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Quark anomalous moments and meson radiative decays

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It is shown within the framework of the naive quark model that the measured rates $\Gamma(\rho\to\pi\gamma)$ and $\Gamma(K^{*0}\to K^0\gamma)$ can be explained if the quarks have anomalous magnetic moments. However, one is led to contradictions with the measured baryon magnetic moments and other measured radiative rates.

I. INTRODUCTION

It is well known that the measured rates for $\rho\to\pi\gamma$ (Ref. 1) and $K^{*0}\to K^0\gamma$ (Ref. 2) are well below the SU(3)-symmetry predictions.[3] Edwards and Kamal[3] have investigated the radiative decays in various broken-SU(3) schemes and demonstrated that the $\rho\to\pi\gamma$ rate cannot be fitted simultaneously with the other known radiative rates.

One possible way to explain the measured $\rho\to\pi\gamma$ rate would be to assign to the quarks an anomalous magnetic moment. However, such an explanation will have repercussions for other radiative rates. The naive quark model also provides a connection between the $M1$ transition operator in meson decays of the kind $1^-\to 0^-\gamma$ and the baryon magnetic moments. The purpose of this note is to investigate the compatibility of the assumption of anomalous magnetic moments of the quarks with the baryon magnetic moments and other measured radiative rates. The connection between the present scheme and the general schemes discussed by Edwards and Kamal[3] is elucidated in Sec. IV.

II. THE MODEL

Restricting ourselves to SU(3) only, the electromagnetic current of the fractionally charged quarks is

$$J_\mu^{\text{em}} = \sum_{i=u,d,s} e_i \bar{q}_i \gamma_\mu q_i\,. \tag{1}$$

The $M1$ transition operator for $1^-\to 0^-\gamma$ meson radiative decays is

$$\theta = \sum_{i=u,d,s} \frac{e_i}{2m_i}\,\vec{\sigma}_i\cdot(\vec{\epsilon}\times\vec{k})\,, \tag{2}$$

where $\vec{\epsilon}$ and $\vec{k}$ are the photon polarization vector and momentum vector. The magnetic-moment operator is

$$\vec{\mathfrak{M}} = \sum_{i=u,d,s} \frac{e_i}{2m_i}\,\vec{\sigma}_i\,. \tag{3}$$

In Eqs. (2) and (3) the entire magnetic moment is assumed to arise from the Dirac moment of the quarks. If we assume the electromagnetic current to be of form $J_\mu^{(3)} + (1/\sqrt{3})J_\mu^{(8)}$, as it is in Eq. (1), then in the SU(3)-symmetry limit the anomalous moments will also be of the form $J_\mu^{(3)} + (1/\sqrt{3})J_\mu^{(8)}$. In particular, no SU(3) singlets will be allowed unless the current itself has an SU(3)-singlet part. Thus in the SU(3)-symmetry limit one would have the anomalous moments of the u, d, and s quarks in the ratio $\kappa_s:\kappa_s:\kappa_u = 2:-1:-1$ and the usual naive quark-model results would be recovered. Consider, however, a picture in which the anomalous moments arise from the electromagnetic vertex modification due to pseudoscalar mesons where the photon interacts with the meson and the quark currents. If one assumes that the meson octet is degenerate in mass and the quarks are also degenerate in mass then the SU(3) limit of the ratio of the quark anomalous moments, just alluded to, is obtained. However, if SU(3) symmetry is broken by having nondegenerate meson octet masses and also nondegenerate quark masses, then one can show (see Appendix) that the SU(3) limit on the ratios of the quark anomalous moments is violated. The view we adopt is that the anomalous moment of the quarks transforms like an arbitrary combination of the generators λ_3 and λ_8. Rather than estimate the anomalous moments even qualitatively we demonstrate that such a freedom can explain the $\rho\to\pi\gamma$ rate. We then investigate other predictions that follow from such an assumption. Labeling the quark anomalous moments as κ_i, the last two equations become

$$\theta = \sum_{i=u,d,s}\left(\frac{e_i}{2m_i}+\kappa_i\mu\right)\vec{\sigma}_i\cdot(\vec{\epsilon}\times\vec{k}) \tag{4}$$

and

$$\vec{\mathfrak{M}} = \sum_{i=u,d,s}\left(\frac{e_i}{2m_i}+\kappa_i\mu\right)\vec{\sigma}_i\,, \tag{5}$$

where $\kappa_i\mu$ is the anomalous magnetic moment of the ith quark and $\mu = e/2m_u$. We shall also assume throughout that $m_u = m_d$. Using the last two equations, one obtains for the ratio of decay amplitudes

$$\frac{A(\omega\to\pi\gamma)}{A(\rho\to\pi\gamma)} = \frac{3(1+\kappa_1-\kappa_2)}{(1+3\kappa_1+3\kappa_2)} \equiv r\,, \tag{6}$$

$$\frac{A(K^{*0}\to K^0\gamma)}{A(\omega\to\pi\gamma)} = -\frac{1}{3}\,\frac{1+\xi-3(\kappa_2+\kappa_3)}{1+\kappa_1-\kappa_2}\,, \tag{7}$$

$$\frac{A(K^{*0}\to K^0\gamma)}{A(\rho\to\pi\gamma)}=-\frac{1+\xi-3(\kappa_2+\kappa_3)}{1+3(\kappa_1+\kappa_2)}\equiv t\,, \qquad (8)$$

where $\xi=m_u/m_s$. Experimentally,[1] r is $\pm(4.9\pm0.75)$. Clearly, given the freedom of κ_1 and κ_2, it is possible to accommodate this value of r. One notices from Eq. (7) or (8) that the $K^{*0}\to K^0\gamma$ rate can also be explained with a suitably chosen κ_3. In the next section we investigate the consequences of such an explanation.

III. IMPLICATIONS

A. Baryon magnetic moments

1. Equation (4) leads to the ratio of proton to neutron magnetic moments

$$\frac{\mu_p}{\mu_n}=-\frac{(3+4\kappa_1-\kappa_2)}{(2+\kappa_1-4\kappa_2)}=\frac{3+5r}{3-5r}\,. \qquad (9)$$

The SU(3) limit $-\frac{3}{2}$ results on using $r=3$. If we use $r=(4.9\pm7.5)$, we get $-(1.28\pm0.05)$ for the ratio μ_p/μ_n; $-r=(4.9\pm0.75)$ leads to $-(0.8\pm.03)$, both well away from experimental value[4] of -1.46.

2. One can also evaluate the ratio

$$\frac{\mu_\Lambda}{\mu_p}=-\frac{\xi+3\kappa_3}{3+4\kappa_1-\kappa_2}=3\,\frac{(2t+r-1)}{(3+5r)}\,. \qquad (10)$$

If we use $r=3$ and $t=-2$, the SU(3) limiting values, one gets $\mu_\Lambda/\mu_p=-\frac{1}{3}$. Using the rates for $K^{*0}\to K^0\gamma$ and $\rho\to\pi\gamma$ one finds $t=-(1.97\pm0.53)$ [notice that both rates are lower than the SU(3) limiting values in such a way that the ratio is consistent with the SU(3) limit]. Using $r=(4.9\pm0.075)$, one finds

$$\frac{\mu_\Lambda}{\mu_p}=+(0.004\pm0.03)\,. \qquad (11)$$

The experimental world average[4] for μ_Λ is $(-0.67\pm0.06)\mu_B$ or $\mu_\Lambda/\mu_p=(-0.24\pm0.02)$. Both μ_p/μ_n and μ_Λ/μ_p are consistent with zero κ_i or at most very small values of anomalous moment with $\xi=0.7$. The radiative rates for $\rho\to\pi\gamma$ and $K^{*0}\to K^0\gamma$, on the other hand, require substantial anomalous magnetic moments.

B. Other radiative decays

Introduction of anomalous quark moments not in the ratio 2 : −1 : −1 is equivalent to a symmetry-breaking scheme which preserves the Okubo-Zweig-Iizuka (OZI) rule.[5]

1. With

$$|\eta\rangle=\cos\theta_P|8\rangle-\sin\theta_P|0\rangle$$

and (12)

$$|\omega\rangle=\cos\theta_V|0\rangle+\sin\theta_V|8\rangle\,,$$

one finds from Eq. (4), with θ_V ideal,

$$\begin{aligned}\frac{A(\omega\to\eta\gamma)}{A(\omega\to\pi\gamma)}&=\frac{1}{3}\,\frac{(1+3\kappa_1+3\kappa_2)}{(1+\kappa_1-\kappa_2)}\left[\frac{1}{\sqrt3}\cos\theta_P-\left(\frac{2}{3}\right)^{1/2}\sin\theta_P\right]\\&=\frac{1}{r}\left[\frac{1}{\sqrt3}\cos\theta_P-\left(\frac{2}{3}\right)^{1/2}\sin\theta_P\right]\\&=\frac{0.71}{r}\,,\quad(\theta_P=-10°)\,.\end{aligned} \qquad (13)$$

Equation (13) results in

$$\frac{\Gamma(\omega\to\eta\gamma)}{\Gamma(\omega\to\pi\gamma)}=0.076\,\frac{\Gamma(\rho\to\pi\gamma)}{\Gamma(\omega\to\pi\gamma)}=(3.0\pm0.8)\times10^{-3}\,. \qquad (14)$$

Experimentally[6] this ratio is $(3.5\pm^{2.9}_{2.1})10^{-3}$, consistent with Eq. (14). Thus no problems result here.

2. Using Eq. (4) one obtains

$$\frac{A(\eta'\to\rho\gamma)}{A(\eta'\to\omega\gamma)}=r \qquad (15)$$

and

$$\frac{\Gamma(\eta'\to\rho\gamma)}{\Gamma(\eta'\to\omega\gamma)}=1.11\,\frac{\Gamma(\omega\to\pi\gamma)}{\Gamma(\rho\to\pi\gamma)}=(28\pm8.2)\,, \qquad (16)$$

which is inconsistent with the measured value[7] (9.9 ± 2).

3. From Eq. (4) one can derive

$$\frac{A(\phi\to\eta\gamma)}{A(\omega\to\pi\gamma)}=\sqrt2\left[\frac{2}{\sqrt3}\cos\theta_P+\left(\frac{2}{3}\right)^{1/2}\sin\theta_P\right]\times\left[\frac{2t-1+r}{2r}\right]. \qquad (17)$$

Therefore, with $\theta_P=-10°$,

$$\frac{\Gamma(\phi\to\eta\gamma)}{\Gamma(\omega\to\pi\gamma)}=1.71\times\left(\frac{2t-1+r}{2r}\right)^2=(0.3\pm2.4)10^{-4}\,. \qquad (18)$$

Equation (18) results in $\Gamma(\phi\to\eta\gamma)\approx0.2$ keV in gross contradiction with the world average of (64 ± 10) keV. Note that $t=-2$ and $r=3$ yield 0.19 for the ratio in Eq. (18), resulting in the SU(3) prediction of $\Gamma(\phi\to\eta\gamma)\approx165$ keV.

4. From Eq. (4) one gets

$$\frac{A(\rho\to\eta\gamma)}{A(\omega\to\pi\gamma}=\left[\frac{1}{\sqrt3}\cos\theta_P-\left(\frac{2}{3}\right)^{1/2}\sin\theta_P\right]=0.71\,, \qquad (19)$$

which yields

$$\frac{\Gamma(\rho\to\eta\gamma)}{\Gamma(\omega\to\pi\gamma)}=0.065 \qquad (20)$$

or

$$\Gamma(\rho\to\eta\gamma)=(57\pm4)\ \text{keV} \qquad (21)$$

in agreement with the lower solution (50 ± 13) keV of Ref. 6.

5. Equation (4) yields

$$\frac{A(\phi\to\eta'\gamma)}{A(\phi\to\eta\gamma)}=\frac{(2\sqrt2\,\sin\theta_P-2\cos\theta_P)}{(2\sin\theta_P+\cos\theta_P)}\,, \qquad (22)$$

which results in

$$\frac{\Gamma(\phi\to\eta'\gamma)}{\Gamma(\phi\to\eta\gamma)}=4.65\times10^{-3}\,. \tag{23}$$

Using Eq. (18) one obtains $\Gamma(\phi\to\eta'\gamma)\simeq 1$ eV. There are no measurements on this rate, but other calculations[3] yield a rate 2–3 orders of magnitude larger.

6. Equation (4) results in

$$\frac{A(K^{*+}\to K^+\gamma)}{A(K^{*+}\to K^0\gamma)}=\frac{t+r}{t}\,. \tag{24}$$

Using $r=(4.9\pm0.75)$ and $t=-(1.97\pm0.53)$, one obtains

$$\frac{\Gamma(K^{*+}\to K^+\gamma)}{\Gamma(K^{*0}\to K^0\gamma)}=2.8\pm2.2\,. \tag{25}$$

The central value of 2.8 would result in $\Gamma(K^{*+}\to K^+\gamma)\simeq 200$ keV well outside the experimental upper bound[4] of 80 keV.

IV. DISCUSSION AND CONCLUSIONS

The decay amplitude for $V^m\to P^i\gamma$ in a λ_8-symmetry-breaking scheme which treats singlets and octets on equal footing (strong nonet symmetry of Edwards and Kamal[3]), may be written as (n refers to the internal symmetry index of the photon)

$$A_{min}=f_1d_{min}+f_2d_{8ik}d_{kmn}+f_3d_{8mk}d_{kin}+f_4d_{8nk}d_{kim} +f_5\delta_{8i}\delta_{mn}+f_6\delta_{8m}\delta_{in}+f_7\delta_{8n}\delta_{im}\quad (k=0,\ldots,8)\,. \tag{26}$$

This amplitude is not symmetric under ($m\rightleftarrows n$). However, if the photon interacts like a hadron, as in vector-meson dominance, then the amplitude will display symmetry under ($m\rightleftarrows n$) which will demand that $f_3=f_4$ and $f_6=f_7$, thereby reducing the number of independent parameters to five. One should also bear in mind that the first term in Eq. (26) obeys the OZI rule.[5]

The scheme proposed in this paper is summarized[8] by writing the $V^m\to P^i\gamma$ decay amplitude in the form

$$A_{min}=g\left[\left(\frac{2}{3}\right)^{1/2}(\kappa_1+\kappa_2+\kappa_3)d_{mi0}+(1+\kappa_1-\kappa_2)d_{mi3} +(1/\sqrt{3})(1+\kappa_1+\kappa_2-2\kappa_3)d_{mi8}\right] \tag{27}$$

Note that if $\kappa_i=0$ then one is led to the first term of Eq. (26) with $n=(3)+(1/\sqrt{3})(8)$ which would be the naive quark-model picture. If $\kappa_1:\kappa_2:\kappa_3=2:-1:-1$ the first term in Eq. (27) is absent and one is still led to the first term of Eq. (26) with $n=(3)+(1/\sqrt{3})(8)$, though with an overall coupling constant which depends on the anomalous-moment parameter. The ratio $A(\omega\to\pi\gamma)/A(\rho\to\pi\gamma)$ is still 3.

It is the violation of $2:-1:-1$ ratio of the anomalous moments that allows the first term of Eq. (27) to survive and allows us to violate the naive quark-model value for the ratio $A(\omega\to\pi\gamma)/A(\rho\to\pi\gamma)$. Note that, as it is linear in d_{min}, the amplitude preserves the OZI rule.[5]

Our experience[3] from the past has been that SU(3) breaking schemes successfully explain the $K^{*0}\to K^0\gamma$ rate but the $\rho\to\pi\gamma$ rate could not be fitted simultaneously with other measured rates. The $K^{*0}\to K^0\gamma$ rate is not very problematic; simply using $m_u/m_s\simeq 0.7$ alone brings the rate down from the SU(3) value of ≈200 keV to 145 keV. It is the $\rho\to\pi\gamma$ rate which poses the problem. In this note we have shown that if one were to invoke anomalous moments of the quarks to explain the $\rho\to\pi\gamma$ rate one runs into problems with μ_p/μ_n, μ_Λ/μ_p, $\Gamma(\eta'\to\rho\gamma)/\Gamma(\eta'\to\omega\gamma)$, and $\Gamma(\phi\to\eta\gamma)$. Thus once again we would emphasize the need for a new measurement of the $\rho\to\pi\gamma$ rate.

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APPENDIX

We evaluate here the induced anomalous moment of the quarks in a model where quarks interact directly with 0^- mesons. The interaction Lagrangian takes the form

$$\mathcal{L}=ig\sum_{i,j}\bar{q}_i\gamma_5M^{ij}q_j\,, \tag{A1}$$

where M is the 0^- octet matrix. In order to satisfy gauge invariance it is necessary to consider, in the lowest order, all the $O(g^2)$ graphs, that is, the vertex modification graphs where the photon interacts with the meson current and the quark current and also the graphs associated with the self-energy insertions and mass renormalizations. This is necessary for charge renormalization. The anomalous moment, however, is generated only through vertex-modification graphs in which the photon interacts with the meson and the quark currents.

In the following calculation, to illustrate how symmetry breaking may induce anomalous magnetic moments which do not obey the $\kappa_u:\kappa_d:\kappa_s=2:-1:-1$ rule, we shall allow a nondegenerate 0^- octet but a degenerate quark triplet, for simplicity.

Meson-current contribution

The anomalous magnetic moment of the u quark arises from π^+ and K^+ currents while that of the d

quark comes from the π^- current and that of the s quark from the K^- current.

The S matrix for $q(p)+\gamma(q)\to q(p')$ has a structure

$$S=\mp eg^2\delta^4(p'-p-q)\int \bar{u}_{p'}\gamma_5\frac{1}{(\not p-\not k)-m_q}\gamma_5 \times\frac{(2k+q)_\mu\epsilon^\mu}{(k+q)^2-m^2}\,\frac{d^4k}{k^2-m^2}\,u_p\,, \tag{A2}$$

where the minus sign is used for the u quark and the plus sign for d and s quarks. m is the 0^- meson mass, in this case π's and K's. Extracting the static anomalous term at $q^2=0$ from the above of form $\epsilon_\mu\sigma^{\mu\nu}q_\nu$, one gets (using $\sigma^{\mu\nu}=(i/2)[\gamma^\mu,\gamma^\nu]$)

$$S\text{ (anomalous part from meson currents)}=\mp eg^2\pi^2 m_q\,\bar{u}_{p'}\epsilon_\mu\sigma^{\mu\nu}q_\nu u_p I\,\delta^4(p'-p-q)\,,$$

where

$$\begin{aligned}I&\equiv\int_0^1\int_0^1\frac{z^2dz\,y^3dy}{[m^2(1-yz)+m_q^2y^2z^2]}\\&=\frac{1}{2m_q^2}\left(1-\frac{2m^2}{m_q^2}\right)+\frac{m^2}{2m_q^4}\left(2-\frac{m^2}{m_q^2}\right)\ln\frac{m_q^2}{m^2}-\frac{m^3(4m_q^2-m^2)^{1/2}}{2m_q^6}\left[\tan^{-1}\frac{m(4m_q^2-m^2)^{1/2}}{2m_q^2-m^2}+\tan^{-1}\frac{(4m_q^2-m^2)^{1/2}}{m}\right]\\&\quad-\frac{m(2m_q^2-m^2)^2}{2m_q^6(4m_q^2-m^2)^{1/2}}\left[\tan^{-1}\frac{m}{(4m_q^2-m^2)^{1/2}}+\tan^{-1}\frac{2m_q^2-m^2}{m(4m_q^2-m^2)^{1/2}}\right],\end{aligned} \tag{A3}$$

where $m=m_\pi$ or m_K. We shall label I as I_π or I_K when $m=m_\pi$ or $m=m_K$, respectively.

Quark-current contribution

The S matrix for $q(p)+\gamma(q)\to q(p')$ is

$$S=-eg^2\delta^4(p'-p-q)C_1C_2\int\bar{u}_{p'}\gamma_5\frac{1}{(\not k+\not q)-m_q}\gamma_\mu\frac{1}{\not k-m_q}\gamma_5\frac{d^4k}{(p-k)^2-m^2}u_p\,, \tag{A4}$$

where $C_1=\frac{2}{3}$ if the struck quark is u type and $-\frac{1}{3}$ if it is d or s type. $C_2=+1$ if the meson involved is $\pi^\pm$ or $K^\pm$, $C_2=\frac{1}{2}$ if it is π^0, $C_2=\frac{1}{6}$ if the meson involved is η^0 coupling to $\bar{u}u$ or $\bar{d}d$, and $C_2=\frac{4}{6}$ if the meson is η^0 coupling to $\bar{s}s$.

Extracting the static anomalous moment term again we get

$$S\text{ (anomalous part from quark currents)}=-eg^2C_1C_2\pi^2m_q\bar{u}_{p'}\epsilon_\mu\sigma^{\mu\nu}q_\nu u_pJ\delta^4(p+q-p')\,, \tag{A5}$$

where

$$\begin{aligned}J&=\int_0^1\int_0^1\frac{y\,dy(1-zy)^2dz}{[m_q^2(1-zy)^2+m^2zy]}\\&=\frac{1}{2m_q^2}\left(1-\frac{2m^2}{m_q^2}\right)+\frac{m^4}{4m_q^6}\ln\frac{m_q^2}{m^2}+\frac{m^3(4m_q^2-m^2)^{1/2}}{2m_q^6}\left[\tan^{-1}\frac{(4m_q^2-m^2)^{1/2}}{m}-\tan^{-1}\frac{m(4m_q^2-m^2)^{1/2}}{m^2-2m_q^2}\right]\\&\quad-\frac{m(m^2-2m_q^2)}{m_q^4(4m_q^2-m^2)^{1/2}}\left\{\tan^{-1}\frac{m^2-2m_q^2}{m(4m_q^2-m^2)^{1/2}}-\int_0^1dy\tan^{-1}\left[\frac{2m_q^2y^2-2m_q^2y+m^2}{ym(4m_q^2-m^2)^{1/2}}\right]\right\}.\end{aligned} \tag{A6}$$

The anomalous moments can now be obtained from (A3) and (A6), resulting in

$$\kappa_1\mu=\frac{eg^2m_q}{32\pi^2}\left(I_\pi+I_K-\tfrac{1}{3}J_K+\tfrac{1}{9}J_\eta\right), \tag{A7}$$

$$\kappa_2\mu=\frac{eg^2m_q}{32\pi^2}\left(-I_\pi+\tfrac{1}{2}J_\pi-\tfrac{1}{18}J_\eta-\tfrac{1}{3}J_K\right), \tag{A8}$$

$$\kappa_3\mu=\frac{eg^2m_q}{32\pi^2}\left(-I_K+\tfrac{1}{3}J_K-\tfrac{2}{9}J_\eta\right). \tag{A9}$$

Note that if $m_\pi=m_K=m_\eta$ then

$$\kappa_1:\kappa_2:\kappa_3=2:-1:-1\,,$$

which is the SU(3)-symmetry limit. The same limit is also obtained if $m_q\gg m$.

[1]B. Gobbi *et al.*, Phys. Rev. Lett. 33, 1450 (1974); 37, 1439 (1976).
[2]W. C. Carithers *et al.*, Phys. Rev. Lett. 35, 349 (1975).
[3]B. J. Edwards and A. N. Kamal, Phys. Rev. Lett. 36, 241 (1976); Ann. Phys. (N.Y.) 102, 252 (1976); Phys. Rev. D 15, 2019 (1977); Phys. Rev. Lett. 39, 66 (1977); P. J. O'Donnell, *ibid.* 36, 177 (1976); Can. J. Phys. 55, 1301 (1977); D. H. Boal, R. H. Graham, and J. W. Moffat, Phys. Rev. Lett. 36, 714 (1976); N. Isgur, *ibid.* 36, 1262 (1976); T. Barnes, Phys. Lett. 63B, 65 (1976); A. Bohm and R. B. Teese, Phys. Rev. Lett. 38, 629 (1977).
[4]T. G. Trippe *et al.*, Rev. Mod. Phys. 48, S51 (1976).
[5]S. Okubo, Phys. Lett. 5, 165 (1963); G. Zweig, CERN Report No. 8419/TH 412, 1964 (unpublished); I. Iizuka, K. Okada, and O. Shito, Prog. Theor. Phys. 35, 1061 (1966).
[6]D. E. Andrews *et al.*, Phys. Rev. Lett. 38, 198 (1977).
[7]C. J. Zanfino *et al.*, Phys. Rev. Lett. 38, 930 (1977).
[8]B. J. Edwards, Ph.D. thesis, Alberta, 1978 (unpublished).