Calculation of the Compton scattering cross section of charged vector mesons

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The Compton scattering cross section of the charged vector meson is calculated and discussed for the first time. It is explicitly shown that the bad unitarity-violating terms are cancelled in the high-energy limit. Also, the contribution of the charged gluons for deep-inelastic Compton scattering is calculated.

I. INTRODUCTION

In the unified gauge models, there appear many charged vector mesons (gauge bosons): $W^{\pm}$ in the Weinberg-Salam model,[1] additional $V_{\rho}^{\pm}$, $V_{K}^{\pm}$ in the Pati-Salam model,[2] etc. Even before the formal development of the gauge theory, charged vector mesons had been hypothesized as possible intermediate quanta in the weak and strong interactions, and many calculations were performed concerning these vector mesons. But such non-gauge-theoretic calculations showed that the production cross section of the vector meson violates unitarity in the high-energy limit; for example, vector-meson production through annihilation[3] of the charged fermions and Compton scattering of the vector meson.[4,5] Such difficulties can be avoided in the frame of gauge theory in which the renormalizability and unitarity are naturally satisfied by the nature of the theory itself. The explicit cancellation of unitarity-violating terms in $W^{\pm}$-boson production in the Weinberg-Salam model has been calculated by many authors.[6] So far, there is no gauge-theoretic calculation on the Compton scattering of the gauge boson. The purpose of this paper is to calculate the Compton scattering cross section of the charged vector-meson gauge theoretically and to investigate the high-energy behavior.

In Sec. II, the calculation of the cross section and the cancellation of the unitarity-violating terms in the high-energy limit is explicitly shown. The color-gluon contribution in the deep-inelastic Compton scattering of a nucleon, via the parton model, is calculated in Sec. III. In Sec. IV, the results are summarized and discussed.

II. COMPTON SCATTERING CROSS SECTION

The lowest-order photon–vector-meson scattering is given by the Feynman diagrams shown in Fig. 1. The kinds of diagrams and corresponding coupling constants are the same in the Weinberg-Salam model (WS) model and in the Pati-Salam (PS) model since the interacting parts of the Lagrangians are given by

$$\mathcal{L}^{\rm WS} = e(A_\mu W_\nu^- i\overleftrightarrow{\partial}^\mu W^{+\nu} + W_\mu^- W^{+\nu} i\partial^\mu A_\nu + W_\mu^+ A^\nu i\partial^\mu W_\nu^-) + e^2[(A_\mu A^\mu)(W_\nu^+ W^{-\nu}) - (A_\mu W^{-\mu})(A_\nu W^{+\nu})]\,, \quad (1)$$

$$\mathcal{L}^{\rm PS} = e(A_\mu V_{\rho\nu}^- i\overleftrightarrow{\partial}^\mu V_\rho^{+\nu} + V_{\rho\mu}^- V_\rho^{+\nu} i\partial^\mu A_\nu + V_{\rho\mu}^+ A^\nu i\partial^\mu V_\rho^-) + e^2[(A_\mu A^\mu)(V_{\rho\nu}^+ V_\rho^{-\nu}) - (V_{\rho\mu}^- A^\mu)(V_\rho^{+\nu} A_\nu)] + (V_\rho^{\pm} \longleftrightarrow V_K^{\pm})\,. \quad (2)$$

Using the Feynman rules for the vector meson,[7] the amplitudes for each diagram are

$$a = \frac{-i}{s-M^2}\{4(\varepsilon\cdot k)[(\epsilon\cdot\epsilon')(\varepsilon'\cdot k') - (\epsilon'\cdot k)(\epsilon\cdot\varepsilon') - (\varepsilon'\cdot\epsilon')(\epsilon\cdot k')] + 4(\epsilon\cdot\varepsilon)(\epsilon'\cdot k)\varepsilon'\cdot(k-k') + 2(\epsilon\cdot\varepsilon)(\epsilon'\cdot\varepsilon')k'\cdot(k-p)\}\,, \quad (3)$$

$$b = \frac{-i}{u-M^2}\{4(\varepsilon\cdot k')[(\epsilon\cdot\epsilon')(\varepsilon'\cdot k) - (\epsilon\cdot k')(\epsilon'\cdot\varepsilon') - (\varepsilon'\cdot\epsilon)(\epsilon'\cdot k)] + 4(\epsilon\cdot k')(\epsilon'\cdot\varepsilon)\varepsilon'\cdot(k'-k) + 2(\epsilon\cdot\varepsilon')(\epsilon'\cdot\varepsilon)k\cdot(k'+p)\}\,, \quad (4)$$

$$c = i[2(\epsilon\cdot\epsilon')(\varepsilon\cdot\epsilon') - (\epsilon\cdot\varepsilon')(\epsilon'\cdot\varepsilon') - (\epsilon\cdot\varepsilon')(\epsilon'\cdot\varepsilon)]\,, \quad (5)$$

where $s=(k+p)^2$ and $u=(-k'+p)^2$.

With the polarization sum of the vector meson[8]

$$\sum_{\rm pol.} \varepsilon^\mu(p)\varepsilon^\nu(p) = -g^{\mu\nu} + p^\mu p^\nu/M^2\,, \quad (6)$$

the square of the total amplitudes $|A|^2$ is calculated as

$$|A|^2=|a|^2+|b|^2+|c|^2+2a^*b+2b^*c+2c^*a, \tag{7}$$

where

$$\begin{aligned}|a|^2=\frac{1}{(s-M^2)^2}\Big(&\frac{16}{M^4}(\epsilon\cdot\epsilon')^2(k\cdot p)^4+\frac{16}{M^2}[(\epsilon'\cdot k)^2+(\epsilon\cdot k')^2](k\cdot p)^2\\&+\frac{4}{M^2}(\epsilon'\cdot k)^2\{9(k\cdot k')^2+6(k\cdot k')(k'\cdot p)+(k'\cdot p)^2-12(k\cdot k')p\cdot(k-k')\\&\qquad+4[p\cdot(k-k')]^2-4p\cdot(k-k')(k'\cdot p)\}\\&-4(\epsilon'\cdot k)^2k'\cdot(k-p)+[k'\cdot(k-p)]^2\Big),\end{aligned}$$

$$\begin{aligned}|b|^2=\frac{1}{(u-M^2)^2}\Big(&\frac{16}{M^4}(\epsilon\cdot\epsilon')^2(k'\cdot p)^4+\frac{16}{M^2}[(\epsilon'\cdot k)^2+(\epsilon\cdot k')^2](k'\cdot p)^2\\&+\frac{4}{M^2}(\epsilon\cdot k')^2\{9(k\cdot k')^2-6(k\cdot k')(k\cdot p)+(k\cdot p)^2-12(k\cdot k')p\cdot(k-k')\\&\qquad+4[p\cdot(k-k')]^2+4p\cdot(k-k')(k\cdot p)\}\\&-4(\epsilon\cdot k')^2k\cdot(k'+p)+[k\cdot(k'+p)]^2\Big),\end{aligned}$$

$$|c|^2=2(\epsilon\cdot\epsilon')^2+2+\frac{1}{M^2}[(\epsilon\cdot k')^2+(\epsilon\cdot k')^2]+\frac{4}{M^4}(\epsilon\cdot\epsilon')^2(p\cdot p')^2-\frac{6}{M^2}(\epsilon\cdot\epsilon')(\epsilon\cdot k')(\epsilon'\cdot k),$$

$$\begin{aligned}2a^*b=&\frac{1}{(s-M^2)(u-M^2)}(\epsilon\cdot\epsilon')(\epsilon\cdot k')(\epsilon'\cdot k)\{-32k\cdot k'+16(k'-k)\cdot p\\&\qquad+\frac{8}{M^2}[-2(k\cdot k')(k'\cdot p)+2(k\cdot k')(k\cdot p')+2(k'\cdot p)(k'\cdot p')\\&\qquad\quad+16(k\cdot p)(k\cdot p')+(k\cdot k')^2-(k\cdot k')(k'\cdot p)+(k\cdot k')(k\cdot p)\\&\qquad\quad-(k'\cdot p)(k\cdot p)]\}\\&+\frac{1}{(s-M^2)(u-M^2)}(\epsilon\cdot\epsilon')^2\{8(k\cdot k')^2-8(k\cdot k')(k'\cdot p)+8(k\cdot k')(k\cdot p)-8(k'\cdot p)(k\cdot p)+32(k\cdot k')^2\\&\qquad-\frac{32}{M^2}[(k\cdot p)(k'\cdot p)(k\cdot k')+(k\cdot k')(k\cdot p)(k'\cdot p')]+\frac{32}{M^4}(k\cdot p)(k'\cdot p)(k\cdot p')(k'\cdot p')\}\\&+\frac{1}{(s-M^2)(u-M^2)}(\epsilon'\cdot k)^2\Big[-(k\cdot k')+\frac{2}{M^2}(k\cdot p)(k'\cdot p)+16(k\cdot p)\Big]\\&+\frac{1}{(s-M^2)(u-M^2)}(\epsilon\cdot k')^2\Big[-(k\cdot k')+\frac{2}{M^2}(k\cdot p)(k'\cdot p)-16(k'\cdot p)\Big],\end{aligned}$$

$$\begin{aligned}2a^*c=&\frac{8}{(s-M^2)}(\epsilon\cdot\epsilon')^2\Big\{\tfrac{1}{2}k'\cdot(k-p)+2k\cdot k'-\frac{2}{M^2}[(k\cdot p')(k'\cdot p')+(k\cdot p)(k'\cdot p)]+\frac{2}{M^4}(k\cdot p)(k'\cdot p')(p\cdot p')\Big\}\\&-\frac{8}{(s-M^2)}(\epsilon\cdot\epsilon')(\epsilon'\cdot k)(\epsilon\cdot k')\Big\{3+\frac{1}{M^2}[(k'\cdot p')-(k-k')\cdot p'-\tfrac{1}{2}(k-p)\cdot k']\Big\}\\&-\frac{8}{(s-M^2)}\frac{(\epsilon'\cdot k)^2}{M^2}[(k'-k)\cdot p'+\tfrac{1}{2}k'\cdot(k-p)]-\frac{4}{(s-M^2)}k'\cdot(k-p),\end{aligned}$$

$$\begin{aligned}2b^*c=&\frac{8}{(u-M^2)}(\epsilon\cdot\epsilon')^2\Big\{\tfrac{1}{2}k\cdot(k'+p)+2(k\cdot k')-\frac{2}{M^2}[(k\cdot p')(k'\cdot p')+(k\cdot p)(k'\cdot p)]+\frac{2}{M^4}(k'\cdot p)(k\cdot p')(p\cdot p')\Big\}\\&-\frac{8}{(u-M^2)}(\epsilon\cdot\epsilon')(\epsilon'\cdot k)(\epsilon\cdot k')\Big\{3+\frac{1}{M^2}[p'\cdot(k'-k)-\tfrac{1}{2}(k'+p)\cdot k-k\cdot p']\Big\}\\&-\frac{8}{(u-M^2)}\frac{(\epsilon\cdot k')^2}{M^2}[p'\cdot(k'-k)+\tfrac{1}{2}k\cdot(k'+p)]-\frac{4}{(u-M^2)}k\cdot(k'+p).\end{aligned} \tag{8}$$

The terms with $(\epsilon\cdot p)$, $(\epsilon'\cdot p)$ are set to zero without practical difficulties, since the form we want to consider is the cross section in the laboratory frame. Since we are interested in the very-high-energy region, the invariant variables are approximated as

$$\begin{aligned} s&=(k+p)^2\simeq 2k\cdot p=(2Mk)_{\mathrm{lab}},\\ t&=(k-k')^2\simeq -2k\cdot k'=-(s+u),\\ u&=(-k'+p)^2\simeq -2k'\cdot p=(-2Mk')_{\mathrm{lab}}. \end{aligned} \tag{9}$$

Using the above approximation Eq. (8) is reduced to

$$|a|^2=(\epsilon\cdot\epsilon')\frac{s^2}{M^4}+\frac{4}{M^4}[(\epsilon'\cdot k)^2+(\epsilon\cdot k')^2]+\frac{1}{M^2}(\epsilon'\cdot k)^2+4\left(\frac{1}{2}+\frac{u}{s}\right)^2-16\frac{(\epsilon'\cdot k)^2}{s}\left(\frac{1}{2}+\frac{u}{s}\right), \tag{10}$$

$$|b|^2=(\epsilon\cdot\epsilon')\frac{u^2}{M^4}+\frac{4}{M^2}[(\epsilon'\cdot k)^2+(\epsilon'\cdot k)^2]+\frac{1}{M^2}(\epsilon\cdot k')^2+4\left(\frac{1}{2}+\frac{s}{u}\right)^2-16\frac{(\epsilon\cdot k')^2}{u}\left(\frac{1}{2}+\frac{s}{u}\right), \tag{11}$$

$$|c|^2=2(\epsilon\cdot\epsilon')^2+(\epsilon\cdot\epsilon')^2\frac{(s+u)^2}{M^4}+\frac{1}{M^2}[(\epsilon\cdot k')^2+(\epsilon'\cdot k)^2]-\frac{6}{M^2}(\epsilon\cdot\epsilon')(\epsilon\cdot k')(\epsilon'\cdot k)+2, \tag{12}$$

$$\begin{aligned} 2a^*b=&\,2(\epsilon\cdot\epsilon')^2\frac{su}{M^4}+(\epsilon\cdot\epsilon')^2\left[26+12\left(\frac{s}{u}+\frac{u}{s}\right)+\frac{8(s+u)}{M^2}\right]-\frac{8}{M^2}[(\epsilon\cdot k')^2+(\epsilon'\cdot k)^2]\\ &-\frac{6}{M^2}(\epsilon\cdot\epsilon')(\epsilon\cdot k')(\epsilon'\cdot k)\left[1+4M^2\left(\frac{1}{s}+\frac{1}{u}\right)\right], \end{aligned} \tag{13}$$

$$\begin{aligned} 2b^*c+2c^*a=&-2(\epsilon\cdot\epsilon')^2\frac{(s+u)^2}{M^4}-(\epsilon\cdot\epsilon')^2\left[20+12\left(\frac{u}{s}+\frac{s}{u}\right)+\frac{8}{M}(s+u)\right]+\frac{8}{M^2}\left[(\epsilon'\cdot k)\frac{u}{s}+(\epsilon\cdot k')\frac{s}{u}\right]+4\\ &+2\left(\frac{s}{u}+\frac{u}{s}\right)+\frac{6}{M^2}[(\epsilon'\cdot k)^2+(\epsilon\cdot k')^2]+24(\epsilon\cdot\epsilon')(\epsilon\cdot k')(\epsilon'\cdot k)\left(\frac{1}{u}+\frac{1}{s}\right)+\frac{12}{M^2}(\epsilon\cdot\epsilon')(\epsilon'\cdot k)(\epsilon\cdot k'). \end{aligned} \tag{14}$$

The badly unitarity-violating terms with $1/M^4$ in Eq. (10)–(14) get canceled out as can be seen easily. After the summation of Eq. (10)–(14), we can get the differential cross section in the high-energy limit,

$$\frac{d\sigma}{k'dk'd\Omega}=\frac{\alpha^2}{3s}\delta(2p\cdot q+q^2)\times\left[24+12\left(\frac{s}{u}+\frac{u}{s}\right)+8\left(\frac{s^2}{u^2}+\frac{u^2}{s^2}\right)+\frac{8}{M^2}\sin^2\theta(k-k')^2\right]. \tag{15}$$

After the dk' integration and using Eq. (9), we get

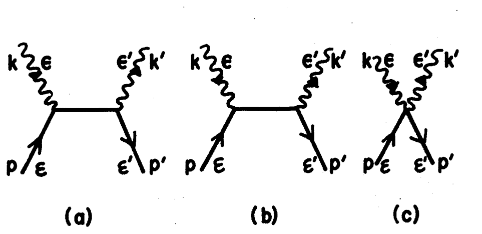

FIG. 1. Feynman diagrams for the Compton scattering of a charged vector meson. k (k') and p (p') represent the incident (outgoing) four-momenta of the photon and vector meson, respectively, and ϵ (ϵ') and ε (ε') are the corresponding polarization vectors.

$$\frac{d\sigma}{d\Omega}=\frac{\alpha^2}{12M^2}\left(\frac{k'}{k}\right)^2\left[24-12\left(\frac{k}{k'}+\frac{k'}{k}\right)+8\left(\frac{k^2}{k'^2}+\frac{k'^2}{k^2}\right)+\frac{8}{M^2}\sin^2\theta(k-k')^2\right]. \tag{16}$$

This result can be compared with the earlier non-gauge-theoretic calculations

$$\frac{d\sigma^{(\mathrm{Ref.\,4})}}{d\Omega}=\frac{\alpha^2}{2M^2}\left(\frac{k'}{k}\right)^2\times\left[1+\cos^2\theta+\frac{kk'}{12M^2}(7-16\cos\theta+3\cos^2\theta)+\frac{(k^2+k'^2)}{48M^2}(29-16\cos\theta+\cos^2\theta)\right], \tag{17}$$

$$\frac{d\sigma^{(\mathrm{Ref.\,5})}}{d\Omega}=\frac{\alpha^2}{2M^2}\left(\frac{k'}{k}\right)^2\left[1+\cos^2\theta-\frac{4}{3}\frac{kk'}{M^2}\cos\theta(1-\cos\theta)+\frac{(k^2+k'^2)}{3M^2}(5+\cos^2\theta)\right]. \tag{18}$$

The possible unitarity-violating terms are the terms with $1/M^2$ in Eqs. (16), (17), and (18),

$$\frac{1}{M^2}\sin^2\theta(k-k')^2,\quad\text{in Eq. (16)} \tag{19}$$

$$\frac{(k^2+k^{2\prime})}{M^2} \text{ and } \frac{kk'}{M^2}, \text{ in Eq. (17) and (18)} \tag{20}$$

where $\cos^2\theta \sim 1$ but $\sin^2\theta$ goes as[9]

$$\sin^2\theta \sim O\left(\frac{M}{k}, \frac{M}{k'}\right) \tag{21}$$

in the high-energy limit. Therefore Eq. (19) does not increase indefinitely in contrast to the Eq. (20) which violates unitarity badly.

Therefore we can see explicitly that the Compton scattering cross section does not violate unitarity as expected by gauge theory.

III. DEEP-INELASTIC COMPTON SCATTERING

It has been well known that the spin-$\frac{1}{2}$ partons have only half of the nucleon momentum and the other half of the momentum must be carried by other types of partons,[10] which are believed to be color gluons. In conventional gauge models such as quantum chromodynamics,[11] the color gluons are massless and neutral. But in the Pati-Salam model the gluons are massive and some of them have charges. For general purposes, however, we do not specify which model the charged gluons come from, but we only assume that the charged gluons are present in hadrons.

In the infinite-momentum frame, the charged gluons have a longitudinal-momentum fraction x of the nucleon momentum. With the replacement of invariant variables in Eq. (15),

$$\begin{aligned} &s \to xs, \quad t \to t \quad \text{and} \quad u \to xu \\ &\delta(2p\cdot q + q^2) \to \delta(2xp\cdot q + q^2), \end{aligned} \tag{22}$$

the Compton scattering cross section of the charged gluon with momentum fraction x can be calculated after integrating over x as

$$\frac{d\sigma^{\text{gluon}}}{d\Omega dk'} = \frac{\alpha^2}{24M_h}\frac{1}{k^2\sin^2(\frac{1}{2}\theta)}\left[24 - 12\left(\frac{k}{k'}+\frac{k'}{k}\right) + 8\left(\frac{k^2}{k'^2}+\frac{k'^2}{k^2}\right) + \frac{8}{M^2}\sin^2\theta(k-k')^2\right]g(x), \tag{23}$$

where $g(x)$ is the gluon distribution function of unit charge in the hadron of mass $m \to M_h$. For the case of many charged gluons, $g(x)$ should be replaced as

$$g(x) \to \sum_i Q_i^4 g_i(x), \tag{24}$$

where Q_i is the electric charge of the ith gluon in the unit of e. Compared with the previous calculation[12] of the Compton scattering of the spin-0 and spin-$\frac{1}{2}$ parton,

$$\frac{d\sigma^0}{d\Omega dk'} = \frac{\alpha^2}{2M_h k^2\sin^2(\frac{1}{2}\theta)} f_0(x), \tag{25}$$

$$\frac{d\sigma^{1/2}}{d\Omega dk'} = \frac{\alpha^2}{4M_h k^2\sin^2(\frac{1}{2}\theta)}\left(\frac{k}{k'}+\frac{k'}{k}\right) f_{1/2}(x), \tag{26}$$

the Compton scattering cross section of the charged gluon has additional terms with k^2, k'^2, and kk'. Now we can calculate the deep-inelastic Compton scattering cross section of the hadron composed of the quarks (spin $\frac{1}{2}$) and gluons (spin 1) as

$$\frac{d\sigma^{\text{Compton}}}{d\Omega dk'} = \frac{d\sigma^{\text{gluon}}}{d\Omega dk'} + \frac{d\sigma^{\text{quark}}}{d\Omega dk'}. \tag{27}$$

IV. SUMMARY

The Compton scattering cross section of the charged vector meson in the gauge theory is calculated in this paper for the first time at the high-energy limit, and the cross section is explicitly shown not to violate unitarity in contrast to earlier calculations. Also the contribution of charged spin-1 partons (charged color gluons) in deep-inelastic Compton scattering is calculated and is found to have quite different kinematic structure compared with the Compton scattering cross section of spin-0 and spin-$\frac{1}{2}$ partons.

The interesting feature of the present experimental situation[13] is that the quark contribution alone cannot explain the experimental data and more contributions from the other types of charged partons are needed. The detailed comparison with the experimental data based on the Pati-Salam model has been made and published elsewhere.[14]

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