{a}}(q^2, \nu) = i \int d^4x e^{iq \cdot x} \langle p | T(J_\mu^a(x) J_\nu^{\bar{a}}(0)) | p \rangle_{\text{spin-averaged}} \\ = e_{\mu\nu} T_L^{a\bar{a}}(q^2, \nu) + d_{\mu\nu} T_2^{a\bar{a}}(q^2, \nu) - i \epsilon_{\mu\nu\alpha\beta} \frac{p_\alpha q_\beta}{\nu} T_3^{a\bar{a}}(q^2, \nu), \quad (2.1)$$

with  $\nu = p \cdot q$ . Here,  $J_\mu^{\bar{a}} = (J_\mu^a)^\dagger$  and  $|p\rangle$  is, for instance, a proton state. The tensors  $e_{\mu\nu}$  and  $d_{\mu\nu}$  are defined as follows:

$$e_{\mu\nu} = g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}, \quad (2.2)$$

$$d_{\mu\nu} = -\frac{p_\mu p_\nu}{\nu^2} q^2 + \frac{p_\mu q_\nu + p_\nu q_\mu}{\nu} - g_{\mu\nu}. \quad (2.3)$$

Using the operator-product expansion for currents,<sup>2</sup> we can write Eq. (2.1) as

$$T_{\mu\nu}^{a\bar{a}}(q^2, \nu) = \sum_n \frac{1}{x^n} \left[ e_{\mu\nu} \sum_i C_{L,n}^{i,a\bar{a}} \left( \frac{Q^2}{\mu^2}, g^2 \right) + d_{\mu\nu} \sum_i C_{2,n}^{i,a\bar{a}} \left( \frac{Q^2}{\mu^2}, g^2 \right) \right. \\ \left. - i \epsilon_{\mu\nu\alpha\beta} \frac{p_\alpha q_\beta}{\nu} \sum_i C_{3,n}^{i,a\bar{a}} \left( \frac{Q^2}{\mu^2}, g^2 \right) \right] A_n^i \left( \frac{p^2}{\mu^2}, g^2 \right), \quad (2.4)$$

where  $Q^2 = -q^2$  and  $p^2$  is the target four-momentum squared. Furthermore,  $x$  is the Bjorken variable ( $Q^2/2\nu$ ),  $g^2$  is the renormalized coupling constant,  $\mu^2$  is the subtraction scale at which the theory is renormalized, and  $A_n^i$  are constants specified in Eq.

(2.8). The sum on the right-hand side of Eq. (2.4) runs over spin- $n$ , twist-2 operators such as<sup>22</sup>

$$O_{\beta}^{\mu_1 \dots \mu_n} = \frac{i^{n-1}}{n!} (\bar{\psi} \lambda_\beta \gamma^{\mu_1} \mathfrak{D}^{\mu_2} \dots \mathfrak{D}^{\mu_n} \psi + \text{permutations}), \quad (2.5)$$

$$O_{\psi}^{\mu_1 \dots \mu_n} = \frac{i^{n-1}}{n!} (\bar{\psi} \gamma^{\mu_1} \mathfrak{D}^{\mu_2} \dots \mathfrak{D}^{\mu_n} \psi + \text{permutations}), \quad (2.6)$$

and

$$O_G^{\mu_1 \dots \mu_n} = \frac{i^{n-2}}{2n!} (F^{\alpha\mu_1} \mathfrak{D}^{\mu_2} \dots \mathfrak{D}^{\mu_{n-1}} F^{\alpha\mu_n} + \text{permutations}). \quad (2.7)$$

$O_{\beta}^{\mu_1 \dots \mu_n}$  are the fermion nonsinglet (under physical symmetries) operators whereas  $O_{\psi}^{\mu_1 \dots \mu_n}$  and  $O_G^{\mu_1 \dots \mu_n}$  are singlet fermion and gluon operators, respectively.  $C_n^i$  are the corresponding coefficient functions of the Wilson expansion. The constants  $A_n^i$  are related to the spin-averaged proton matrix elements of  $O_{\psi}^{\mu_1 \dots \mu_n}$  as follows:

$$\langle p | O_{\psi}^{\mu_1 \dots \mu_n} | p \rangle = A_n^i p_{\mu_1} \dots p_{\mu_n} + \text{trace terms}. \quad (2.8)$$

In our field-theoretical calculations we shall deal with the matrix elements of the operators in question between quark and gluon states rather than between proton states and therefore it is convenient to generalize (2.8) to

$$\langle p; j | O_{\psi}^{\mu_1 \dots \mu_n} | p; j \rangle = A_{nj}^i p_{\mu_1} \dots p_{\mu_n} + \text{trace terms} \quad (2.9)$$

with  $i, j = \beta, \psi, G$ .

The coefficients  $C_{2,n}^{i,a\bar{a}}$ ,  $C_{L,2,n}^{i,a\bar{a}}$ , and  $C_{3,n}^{i,a\bar{a}}$  as defined in Eq. (2.4) are related to the moments of the standard structure functions

$$\begin{aligned} F_1^{a\bar{a}}(x, Q^2) &\equiv W_1^{a\bar{a}}(x, Q^2), \\ F_2^{a\bar{a}}(x, Q^2) &\equiv \nu W_2^{a\bar{a}}(x, Q^2), \\ F_3^{a\bar{a}}(x, Q^2) &\equiv \nu W_3^{a\bar{a}}(x, Q^2) \end{aligned} \quad (2.10)$$

as follows:<sup>23</sup>

$$\int_0^1 dx x^{n-1} F_1^{a\bar{a}}(x, Q^2) = \sum_i A_n^i \left[ C_{2,n}^{i,a\bar{a}} \left( \frac{Q^2}{\mu^2}, g^2 \right) - C_{L,2,n}^{i,a\bar{a}} \left( \frac{Q^2}{\mu^2}, g^2 \right) \right], \quad (2.11)$$

$$\int_0^1 dx x^{n-2} F_2^{a\bar{a}}(x, Q^2) = 2 \sum_i A_n^i C_{2,n}^{i,a\bar{a}} \left( \frac{Q^2}{\mu^2}, g^2 \right), \quad (2.12)$$

and

$$\int_0^1 dx x^{n-1} F_3(x, Q^2) = 2 \sum_i A_n^i C_{3,n}^{i,a\bar{a}} \left( \frac{Q^2}{\mu^2}, g^2 \right). \quad (2.13)$$

In spite of the fact that there is a set of nonsinglet operators corresponding to various  $\lambda_{\beta}$ , the  $Q^2$  dependence of their coefficient functions in the Wilson expansion is in common since they neither mix under renormalization with each other nor with the singlet operators. Therefore it will be convenient to factor out the dependence on the  $SU(n)$  nature of the operator as well as the current involved and write

$$C_{r,n}^{a\bar{a},\beta} \left( \frac{Q^2}{\mu^2}, g^2 \right) = K^{a\bar{a},\beta} C_{r,n}^{\text{NS}} \left( \frac{Q^2}{\mu^2}, g^2 \right), \quad r=1, 2, 3 \quad (2.14)$$

with  $C_{r,n}^{\text{NS}}(Q^2/\mu^2, g^2)$  being common for all nonsinglet operators. In what follows we shall directly work with  $C_{r,n}^{\text{NS}}(Q^2/\mu^2, g^2)$ . In addition, we shall choose the overall normalization of  $C_{r,n}^{\text{NS}}$  in such a way that in the  $g^0$  order

$$C_{r,n}^{(0)\text{NS}} = 1. \quad (2.15)$$

The relative normalization of the Born term and the  $g^2$  correction is then fixed by the minimal-subtraction scheme used.

Correspondingly, for the coefficient function of the singlet fermion operator we shall write

$$C_{r,n}^{a\bar{a},\psi} \left( \frac{Q^2}{\mu^2}, g^2 \right) = K^{a\bar{a},\psi} C_{r,n}^{\psi} \left( \frac{Q^2}{\mu^2}, g^2 \right), \quad r=1, 2, 3 \quad (2.16)$$

where the overall normalization of  $C_{r,n}^{\psi}$  is chosen in such a way that in the  $g^0$  order

$$C_{r,n}^{(0)\psi} = 1. \quad (2.17)$$

Similarly, we shall choose [see (2.5)–(2.7)] the overall normalization for the matrix elements of the local operators so that in  $g^0$  order

$$A_{nj}^{(0)i} = \delta_{ij}. \quad (2.18)$$

As in the case of coefficient functions, the relative normalization of the Born term and the  $g^2$  corrections to the matrix element of local operators is fixed by the minimal-subtraction scheme.

We shall now present in simple terms the general structure of our calculations. To simplify matters we shall neglect Lorentz indices whenever possible and only write formulas for structure functions which do not vanish in the leading order (e.g.,  $\nu W_2$ ). For completeness, however, we shall comment from time to time on how the formulas change when applied to the longitudinal structure function which vanishes in the leading order. We shall keep all Lorentz indices in Sec. III.

## B. Wilson coefficient function to order $\bar{g}^2$

For the nonsinglet combinations of the structure functions (e.g.  $F_2^{e\bar{p}} - F_2^{en}$ ,  $F_2^{\bar{\nu}p} - F_2^{\nu p}$ , etc.), Eqs. (2.11)–(2.13) simplify to the following general expression:

$$\begin{aligned} M_n^{\text{NS}}(Q^2) &= \int_0^1 dx x^{n-2} F_{\text{NS}}(x, Q^2) \\ &= C_n^{\text{NS}} \left( \frac{Q^2}{\mu^2}, g^2 \right) A_n^{\text{NS}}. \end{aligned} \quad (2.19)$$

The  $Q^2$  dependence of  $C_n^{\text{NS}}(Q^2/\mu^2, g^2)$  is governed by the renormalization-group equation<sup>2</sup>

$$\left(\mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g} - \gamma_n(g)\right) C_n^{\text{NS}}\left(\frac{Q^2}{\mu^2}, g^2\right) = 0, \quad (2.20)$$

which has the following solution:

$$C_n^{\text{NS}}\left(\frac{Q^2}{\mu^2}, g^2\right) = C_n^{\text{NS}}(1, \bar{g}^2) \exp\left[-\int_{\bar{g}(\mu^2)}^{\bar{g}(Q^2)} d\tau \frac{\gamma_n(\tau)}{\beta(\tau)}\right]. \quad (2.21)$$

Here  $\bar{g}^2(Q^2)$  is the effective coupling constant which satisfies the equation

$$\frac{d\bar{g}^2}{dt} = \bar{g}\beta(\bar{g}), \quad \bar{g}(t=0) = g \quad (2.22)$$

where  $t = \ln Q^2/\mu^2$ .  $\gamma_n(\bar{g})$  is the anomalous dimension of the operator  $O_{\text{NS}}^{\mu_1 \dots \mu_n}$ .

In order to find explicit expressions for  $f_n^{(1)}$  and  $f_n^{(2)}$  in Eq. (1.2) we follow Ref. 7 and expand first  $\gamma_n(\bar{g})$ ,  $\beta(\bar{g})$ , and  $C_n^{\text{NS}}(1, \bar{g}^2)$  in powers of  $\bar{g}$ :

$$\gamma_n(\bar{g}) = \gamma_0^n \frac{\bar{g}^2}{16\pi^2} + \gamma_1^n \left(\frac{\bar{g}^2}{16\pi^2}\right)^2 + \dots, \quad (2.23)$$

$$\beta(\bar{g}) = -\beta_0 \frac{\bar{g}^3}{16\pi^2} - \beta_1 \frac{\bar{g}^5}{(16\pi^2)^2} + \dots, \quad (2.24)$$

$$C_n^{\text{NS}}(1, \bar{g}^2) = 1 + B_n^{\text{NS}} \frac{\bar{g}^2}{16\pi^2} + \dots \quad (2.25)$$

For the longitudinal structure function the first term on the right-hand side of Eq. (2.25) is zero.

We next expand  $\bar{g}(Q^2)$ , the solution of Eq. (2.22) with  $\beta(g)$  given by (2.24), in powers of  $\bar{g}_0^2(Q^2)$ , the effective coupling constant calculated in the one-loop approximation with the result

$$\bar{g}^2(Q^2) = \bar{g}_0^2(Q^2) - \frac{\beta_1 \bar{g}_0^4(Q^2)}{\beta_0 16\pi^2} \ln \ln \frac{Q^2}{\Lambda^2} + O(\bar{g}_0^6), \quad (2.26)$$

where

$$\bar{g}_0^2(Q^2) = \frac{48\pi^2}{(33-2f)\ln(Q^2/\Lambda^2)} \quad (2.27)$$

with  $f$  being the number of flavors.

In Eq. (2.26) and following Ref. 7, the constant  $\Lambda$  has been arbitrarily chosen so that there are no further terms of order  $\bar{g}_0^4$ . A little algebra shows that  $\mu^2$ ,  $\Lambda^2$ , and  $g^2$  are related to each other by

$$\Lambda^2 = \mu^2 \exp\left[-\frac{16\pi^2}{\beta_0 g^2} + \frac{\beta_1}{\beta_0} \ln(\beta_0 g^2)\right]. \quad (2.28)$$

Clearly this choice of  $\Lambda$  is not unique and we shall discuss in detail in Sec. II D definitions for  $\Lambda$  which lead to additional terms of  $O(\bar{g}_0^4)$  in Eq. (2.26).

Inserting (2.23)–(2.26) into (2.21) and expanding in powers of  $\bar{g}_0^2$ , we obtain<sup>7</sup>

$$C_n^{\text{NS}}\left(\frac{Q^2}{\mu^2}, g^2\right) = C_n \left[1 + \frac{1}{\beta_0 \ln(Q^2/\Lambda^2)} (B_n^{\text{NS}} + P_n + L_n)\right] \times [\ln(Q^2/\Lambda^2)]^{-\gamma_0^n/2\beta_0}, \quad (2.29)$$

where

$$P_n = \frac{\gamma_1^n}{2\beta_0} - \frac{\beta_1 \gamma_0^n}{2\beta_0^2}, \quad (2.30)$$

$$L_n = -\frac{\beta_1 \gamma_0^n}{2\beta_0^2} \ln \ln \frac{Q^2}{\Lambda^2}, \quad (2.31)$$

and  $C_n$  is an overall  $Q^2$  independent constant. In the case of the longitudinal structure function,  $P_n$ ,  $L_n$ , and the leading term 1 are absent and  $B_n^{\text{NS}}$  is replaced by  $B_{nL}^{\text{NS}}$ .

The parameters  $\gamma_0^n$ ,  $\beta_0$ , and  $\beta_1$  are gauge and renormalization-prescription independent and are given for an SU(3) gauge theory with  $f$  flavors as follows<sup>2,21</sup>:

$$\gamma_0^n = \frac{8}{3} \left[1 - \frac{2}{n(n+1)} + 4 \sum_{j=1}^n \frac{1}{j}\right], \quad (2.32)$$

$$\beta_0 = 11 - \frac{2}{3}f, \quad (2.33)$$

$$\beta_1 = 102 - \frac{38}{3}f. \quad (2.34)$$

The quantities  $B_n$  and  $\gamma_1^n$  are renormalization-prescription dependent (and in principle also gauge dependent). However, as pointed out in Ref. 15, the renormalization-prescription dependence of  $B_n$  is related to that of  $\gamma_1^n$  and when these quantities are inserted in Eq. (2.29) a physical, renormalization-prescription independent answer is obtained. In other words, the calculation of  $B_n$  and  $\gamma_1^n$  may be performed in any renormalization scheme *but* care must be taken that these two quantities are calculated in the same scheme.

The  $\gamma_1^n$  for nonsinglet operators has been calculated by Floratos, Ross, and Schrajda,<sup>15</sup> who have used 't Hooft's<sup>18</sup> minimal-subtraction scheme to renormalize the amplitudes. In this scheme the Feynman diagrams are evaluated, using dimensional regularization, in  $d=4-\epsilon$  dimensions and singularities are extracted as poles  $1/\epsilon$ ,  $1/\epsilon^2$ , etc. The minimal subtraction then means that the amplitudes are renormalized by simply subtracting the pole parts  $1/\epsilon$ ,  $1/\epsilon^2$ , etc.

The coefficients  $B_n^{\text{NS}}$  existing in the literature<sup>6,11</sup> have been calculated in schemes which differ from the one above and cannot be combined with the results of Ref. 15. Therefore, in our paper we shall calculate them in the minimal-subtraction scheme. We now outline a method for calculating  $B_n^{\text{NS}}$ .

### C. Procedure for the calculation of $B_n^{\text{NS}}$

We first remark that in order to find  $B_n$  as defined in Eq. (2.25) it is sufficient to calculate  $C_n(Q^2/\mu^2, g^2)$  in perturbation theory to order  $g^2$  and set  $Q^2 = \mu^2$ . This is obvious from Eqs. (2.21) and (2.22).

Writing next the nonsinglet version of (2.4) symbolically as

$$T^{\text{NS}}(Q^2, \nu) = \sum_n \left(\frac{1}{x}\right)^n C_n^{\text{NS}}\left(\frac{Q^2}{\mu^2}, g^2\right) A_n^{\text{NS}}\left(\frac{p^2}{\mu^2}, g^2\right), \quad (2.35)$$

we see that in order to find  $C_n^{\text{NS}}(Q^2/\mu^2, g^2)$  we generally have to calculate both  $T^{\text{NS}}(Q^2, \nu)$  and  $A_n^{\text{NS}}(p^2/\mu^2, g^2)$ . Since the coefficient functions do not depend on the target, we can choose as a target any state for which calculations can be easily performed. In what follows we shall choose either quarks or gluons as targets.<sup>24</sup> In order to be consistent with the calculations of Ref. 15, we shall deal with massless, off-shell quarks or gluons with spacelike momenta  $p^2 < 0$ .

Having all information at hand, we can now state the procedure for extracting  $C_n^{\text{NS}}(Q^2/\mu^2, g^2)$  and specifically the constants  $B_n^{\text{NS}}$ .

The procedure is as follows:

(i) Calculate  $T^{\text{NS}}(q^2, \nu)$  (forward amplitude for scattering off quarks) in perturbation theory to order  $g^2$  and expand in powers  $(1/x)^n$  for  $x > 1$ . The coefficients of this expansion will have the form

$$T_n^{\text{NS}} = 1 + \frac{g^2}{16\pi^2} \left( -\frac{1}{2}\gamma_0^n \ln \frac{Q^2}{-p^2} - \frac{1}{2}\gamma_F \ln \frac{-p^2}{\mu^2} + T_n^{(2)\text{NS}} \right), \quad (2.36)$$

with  $\gamma_0^n$  given by (2.32) and  $\gamma_F$  being the anomalous dimension of the quark field. The overall normalization has been chosen in accordance with Eqs. (2.15)–(2.18).

(ii) Calculate  $A_n^{\text{NS}}(p^2/\mu^2, g^2)$  by considering the matrix elements of nonsinglet operator between quark states with the result

$$A_n^{\text{NS}} = 1 + \frac{g^2}{16\pi^2} \left( \frac{1}{2}\gamma_0^n \ln \frac{-p^2}{\mu^2} - \frac{1}{2}\gamma_F \ln \frac{-p^2}{\mu^2} + A_n^{(2)\text{NS}} \right). \quad (2.37)$$

(iii) Insert (2.37) and (2.36) into (2.35) to obtain

$$C_n^{\text{NS}}\left(\frac{Q^2}{\mu^2}, g^2\right) = 1 + \frac{g^2}{16\pi^2} \left( -\frac{1}{2}\gamma_0^n \ln \frac{Q^2}{\mu^2} + T_n^{(2)\text{NS}} - A_n^{(2)\text{NS}} \right). \quad (2.38)$$

(iv) Finally, evaluate (2.38) for  $Q^2 = \mu^2$  and compare with (2.25) to find

$$B_n^{\text{NS}} = T_n^{(2)\text{NS}} - A_n^{(2)\text{NS}}. \quad (2.39)$$

This equation applies to the nonsinglet components of the structure functions  $F_1$ ,  $F_2$ , and  $F_3$ . In the case of the longitudinal structure function, which vanishes in  $g^0$  order, the corresponding expression is

$$B_{L,n}^{\text{NS}} = T_{L,n}^{(2)\text{NS}}. \quad (2.40)$$

The above procedure applies for any renormalization scheme. However, as we already stated above, different renormalization schemes lead generally to different values of  $B_n$ . If  $B_n$  and  $\gamma_1^n$  are calculated in the same scheme, a renormal-

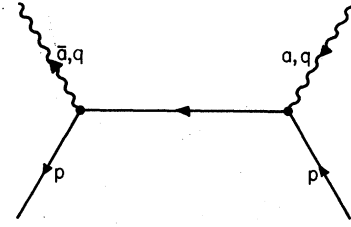


FIG. 1. The Born term for virtual Compton scattering. Inclusion of the crossed diagram is understood.

ization-prescription-independent result for the moments of the structure functions is obtained.

In the scheme of De Rújula *et al.*,<sup>6</sup> the matrix elements of operators are normalized so that  $A_n^{(2)} = 0$ . Therefore, in that scheme we have simply  $B_n^{\text{NS}} = T_n^{(2)\text{NS}}$ . Unfortunately, we cannot use this scheme for our calculations since the only existing results for  $\gamma_1^n$  have been obtained<sup>15</sup> using the minimal-subtraction scheme. As we shall see in the next section, in the latter scheme,  $A_n^{(2)\text{NS}} \neq 0$  and must be calculated in order to extract the coefficient function.

### III. $C_n(Q^2/\mu^2, g^2)$ TO ORDER $g^2$ FOR ELECTROMAGNETIC CURRENTS (NONSINGLET CONTRIBUTIONS)

For electromagnetic currents, the calculation of the nonsinglet contributions using Eq. (2.4) simplifies to

$$T_{\mu\nu}^{\text{NS}}(q^2, \nu, p^2) = \sum_n \left(\frac{1}{x}\right)^n \left[ e_{\mu\nu} C_{L,n}^{\text{NS}}\left(\frac{Q^2}{\mu^2}, g^2\right) + d_{\mu\nu} C_{2,n}^{\text{NS}}\left(\frac{Q^2}{\mu^2}, g^2\right) \right] A_n^{\text{NS}}\left(\frac{p^2}{\mu^2}, g^2\right). \quad (3.1)$$

$T_{\mu\nu}^{\text{NS}}$  is the forward spin-averaged amplitude for scattering of photons off off-shell massless quarks with spacelike momentum  $p^2 < 0$ , and  $A_n^{\text{NS}}$  are the matrix elements of the nonsinglet operators of Eq.

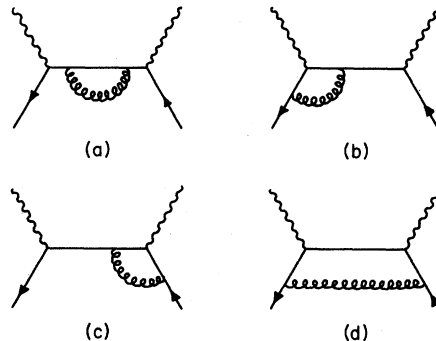


FIG. 2. Diagrams contributing in order  $g^2$  to the virtual Compton scattering. Inclusion of the crossed diagrams is understood.

(2.5) between the quark states in question.  $C_{L,n}^{NS}$  and  $C_{2,n}^{NS}$  are the coefficient functions which we want to calculate in perturbation theory to order  $g^2$ . To this end, according to the procedure of Sec. II C, we begin by calculating  $T_{\mu\nu}^{NS}$  to order  $g^2$ .

#### A. Calculation of the virtual Compton amplitude to order $g^2$

We first recall that the calculation of  $T_{\mu\nu}^{NS}$  in  $g^0$  order involves diagrams of Fig. 1 and when the result is expanded in powers of  $1/x$  we find in the normalization of (2.15)–(2.18)

$$T_{2,n}^{(2)NS} = C_2(R) \left[ 3 \sum_{j=1}^n \frac{1}{j} - 8 \sum_{j=1}^n \frac{1}{j^2} + \frac{6}{n+1} + \frac{4}{n^2} - \frac{4}{(n+1)^2} - 1 - \frac{1}{n} + (\ln 4\pi - \gamma_E) + (1-\alpha) \left( \frac{1}{n} + \sum_{j=1}^n \frac{1}{j} - 1 - \ln 4\pi + \gamma_E \right) \right], \quad n \text{ even} \quad (3.4)$$

and

$$T_{L,n}^{(2)NS} = C_2(R) \frac{4}{n+1}, \quad C_2(R) = \frac{4}{3}. \quad (3.5)$$

The contributions from individual diagrams are collected in the Appendix. The gauge dependence and the presence of the term  $(\ln 4\pi - \gamma_E)$  where  $\gamma_E$  is the Euler-Mascheroni constant are due to the normalization procedure and will be discussed in Sec. III D.

#### B. Matrix elements of nonsinglet operators to order $g^2$

The calculation involves the diagrams of Fig. 3. Using again the minimal-subtraction scheme, we determine the constants  $A_n^{(2)NS}$  defined in Eq. (2.37) with the result

$$A_n^{(2)NS} = C_2(R) \left[ 8 - \frac{4}{n} + \frac{2}{n+1} + \frac{2}{n^2} - \frac{4}{(n+1)^2} - 4 \sum_{j=1}^n \frac{1}{j^2} + \frac{2}{n(n+1)} \sum_{j=1}^n \frac{1}{j} - 4 \sum_{s=1}^n \frac{1}{s} \sum_{j=1}^s \frac{1}{j} + (\ln 4\pi - \gamma_E) \left( \frac{2}{n(n+1)} - 4 \sum_{j=2}^n \frac{1}{j} \right) + (1-\alpha) \left( \frac{1}{n} + \sum_{j=1}^n \frac{1}{j} - 1 - \ln 4\pi + \gamma_E \right) \right], \quad n \text{ even}. \quad (3.6)$$

Details of the calculation are given in the Appendix.

#### C. Final result for $C_{L,n}^{NS}(1, \bar{g}^2)$ and $C_{2,n}^{NS}(1, \bar{g}^2)$

Inserting (3.4)–(3.6) into Eqs. (2.39) and (2.40) and using the definition (2.25), we obtain the following results for  $C_{L,n}^{NS}(1, \bar{g}^2)$  and  $C_{2,n}^{NS}(1, \bar{g}^2)$  to order  $\bar{g}^2$ :

$$C_{2,n}^{NS}(1, \bar{g}^2) = 1 + \frac{\bar{g}^2}{16\pi^2} B_{2,n}^{NS} \quad (3.7)$$

and

$$C_{L,n}^{NS}(1, \bar{g}^2) = \frac{\bar{g}^2}{16\pi^2} C_2(R) \frac{4}{n+1}, \quad (3.8)$$

where

$$B_{2,n}^{NS} = C_2(R) \left[ 3 \sum_{j=1}^n \frac{1}{j} - 4 \sum_{j=1}^n \frac{1}{j^2} - \frac{2}{n(n+1)} \sum_{j=1}^n \frac{1}{j} + 4 \sum_{s=1}^n \frac{1}{s} \sum_{j=1}^s \frac{1}{j} + \frac{3}{n} + \frac{4}{n+1} + \frac{2}{n^2} - 9 + \bar{\gamma}_0^n (\ln 4\pi - \gamma_E) \right], \quad n \text{ even} \quad (3.9)$$

$$T_{L,n}^{(0)NS} = 0, \quad T_{2,n}^{(0)NS} = 1 \quad (n \text{ even}). \quad (3.2)$$

The calculation of  $T_{\mu\nu}^{NS}$  in  $g^2$  order involves diagrams of Fig. 2. We use an arbitrary covariant gauge  $\alpha$  where the gluon propagator is given by

$$-\frac{i}{k^2} \left[ g_{\mu\nu} - (1-\alpha) \frac{k_\mu k_\nu}{k^2} \right]. \quad (3.3)$$

Using the minimal-subtraction scheme and dropping terms of order  $p^2$  we obtain the following results for the coefficients  $T_n^{(2)NS}$  defined in Eq. (2.36):

and

$$\bar{\gamma}_0^n = 1 - \frac{2}{n(n+1)} + 4 \sum_{j=2}^n \frac{1}{j} = \frac{3}{8} \gamma_0^n. \quad (3.10)$$

The result (3.8) is well known.<sup>11-14</sup> On the other hand, the result for  $B_{2,n}^{NS}$  in the particular renormalization scheme considered is new.

Notice that the gauge dependence of  $A_n^{(2)NS}$  canceled that of  $T_n^{(2)NS}$  leaving  $B_{2,n}^{NS}$  gauge independent in the minimal-subtraction scheme. Numerical values of  $B_{2,n}^{NS}$  together with those for  $\gamma_1^n$  and  $\gamma_0^n$  are collected in Table I.

There are only two calculations of  $B_{2,n}^{NS}$  in the present literature: one by Calvo<sup>11</sup> and another by De Rújula, Georgi, and Politzer.<sup>6</sup> None of these results for  $B_{2,n}^{NS}$  can be directly compared with Eq. (3.9) because the renormalization schemes used in Refs. 6 and 11 are different from ours. On the other hand, we can directly compare our results for  $T_{2,n}^{(2)NS}$  with that obtained by De Rújula

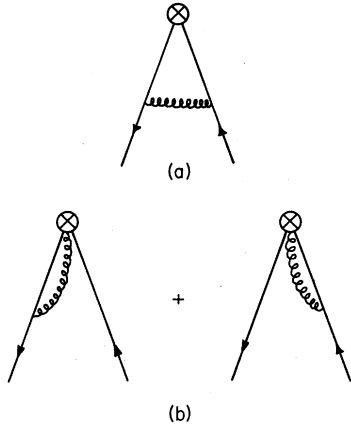


FIG. 3. Diagrams contributing in order  $g^2$  to the matrix elements of the nonsinglet operator between quark states.

*et al.*<sup>6</sup> Setting  $\alpha=0$  in Eq. (3.4), we obtain their result except for the numerator of the term  $1/(n+1)$  which in our case is 6 and theirs 10.

#### D. Discussion of the results for nonsinglet structure functions

In this section we discuss some technical aspects of our calculations including infrared sensitivity, gauge dependence, and normalization dependence. We defer discussion of phenomenological aspects to Sec. V.

In the previous sections we have calculated the coefficient functions  $C_n$ , using 't Hooft's minimal-subtraction scheme. This normalization procedure is insensitive to the infrared structure of the theory for large  $Q^2$ ; corrections are of order  $p^2/Q^2$  or  $m_q^2/Q^2$  for the coefficient functions. The amplitudes  $T_n$  and  $A_n$ , which were used for our cal-

TABLE I. The values of various quantities which enter Eq. (2.29) for  $f=4$ .  $\bar{B}_{2,n}^{NS} = B_{2,n}^{NS} - \frac{1}{2}\gamma_0^n(\ln 4\pi - \gamma_E)$ . The values for odd  $n$  contribute to  $B_{2,n}^{NS}$  in  $\nu, \bar{\nu}$  reactions.

$n$	$B_{2,n}^{NS}$	$\bar{B}_{2,n}^{NS}$	$\frac{\gamma_1^n}{2\beta_0}$	$-\frac{\gamma_0^n\beta_1}{2\beta_0^2}$
1	0	0	0	0
2	7.39	0.44	4.28	-2.63
3	14.08	3.22	6.05	-4.11
4	19.70	6.07	7.21	-5.16
5	24.53	8.73	8.09	-5.98
6	28.77	11.18	8.82	-6.66
7	32.55	13.44	9.44	-7.23
8	35.96	15.53	9.99	-7.73
9	39.08	17.48	10.46	-8.17
10	41.96	19.30	10.91	-8.57
11	44.63	21.01	11.31	-8.93
12	47.12	22.63	11.68	-9.27

ulation of  $C_n$ , are sensitive to the infrared behavior since they involve matrix elements in particular states, free quarks in our case. Although our states are only logarithmically off shell ( $p^2 \rightarrow 0$ ,  $\ln p^2$  finite), numerator factors of order  $p^2$  may still contribute as the Feynman integrals can yield terms of order  $1/p^2$ . Therefore, one must be careful when dropping  $p^2$  terms. Such terms are responsible for the gauge dependence of the amplitudes  $T_n$ (3.4) and  $A_n$ (3.6).

The question of gauge dependence involves a number of aspects. We have noted that the spin-averaged matrix elements of the correlation function  $T_n$  and the local operators  $A_n$  are found to be gauge dependent. On the other hand, the coefficient functions  $C_n$ (3.9) are found to be gauge invariant. This result is expected as the  $C_n$  appear as coefficients in the Wilson expansion involving gauge-invariant operators. The coefficient functions are not automatically gauge invariant as the local operators may be given a gauge-dependent normalization in some renormalization schemes.

The gauge dependence of the theory also affects the renormalization-group equations which are used to convert the perturbation-theory results into the true asymptotic behavior of the theory. Except in the Landau gauge, the usual renormalization-group equations involve a derivative with respect to the renormalized gauge parameter.<sup>19</sup> The solution of the renormalization-group equations usually requires full knowledge of the gauge dependence of the renormalized group parameters. However, the two-loop anomalous dimensions have only been computed in the Feynman gauge.<sup>15</sup> The renormalization-group equations for amplitudes in the Feynman gauge must be modified by the inclusion of inhomogeneous terms. The Feynman-gauge amplitudes for  $T_n$  and  $A_n$  will satisfy renormalization-group equations of the forms

$$\begin{aligned} [\mu \partial_\mu + \beta(g) \partial_g - 2\gamma_F] T_n &= \Delta T_n, \\ [\mu \partial_\mu + \beta(g) \partial_g + \gamma_n - 2\gamma_F] A_n &= \Delta A_n. \end{aligned} \quad (3.11)$$

The inhomogeneous terms result from the variation of the bare gauge parameter as we change the normalization scale while remaining in the Feynman gauge. In the minimal-subtraction scheme, these terms may be computed for the relations in (3.11). However, the analogous relations for the coefficient functions  $C_n$  will not involve inhomogeneous terms. Hence the naive renormalization-group equations used in (2.20) for the coefficient functions are correct for Feynman-gauge calculations in the minimal-subtraction scheme.

We now turn to problems associated with the solution of renormalization-group equations which relate to the significance of the parameter  $\Lambda$  and

the presence of terms such as  $\ln 4\pi$  and  $\gamma_E$  in our expressions for the coefficient functions. The coefficient functions for nonsinglet operators satisfy homogeneous renormalization-group equations in the form

$$[\mu \partial_\mu + \beta(g) \partial_g - \gamma_n(g)] C_n \left( \frac{Q^2}{\mu^2}, g^2 \right) = 0, \quad (3.12)$$

where  $\mu$  is the normalization scale and  $g$  the renormalized coupling constant. These equations may be solved in a standard manner by introducing a renormalization-group-invariant coupling constant  $\bar{g}(Q^2/\mu^2, g)$ . The result (2.21) and (2.22) may be expressed as

$$C_n \left( \frac{Q^2}{\mu^2}, g \right) = C_n(1, \bar{g}^2) \exp \left[ \int_{\bar{g}}^g d\tau \gamma^n(\tau) / \beta(\tau) \right], \quad (3.13)$$

$$-\frac{1}{2} \ln \frac{Q^2}{\mu^2} = \int_{\bar{g}}^g d\tau [1/\beta(\tau)]. \quad (3.14)$$

As noted in Sec. II,  $C_n(1, \bar{g}^2)$  is simply evaluated in perturbation theory by computing  $C_n(Q^2/\mu^2, g^2)$  at the normalization point  $Q^2 = \mu^2$ , as the exponential factor in (3.13) is 1 if  $\bar{g}$  is defined as in (3.14).

The leading-order results are obtained by evaluating  $C_n(1, \bar{g}^2)$  in zeroth order and truncating  $\gamma^n(g)$  and  $\beta(g)$  in one-loop order. Using the expansions in Eqs. (2.23) and (2.24), Eqs. (3.14) and (3.13) may be integrated to obtain

$$\frac{16\pi^2}{\beta_0} \frac{1}{\bar{g}^2} = \frac{16\pi^2}{\beta_0} \frac{1}{g^2} + \ln \frac{Q^2}{\mu^2} \equiv \ln \frac{Q^2}{\Lambda^2} \quad (3.15)$$

and

$$C_n \left( \frac{Q^2}{\mu^2}, g^2 \right) = C_n \left\{ 1 + \frac{1}{\beta_0 \ln Q^2 / \Lambda^2} \left[ B_n + \frac{1}{2} \left( \frac{\gamma_1^n}{\beta_0} - \frac{\beta_1 \gamma_0^n}{\beta_0^2} \right) - \frac{\gamma_0^n \beta_1}{2\beta_0^2} \ln \ln \frac{Q^2}{\Lambda^2} \right] \right\} \left( \ln \frac{Q^2}{\Lambda^2} \right)^{-\gamma_0^n / 2\beta_0}. \quad (3.17)$$

The Eq. (3.17) is exact through order  $O(1/\ln Q^2 / \Lambda^2)$  in the expansion for large  $Q^2$  with the scale  $\Lambda$ . Asymptotic expansions of this type are also possible for some other choice of scale, say  $\Lambda_0$ . To be specific, take  $\Lambda_0 = 1$  GeV and reexpand (3.17) in powers of  $1/\ln Q^2$  to find

$$C_n \left( \frac{Q^2}{\mu^2}, g \right) = C_n \left\{ 1 + \frac{1}{\beta_0 \ln Q^2} \left[ B_n^{\text{NS}} + \frac{1}{2} \gamma_0^n \ln \Lambda^2 + \frac{1}{2} \left( \frac{\gamma_1^n}{\beta_0} - \frac{\beta_1 \gamma_0^n}{\beta_0^2} \right) - \frac{\gamma_0^n \beta_1}{2\beta_0^2} \ln \ln Q^2 \right] \right\} (\ln Q^2)^{-\gamma_0^n / 2\beta_0}, \quad (3.18)$$

through order  $O(1/\ln Q^2)$ . An important observation is that the effect of the change of scale is equivalent through order  $1/\ln Q^2$  to the shift of the constant  $B_n^{\text{NS}}$  by an amount proportional to the one-loop anomalous dimension  $\gamma_0^n$ . This may be seen in (3.18) explicitly. Thus any term proportional to  $\gamma_0^n$  in  $B_n^{\text{NS}}$  can be absorbed by the redefinition of the scale  $\Lambda$ . In particular, the term  $\ln 4\pi - \gamma_E$  in (3.9) may be absorbed by choosing the new scale  $\Lambda \exp[\frac{1}{2}(\ln 4\pi - \gamma_E)]$ .

From the point of view of the renormalization-

$$C_n \left( \frac{Q^2}{\mu^2}, \bar{g}^2 \right) = e^{-(\gamma_0^n / \beta_0) \ln(Q^2/\mu^2)} = \left( \frac{\ln Q^2 / \Lambda^2}{\ln \mu^2 / \Lambda^2} \right)^{-\gamma_0^n / 2\beta_0}. \quad (3.16)$$

For asymptotically free theories ( $\beta_0 > 0$ ), this result represents the true asymptotic behavior of the coefficient functions as  $Q^2 \rightarrow \infty$ . If the full theory were precisely given by the truncated theory, then the parameter  $\Lambda$  would have significance as the scale of strong interactions. However, in the full theory there are corrections of order  $(1/\ln Q^2)$ . These corrections cannot be uniquely specified independent of the scale  $\Lambda$ , as changes in the value of  $\Lambda$  may also be represented as order  $(1/\ln Q^2)$  for large  $Q^2$ . This fact becomes more apparent when considering the first systematic corrections to the behavior of the coefficient function.

This next-order calculation requires the knowledge of the one- and two-loop contributions to the anomalous dimension  $\gamma^n$  and the  $\beta$  function as well as the entire one-loop contributions to the coefficient functions. As discussed in Sec. II, the calculation of these quantities must be done in a consistent manner. The two-loop  $\beta$  function<sup>2,21</sup> and the two-loop anomalous dimension for nonsinglet operators<sup>15</sup> have been computed using 't Hooft's minimal-subtraction scheme.<sup>18</sup> We have presented our results for the full one-loop coefficient function using this same scheme in Sec. III C.

These results may be combined to obtain a solution to the renormalization group consistent to second order. One such solution is discussed in Sec. II Eqs. (2.26)–(2.29) with the result

group equation, the ambiguity of  $\bar{g}^2$  corrections in terms proportional to  $\gamma_0^n$  as well as in the redefinition of  $\Lambda$  are related to the freedom of defining the effective coupling constant in solving the renormalization-group Eq. (3.12). In fact, a new solution for  $\bar{g}$  may be obtained by adding a constant  $b$  to the left-hand side of (3.14), yielding

$$-\frac{1}{2} \ln \frac{Q^2}{\mu^2} + b = \int_{\bar{g}}^g d\tau \frac{1}{\beta(\tau)}. \quad (3.19)$$

With this normalization,  $\bar{g}(Q^2)$  is equal to the re-



normalized coupling constant  $g$  at  $Q^2 = \mu^2 e^{-2b}$  ( $= \mu'^2$ ) and not at  $Q^2 = \mu^2$  as defined in (3.14). Since  $\Lambda$  is proportional to  $\mu$  [see (2.28) and (3.15)], we observe that any redefinition of the scale  $\Lambda$  corresponds to redefinition of the effective coupling constant. If the new renormalization scale  $\mu'$  is used, the solution of the renormalization-group equations takes the following form:

$$C_n \left( \frac{Q^2}{\mu^2}, g^2 \right) = C_n \left( \frac{\mu'^2}{\mu^2}, \bar{g}^2 \right) \exp \left( \int_{\bar{g}}^g d\tau \frac{\gamma_n(\tau)}{\beta(\tau)} \right), \quad (3.20)$$

with  $\bar{g}(\mu'^2) = g$ . Although the expression of  $C_n(Q^2/\mu^2, g^2)$  in terms of  $g$  and  $\mu$  in (3.20) is different from that in (2.21), the physical result is identical.

It is clear that the addition of the constant  $b$  in the definition of  $\bar{g}^2$  (3.19) just reproduces the effect of shifting  $\Lambda$  to absorb terms proportional to  $\gamma_0^n$  in  $B_n$ . The complete second-order contributions to the coefficient function given in Eq. (3.18) are, of course, independent of this redefinition, as would be full all order calculations. However, if we truncate the calculation in second order in  $\bar{g}^2$ , different choices for  $b$  correspond to different estimates of the higher-order terms in  $\bar{g}^2$  for the coefficient functions. The choice for  $b$  must give results consistent with the asymptotic perturbation expansion and the normalization scheme used.

This freedom in choosing  $b$  can be exercised separately for each  $n$  when solving the renormalization-group equation. One possible procedure, therefore, involves the introduction of a  $b_n$  in Eq. (3.19) such that in Eq. (3.20)

$$C_n \left( \frac{\mu_n'^2}{\mu^2}, \bar{g}_n^2 \right) = 1,$$

with  $\mu_n' = \mu e^{-b_n}$  and  $\bar{g}_n^2$  depending on  $n$  through

$$\Lambda_n = \Lambda \exp(B_n^{\text{NS}}/\gamma_0^n),$$

so the  $n$  dependence of  $\Lambda_n$  is calculable. This procedure has the nice property that the relative  $n$  dependence between  $B_n^{\text{NS}}$  and  $\gamma_0^n$ , i.e., the difference in  $n$  dependence between order  $\bar{g}^2$  and leading order, is isolated in  $\Lambda_n$  and in the two-loop anomalous dimensions (the effect of the latter is small for  $n < 12$ ). This procedure is similar to one proposed by Bacé.<sup>25</sup> The phenomenological implications of our calculations are discussed in Sec. V.

#### IV. $g^2$ CORRECTIONS TO $\nu$ AND $\bar{\nu}$ SCATTERING (NON-SINGLET CONTRIBUTIONS)

##### A. Calculation of $g^2$ corrections

The evaluation of  $g^2$  corrections to  $\nu$  and  $\bar{\nu}$  deep-inelastic scattering proceeds as in Sec. II and III

except that now we must also deal with axial-vector currents. It is convenient to consider certain combinations of the  $\nu, \bar{\nu}$  structure functions which have simple properties under crossing. These are

$$F_2^{\bar{\nu}p} - F_2^{\nu p}, \quad (4.1)$$

$$F_2^{\bar{\nu}p} + F_2^{\nu p}, \quad (4.2)$$

$$F_3^{\bar{\nu}p} - F_3^{\nu p}, \quad (4.3)$$

and

$$F_3^{\bar{\nu}p} + F_3^{\nu p}. \quad (4.4)$$

The remaining structure functions for scattering off neutron or nuclear targets can be directly obtained from (4.1)–(4.4) using charge symmetry. For instance,

$$F_2^{\bar{\nu}p} - F_2^{\nu p} = F_2^{\nu n} - F_2^{\nu p} = F_2^{\nu n} - F_2^{\bar{\nu}n} = F_2^{\bar{\nu}p} - F_2^{\bar{\nu}n}. \quad (4.5)$$

In order to calculate  $g^2$  corrections to  $F_2$ , one considers again the diagrams of Fig. 2 except that now diagrams with both vector currents replaced by axial-vector currents also contribute. But since we have put masses to zero the axial-vector–axial-vector contributions are equal to vector–vector contributions. Obviously, calculation of the  $g^2$  corrections to the combinations (4.1) and (4.2) corresponds to subtracting and adding crossed diagrams respectively.

The structure function  $F_3$  corresponds to the vector–axial-vector interference and therefore the diagrams contributing to it are obtained from Fig. 2 by replacing one of the vector currents by an axial-vector current. Again the calculation of the  $g^2$  corrections to the combinations (4.3) and (4.4) corresponds to subtracting and adding crossed diagrams, respectively.

By inspecting the diagrams directly or by considering the decomposition (2.1) and taking into account known properties of various structure functions under the transformations  $\mu \rightarrow \nu, x \rightarrow -x$ , one can easily find whether even or odd spin operators contribute to each of the combinations (4.1)–(4.4). It turns out<sup>2,26</sup> that to  $F_2^{\bar{\nu}-\nu}$  and  $F_3^{\bar{\nu}+\nu}$  only odd-spin and to  $F_2^{\bar{\nu}+\nu}$  and  $F_3^{\bar{\nu}-\nu}$  only even-spin operators contribute.

Finally we have to determine which combinations are independent of gluon operators and therefore satisfy simple renormalization-group equations as given in Eq. (2.20). The combinations (4.1) and (4.3) transform obviously as nonsinglets under flavor symmetry and therefore satisfy equations like (2.20).  $F_2^{\bar{\nu}p} + F_2^{\nu p}$  is a singlet combination which can be seen, for instance, by writing it in terms of quark distributions. Therefore, because of mixing between gluon and fermion singlet operators this combination will satisfy more compli-

cated renormalization-group equations, which we shall discuss in Sec. VI. On the other hand,  $F_3^{\bar{\nu}+\nu}$  still satisfies Eq. (2.20) in spite of having contributions from singlet fermion operators. This is because the gluon operators of odd spin<sup>27</sup> transform differently under charge conjugation than the corresponding singlet fermion operators and therefore there is no mixing.<sup>26</sup>

In this section we shall restrict the calculation to the combinations (4.1), (4.3), and (4.4) and come back to the combination (4.2) in the next section.

For  $F_2^{\bar{\nu}+\nu} - F_2^{\nu p}$  the calculation is exactly as in Sec. III and we obtain for the corresponding coefficient functions defined in Eq. (2.4) the following final result:

$$C_{2,n}^{\bar{\nu}+\nu}(1, \bar{g}^2) = C_{2,n}^{\text{NS}}(1, \bar{g}^2), \quad n \text{ odd} \quad (4.6)$$

$$C_{L,n}^{\bar{\nu}+\nu}(1, \bar{g}^2) = C_{L,n}^{\text{NS}}(1, \bar{g}^2), \quad n \text{ odd} \quad (4.7)$$

where  $C_{2,n}^{\text{NS}}$  and  $C_{L,n}^{\text{NS}}$  are given by Eqs. (3.7)–(3.10).

On the other hand, we find

$$C_{3,n}^{\bar{\nu}+\nu}(1, \bar{g}^2) = C_{2,n}^{\text{NS}}(1, \bar{g}^2) - \frac{\bar{g}^2}{16\pi^2} C_2(R) \frac{4n+2}{n(n+1)}, \quad \begin{array}{l} n \text{ odd} \\ n \text{ even} \end{array} \quad (4.8)$$

#### B. Corrections to sum rules and parton-model relations

It is well known<sup>2</sup> that in the leading order of asymptotic freedom parton-model relations and sum rules are satisfied. The  $\bar{g}^2$  corrections calculated in this section can generally introduce violations of the sum rules and relations in question.

Evaluating the formulas (4.6)–(4.8) for  $n=1$  and

recalling that  $\gamma_{n-1}=0$  due to current conservation, so that, except for  $B_n^{\text{NS}}$  calculated here, all contributions in Eq. (2.29) vanish, we obtain

$$\int_0^1 \frac{dx}{x} (F_2^{\bar{\nu}p} - F_2^{\nu p}) = 2, \quad (4.9)$$

$$\int_0^1 dx (F_3^{\bar{\nu}p} + F_3^{\nu p}) = -6 \left[ 1 - \frac{12}{(33-2f)\ln(Q^2/\Lambda^2)} \right], \quad (4.10)$$

and

$$\int_0^1 dx (F_1^{\bar{\nu}p} - F_1^{\nu p}) = \left[ 1 - \frac{8}{(33-2f)\ln(Q^2/\Lambda^2)} \right]. \quad (4.11)$$

These results disagree with those obtained by Calvo.<sup>11</sup> Notice that the Adler sum rule<sup>28</sup> (4.9) is exactly satisfied, whereas both the Gross–Llewellyn Smith<sup>29</sup> sum rule (4.10) and the Bjorken sum rule<sup>30</sup> (4.11) are violated. In Fig. 4 we have plotted predictions of (4.10) and (4.11) versus  $Q^2/\Lambda^2$ . We observe that the deviations from the two sum rules in question are predicted to be non-negligible and accurate measurements should detect them.

Equations (3.8) and (4.1) imply violation of the Callan–Gross relation  $2xF_1 = F_2$ . Previous investigation<sup>6, 7, 14</sup> has shown, however, that the violations of the relations in question seen in the data at large  $x$  are larger than predicted by the theory.

#### V. PHENOMENOLOGY OF THE ORDER- $\bar{g}^2$ CORRECTIONS

We discuss in this section the phenomenological application of our calculation. The theory predicts directly the  $Q^2$  dependence of the moments of

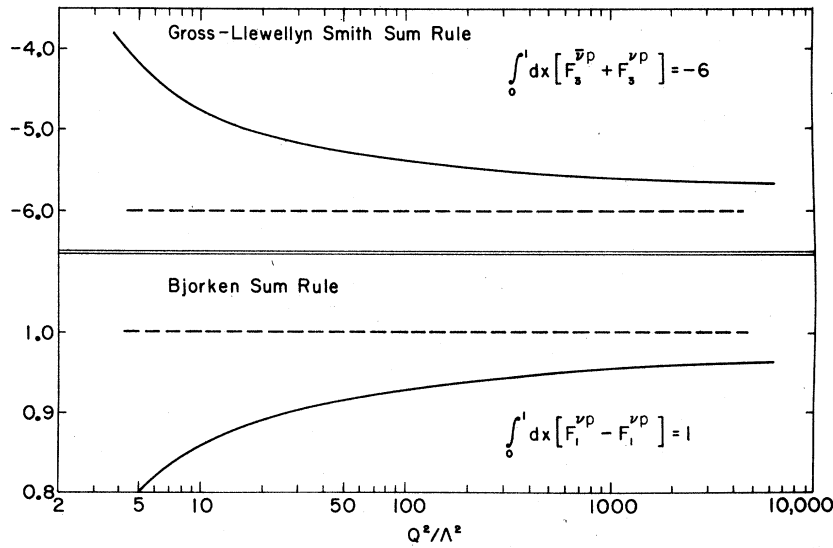


FIG. 4. Order- $\bar{g}^2$  deviations from the Gross–Llewellyn Smith and Bjorken sum rules. The dashed lines are parton-model predictions. The solid lines follow from Eqs. (4.10) and (4.11).

structure functions; we will use in our analysis the combined Gargamelle-BEBC data<sup>10</sup> for the non-singlet structure function  $x F_3(x, Q^2)$  to obtain the experimental values of the moments, thereby avoiding the somewhat involved problem of inverting the moments. Bosetti *et al.*<sup>10</sup> have already analyzed the moments of  $x F_3$  using only the leading-order effects of asymptotic freedom. It is our purpose here to investigate the effect of including the order- $\bar{g}^2$  corrections in the analysis.

The moments of  $x F_3$  most appropriate for comparison with our calculation are the Nachtmann moments<sup>5</sup>

$$M_n(Q^2) = \int_0^1 dx \frac{\xi^{n+1}}{x^3} x F_3(x, Q^2) \left[ 1 + \frac{n+1}{n+2} 2 \left( \frac{x}{\xi} - 1 \right) \right] \quad (5.1)$$

with

$$\xi = 2x / [1 + (1 + 4M^2 x^2 / Q^2)^{1/2}]$$

and  $M$  the nucleon mass. The moments are obtained by straightforward numerical evaluation of the integral using the data of Ref. 10.

In order to investigate the effect of the order- $\bar{g}^2$  corrections, we have chosen four different schemes with which to compare theory and experiment. The first scheme, denoted LO, uses simply the leading-order prediction of asymptotic freedom, which for the nonsinglet moments is

$$M_n(Q^2) = A_n \left( \ln \frac{Q^2}{\Lambda_{LO}^2} \right)^{-\gamma_0^n / 2\beta_0}, \quad (5.2)$$

where the  $A_n$  are related to matrix elements of the relevant operators between target states and are taken as free parameters,  $\gamma_0^n$  and  $\beta_0$  are defined in Eqs. (2.32) and (2.33), and  $\Lambda_{LO}$  is the scale parameter defined by the leading-order expression (2.27) for  $\bar{g}^2(Q^2)$ .

The second and third schemes include the order- $\bar{g}^2$  corrections which multiply (5.2) by an additional term, yielding

$$M_n(Q^2) = A_n \left[ 1 + \frac{1}{\beta_0 \ln(Q^2 / \Lambda_{MS}^2)} (B_n + P_n + L_n) \right] \times \left( \ln \frac{Q^2}{\Lambda_{MS}^2} \right)^{-\gamma_0^n / 2\beta_0}, \quad (5.3)$$

where

$$P_n = \frac{\gamma_1^n}{2\beta_0} - \frac{\beta_1 \gamma_0^n}{2\beta_0^2},$$

$$L_n = - \frac{\beta_1 \gamma_0^n}{2\beta_0^2} \ln \ln \frac{Q^2}{\Lambda_{MS}^2},$$

and  $B_n$  is obtained for  $x F_3$  from Eqs. (3.7), (3.9), and (4.8). This expression uses a scale parameter  $\Lambda_{MS}$  (MS for minimal scheme) corresponding

to the definition (2.26) of  $\bar{g}^2(Q^2)$  introduced in Ref. 7. As discussed in Sec. III D, however,  $B_n$  is actually determined only up to a term proportional to  $\gamma_0^n$ , corresponding to an arbitrariness in the normalization involved in introducing  $\bar{g}$  when solving the renormalization-group equation. For the purposes of illustration, therefore, we will also use (5.3) with  $B_n$  replaced by  $\bar{B}_n = B_n - \frac{1}{2} \gamma_0^n (\ln 4\pi - \gamma_E)$  and  $\Lambda_{MS}$  replaced by  $\Lambda_{\overline{MS}}$ , thereby defining the scheme  $\overline{MS}$ . It is easy to see that the schemes MS and  $\overline{MS}$  are equivalent (through order  $\bar{g}^2$ ) provided

$$\Lambda_{\overline{MS}} = \Lambda_{MS} e^{(\ln 4\pi - \gamma_E)/2}. \quad (5.4)$$

Finally, we introduce a fourth scheme which, as discussed in Sec. III D, treats the  $n$  dependence of  $\gamma_1^n$  exactly but absorbs the  $n$  dependence of  $B_n$  into  $\Lambda$ . Thus using Eqs. (2.23) and (2.24), we evaluate the integral in Eq. (1.1) to obtain

$$M_n(Q^2) = A_n \left( 1 + \frac{\beta_1}{\beta_0} \frac{\bar{g}^2}{16\pi^2} \right)^{\gamma_1^n / 2\beta_1 - \gamma_0^n / 2\beta_0} \left( \frac{\bar{g}^2}{16\pi^2} \right)^{\gamma_0^n / 2\beta_0}, \quad (5.5)$$

where  $\bar{g}^2$  is the exact solution of Eq. (2.22) appropriate to our choice of  $\Lambda_n$

$$\frac{16\pi^2}{\beta_0 \bar{g}^2} - \frac{\beta_1}{\beta_0^2} \ln \frac{16\pi^2 + (\beta_1 / \beta_0) \bar{g}^2}{\beta_0 \bar{g}^2} = \ln \frac{Q^2}{\Lambda_n^2}, \quad (5.6)$$

$$\Lambda_n = \Lambda e^{\bar{B}_n / \gamma_0^n}. \quad (5.7)$$

This fourth scheme, which we denote as the  $\Lambda_n$  scheme, is clearly equivalent (through order  $\bar{g}^2$ ) to the schemes MS and  $\overline{MS}$ . This scheme has the particularly nice property that if Eq. (5.5) is used to determine "experimental"  $\Lambda_n$ 's separately for each  $n$ , then the resulting  $\Lambda_n$ 's should follow the pattern of  $n$  dependence predicted by Eq. (5.7).

As a corollary to this scheme, we remark that the second factor in Eq. (5.5) is always very near unity (see Table I) in the region of interest, hence Eq. (5.5) has essentially the same form as the leading order Eq. (5.2), and therefore this scheme is quite similar in spirit to one proposed by Bacé.<sup>25</sup>

We have used each of the four schemes LO, MS,  $\overline{MS}$ , and  $\Lambda_n$  discussed above to determine the unknown scale parameter in each scheme (the constants  $A_n$  are also fitted but their values are uninteresting and are not given). We find

$$\Lambda_{LO} = 0.73 \text{ GeV},$$

$$\Lambda_{MS} = 0.40 \text{ GeV},$$

$$\Lambda_{\overline{MS}} = 0.52 \text{ GeV},$$

and

$$\Lambda = 0.40 \text{ GeV}.$$

These values are obtained using  $\beta_0, \beta_1, \gamma_0^n, \gamma_1^n$  appro-

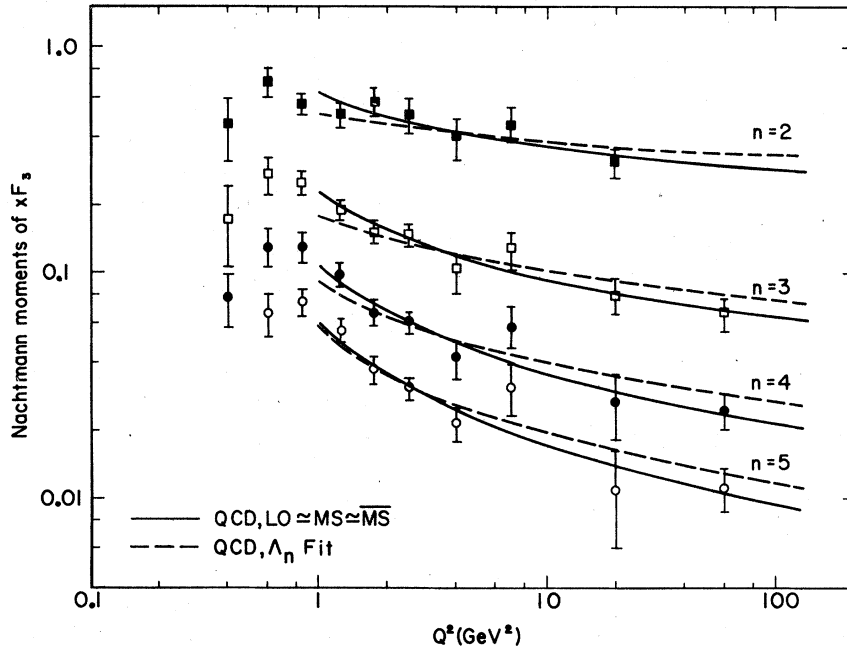


FIG. 5. Nachtmann moments of  $xF_3(w, Q^2)$  vs  $Q^2$ . The data are from Ref. 10. The solid lines represent the LO, MS, and  $\overline{\text{MS}}$  schemes; the dashed lines represent the  $\Lambda_n$  scheme.

priate to four flavors (using three flavors shifts each  $\Lambda$  to a slightly larger value). The fitting program estimates the errors on each  $\Lambda$  at about ten percent. In each case we used moments for  $n \leq 8$  and  $Q^2 > 1 \text{ GeV}^2$ . The results of the fits do not change significantly if we restrict our considerations to  $Q^2 > 2 \text{ GeV}^2$ . In all cases the quality of the fit was very good ( $\chi^2/\text{D.F.} \leq 1$ ).

We now proceed to discuss several important points related to the phenomenology: (1) Our fits for the four schemes are shown in Fig. 5. Somewhat suprisingly (and disappointingly), we find that the LO, MS, and  $\overline{\text{MS}}$  schemes are virtually indistinguishable for  $Q^2 > 1.5 \text{ GeV}^2$ ; thus they are represented by the same (solid) line in the figure. Of course, the fits differ for  $Q^2 < 1 \text{ GeV}^2$  but then  $\bar{g}^2$  is large and the perturbation theory is meaningless. The fit using the  $\Lambda_n$  scheme does not fit quite as well as the others, but the quality of the fit is still fairly good. The similarity of the LO, MS, and  $\overline{\text{MS}}$  fits simply indicates that it is possible for the  $\Lambda_n$ 's and  $\Lambda$  in each case to conspire to mask the combined  $n$  and  $Q^2$  dependence of the order- $\bar{g}^2$  corrections. This is in spite of the fact that the corrections in question are not necessarily small for low values of  $Q^2$  and  $n > 3$ . For instance, for  $Q^2 \approx 10 \text{ GeV}^2$  and  $n = 5$  the second factor in Eq. (5.3) is roughly 1.2 and 1.5 for  $\overline{\text{MS}}$  and MS, respectively, as compared to 1 in the leading order. The situation is much worse for higher moments since for fixed  $Q^2$  the corrections in question grow due

to  $B_n$  like  $(lm)^2$  [see Eq. (3.9)] and perturbation theory breaks down.

(2) The most important result of the analysis of Bosetti *et al.*<sup>10</sup> is the quantitative verification of quantum chromodynamics (QCD) based on the leading-order prediction

$$\frac{d \ln M_n}{d \ln M_m} = \frac{\gamma_n^n}{\gamma_0^m}. \quad (5.8)$$

It is demonstrated in Ref. 10 that the experimental values of the left-hand side of (5.8) agree remarkably well with the QCD predictions for the ratios  $\gamma_n^n/\gamma_0^m$ . We find that the agreement between theory and experiment is not disturbed even when the order- $\bar{g}^2$  corrections are large, i.e., as discussed in point (1) above, the schemes LO and MS,  $\overline{\text{MS}}$  are indistinguishable (except that the  $\Lambda_n$ 's and  $\Lambda$  are very different in each case).

(3) Finally, we have fitted the moments for each  $n$  separately using the  $\Lambda_n$  scheme, and the "experimental" results for  $\Lambda_n$  are shown in Fig. 6, along with the prediction of Eq. (5.7) using the value  $\Lambda = 0.4 \text{ GeV}$  determined previously in the  $\Lambda_n$  scheme by fitting together all moments with  $n \leq 8$ . The "data" points do not seem to follow the theoretical prediction, but we cannot claim that the disagreement is significant since, after all, the  $\Lambda_n$  scheme is equivalent (through the order  $\bar{g}^2$  we have computed) with the MS,  $\overline{\text{MS}}$  schemes, and the latter fit the data quite well. We believe, however, that the  $\Lambda_n$  scheme is particularly well suited to compari-

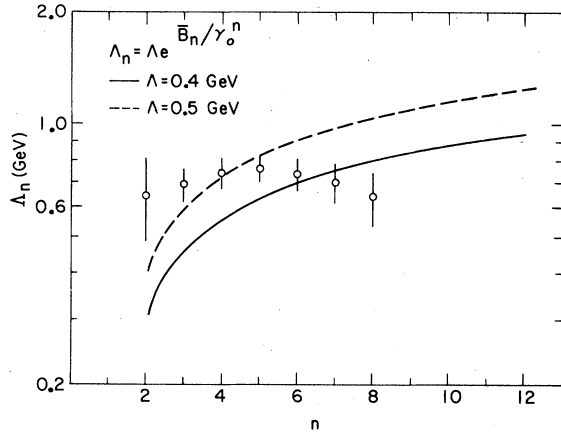


FIG. 6. The  $n$  dependence of  $\Lambda_n$  in the  $\Lambda_n$  scheme. The "data" points come from fitting each  $n$  separately. The curves are the predictions of Eq. (5.7).

son with experiment because the  $n$  dependence predicted by the theory is strictly enforced in the fit (thus prohibiting the kind of conspiracy between the  $\Lambda_n$ 's and  $\Lambda$  which occurs in the  $\overline{\text{MS}}$ ,  $\overline{\text{MS}}$  schemes).

(4) We have also shown (as a dashed line) in Fig. 6 the prediction of Eq. (5.7) for  $\Lambda=0.5$  GeV. This value of  $\Lambda$  gives good agreement with the  $\Lambda_n$  "data" for  $n \leq 5$  and isolates the disagreement at large  $n$ . We are motivated here by the conjecture of De Rújula *et al.*,<sup>6</sup> which claims that the effect of higher twist operators in the operator product expansion is most strongly felt at large  $n$ , or more specifically whenever

$$n \frac{M_0^2}{Q^2} \sim O(1),$$

with  $M_0^2 \sim O(\Lambda^2)$  an appropriate (but unknown) scale. Mainly as a curiosity, we have included a term of the form

$$1 + n \frac{M_0^2}{Q^2}$$

as an additional factor in (5.5) and redetermined the "experimental"  $\Lambda_n$ 's. With  $M_0^2 \sim -0.16$  GeV<sup>2</sup>, the resulting  $\Lambda_n$  "data" do in fact follow the trend predicted by Eq. (5.7), i.e.,  $\Lambda_n$  increases with  $n$ . Owing to large theoretical uncertainties in the higher twist corrections, it is not appropriate to pursue this farther.

We conclude this section with the following words of caution and recommendation. If the  $n$  dependence of  $\Lambda_n$  as predicted by Eq. (5.7) is not eventually found in higher-statistics experimental data, then one must conclude (short of abandoning QCD) that certain possibly strong effects not included in the analysis are present in the data. Possible sources for these effects would be higher-twist

operators, higher-order gluon corrections to twist-2 operators, or perhaps even nonperturbative (instanton) effects. It will be very helpful to isolate these effects phenomenologically, since the theoretical calculations are apparently rather difficult. Finally, it should be apparent by now that due to the fact that  $B_n$  is determined only up to a term proportional to  $\gamma_0^n$ , any attempts to extract the scale parameter  $\Lambda$  of the theory from the experimental data must be viewed with a certain amount of reservation.

## VI. $C_n(Q^2/\mu^2, g^2)$ TO ORDER $g^2$ FOR SINGLET OPERATORS

### A. Preliminaries

In Secs. III and IV we have calculated the coefficient function to order  $\bar{g}^2$  for nonsinglet operators relevant for  $e p$  and  $\nu$  scattering. Here we extend our analysis to singlet operators. We begin with electromagnetic currents.

The  $Q^2$  dependence of the Wilson coefficient functions  $C_n^\psi$  and  $C_n^G$  corresponding to singlet fermion  $O_n^\psi$  and  $O_n^G$  operators is governed by the following renormalization-group equations<sup>2</sup>:

$$\left[ \mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g} \right] C_n^i \left( \frac{Q^2}{\mu^2}, g^2 \right) = \sum_j \gamma_{ji}^n(g^2) C_n^j \left( \frac{Q^2}{\mu^2}, g^2 \right), \quad (6.1)$$

where  $i, j = \psi, G$  and  $\gamma_{ij}^n$  is the anomalous-dimension matrix.

The solution to (6.1) is given as follows:

$$C_n^i \left( \frac{Q^2}{\mu^2}, g^2 \right) = \sum_j \left\{ T_g \exp \left[ + \int_{\bar{g}}^g \frac{\gamma_{ji}^n(g')}{\beta(g')} dg' \right] \right\}_{ji} \times C_n^j(1, \bar{g}^2), \quad (6.2)$$

with  $C_n^j(1, \bar{g}^2)$  having the expansion in powers of  $\bar{g}^2$

$$C_n^i(1, \bar{g}^2) = C_n^{(0)i} + B_n^i \frac{\bar{g}^2}{16\pi^2} + O(\bar{g}^4). \quad (6.3)$$

It is instructive to expand  $C_n^i(Q^2/\mu^2, g^2)$  as given by (6.2) in powers of  $g$ . Recalling that in the  $g^0$  order in the normalization of Eq. (2.17)

$$C_n^{(0)\psi} = 1, \quad C_n^{(0)G} = 0, \quad (6.4)$$

we obtain

$$C_n^\psi \left( \frac{Q^2}{\mu^2}, g^2 \right) = 1 + \frac{g^2}{16\pi^2} \left( -\frac{1}{2} \gamma_{\psi\psi}^n \ln \frac{Q^2}{\mu^2} + B_n^\psi \right) \quad (6.5)$$

and

$$C_n^G \left( \frac{Q^2}{\mu^2}, g^2 \right) = \frac{g^2}{16\pi^2} \left( -\frac{1}{2} \gamma_{\psi G}^n \ln \frac{Q^2}{\mu^2} + B_n^G \right). \quad (6.6)$$

Here  $\gamma_{\psi\psi}^n$  is equal to  $\gamma_0^n$  of Eq. (2.32).  $\gamma_{\psi G}^n$  is the nondiagonal element of the anomalous-dimension matrix given by<sup>2</sup>

$$\gamma_{\psi G}^n = -4f \frac{n^2 + n + 2}{n(n+1)(n+2)}, \quad (6.7)$$

$f$  being the number of flavors. In order to calculate  $B_n^i$  in perturbation theory we consider the two forward Compton amplitudes

$$T^\psi(Q^2, \nu) = i \int d^4x e^{i\alpha \cdot x} \langle \psi; p | T[j(x)j(0)] | \psi; p \rangle \quad (6.8)$$

and

$$T^G(Q^2, \nu) = i \int d^4x e^{i\alpha \cdot x} \langle G; p | T[j(x)j(0)] | G; p \rangle \quad (6.9)$$

where  $p^2 < 0$ .

Using the operator-product expansion for the appropriate currents, we obtain the following generalizations of Eq. (2.35):

$$T^\psi(Q^2, \nu) = \sum_n \left( \frac{1}{x} \right)^n \left[ C_n^\psi \left( \frac{Q^2}{\mu^2}, g^2 \right) A_{n\psi}^\psi \left( \frac{p^2}{\mu^2}, g^2 \right) + C_n^G \left( \frac{Q^2}{\mu^2}, g^2 \right) A_{n\psi}^G \left( \frac{p^2}{\mu^2}, g^2 \right) \right] \quad (6.10)$$

$$T^G(Q^2, \nu) = \sum_n \left( \frac{1}{x} \right)^n \left[ C_n^\psi \left( \frac{Q^2}{\mu^2}, g^2 \right) A_{nG}^\psi \left( \frac{p^2}{\mu^2}, g^2 \right) + C_n^G \left( \frac{Q^2}{\mu^2}, g^2 \right) A_{nG}^G \left( \frac{p^2}{\mu^2}, g^2 \right) \right]. \quad (6.11)$$

The reduced matrix elements  $A_{nj}^i$  defined in Eq. (2.9) can be calculated in perturbation theory and the result can be written as

$$A_{nj}^i \left( \frac{p^2}{\mu^2}, g^2 \right) = A_{nj}^{(0)i} + \frac{g^2}{16\pi^2} \left[ \frac{1}{2} \gamma_{ij}^n \ln \frac{-p^2}{\mu^2} + A_{nj}^{(2)i} \right], \quad (6.12)$$

where  $A_{nj}^{(0)i}$  in the normalization of Eq. (2.18) take the following values:

$$A_{n\psi}^{(0)\psi} = A_G^{(0)G} = 1, \quad A_{n\psi}^{(0)G} = A_{nG}^{(0)\psi} = 0. \quad (6.13)$$

Inserting (6.5), (6.6), and (6.12) into (6.10) and (6.11), and using (6.4) and (6.13), we obtain in an obvious notation

$$T_n^\psi = 1 + \frac{g^2}{16\pi^2} \left( -\frac{1}{2} \gamma_{\psi\psi}^n \ln \frac{Q^2}{-p^2} + B_n^\psi + A_{n\psi}^{(2)\psi} \right) \quad (6.14)$$

and

$$T_n^G = \frac{g^2}{16\pi^2} \left( -\frac{1}{2} \gamma_{\psi G}^n \ln \frac{Q^2}{-p^2} + B_n^G + A_{nG}^{(2)\psi} \right). \quad (6.15)$$

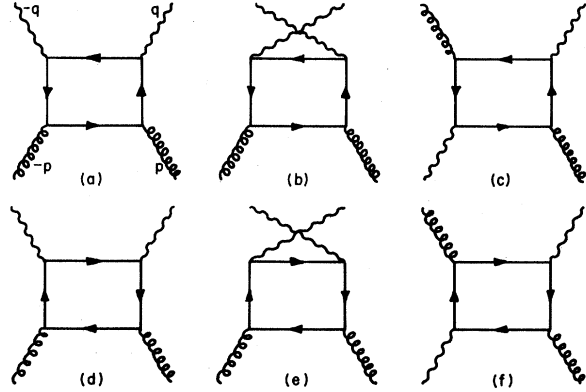


FIG. 7. Diagrams contributing in order  $g^2$  to the virtual photon-gluon scattering.

Consequently,

$$B_n^\psi = T_n^{(2)\psi} - A_{n\psi}^{(2)\psi} \quad (6.16)$$

$$B_n^G = T_n^{(2)G} - A_{nG}^{(2)\psi}, \quad (6.17)$$

where  $T_n^{(2)\psi}$  and  $T_n^{(2)G}$  are the constant parts of order  $g^2$  in the Compton amplitudes (6.8) and (6.9). For longitudinal structure functions which vanish in zeroth order only the first terms on the rhs of Eqs. (6.16) and (6.17) are present.

Equation (6.16) is equivalent to (2.39) and using (6.3) we obtain to order  $\bar{g}^2$

$$C_{2,n}^\psi(1, \bar{g}^2) = C_{2,n}^{NS}(1, \bar{g}^2) \quad (6.18)$$

and

$$C_{L,n}^\psi(1, \bar{g}^2) = C_{L,n}^{NS}(1, \bar{g}^2) \quad (6.19)$$

with the nonsinglet structure functions calculated in Sec. III C.

On the other hand, Eq. (6.17) tells us that in order to calculate  $B_n^G$  we have to find the forward Compton amplitude for a photon scattering off a gluon and subtract from it to the matrix element of the fermion singlet operator (2.6) between gluon states.

### B. Calculation of $T_n^{(2)G}$

The calculation of  $T_n^{(2)G}$  involves diagrams of Fig. 7. We use again the dimensional regularization scheme and keep the gluons offshell with space-like momentum  $p^2 < 0$ . Both gluons and quarks are kept massless. The diagrams separately are divergent but when they are combined the divergences cancel and no renormalization is needed. The result for  $T_{L,n}^{(2)G}$  and  $T_{2,n}^{(2)G}$  is

$$T_{L,n}^{(2)G} = \frac{g^2}{16\pi^2} T(R) \frac{16}{(n+1)(n+2)}, \quad n \text{ even} \quad (6.20)$$

and

$$T_{2,n}^{(2)G} = \frac{g^2}{16\pi^2} T(R) 4 \left( -\frac{2}{n} + \frac{6}{n+1} - \frac{6}{n+2} + \frac{2}{n^2} - \frac{4}{(n+1)^2} + \frac{4}{(n+2)^2} \right), \quad n \text{ even} \quad (6.21)$$

where  $T(R) = f/2$ ,  $f$  being the number of flavors.

### C. Calculation of $A_{nG}^{(2)\psi}$

The calculation of  $A_{nG}^{(2)\psi}$  involves diagrams of Fig. 8. Using the minimal-subtraction scheme we obtain

$$A_{nG}^{(2)\psi} = \frac{g^2}{16\pi^2} T(R) 4 \left[ \frac{2}{n+1} - \frac{2}{n} - \frac{2}{n+2} + \frac{1}{n^2} - \frac{4}{(n+1)^2} + \frac{4}{(n+2)^2} + \frac{n^2+n+2}{n(n+1)(n+2)} \left( 1 + \sum_{j=1}^n \frac{1}{j} \right) + \tilde{\gamma}_{\psi G}^n (\ln 4\pi - \gamma_E) \right], \quad n \text{ even} \quad (6.22)$$

where

$$\tilde{\gamma}_{\psi G}^n = \frac{n^2+n+2}{n(n+1)(n+2)} = -\frac{1}{4f} \gamma_{\psi G}^n. \quad (6.23)$$

### D. Final result for $C_{L,n}^G(1, \bar{g}^2)$ and $C_{2,n}^G(1, \bar{g}^2)$

From Eqs. (6.20)–(6.22) we finally obtain, using (6.17) and (6.3), the following expressions for  $C_{L,n}^G(1, \bar{g}^2)$  and  $C_{2,n}^G(1, \bar{g}^2)$  to order  $\bar{g}^2$  calculated in the minimal-subtraction scheme:

$$C_{L,n}^G(1, \bar{g}^2) = \frac{\bar{g}^2}{16\pi^2} T(R) \frac{16}{(n+1)(n+2)}, \quad n \text{ even} \quad (6.24)$$

and

$$C_{2,n}^G(1, \bar{g}^2) = \frac{\bar{g}^2}{16\pi^2} T(R) 4 \left[ \frac{4}{(n+1)} - \frac{4}{(n+2)} + \frac{1}{n^2} - \frac{n^2+n+2}{n(n+1)(n+2)} \left( 1 + \sum_{j=1}^n \frac{1}{j} \right) - \tilde{\gamma}_{\psi G}^n (\ln 4\pi - \gamma_E) \right], \quad n \text{ even}. \quad (6.25)$$

The treatment of the  $\ln 4\pi - \gamma_E$  term is exactly the same as in the nonsinglet case.

### E. Results for $\nu, \bar{\nu}$ scattering

The results (6.18), (6.19) and (6.24) and (6.25) apply also to the structure function  $F_2^{\nu+\bar{\nu}}$ . On the other hand,  $F_3^{\nu+\bar{\nu}}$  does not receive contributions from gluon operators by the discussion of Sec. IV. The result for  $C_3^{\nu+\bar{\nu}}(1, \bar{g}^2)$  is given in Eq. (4.9).

### F. Discussion of results for singlet parts

At present we are not ready to apply (6.25) to phenomenological analyses because the anomalous dimensions of singlet operators to order  $g^4$  are not yet available. However, the longitudinal coefficient function (6.24) does not involve two-loop contributions and hence may be used directly in phenomenology. Such analyses have been performed in Ref. 14. It should be remarked that in (6.24) and (6.25) the  $g^2$  corrections are small and vanish as  $n \rightarrow \infty$ . Therefore, for large  $x$  the gluon contributions and their mixing with quark operators are of little importance.

To compare our results with those of other groups, we first note that the calculation of virtual photon-gluon scattering is equivalent to that of virtual photon-photon scattering if one replaces  $g^2 T(R)$  by  $e^4$ . Accordingly, the coefficients of the expansion in  $1/x$ ,  $T_{L,n}^G$  and  $T_{2,n}^G$ , are equal to moments of

the imaginary part of the virtual photon-photon scattering amplitude. Our result for  $T_{L,n}^G$  (6.20) agrees with the results of Refs. 14, 31, and 32, but disagrees in minor respects with Refs. 11, 12, 13, 33, and 34.

Special care is needed in comparing our result for  $T_{2n}^G$  with others. In the calculation of  $T_{2n}^G$  there exists an infrared divergence coming from mass singularities, while no ultraviolet divergence appears owing to the gauge invariance. We

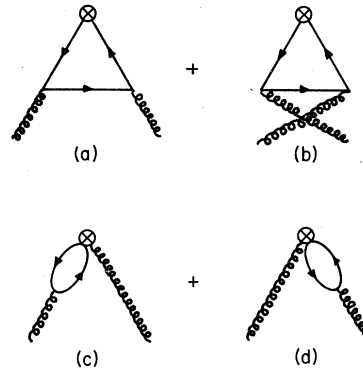


FIG. 8. Diagrams contributing in order  $g^2$  to the matrix elements of the quark operator between gluon states.

have kept the gluon mass  $p^2$  finite (spacelike) to circumvent the mass singularities. Another way of avoiding the mass singularity is to introduce the quark mass  $m$  while  $p^2=0$ . This latter method has been used by other groups.<sup>11, 14, 32-34</sup> Naturally, the results for  $T_{2n}^G$  are different in these two methods and should not be directly compared.<sup>35</sup> On the other hand, the coefficient function  $C_{2n}^G$  should be insensitive to mass singularities which depend on the particular gluon or quark matrix element considered. Calculations of the coefficient function must agree in two methods up to the normalization scale. Our normalization scheme of the operator matrix element is different from that of other groups where the operator matrix element is normalized on the mass shell so that  $T_{2n}^G = C_{2n}^G$ . In order to translate our  $C_{2n}^G$  into theirs we must simply add to our  $C_{2n}^G$  the operator matrix element  $A_{2n}^G$  in the minimal-subtraction scheme with  $m \neq 0$  and  $p^2=0$ . We find

$$A_{2n}^G = \frac{g^2}{4\pi^2} T(R) \tilde{\gamma}_{\psi G}^n \left( \ln \frac{4\pi\mu^2}{m^2} - \gamma_E \right). \quad (6.26)$$

We can now recast our result (6.25) into the form obtained by other groups

$$\int_0^1 dx x^{n-2} F_2^G(x, Q^2) = \frac{g^2}{4\pi^2} T(R) \left[ \tilde{\gamma}_{\psi G}^n \ln \frac{Q^2}{m^2} - 1 - \sum_{j=1}^n \frac{1}{j} + \frac{4}{n+1} - \frac{4}{n+2} + \frac{1}{n^2} \right], \quad (6.27)$$

or equivalently,

$$F_2^G(x, Q^2) = \frac{g^2}{4\pi^2} T(R) x \left[ (1-2x+2x^2) \ln \frac{Q^2(1-x)}{m^2 x} - 1 + 8x(1-x) \right]. \quad (6.28)$$

This result agrees with that of Witten<sup>34</sup> (if we correct the factor  $\frac{1}{4}$  and Kingsley,<sup>31</sup> but disagrees with the result of Hinchliffe and Llewellyn Smith.<sup>14</sup>

## VII. SUMMARY AND CONCLUSIONS

In this paper we have presented a general procedure (see Secs. II and VI) for the calculation of the Wilson coefficient functions  $C_n(Q^2/\mu^2, g^2)$  to order  $g^2$ . An important step in this procedure, not previously discussed in the literature, involves the necessity of calculating both  $g^2$  corrections to the virtual Compton amplitude and  $g^2$  corrections to the matrix elements of local operators in order to find  $C_n(Q^2/\mu^2, g^2)$  in a general renormalization scheme. Using this procedure we have calculated quark and gluon coefficient functions as predicted

by asymptotically free gauge theories. This we have done for both nonsinglet and singlet structure functions ( $F_2, F_3, F_L$ ) relevant for electromagnetic and  $\nu$  processes. Our results when combined with renormalization-group equations give  $\bar{g}^2$  corrections to the functions  $C_n^i(1, \bar{g}^2)$  ( $i=q, G$ ) which enter the solution of the equations in question. The results may be found in Eqs. (3.7)–(3.10), (4.6)–(4.8), (6.24), (6.25). We have emphasized following Ref. 15 that the functions  $C_n^i(1, \bar{g}^2)$  relevant to  $\nu W_2$  and  $\nu W_3$  are renormalization-prescription dependent and that this renormalization-prescription dependence is canceled by that of two-loop anomalous dimensions  $\gamma_1^n$  when the full  $\bar{g}^2$  corrections to the moments of various structure functions are computed. Of course, in order for the cancellation to occur, both  $C_n(1, \bar{g}^2)$  and  $\gamma_1^n$  should be calculated in the same renormalization scheme. As a renormalization scheme we have chosen 't Hooft's minimal-subtraction scheme since the only existing results for  $\gamma_1^n$  have been obtained in this scheme. The longitudinal structure functions to order  $\bar{g}^2$  do not depend on two-loop anomalous dimensions and therefore  $C_{n,L}(1, \bar{g}^2)$  is automatically renormalization-prescription independent.

In the course of the calculation of the quark coefficient function  $C_n^q(1, \bar{g}^2)$ , we have found that the relevant Compton amplitude and the relevant matrix elements of local operators were separately gauge dependent. We have demonstrated however that in the renormalization scheme considered these gauge dependences cancel each other leaving  $C_n(1, \bar{g}^2)$  gauge independent.

Another feature of the  $\bar{g}^2$  corrections to  $C_n(1, \bar{g}^2)$  concerns the ambiguity of these corrections in the term proportional to  $\gamma_0^n$ . This ambiguity and corresponding ambiguity in parameter  $\Lambda$  is related to the freedom of defining the effective coupling constant when solving renormalization-group equations. We have discussed it in detail in Sec. III D.

In Sec. V we have combined our results for nonsinglet structure functions with those for two-loop anomalous dimensions of Ref. 15 and two-loop  $\beta$  function of Ref. 21 to obtain the full  $\bar{g}^2$  correction to the leading-order formula for nonsinglet structure functions. We have compared our results with the recently measured<sup>10</sup> moments of  $\nu W_3^{\nu, \bar{\nu}}$ . In order to demonstrate the ambiguity of  $\bar{g}^2$  corrections mentioned above, we have considered various schemes corresponding to various definitions of the effective coupling constant. All schemes gave a good agreement with experimental data although, using one of them ( $\Lambda_n$  scheme) which is particularly suited for testing the  $n$  dependence of  $\bar{g}^2$  corrections, we have found that for  $n > 5$  some indication of other effects (e.g., higher



twist operators) not included in our analysis may be present in the data. It is important to check for these effects in experiments with high statistics.

Our results for singlet structure functions can be combined with two-loop singlet anomalous dimensions calculated in the minimal-subtraction scheme once such a calculation is completed.<sup>16</sup>

Finally we have calculated  $\bar{g}^2$  corrections to the Gross-Llewellyn Smith and Bjorken sum rules. Our results for these sum rules disagree with those of Calvo.<sup>11</sup> It turns out that the violations of the sum rules in question are of the order of 15% at presently available values of  $Q^2$  and experiments with high statistics should detect them.

*Note added in proof.* The results in Eqs. (6.22), (6.25), (6.26), (A20), and (A22) differ from our results in Fermilab-Pub-78/42-THY (unpublished) by the replacement  $\gamma_E \rightarrow \gamma_E - 1$ . This is due to the fact that in our previous paper we have applied the minimal subtraction scheme to the spin averaged matrix elements of the local operators whereas the authors of Ref. 15 applied the scheme in question directly to the operators. To be consistent with the result of Refs. 15 and 16 we have modified our previous results as indicated above. We thank the authors of Ref. 16 for pointing out the discrepancy between their procedure and ours.

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#### APPENDIX

The details of our calculation of the coefficient functions in electroproduction and neutrino reactions are presented here. The results of the calculation are given for the minimal-subtraction scheme in the dimensional regularization. The calculation is performed in an arbitrary covariant gauge, and the gauge term is given separately. The nonsinglet Born amplitude is normalized to 1. The following projection tensors are used to project out invariant amplitudes:

$$d_{\mu\nu} = -\frac{p_\mu p_\nu}{(p \cdot q)^2} q^2 + \frac{p_\mu p_\nu + p_\nu q_\mu}{p \cdot q} - g_{\mu\nu}, \quad (\text{A1})$$

$$e_{\mu\nu} = g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}. \quad (\text{A2})$$

The constant  $\gamma_E$  appearing in the text is the Euler-Macheroni constant  $\gamma_E = 0.5772\dots$ , and  $g$  is a dimensionless renormalized coupling constant in the minimal-subtraction scheme.

#### A. Electroproduction: Nonsinglet

##### 1. Current correlation functions

The internal self-energy diagram [Fig. 2(a)] gives

$$\frac{g^2}{16\pi} C_2(R) \sum_{n=2,4,\dots} \left(\frac{1}{x}\right)^n \left[ d_{\mu\nu} \left( \gamma_E + \ln \frac{Q^2}{4\pi\mu^2} - 1 - \sum_{j=1}^n \frac{1}{j} + \frac{1}{n} \right) - \frac{p_\mu p_\nu}{(p \cdot q)^2} q^2 \frac{1}{n-1} \right]. \quad (\text{A3})$$

The vertex-correction diagrams [Fig. 2(b) and 2(c)] give

$$\begin{aligned} \frac{g^2}{16\pi} C_2(R) \sum_{n=2,4,\dots} \left(\frac{1}{x}\right)^n & \left[ d_{\mu\nu} \left( -4 \sum_{j=2}^n \frac{1}{j} \ln \frac{Q^2}{-p^2} - 2 \left( \gamma_E + \ln \frac{Q^2}{4\pi\mu^2} \right) + \frac{4}{n} + 4 \sum_{j=1}^n \frac{1}{j} - 8 \sum_{j=1}^n \frac{1}{j^2} \right) \right. \\ & \left. + \frac{p_\mu p_\nu}{(p \cdot q)^2} q^2 \left( \frac{2}{n-1} - \frac{2}{n} \right) + g_{\mu\nu} \frac{2}{n} \right]. \quad (\text{A4}) \end{aligned}$$

The box diagram [Fig. 2(d)] gives

$$\begin{aligned} \frac{g^2}{16\pi} C_2(R) \sum_{n=2,4,\dots} \left(\frac{1}{x}\right)^n & \left[ e_{\mu\nu} \frac{4}{n+1} + d_{\mu\nu} \left( \frac{2}{n(n+1)} \ln \frac{Q^2}{-p^2} - \frac{6}{n} + \frac{6}{n+1} + \frac{4}{n^2} - \frac{4}{(n+1)^2} \right) \right. \\ & \left. + \frac{p_\mu p_\nu}{(p \cdot q)^2} q^2 \left( \frac{2}{n} - \frac{1}{n-1} \right) - g_{\mu\nu} \frac{2}{n} \right]. \quad (\text{A5}) \end{aligned}$$

The gauge term for the whole amplitude is

$$(1-\alpha) \frac{g^2}{16\pi^2} C_2(R) d_{\mu\nu} \sum_{n=2,4,\dots} \left(\frac{1}{x}\right)^n \left( \frac{1}{n} + \sum_{j=1}^n \frac{1}{j} - 1 + \gamma_E + \ln \frac{-p^2}{4\pi\mu^2} \right). \quad (\text{A6})$$

The definition of the gauge parameter  $\alpha$  is given in (3.3). The whole expression is

$$\frac{g^2}{16\pi^2} C_2(R) \sum_{n=2,4,\dots} \left(\frac{1}{x}\right)^n \left\{ e_{\mu\nu} \frac{4}{n+1} + d_{\mu\nu} \left[ -\tilde{\gamma}_0^n \ln \frac{Q^2}{-p^2} - \gamma_E - \ln \frac{-p^2}{4\pi\mu^2} - 1 - \frac{1}{n} + \frac{6}{n+1} + \frac{4}{n^2} - \frac{4}{(n+1)^2} \right. \right. \\ \left. \left. + 3 \sum_{j=1}^n \frac{1}{j} - 8 \sum_{j=1}^n \frac{1}{j^2} + (1-\alpha) \left( \frac{1}{n} + \sum_{j=1}^n \frac{1}{j} - 1 + \gamma_E + \ln \frac{-p^2}{4\pi\mu^2} \right) \right] \right\}, \quad (\text{A7})$$

where

$$\tilde{\gamma}_0^n = 1 - \frac{2}{n(n+1)} + 4 \sum_{j=2}^n \frac{1}{j}, \quad (\text{A8})$$

### 2. Operator matrix element

We present the product of the operator matrix element and the lowest-order coefficient function. The triangle diagram [Fig. 3(a)] gives

$$\frac{g^2}{16\pi^2} C_2(R) d_{\mu\nu} \sum_{n=2,4,\dots} \left(\frac{1}{x}\right)^n \left[ -\frac{2}{n(n+1)} \left( \gamma_E + \ln \frac{-p^2}{4\pi\mu^2} \right) + \frac{2}{n(n+1)} \sum_{j=1}^n \frac{1}{j} - \frac{4}{n} + \frac{2}{n+1} + \frac{2}{n^2} - \frac{4}{(n+1)^2} \right]. \quad (\text{A9})$$

The vertex-correction diagram [Fig. 3(b)] gives

$$\frac{g^2}{16\pi^2} C_2(R) d_{\mu\nu} \sum_{n=2,4,\dots} \left(\frac{1}{x}\right)^n \left[ 4 \sum_{j=2}^n \frac{1}{j} \left( \gamma_E + \ln \frac{-p^2}{4\pi\mu^2} \right) + 8 - 4 \sum_{j=1}^n \frac{1}{j^2} - 4 \sum_{s=1}^n \frac{1}{s} \sum_{j=1}^s \frac{1}{j} \right]. \quad (\text{A10})$$

The gauge term for the whole amplitude is

$$(1-\alpha) \frac{g^2}{16\pi^2} C_2(R) d_{\mu\nu} \sum_{n=2,4,\dots} \left(\frac{1}{x}\right)^n \left( \frac{1}{n} + \sum_{j=1}^n \frac{1}{j} - 1 + \gamma_E + \ln \frac{-p^2}{4\pi\mu^2} \right). \quad (\text{A11})$$

The whole expression is

$$\frac{g^2}{16\pi^2} C_2(R) d_{\mu\nu} \sum_{n=2,4,\dots} \left(\frac{1}{x}\right)^n \left[ (\tilde{\gamma}_0^n - 1) \left( \gamma_E + \ln \frac{-p^2}{4\pi\mu^2} \right) + 8 - \frac{4}{n} + \frac{2}{n+1} + \frac{2}{n^2} - \frac{4}{(n+1)^2} \right. \\ \left. - 4 \sum_{j=1}^n \frac{1}{j^2} + \frac{2}{n(n+1)} \sum_{j=1}^n \frac{1}{j} - 4 \sum_{s=1}^n \frac{1}{s} \sum_{j=1}^s \frac{1}{j} + (1-\alpha) \left( \frac{1}{n} + \sum_{j=1}^n \frac{1}{j} - 1 + \gamma_E + \ln \frac{-p^2}{4\pi\mu^2} \right) \right]. \quad (\text{A12})$$

### 3. Coefficient functions

The coefficient functions are given by

$$\frac{g^2}{16\pi^2} C_2(R) \sum_{n=2,4,\dots} \left(\frac{1}{x}\right)^n (e_{\mu\nu} \tilde{C}_{Ln} + d_{\mu\nu} \tilde{C}_{2n}) = (\text{A7}) - (\text{A12}), \quad (\text{A13})$$

$$\tilde{C}_{Ln} = \frac{4}{n+1},$$

$$\tilde{C}_{2n} = -\tilde{\gamma}_0^n \left( \gamma_E + \ln \frac{Q^2}{4\pi\mu^2} \right) - 9 + \frac{3}{n} + \frac{4}{n+1} + \frac{2}{n^2} + 3 \sum_{j=1}^n \frac{1}{j} - 4 \sum_{j=1}^n \frac{1}{j^2} - \frac{2}{n(n+1)} \sum_{j=1}^n \frac{1}{j} + 4 \sum_{s=1}^n \frac{1}{s} \sum_{j=1}^s \frac{1}{j}, \quad (\text{A14})$$

where  $\tilde{C}_{Ln}$  and  $\tilde{C}_{2n}$  are related to  $C_{Ln}^{\text{NS}}$  and  $C_{2n}^{\text{NS}}$  defined in (3.7) and (3.8) such that  $C_{Ln}^{\text{NS}} = (\bar{g}^2/16\pi^2) C_2(R) \tilde{C}_{Ln}$  and  $C_{2n}^{\text{NS}} = 1 + (\bar{g}^2/16\pi^2) C_2(R) \tilde{C}_{2n}$  at  $Q^2 = \mu^2$ .

## B. Electroproduction: Singlet

Since the quark contribution to the singlet part is exactly the same as that of the nonsinglet case, we discuss here only the gluon contributions.

### 1. Current correlation functions

The contributions of diagrams Fig. 7(d)–7(f) are equal to those of Fig. 7(a)–7(c), respectively, and Fig. 7(b) is a crossed diagram of Fig. 7(a). The contribution of 7(a) + 7(b) + 7(d) + 7(e) is

$$\frac{g^2}{4\pi^2} T(R) \sum_{n=2,4,\dots} \frac{1}{x} \left[ d_{\mu\nu} \left( \frac{1}{n} \ln \frac{Q^2}{-p^2} - \frac{2}{n} + \frac{2}{n^2} \right) - \frac{p_\mu p_\nu}{(p \cdot q)^2} q^2 \left( \frac{1}{n-1} - \frac{1}{n} \right) \right] + \frac{g^2}{4\pi^2} T(R) \left[ -4e_{\mu\nu} + 2g_{\mu\nu} \left( 1 - \gamma_E - \ln \frac{Q^2}{4\pi\mu^2} \right) \right]. \quad (\text{A15})$$

The contribution of 7(c)+7(f) is

$$\frac{g^2}{4\pi^2} T(R) \sum_{n=2,4,\dots} \left( \frac{1}{x} \right)^n \left\{ e_{\mu\nu} \frac{4}{(n+1)(n+2)} + d_{\mu\nu} \left[ -\frac{2}{(n+1)(n+2)} \ln \frac{Q^2}{-p^2} + \frac{6}{n+1} - \frac{6}{n+2} - \frac{4}{(n+1)^2} + \frac{4}{(n+2)^2} \right] \right. \\ \left. + \frac{p_\mu p_\nu}{(p \cdot q)^2} q^2 \left( \frac{1}{n-1} - \frac{1}{n} \right) \right\} + \frac{g^2}{4\pi^2} T(R) \left[ 4e_{\mu\nu} - 2g_{\mu\nu} \left( 1 - \gamma_E - \ln \frac{Q^2}{4\pi\mu^2} \right) \right]. \quad (\text{A16})$$

The whole expression is

$$\frac{g^2}{4\pi^2} T(R) \sum_{n=2,4,\dots} \left( \frac{1}{x} \right)^n \left\{ e_{\mu\nu} \frac{4}{(n+1)(n+2)} + d_{\mu\nu} \left[ \frac{n^2+n+2}{n(n+1)(n+2)} \ln \frac{Q^2}{-p^2} \right. \right. \\ \left. \left. - \frac{2}{n} + \frac{6}{n+1} - \frac{6}{n+2} + \frac{2}{n^2} - \frac{4}{(n+1)^2} + \frac{4}{(n+2)^2} \right] \right\}. \quad (\text{A17})$$

### 2. Operator matrix element

The contribution of Fig. 8(a) and 8(b) is

$$\frac{g^2}{4\pi^2} T(R) d_{\mu\nu} \sum_{n=2,4,\dots} \left( \frac{1}{x} \right)^n \left[ -\frac{1}{n} \left( \gamma_E + \ln \frac{-p^2}{4\pi\mu^2} \right) - \frac{2}{n} + \frac{1}{n^2} + \frac{1}{n} \sum_{j=1}^n \frac{1}{j} + \frac{n^2+n+2}{n(n+1)(n+2)} \right]. \quad (\text{A18})$$

The contribution of Fig. 8(c) and 8(d) is

$$\frac{g^2}{4\pi^2} T(R) d_{\mu\nu} \sum_{n=2,4,\dots} \left( \frac{1}{x} \right)^n \left[ \frac{2}{(n+1)(n+2)} \left( \gamma_E + \ln \frac{-p^2}{4\pi\mu^2} \right) + \frac{2}{n+1} - \frac{2}{n+2} \right. \\ \left. - \frac{4}{(n+1)^2} + \frac{4}{(n+2)^2} - \frac{2}{(n+1)(n+2)} \sum_{j=1}^n \frac{1}{j} \right]. \quad (\text{A19})$$

The whole expression is

$$\frac{g^2}{4\pi^2} T(R) d_{\mu\nu} \sum_{n=2,4,\dots} \left( \frac{1}{x} \right)^n \left[ -\frac{n^2+n+2}{n(n+1)(n+2)} \left( \gamma_E + \ln \frac{-p^2}{4\pi\mu^2} \right) - \frac{2}{n} + \frac{2}{n+1} - \frac{2}{n+2} \right. \\ \left. + \frac{1}{n^2} - \frac{4}{(n+1)^2} + \frac{4}{(n+2)^2} + \frac{n^2+n+2}{n(n+1)(n+2)} \left( 1 + \sum_{j=1}^n \frac{1}{j} \right) \right]. \quad (\text{A20})$$

### 3. Coefficient functions

The coefficient functions are given by

$$\frac{g^2}{4\pi^2} T(R) \sum_{n=2,4,\dots} \left( \frac{1}{x} \right)^n (e_{\mu\nu} \tilde{C}_{Ln} + d_{\mu\nu} \tilde{C}_{2n}) = (\text{A17}) - (\text{A20}), \\ \tilde{C}_{Ln} = \frac{4}{(n+1)(n+2)}, \quad (\text{A21})$$

$$\tilde{C}_{2n} = \frac{n^2+n+2}{n(n+1)(n+2)} \left( \gamma_E + \ln \frac{Q^2}{4\pi\mu^2} \right) + \frac{4}{n+1} - \frac{4}{n+2} + \frac{1}{n^2} - \frac{n^2+n+2}{n(n+1)(n+2)} \left( 1 + \sum_{j=1}^n \frac{1}{j} \right), \quad (\text{A22})$$

where  $\tilde{C}_{Ln}$  and  $\tilde{C}_{2n}$  are related to  $C_{Ln}^C$  and  $C_{2n}^C$  in (6.24) and (6.25) such that  $C_{Ln}^C = (\bar{g}^2/4\pi^2) T(R) \tilde{C}_{Ln}$  and  $C_{2n}^C = (\bar{g}^2/4\pi^2) T(R) \tilde{C}_{2n}$  at  $Q^2 = \mu^2$ .

### C. Neutrino reactions

For the combinations  $T_2^{\bar{\nu}} \pm T_2^{\nu}$  and  $T_L^{\bar{\nu}} \pm T_L^{\nu}$  defined in Sec. IV A the results are exactly the same as in the case of electroproduction with even  $n$  for  $T_{2,L}^{\bar{\nu}} + T_{2,L}^{\nu}$  and with odd  $n$  for  $T_{2,L}^{\bar{\nu}} - T_{2,L}^{\nu}$ . The operator matrix elements are also the same as (A12) with  $n$  even and odd, respectively.

For  $T_3^{\bar{\nu}} \pm T_3^{\nu}$ , which correspond to the VA interference terms, we obtain

$$\frac{g^2}{16\pi^2} C_2(R) \sum_{n=1,3,4,\dots} \left(\frac{1}{x}\right)^n \left[ -\tilde{\gamma}_0^n \ln \frac{Q^2}{-p^2} - \gamma_E - \ln \frac{-p^2}{4\pi\mu^2} - 1 - \frac{3}{n} + \frac{4}{n+1} + \frac{4}{n^2} - \frac{4}{(n+1)^2} \right. \\ \left. + 3 \sum_{j=1}^n \frac{1}{j} - 8 \sum_{j=1}^n \frac{1}{j^2} + (1-\alpha) \left( \frac{1}{n} + \sum_{j=1}^n \frac{1}{j} - 1 + \gamma_E + \ln \frac{-p^2}{4\pi\mu^2} \right) \right]. \quad (A23)$$

The corresponding operator matrix elements are identical to (A12) with  $n$  odd or even. Consequently, the coefficient function is given by

$$C_{3n}^{\bar{\nu}\pm\nu} = 1 + \frac{g^2}{16\pi^2} C_2(R) \left[ -\tilde{\gamma}_0^n \left( \gamma_E + \ln \frac{Q^2}{4\pi\mu^2} \right) - 9 + \frac{1}{n} + \frac{2}{n+1} + \frac{2}{n^2} \right. \\ \left. + 3 \sum_{j=1}^n \frac{1}{j} - 4 \sum_{j=1}^n \frac{1}{j^2} - \frac{2}{n(n+1)} \sum_{j=1}^n \frac{1}{j} + 4 \sum_{s=1}^n \frac{1}{s} \sum_{j=1}^s \frac{1}{j} \right], \quad (A24)$$

with  $n$  odd for  $\bar{\nu}+\nu$  and  $n$  even for  $\bar{\nu}-\nu$ .

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