PHYSICAL REVIEW D VOLUME 18, NUMBER 9 1 NOVEMBER 1978

Spin structure in meson spectroscopy with an effective scalar confinement of quarks

Howard J. Schnitzer
Department of Physics, Brandeis University, Waltham, Massachusetts 02154
(Received 31 March 1978)

Evidence from meson spectroscopy is presented to support the view that the effective interaction between quark-antiquark pairs is a long-range scalar confining force, together with a short-range Coulomb-type vector exchange governed by the underlying quantum chromodynamics. As a result, it is prediced that the ^3P-wave charmed mesons and mesons of the type $(b\bar{u})$ and $(b\bar{s})$ will be inverted multiplets. Discovery of such an effect would provide strong evidence for an effective scalar confining force. It is argued, on the basis of the systematics of the tensor force in the ordinary mesons, that there are important *induced* tensor forces in the $I = 1$ and $I = 1/2$ P-wave meson multiplets. A qualitative discussion suggests that the induced tensor forces are due to couplings of the $J = 1$, $I = 1$ and $I = 1/2$ mesons to open decay channels. As a result, the A_1, B, Q_A, and Q_B mesons appear to have substantial mixings with $(q\bar{q})(q\bar{q})$ configurations.

I. INTRODUCTION

The study of hadron spectroscopy presently emphasizes dynamics, which represents a shift from earlier group-theoretic techniques. Although a theory of hadron stationary states still cannot be constructed from first principles, considerable success has been achieved from a mixture of phenomenological and theoretical considerations based on quantum chromodynamics (QCD). The discovery of several levels of charmonium[1] has provided increased confidence in the usefulness of the dynamical approach to hadron spectroscopy based on atomic-type models for valence quarks.

The first step in this direction was the construction of spin-independent, confining potential models,[1,2] which successfully predicted the main features of the spin-averaged levels of charmonium. In the nonrelativistic limit the spin-independent potential does not depend on the Lorentz-transformation properties of the effective (color-averaged) quark forces; however, this issue becomes relevant when spin-dependent corrections to level structures are considered. In order to probe for this information, models for the spin-dependent quark interactions have been considered[3-5] which have provided a qualitative understanding of the features of charmonium. However, those models based on pure vector exchange for the confining force do not appear to work in detail.[1,6,7]

Many of the workers interested in quark dynamics have shifted their attention to the recently discovered Υ, Υ', and Υ'' (?) states in the hope that this will help resolve some of the unsettled and controversial issues of quark spectroscopy. However, we feel that there is still a great deal of information to be extracted from ordinary hadron spectroscopy, as we shall attempt to demonstrate in this paper. Therefore, rather than looking to the sparse, controversial data relating to new meson states for clues to the behavior of quark interactions, we will focus on the ordinary mesons, which are more numerous and less controversial in their interpretation. Our considerations will also be extended to the P states of charmonium, which seem relatively free of controversy. Some remarks relevant to Υ states will also be presented.

Typically one deals with this class of problems by constructing a Breit-type model Hamiltonian[3-5] for the spin-dependent quark-antiquark $(q\bar{q})$ interactions. This Hamiltonian represents an *effective* Abelian interaction, which in some sense characterizes the effective behavior of the underlying QCD *after* color sums have been performed. Thus, although the fundamental interaction is presumably vector in character, there is no guarantee that the phenomenological potential will have this behavior. In fact, one of the purposes of this paper will be to persuade the reader that the long-range (confining) part of the $q\bar{q}$ interaction behaves as an effective *scalar* exchange.[8] This point has been made in connection with charmonium by other workers[9-11]; we will argue that this conclusion holds in ordinary meson spectroscopy as well.

Given the effective Hamiltonian, one may proceed directly with a comparison of the predictions of the model with experimental data. Since one frequently encounters only partial success in this approach, one usually does not gain sufficient insights into those features of the model which are valid, and those which must be modified or discarded, unless one analyzes the data and theory in a sufficiently diagnostic manner. Accordingly, we will dissect both the experimental data and quark-interaction models in terms of the irreducible elements: spin-spin, spin-orbit, and tensor forces. Although we may not as yet have complete control of all elements of the spin-dependent interactions, at least we will be able to sharpen the focus on existing problems and successes of our approach.

Since we would like our conclusions to be as free from dispute as possible, we will attempt to minimize model-dependent features of our analysis by presenting when possible a *qualitative* basis for our main conclusions, and by presenting numerical results of other workers when relevant to our work. We hope that this will provide the reader with some sense of the variation of numerical values to be expected as one varies the underlying assumptions and input parameters. Throughout this paper we will work to lowest nontrivial order in $(v/c)^2$, with the assumption that higher-order corrections may modify numerical results somewhat, but will not undermine our main conclusions.

In Sec. II we discuss the relevant experimental data for the organization of the P-wave meson multiplets. The next section presents a discussion of some of our qualitative results which will provide the reader with an introduction to the more detailed analyses of subsequent sections. The main conclusion of that section is that within the framework of our methodology, the effective $q\bar{q}$ spin-dependent interaction is best described by a short-ranged $1/r$ vector interaction, and a long-range (confining) scalar force. Section IV presents the Breit-type Hamiltonian for[3-7,9-12] scalar + vector exchange (including quark-gluon anomalous color moment).[6,11] This is the starting point for all numerical studies in this paper. In Sec. V we review our previous analysis of the S-wave spin-spin interactions,[12] where we presented evidence from the S-wave meson hyperfine splittings for a long-range scalar exchange and/or a long-range quark-gluon anomalous moment $\kappa \simeq -1$ for mesons containing at least one u-, d-, or s-type quark. Furthermore, in that paper[12] we showed that excellent numerical results were obtained if the short-ranged vector portion of the $q\bar{q}$ potential was written as

$$V_s(r) = -\tfrac{4}{3}\alpha_s(M^2)/r\,, \tag{1.1}$$

with $\alpha_s(M^2)$ normalized by deep-inelastic scattering, with the mass scale M chosen to be the $^3S_1$ ground-state mass of the meson system considered. This procedure fixes α_s at the ψ mass to be larger than that obtained from analysis of $\psi \to 3$ gluons $\to$ hadrons (say).[1] However, it is not obvious to us that α_s should be the same in these two different kinds of problems.

Sections VI–VII will be devoted respectively to a detailed discussion of the spin-orbit, tensor, and spin-orbit mixing forces in the P-state mesons. We argue that an atomic-type $(q\bar{q})$ model cannot explain the $I=1$ and $I=\frac{1}{2}$ tensor forces determined from experiment, which suggests that an induced tensor force (due to coupling to decay channels) becomes relevant for the ordinary $I=1$ and $I=\frac{1}{2}$ P-wave mesons. We emerge with a picture of the A_1 and B mesons as

$$\begin{aligned} A_1 &= (q\bar{q}) + (\rho\pi)_{S\ \text{wave}} \\ &= (q\bar{q}) + (q\bar{q})(q\bar{q}) \end{aligned}$$

and (1.2)

$$\begin{aligned} B &= (q\bar{q}) + (\omega\pi)_{S\ \text{wave}} \\ &= (q\bar{q}) + (q\bar{q})(q\bar{q})\,. \end{aligned}$$

On the other hand, we argue that the $^3P_2$ and $^3P_0$ mesons are well described as valence states; i.e.,

$$A_2 = (q\bar{q})$$

and (1.3)

$$\delta(970) = (q\bar{q})\,.$$

Thus, only the $^3P_1$ and $^1P_1$ ordinary P-wave mesons have important "molecular"-type configuration mixing. This conclusion might serve to explain why it is so difficult to establish the existence of the axial-vector mesons. A similar conclusion holds for the $I=\frac{1}{2}$ P-wave mesons as well as well but the ordinary $I=0$, P-wave mesons seem to have no important configuration mixings.

We consider the charmonium spectrum in Sec. IX. It is argued that the P-wave level structure is compatible with a large charmed-quark chromomagnetic moment, coupled to a small fraction of long-range vector exchange, along with Coulomb and a dominant long-range scalar exchange. A fit to the ^3P levels of charmonium results in the *prediction* that $M(\eta_c) \simeq 2870$ MeV. Therefore, our analysis of the undisputed P-wave levels of charmonium leads to a prediction of ^1S states near the observed $\eta_c(2830)$ and $\chi(3450)$. This result only sharpens the mystery as to the nature of these states.

In Sec. X we discuss the limit of large quark mass, and come to the conclusion[13] that a long-range scalar exchange with a short-range vector exchange requires the spin-orbit force to have *opposite sign* for the case $m_1 \gg m_2$ relative to the case $m_1 = m_2$, where m_1 and m_2 are th e quark constituent masses; a conclusion which depends predominantly on kinematics. Thus the P-wave charmed D and F mesons and mesons of the type (bu) and (bs) should have a spin-orbit force opposite in sign to that found in the ordinary $I=1, \frac{1}{2}$, and 0 mesons. This would have the dramatic effect of making $^3P_2$ lie lower than $^3P_1$ and $^3P_0$ in these systems if the tensor force is sufficiently small. In the absence of significant tensor forces one should observe inverted multiplets with $E(^3P_2) < E(^3P_1) < E(^3P_0)$ for the charmed mesons and mesons of the type ($b\bar{u}$) and ($b\bar{s}$). The discovery of such inverted multiplets would give strong confirmation of our views. We close the paper with a summary of our results and principal conclusions.

II. THE P-WAVE MESONS

We will analyze the experimental data in terms of spin-orbit, spin-spin, and tensor forces. For this purpose we define the Hamiltonian for spin-dependent $q\bar{q}$ interactions, to leading order in $(v/c)^2$, as

$$H+H_0+A(r)\vec{L}\cdot\vec{S}+B(r)S_{12}+C(r)\vec{\sigma}_1\cdot\vec{\sigma}_2 + D(r)(\vec{S}_1-\vec{S}_2)\cdot\vec{L}\,, \tag{2.1}$$

where H_0 denotes the spin-independent part of the Hamiltonian, $S_{12}=3\vec{\sigma}_1\cdot\hat{r}\,\vec{\sigma}_2\cdot\hat{r}-\vec{\sigma}_1\cdot\vec{\sigma}_2$, $\vec{S}_i=\frac{1}{2}\vec{\sigma}_i$, $\vec{S}=\vec{S}_1+\vec{S}_2$, and A, B, C, and D are radially dependent potential functions for the spin-orbit, tensor, spin-spin, and spin-orbit mixing interactions, respectively. (Spin-orbit mixing is relevant for mesons which are not self-conjugate.)

Some features of P-wave meson spectroscopy have been clarified during the last year.[14,15] The main advances involve the improved status of the A_1 meson, and the existence of a detailed phase-shift analysis of the $J=1$, $I=\frac{1}{2}$ mesons,[14] establishing both the Q_A and Q_B states from the observed Q_1 and Q_2 resonances. Given this new information, the P-wave multiplets for the $I=1$ and $I=\frac{1}{2}$ mesons appear complete. All ^3P states for the $I=0$ mesons with nonstrange quarks may also have been identified. We suggest that all ^3P states of the ($s\bar{s}$) $I=0$ multiplet have also been found, although our identification of $E(1420)$ as the appropriate $^3P_1$ state might be challenged. Nevertheless, our assignments fit well with the systematics exposed in this paper. Our multiplet assignments for the P-wave mesons are summarized in Table I. Notice that we enter both the experimentally observed Q_1 and Q_2 as well as the unmixed states Q_A and Q_B, as determined from the SLAC analysis.[14] Let us now comment further on our assignments.

$I=1$ states. There do not appear to be major uncertainties remaining in this multiplet. The equal spacing of all four states implies extremely small spin-spin-spin tensor forces for the $I=1$, P-wave mesons.

$I=0$ ($u\bar{u}+d\bar{d}$) mesons. We assign $D(1285)$ as the $^3P_1$ state of this multiplet, since $J^P=1^+$ is favored by the data.[16] Furthermore, the dominant decay modes of $D(1285)$ all involve nonstrange mesons.[16] Although $D\to K\bar{K}\pi$ has been seen, it is readily interpreted as the sequential decay

$$D(1285)\to\delta\pi\to K\bar{K}\pi$$

taking place initially to the δ meson via nonstrange quarks. Accordingly, there is no justification for placing $D(1285)$ in the ($s\bar{s}$) multiplet. The approximate degeneracy of $f(1270)$ and $D(1285)$ is enough to imply a large tensor force in this multiplet.

We place $\epsilon(700)$ as the $^3P_0$ state of this multiplet. The alternative of choosing $S^*(993)$ does not seem to be acceptable in view of the substantial $K\bar{K}$ coupling to S^*. Further, $\epsilon(700)$ is now compatible with new high-statistics experiments.[14,17] The assignment of a $\pi\pi$ state at 1250 MeV is at complete variance with a *systematic* behavior of the spin-orbit force, as one moves from one multiplet to another. We do not have a role for an $I=0$ state at 1250 MeV in the context of our analysis.

$I=\frac{1}{2}$ multiplet. All four P-states have been identified.[14] The spacing of the observed levels rules out substantial spin-spin or tensor forces. The splitting $K^*(1420)$-$\kappa(1250)$ sets the energy scale of the spin-orbit force to be ~50 MeV, which is approximately half the $I=1$ or $I=0$ spin-orbit force. This suppression of the $I=\frac{1}{2}$ spin-orbit force will be attributed to the partial enhancement of a long-range scalar exchange, relative to a short-range vector exchange, owing to the (m_s/m_u) constituent quark mass ratio. In the extreme, this sort of enhancement leads to our prediction of inverted P-wave multiplets for D and F charmed mesons.

TABLE I. Level assignments of P-wave mesons, and predictions for masses of the missing $^1P_1$ states.

State	$I=1$	$I=0$ ($u\bar{u}+d\bar{d}$)	$I=\frac{1}{2}$		$I=0$ ($s\bar{s}$)	($c\bar{c}$)
$^3P_2$	$A_2(1310)$	$f(1270)$	$K^*(1420)$		$f'(1514)$	$\chi(3545)$
$^1P_1$	$B(1235)$	predict 1212	$Q_B(1355)$	$Q_2(1400)$	predict 1428	predict 3417
$^3P_1$	$A_1(1100)$	$D(1285)$	$Q_A(1340)$	$Q_1(1290)$	$E(1420)$	$\chi(3505)$
$^3P_0$	$\delta(976)$	$\epsilon(700)$	$\kappa(1250)$		$S^*(993)$	$\chi(3410)$

I = 0 (ss) mesons. $S^*(993)$ fits well with this multiplet because of its $K\bar{K}$ coupling. The assignment of $E(1420)$ as the $^3P_1$ state of the multiplet is somewhat speculative. However, the observed decays[16] $E\to K\bar{K}\pi$ and $E\to K^*\bar{K}$ demonstrate a substantial $(s\bar{s})$ quark content for the E. The decay $E\to\eta\pi\pi$ is also observed, but this decay presumably proceeds via the $(s\bar{s})$ content of the η meson, contrary to the situation for the D, its $I=0$, $(u\bar{u}+d\bar{d})$ analog. Thus we suggest that

$$E\to\eta\pi\pi$$

only involves the $(s\bar{s})$ quarks of the E meson. The spin and parity of the E is not well established, but it has charge conjugation $C=+$, and the assignment $^3P_1$ is compatible with its possible spin and parity.

Even if our assignment of $E(1420)$ as the $^3P_1$ state is not correct, the splitting $f'(1514)-S^*(993)$ establishes the spin-orbit matrix element $\langle A\rangle\simeq 100$–150 MeV for this multiplet. Our assignment of $E(1420)$ as $^3P_1$ only serves to refine this estimate, and establish a tensor force of the same order of magnitude as the $I=0$ multiplet withe nonstrange quarks.

We may combine the information contained in Table I with Eq. (2.1) to provide the experimentally determined P-wave matrix elements $\langle A\rangle$, $\langle B\rangle$, $\langle C\rangle$, and $\langle D\rangle$. These are calculated by means of the relations

$$E(^3P_2)-E(^3P_1)=2\langle A\rangle-\tfrac{12}{5}\langle B\rangle\,, \qquad (2.2)$$

$$E(^1P_1)-E(^3P_1)=\langle A\rangle-2\langle B\rangle-4\langle C\rangle\,, \qquad (2.3)$$

$$E(^3P_1)-E(^3P_0)=\langle A\rangle+6\langle B\rangle\,, \qquad (2.4)$$

and

$$[E(Q_1)-E(Q_A)][E(Q_A)-E(Q_2)]=2(\langle D\rangle)^2\,. \qquad (2.5)$$

We have written (2.5) with notation appropriate to the $I=\frac{1}{2}$ multiplet, but of course it applies to any multiplet for which $m_1\neq m_2$. The result of converting experimental mass differences to spin-orbit, tensor, and spin-spin forces is summarized in Table II. (The sign convention chosen for $\langle D\rangle$ is that of Ref. 18 in order to facilitate comparison of our results.)

It is evident from Table II that the P-wave spin-spin force is negligible for the $I=1$ and $I=\frac{1}{2}$ states. We assume that the P-wave spin-spin force also vanishes for the $I=0$ mesons as well, and predict the yet as unobserved $^1P_1$ from the values of $\langle A\rangle_{I=0}$ and $\langle B\rangle_{I=0}$ entered in Table II. Our findings are

$$I=0,\ (u\bar{u}+d\bar{d}):\ E(^1P_1)=1212\ \text{MeV} \qquad (2.6)$$

and

$$I=0,\ (s\bar{s}):\ E(^1P_1)=1428\ \text{MeV}\,. \qquad (2.7)$$

There are mesons listed in the Particle Data Sheets[16] as $M(1150)$ and $X(1430)$, the latter with $I=0$; however, since they are not at all well established, it would be presumptuous to identify them as candidates for the missing $I=0$, $^1P_1$ states.

It is amusing to observe the systematic behavior of the masses of the $^1P_1$ states if our predictions (2.6) and (2.7) are correct. That is, the $^1P_1$ states

$$\begin{array}{ll} I=1: & B(1235) \\ I=0,\ (u\bar{u}+d\bar{d}): & 1212\ \text{MeV (predicted)} \\ I=\frac{1}{2}: & Q_B(1355) \\ I=0,\ (s\bar{s}): & 1428\ \text{MeV (predicted)} \end{array} \qquad (2.8)$$

follow the pattern of the $^3P_2$ states, as well as the $^3S_1$ states

$$\begin{array}{ll} I=1: & \rho(770) \\ I=0,\ (u\bar{u}+d\bar{d}): & \omega(783) \\ I=\frac{1}{2}: & K^*(892) \\ I=0:\ (s\bar{s}) & \phi(1020)\,. \end{array} \qquad (2.9)$$

III. QUALITATIVE CONSIDERATIONS

We now consider some systematics of the $I=1$, $\frac{1}{2}$, and 0 mesons, and extract conclusions con-

TABLE II. Experimentally determined P-wave matrix elements in MeV of the spin-dependent interactions from the assignments of Table I. The sign convention for $\langle D\rangle$ is that of Ref. 18.

Multiplet	$\langle A\rangle$ spin-orbit	$\langle B\rangle$ tensor	$\langle C\rangle$ spin-spin	$\langle D\rangle$ spin-orbit mixing
$I=1$	108	2.6	−8.1	···
$I=0$ $(u\bar{u}+d\bar{d})$	91.4	82	?	···
$I=\frac{1}{2}$	49	6.9	5	−12.2
$I=0$ $(s\bar{s})$	112.6	52.4	?	···
$(c\bar{c})$	32.5	10.4	?	···

cerning the spin-orbit and spin-spin forces of the P-wave mesons, which then can be phrased in the language of the Breit Hamiltonian. In this section we present some qualitative results which point to an effective long-range scalar exchange. Detailed justification of these claims are reversed for subsequent sections.

From the known level spacings of the $I=1$ and $I=\frac{1}{2}$ P-wave mesons, one obtains the P-wave matrix elements of the spin-spin force

$$\langle C\rangle_{I=1,\ P\ \mathrm{wave}}=-8\ \mathrm{MeV}$$

and (3.1)

$$\langle C\rangle_{I=1/2,\ P\ \mathrm{wave}}=5\ \mathrm{MeV}.$$

These values are striking in view of the fact that corresponding S-wave matrix elements obtained from the ρ-π and K^*-K mass differences give

$$\langle C\rangle_{I=1,\ S\ \mathrm{wave}}\simeq 150\ \mathrm{MeV}$$

and (3.2)

$$\langle C\rangle_{I=1/2,\ S\ \mathrm{wave}}\simeq 100\ \mathrm{MeV}.$$

Thus the S-wave matrix elements of the spin-spin force are approximately 20 times larger than their P-wave counterparts. We are obviously justified in constructing models of ordinary mesons for which the P-wave spin-spin force is absent.

Within the framework of the Breit Hamiltonian, this conclusion allows two possibilities: Either

(1) Pure (short-range) vector exchange with $1/r$ radial dependence (we denote this interaction as Coulomb exchange for convenience) or

(2) Coulomb + scalar exchange.

However, to either of these cases we may add a long-range vector exchange interacting with quark-gluon anomalous magnetic moment fixed at $\kappa=-1$. If *all* quarks have $\kappa=-1$, the physical predictions of this possibility are identical to (1) and (2). That is, one may also consider the variants

(1′) Coulomb + long-range vector interacting with quark–gluon anomalous moment $\kappa=-1$ or

(2′) Coulomb + long-range scalar exchange + long-range vector exchange interacting with quark-gluon anomalous moment $\kappa=-1$.

Usually we shall not distinguish between (1) and (1′) or between (2) and (2′). (See Sec. IV for a precise formulation of these choices.) Note that *pure* scalar exchange is ruled out by the level ordering of the ordinary mesons, since

$$E(^3P_2)>E(^3P_1)>E(^3P_0)$$

is observed in the $I=1$ multiplet, for example. Of course *pure* vector exchange with other than Coulomb radial dependence is ruled out by (3.1) and our assumption that $\langle C\rangle=0$ for all P states.

Possibility (1) is in accord with the philosophy enunciated by De Rújula, Georgi, and Glashow (DGG),[5] abstracted from an interpretation of lattice gauge theory by Georgi.[19] These authors speculate that the long-range confining forces do not contribute to the spin-dependent quark forces. On the other hand, possibility (2) is closer to the conventional point of view in that *all* contributions to the spin-independent potential play a role in the spin-dependent Hamiltonian. The evidence we now present, derived from the P-state spin-orbit forces gives preference to (2), i.e., to the view that the spin-dependence originates via Coulomb force + long-range (confining) *scalar* exchange.

We present three arguments to support the claim that the effective $q\bar{q}$ can be understood in terms of Coulomb + long-range scalar exchanges. These arguments are based on

(1) the *relative* magnitude of $\langle A\rangle$ for the $I=1$, $\frac{1}{2}$, and 0 multiplets;

(2) the *relative* magnitude of $\langle A\rangle$ for the ordinary mesons as compared to charmonium; and

(3) the *absolute* energy scale of the spin-orbit force in the ordinary mesons, respectively.

This discussion is based on the experimentally determined P-wave matrix elements of the spin-orbit force listed in Table II. These matrix elements have the approximate magnitudes

$$\langle A\rangle_{I=1}\simeq\langle A\rangle_{I=0;\,u\bar{u}}\simeq\langle A\rangle_{I=0,\,s\bar{s}}\simeq 100\ \mathrm{MeV},$$

$$\langle A\rangle_{I=1/2}\simeq 50\ \mathrm{MeV},\qquad \text{(experiment)}\quad (3.3)$$

and

$$\langle A\rangle_{c\bar{c}}\simeq 33\ \mathrm{MeV}.$$

(It should be emphasized that $\langle A\rangle$ for charmonium only involves the apparently well-determined ^3P states.) Let us now proceed with the qualitative discussion.

A. Relative magnitude of $\langle A\rangle$ for ordinary mesons

If possibility (1) held, i.e., if the effective $q\bar{q}$ kernal was *pure* Coulomb, then one would expect the *relative* ordering

$$\langle A\rangle_{I=1}\simeq\langle A\rangle_{I=0,\,u\bar{u}}>\langle A\rangle_{I=1/2}>\langle A\rangle_{I=0,\,s\bar{s}}\qquad \text{(Coulomb)}\quad (3.4)$$

from the kinematical structure of the spin-orbit force, which behaves roughly as $(m_1m_2)^{-1}$, where m_1 and m_2 are constituent quark masses. Numerical results obtained in several models[20] justify identifying (3.4) as a property of pure Coulomb exchange. In particular, DGG[5] find

$$\langle A\rangle_{I=1}=\langle A\rangle_{I=0,u\bar{u}}\simeq 2\langle A\rangle_{I=1/2}$$

and (DGG) (3.5)

$$\langle A\rangle_{I=1}\simeq 4\langle A\rangle_{I=0,s\bar{s}}\,.$$

Similarly, in the model of Barbieri *et al.* (BG),[5]

$$\langle A\rangle_{I=1}=\langle A\rangle_{I=0,u\bar{u}}\simeq 1.3\,\langle A\rangle_{I=1/2}$$

and (BG) (3.6)

$$\langle A\rangle_{I=1}\simeq 1.7\langle A\rangle_{I=0,s\bar{s}}\,.$$

Finally, one can abstract from Sec. IV of this paper

$$\langle A\rangle_{I=1}=\langle A\rangle_{I=0,u\bar{u}}\simeq 1.6\langle A\rangle_{I=1/2}$$

and (this paper, Coulomb) (3.7)

$$\langle A\rangle_{I=1}\simeq 1.7\langle A\rangle_{I=0,s\bar{s}}$$

for pure Coulomb exchange, with our choice of parametrizations. Although there are numerical differences between (3.5), (3.6), and (3.7), each of these calculations satisfies (3.4), as expected. However, the *experimental* values for $\langle A\rangle$ presented in (3.3) are *not* ordered according to (3.4), suggesting that pure Coulomb exchange is not favored in ordinary mesons.

B. Magnitude of $\langle A\rangle$ for ordinary mesons relative to charmonium

Consider the ratio

$$R_1=\langle A\rangle_{I=1}/\langle A\rangle_{c\bar{c}}\,. \tag{3.8}$$

One finds from experiment

$$R_1\simeq 3 \qquad \text{(experiment)} \quad (3.9)$$

as compared to the results of model calculations with pure Coulomb exchange, which obtain

$$R_1\simeq 25\,, \qquad \text{(DGG)} \quad (3.10)$$

or

$$R_1\simeq 7\,, \qquad \text{(BG)} \quad (3.11)$$

or

$$R_1\simeq 18\,, \qquad \text{(Coulomb, this paper)} \quad (3.12)$$

respectively. One can understand Eqs. (3.10)–(3.12) as a scaling effects, for which pure Coulomb exchange makes the approximate prediction

$$R_1\simeq\left(\frac{m_c}{m_u}\right)^2\simeq 25 \tag{3.13a}$$

or

$$R_1\simeq\left(\frac{m_c}{m_u}\right)^{5/3}\simeq 15\,, \tag{3.13b}$$

depending on whether one considers the dynamical part of the spin-orbit matrix element an SU(4) invariant, or instead scales the matrix elements as appropriate to a linear potential. Thus, pure Coulomb exchange overestimates the ratio R_1 by a factor of 2.5 to 8, depending on the model considered. Thus, the spin-orbit force in ordinary mesons is much smaller than naively expected, since the spin-orbit force in charmonium is $O((v/c)^2)[E(\psi')-E(\psi)]$, as expected.

C. Absolute energy scale of spin-orbit force

The models with pure Coulomb exchange fail to give an understanding of the absolute energy scale of the spin-orbit force in ordinary mesons. We may compare the experimental value

$$\langle A\rangle_{I=1}\simeq 100\text{ MeV} \qquad \text{(experiment)} \quad (3.14)$$

with the predictions

$$\langle A\rangle_{I=1}=141\text{ MeV}\,,\quad \text{with } \alpha_s=0.32 \qquad \text{(BG)} \quad (3.15)$$

and

$$\langle A\rangle_{I=1}=364\pm 60\text{ MeV},\quad \text{with } \alpha_s=1.3\pm 0.2\,.$$

(Coulomb, this paper) (3.16)

(Recall that DGG do not predict absolute energies.) Although we advocate the larger value of α_s, consistent with deep-inelastic scattering, even the model of BG overestimates $\langle A\rangle_{I=1}$ by 50%, predicting $[E(^3P_2)-E(^3P_0)]_{I=0}$ to be 120 MeV too large.

Taken separately, each one of our three arguments may be challenged, but taken together, the evidence strongly indicates that pure Coulomb exchange in ordinary mesons is not favored. On the positive side, we shall show in subsequent sections of this paper that Coulomb+ scalar exchange gives considerable improvement in our understanding of $q\bar{q}$ spin-dependent forces. For the spin-orbit force, the mechanism at work is a partial cancellation of the attractive spin-orbit force obtained from the Coulomb force be a repulsive spin-orbit interaction originating with the scalar exchange. The relative contribution of these two effects depends on the constituent-quark mass ratio m_2/m_1, which in the extreme predicts inverted multiplets for mesons for which $m_2\gg m_1$.

IV. THE MODEL

In the absence of a derivation of the $q\bar{q}$ interactions from first principles, one has to approach this subject with more modest aims in mind. One frequently phrases the question of quark-antiquark forces in terms of a phenomenological model which attempts to represent the actual (color-averaged) $q\bar{q}$ spin-dependent forces by means of a Breit-type Hamiltonian[3-7,9-13] with effective Abelian exchanges. By its very nature, this construction ensures that spin-dependent effects are $(v/c)^2$ corrections to the spin-averaged spectrum. Although the fundamental interaction is presumably vector in character, the phenomenological interactions exhibit more complicated behavior, due to the nonperturbative nature of Yang-Mills interactions at long distances.

Consistent with the discussion of Sec. III, we assume that the effective $q\bar{q}$ interaction kernel is described by a combination of Coulomb + long-range scalar + long-range vector exchange coupled with a quark-gluon anomalous moment interaction.[6] That is, schematically we have[11,13]

$$K = V_{\text{Coul}}\gamma_1^{\mu}\gamma_{2\mu} + fV_{\text{lin}}\Gamma_1^{\mu}\Gamma_{2\mu} + (1-f)V_{\text{lin}}I_1\otimes I_2\,, \quad (4.1)$$

where the effective vector vertex Γ_μ is

$$\Gamma_\mu(q) = \gamma_\mu - \frac{i\kappa}{2m}\sigma_{\mu\nu}q^{\nu}\,. \quad (4.2)$$

In the above, f represents the fraction of long-range vector exchange, and κ is the long-range quark-gluon anomalous chromomagnetic moment.[6] This kernel is inserted in the Bethe-Salpeter equation, where to lowest nontrivial order in $(v/c)^2$ one obtains a Breit-type equation. The Fourier transform of the (instantaneous) exchanges gives the coordinate-space potential functions, which are assumed to be of the form

$$V_{\text{Coul}}(r) = -\frac{4}{3}\frac{\alpha_s}{r} \quad (4.3)$$

and

$$V_{\text{lin}}(r) = ar + b\,. \quad (4.4)$$

Note that the same function V_{lin} appears in the scalar and long-range vector exchanges, since these two terms sum to the spin-independent confirming potential.

The result of this program is[21]

$$\begin{aligned} H = H_0 &- \frac{4}{3}\frac{\alpha_s}{r} + ar + b + \text{spin-independent corrections} \\ &+ \frac{1}{m_1m_2}\left\{\left[1+\frac{(m_2{}^2+m_1{}^2)}{4m_1m_2}\right]\left(\frac{4}{3}\frac{\alpha_s}{r^3}\right) - \frac{(m_2{}^2+m_1{}^2)}{4m_1m_2}\left(\frac{a}{r}\right) + \frac{f}{2}\frac{(m_2+m_1)}{m_1m_2}\left[(1+\kappa_1)m_2+(1+\kappa_2)m_1\right]\left(\frac{a}{r}\right)\right\}\vec{L}\cdot\vec{S} \\ &+ \frac{1}{m_1m_2}\left\{\frac{1}{4}\frac{(m_2{}^2-m_1{}^2)}{m_1m_2}\left(\frac{4}{3}\frac{\alpha_s}{r^3}\right) - \frac{(m_2{}^2-m_1{}^2)}{4m_1m_2}\left(\frac{a}{r}\right) + \frac{f}{2}\frac{(m_2+m_1)}{m_1m_2}\left[(1+\kappa_1)m_2-(1+\kappa_2)m_1\right]\frac{a}{r}\right\}(\vec{S}_1-\vec{S}_2)\cdot\vec{L} \\ &+ \frac{1}{12m_1m_2}\left\{3\left(\frac{4}{3}\frac{\alpha_s}{r^3}\right) + f(1+\kappa_1)(1+\kappa_2)\left(\frac{a}{r}\right)\right\}S_{12} + \frac{1}{6m_1m_2}\left\{4\pi\left(\frac{4}{3}\alpha_s\right)\delta^3(\vec{r}) + 2f(1+\kappa_1)(1+\kappa_2)\left(\frac{a}{r}\right)\right\}\vec{\sigma}_1\cdot\vec{\sigma}_2\,, \end{aligned} \quad (4.5)$$

where we have made use of (4.3) and (4.4). Our predictions for the potential functions A, B, C, and D are obtained by comparing (4.5) with (2.1).

Observe that the case $f=0$ gives predictions which are identical to the case $f\neq 0$, $(1+\kappa_1) = (1+\kappa_2)=0$, so that for our purposes the situation in which the entire long-range exchange is scalar results in the same physics as that for which *both* quark (long-range) total chromomagnetic moments vanish.[12,22] One can easily understand this result, since $(1+\kappa_1)=(1+\kappa_2)=0$ means all chromomagnetic interactions vanish at long distances, and all that remains is the Thomas contribution to the (long-range) spin-orbit force. However, scalar exchanges only produce a Thomas term, hence the identity of the two situations. The distinction between these two cases should be regarded as a semantic difference only. That is, if $f=0$, one attributes the absence of long-ranged chromomagnetic interactions to a property of the effective gluon field. On the other hand if one has $(1+\kappa_1) = (1+\kappa_2)=0$ at long distances, the absence of chromomagnetic interactions is considered to be a property of the quarks. However these two limits are just different aspects of the same physics.

A weaker case results if $(1+\kappa_1)=0$, but $(1+\kappa_2)\neq 0$. This may be distinguished from $f=0$ by an analysis of the spin-orbit forces, but not from the spin-spin or tensor forces. As a practical matter, $f\simeq 0$ (accompanied by Coulomb exchange) gives an excellent first approximation to meson spectroscopy. However, the P-state spacings of charmonium suggest that $(1+\kappa_c)\neq 0$ even though f_c is close to zero in that system.

In order to convert (4.5) to predictions which may be compared with Table II, one must evaluate the matrix elements appearing in the Hamiltonian. Using the estimates of Refs. 3 and 4, we have

$$\begin{aligned} \langle r^{-1}\rangle_{P\text{ wave}} &\simeq \frac{2}{3}\left(\frac{256}{15\pi^2}\right)^{1/3}\left[a\left(\frac{m_1m_2}{m_1+m_2}\right)\right]^{1/3} \\ &\simeq 0.8\left[a\left(\frac{m_1m_2}{m_1+m_2}\right)\right]^{1/3} \end{aligned} \quad (4.6)$$

and

$$\langle r^{-3}\rangle_{P\text{ wave}} \simeq \frac{64}{45\pi}\left[a\left(\frac{m_1 m_2}{m_1+m_2}\right)\right]$$
$$\simeq 0.45\left[a\left(\frac{m_1 m_2}{m_1+m_2}\right)\right]. \qquad (4.7)$$

In our numerical work we choose the parameters

$$a = 0.194\ (\text{GeV})^2,$$
$$m_u = 313\ \text{MeV},$$
$$m_s = 490\ \text{MeV},$$
$$m_c = 1.6\ \text{GeV},$$
and (4.8)
$$m_b = 5\ \text{GeV}$$

as obtained from spin-averaged meson spectra.[1,2] [The values (4.8) are representative, and have not been selected to optimize the fits of our model to data.] Note that we are assuming that the slope of the linear potential is flavor independent, as expected from theoretical considerations.[23]

In Ref. 12 we argued that our model gives excellent numerical agreement with the ρ-π, K^*-K, D^*-, and F^*-F hyperfine splittings if $f=0$ and/or $(1+\kappa_u)=(1+\kappa_s)=0$, together with $\alpha_s(M^2)$ chosen to have a magnitude consistent with deep-inelastic scattering, evaluated at the mass scale $M^2 \simeq$ (centroid of multiplet)^2. On the basis of this evidence we shall also evaluate the P-wave matrix elements using the same criterion for α_s. Analyses of charmonium P states by other workers show that $f\simeq 0$ (Refs. 9–11) for the ψ system, with[11] $(1+\kappa_c)\neq 0$. The ρ, K^*, D^*, and F^* hyperfine data are also consistent with $f=0$, demonstrating approximate flavor independence of this parameter from the ρ multiplet all the way to charmonium. Since spin-averaged spectroscopy has fixed the parameters (4.8) within reasonable limits, and since meson hyperfine splittings suggest that $f=0$ and α_s is scaled as in deep-inelastic scattering, we now have a model for the spin structure of $q\bar{q}$ interactions with parameters chosen from considerations which make no reference to spin-dependent spectroscopy. The dependence of our predictions are most sensitive to our choice of α_s, as can be seen from the Tables. Reasonable variations of (4.8) are less significant by comparison.

The evaluation of Eq. (4.5), using (4.6)–(4.8) is exhibited in Table III. The errors shown for the spin-orbit and tensor forces are due to the range of values for α_s shown in the last column of the table. This error is a large fraction of the spin-orbit force because of the substantial cancellation of the attractive Coulomb contribution by the scalar-exchange contribution of opposite sign. The entries in Table III may be compared with the experimentally determined quantities in Table II. However, since the agreement is not uniformly good, we will dissect our results in order to understand the successes and failures of the model. In order to provide a comparison, we exhibit the contribution of pure Coulomb exchange in Table IV. Comparison of Table III with Table IV shows that the addition of scalar exchange makes a substantial improvement in the predictions for the spin-orbit force, as already argued in Sec. III of this paper.

In the next section we review our previously published[12] treatment of the S-wave spin-spin interactions. This analysis provides the support for the choice of $\alpha_s(M)$ made in this paper.

TABLE III. Predictions for the P-wave matrix elements in MeV of the spin-dependent forces for the Coulomb + scalar-exchange model of this paper. See Eq. (4.5) with $\kappa=0$, and parameters chosen as in text.

Multiplet	$\langle A\rangle$	$\langle B\rangle$	$\langle C\rangle$	$\langle D\rangle$	α_s
$I=1$	118 ±56	61 ±10	0	···	1.3 ±0.2
$I=0$					
$(u\bar{u}+d\bar{d})$	118 ±56	61 ±10	0	···	1.3 ±0.2
$I=\frac{1}{2}$	40 ±40	37 ± 7	0	−44 ±6	1.2 ±0.2
$I=0$					
$(s\bar{s})$	98 ±45	36 ± 8	0	···	1.2 ±0.2
$(c\bar{c})$	6.3 ± 2.2	3.7 ± 0.4	0	···	0.45 ± 0.05
$C=1$					
D mesons					
$(c\bar{u})$	−80.5 ± 15	7.7 ± 1.5	0	−103 ±8	0.5 ±0.1
F mesons					
$(c\bar{s})$	−20.6 ± 11	7.0 ± 1.4	0	−40.4 ± 4.2	0.5 ±0.1

TABLE IV. Same as Table III, but only Coulomb exchange is considered.

Multiplet	$\langle A\rangle$	$\langle B\rangle$	$\langle C\rangle$	$\langle D\rangle$	α_s
$I=1$	364 ±56	61 ±10	0	···	1.3 ±0.2
$I=0$ $(u\bar{u}+d\bar{d})$	364 ±56	61 ±10	0	···	1.3 ±0.2
$I=\frac{1}{2}$	244 ±40	37 ± 7	0	40 ±6	1.2 ±0.2
$I=0$ $(s\bar{s})$	214 ±45	36 ± 8	0	···	1.2 ±0.2
$(c\bar{c})$	22.6 ± 2.2	3.7 ± 0.4	0	···	0.45 ± 0.05
$C=1$ $(c\bar{u})$	71.5 ±15	7.7 ± 1.5	0	38 ±8	0.5 ±0.1
$(c\bar{s})$	53.1 ±11	7.0 ± 1.4	0	20.8 ±4.2	0.5 ±0.1

V. SPIN-SPIN FORCES

A. *P* waves

The P-wave hyperfine matrix element predicted by (4.5) is

$$\langle C\rangle = \frac{1}{3} f \frac{(1+\kappa_1)(1+\kappa_2)}{m_1 m_2} a \langle r^{-1}\rangle_{P\text{ wave}}, \tag{5.1}$$

where $\langle r^{-1}\rangle_{P\text{ wave}}$ is given by Eq. (4.6). Using the parameters of (4.8), we obtain

$$\langle C\rangle_{I=1} = 166[f(1+\kappa_u)^2]\ \text{MeV}$$

and (5.2)

$$\langle C\rangle_{I=1/2} = 112[f(1+\kappa_u)(1+\kappa_s)]\ \text{MeV}.$$

Comparison with the corresponding entries in Table II shows that we are justified in setting $f=0$ in the ordinary mesons. [Equivalently we may set $(1+\kappa_u)=0$, as discussed in Sec. IV.]

B. *S* waves

From Eq. (4.5) one obtains the S-wave hyperfine splittings

$$\Delta E = \frac{32\pi}{9m_1 m_2}\alpha_s |\psi(0)|^2 + \frac{4}{3}\frac{f(1+\kappa_1)(1+\kappa_2)}{m_1 m_2} a\langle r^{-1}\rangle_{S\text{ wave}}, \tag{5.3}$$

where $|\psi(0)|^2$ is the $q\bar{q}$ wave function at the origin. One could evaluate $|\psi(0)|^2$ from the potential model, but this has not proved to be an accurate prediction of the potential models.[24] It is far more reliable to obtain this information from experiment using the rates of vector meson decay to leptons given by the quark model as

$$\Gamma(V\to e^+e^-) = \frac{16\pi^2 e_q^{\,2}}{M_V^{\,2}}|\psi(0)|^2. \tag{5.4}$$

When this information is not available, one may obtain $|\psi(0)|^2$ from current algebra,[25] from sum rules,[26] and by interpretation[27] between known values of $|\psi(0)|^2$. This information was given in Table I of Ref. 12, but we summarize our results for the convenience of the reader. We found

$$|\psi(0)|^2 = (2.9\pm0.35)\times10^{-3}\ (\text{GeV})^3\quad (\rho\text{ meson}) \tag{5.5}$$

$$|\psi(0)|^2 = (3.7\pm0.7)\times10^{-3}\ (\text{GeV})^3\quad (K^*\text{ meson}) \tag{5.6}$$

$$|\psi(0)|^2 = (13\pm4)\times10^{-3}\ (\text{GeV})^3\quad (D^*\text{ meson}) \tag{5.7}$$

$$|\psi(0)|^2 = (15\pm5)\times10^{-3}\ (\text{GeV})^3\quad (F^*\text{ meson}). \tag{5.8}$$

We cannot give a direct experimental determination of $\langle r^{-1}\rangle$, however, the methods[3,4] which give (4.6) and (4.7) also imply that

$$\langle r^{-1}\rangle_{S\text{ wave}} \simeq \left(\frac{32}{3\pi^2}\right)^{1/3}\left[\left(a\frac{m_1 m_2}{m_1+m_2}\right)\right]^{1/3} \simeq 1.03\left[a\left(\frac{m_1 m_2}{m_1+m_2}\right)\right]^{1/3}. \tag{5.9}$$

Recall that this value has received some verification through its role in model calculations of the D^+-D^0 mass difference.[28]

One combines (5.3), and (5.5)–(5.9) to obtain the S-wave hyperfine splittings

$$(\Delta E)_{\rho-\pi} = \{[330\pm40]\alpha_s + 839[f(1+\kappa_u)^2]\}\ \text{MeV}, \tag{5.10}$$

$$(\Delta E)_{K^*-K} = \{[269\pm51]\alpha_s + 576[f(1+\kappa_u)(1+\kappa_s)]\}\ \text{MeV}, \tag{5.11}$$

$$(\Delta E)_{D^*-D} = \{[290\pm89]\alpha_s + 215[f(1+\kappa_a)(1+k_c)]\ \text{MeV}, \tag{5.12}$$

$$(\Delta E)_{F^*-F} = \{[213\pm71]\alpha_s + 144[f(1+\kappa_s)(1+\kappa_c)]\}\ \text{MeV}. \tag{5.13}$$

(Notice that the energy associated with the long-range vector exchange is large.) As discussed in previous sections, the data on the P-wave spin-dependent forces supports the hypothesis that long-range vector exchange is absent. Thus we evaluate (5.10)–(5.13) with

$$f \simeq 0 . \tag{5.14}$$

With $f=0$ in (5.10)–(5.13), the only remaining parameter to be fixed is $\alpha_s(M^2)$, which varies according to mass scale M^2 as

$$\alpha_s(M^2) = \tfrac{12}{25}\pi[\ln(M^2/\Lambda)^2]^{-1} \tag{5.15}$$

for an asymptotically free theory with four flavors. In a *fundamental* calculation, the mass scale M^2 should be chosen to minimize infrared logarithms in a perturbative calculation. In a phenomenological calculation the choice is not quite as obvious, but in order to remain close to the spirit of QCD, we hypothesize[12] that for the S-wave ground state one should choose

$$M^2 = M_V{}^2 = (\text{vector-meson mass})^2 \tag{5.16}$$

and

$$\Lambda = 500 \text{ MeV} , \tag{5.17}$$

as the value which[29] best describes the variation of $\alpha_s(M^2)$ with M^2 in deep-inelastic scattering.

Given (5.15)–(5.17) one obtains

$$\begin{aligned} &\alpha_s(M_\rho{}^2) = 1.8 , \\ &\alpha_s(M_{K^*}{}^2) = 1.43 , \\ &\alpha_s(M_{D^*}{}^2) = 0.545 , \end{aligned} \tag{5.18}$$

and

$$\alpha_s(M_{F^*}{}^2) = 0.525 .$$

Combining (5.10)–(5.14) with (5.18) we obtain the *predictions* (with no adjustable parameters once long-range scalar exchange is assumed)

$$(\Delta E)_{\rho-\pi} = 594 \pm 72 \text{ MeV} , \quad \text{expt: } 630 \text{ MeV} , \tag{5.19}$$

$$(\Delta E)_{K^*-K} = 385 \pm 73 \text{ MeV} , \quad \text{expt: } 395 \text{ MeV} , \tag{5.20}$$

$$(\Delta E)_{D^*-D} = 158 \pm 48 \text{ MeV} , \quad \text{expt: } 140 \text{ MeV} , \tag{5.21}$$

and

$$(\Delta E)_{F^*-F} = 112 \pm 37 \text{ MeV} , \quad \text{expt: } 120 \pm 40 \text{ MeV} . \tag{5.22}$$

This information was also displayed in Fig. 1 of Ref. 12. (Recall that DGG[5] successfully predict relative but *not* absolute energies.)

The agreement of the predictions (5.29)–(5.22) is excellent, giving support to our choice of $\alpha_s(M^2)$, characterized by Eqs. (5.16)–(5.18). Our conclusion is that in $q\bar{q}$ spectroscopy, and in Eq. (4.5) in particular, one should choose $\alpha_s(M^2)$ compatible with the values obtained in deep-inelastic scattering. Throughout this paper we make this choice, with results entered in Tables III and IV. The spin-spin force observed in ordinary mesons and the charmed D and F mesons are in excellent agreement with the model developed in this paper.

With long-range scalar exchange assumed, and $\alpha_s(M^2)$ fixed as in (5.15)–(5.17), there are no other free parameters to choose.

It should be emphasized that the success of this section is not a detailed check of the model developed in Sec. IV. It does seem to verify the general features, in that (a) scalar exchange is compatible with absence of P-wave spin-spin forces, and (b) the absolute energy scale of the S-wave hyperfine splittings is compatible with the suggestion that one-gluon exchange is the sole contributor to this effect. (See also the Note added to Sec. XI.)

VI. SPIN-ORBIT FORCE

We gave a qualitative discussion of the systematics of the $q\bar{q}$ spin-orbit force in Sec. III, which indicated that Coulomb + scalar exchange was favored over pure Coulomb exchange. Here we focus on some quantitative details. First we remark that we have chosen the mass scale M^2 in $\alpha_s(M^2)$ for the P-wave multiplets at the centroid of the P states, which, with Eqs. (5.15) and (5.17), leads to the entries shown in Tables III and IV. The predictions of Coulomb + scalar exchange for the spin-orbit force are considerably better than those of pure Coulomb exchange with regard to both absolute energy scale, and systematic variation of the spin-orbit force from multiplet to multiplet.

It is profitable to compare our predictions for the spin-orbit force with that of other workers,[20] which we do in Table V. The reader should note that the most important difference between the evaluation of Barbieri *et al.*[5] and ours is in the choice of α_s, with their's approximately three times smaller. These authors also choose parameters which differ slightly from those of (4.8). Our choice of α_s is of course strongly supported by the discussion in Sec. V.

The reader may wish to reread Sec. III in conjunction with Table V for qualitative interpretation. Here we emphasize that the approximate equality

$$\langle A\rangle_{I=1} \simeq \langle A\rangle_{I=0,\bar{u}u} \simeq \langle A\rangle_{I=0,\bar{s}s} \simeq 100 \text{ MeV}$$

is due to a delicate partial cancellation of the spin-orbit force owing to Coulomb exchange by the scalar exchange, as can be observed in the first two columns of Table V. Note that none of the models with pure Coulomb exchange have this

TABLE V. A comparison of the P-wave matrix elements in MeV of the spin-orbit force from various models with the experimental values of Table II.

Multiplet	This paper Coulomb + scalar	This paper pure Coulomb	Pure Coulomb Barbieri *et al.* (Ref. 5)	Pure Coulomb DGG (Ref. 5)	Experiment
$I=1$	118 ±56	364 ±56	141 ($\alpha_s=0.32$)	108 (input)	108
$I=0$ ($u\bar{u}+d\bar{d}$)	118 ±56	364 ±56	141 ($\alpha_s=0.32$)	108	91.4
$I=\frac{1}{2}$	40 ±40	224 ±40	107 ($\alpha_s=0.30$)	58	49
$I=0$ ($s\bar{s}$)	98 ±45	214 ±45	84 ($\alpha_s=0.27$)	23	112.6
$C=1$ ($c\bar{u}$)	−80.5 ±15	71.5 ±15	50.4 ($\alpha_s=0.25$)	...	...
($c\bar{s}$)	−20.6 ±11	53.1 ±11	41.8 ($\alpha_s=0.25$)	...	...

property. The fact that

$$\langle A\rangle_{I=1/2}\simeq \tfrac{1}{2}\langle A\rangle_{I=1} \simeq 50 \text{ MeV}$$

is then interpreted as a kinematical enhancement of scalar exchange relative to Coulomb exchange owing to the fact that $m_s > m_u$. [See also Eq. (4.5) and Sec. X.] In the extreme we predict[13] inverted P-wave multiplets for the charmed D and F mesons, contrary to the case for ordinary meson multiplets.

Our conclusion is that the spin-orbit force of ordinary mesons is very well accounted for by an atomic-type quark model dominated by Coulomb + scalar exchange.

VII. TENSOR FORCE

A. Valence quarks

Let us consider the implications of the tensor forces presented in Table II. Notice the contrast between

$$\langle B\rangle_{I=1,u\bar{u}}\simeq 3 \text{ MeV} \tag{7.1}$$

and

$$\langle B\rangle_{I=0,u\bar{u}}\simeq 80 \text{ MeV}, \tag{7.2}$$

which involve the same valence quarks. Equation (7.1) reflects the equal spacing of the A_2, B, A_1, and δ mesons, while (7.2) is essentially a consequence of the near degeneracy of $f(1270)$ and $D(1285)$. It is obvious that no atomic ($q\bar{q}$) model, with a flavor-*independent* two-body potential can accommodate a difference as large as that between (7.1) and (7.2). A flavor-*dependent* two-body tensor force for the $I=1$ and $I=0$ mesons with nonstrange quarks is extremely unlikely since (a) the two-body potentials are believed to arise from flavor-independent colored-gluon interactions, (b) the valence-quark content of the $I=1$ and $I=0$ states are identical, and (c) the spin-orbit force for these same $I=1$ and $I=0$ multiplets is flavor independent. It must be concluded that if the difference between (7.1) and (7.2) is genuine, then one must have configurations which are not described as valence states, and which make important contributions to the effective tensor force in mesons.

Before turning to a possible resolution of this problem, it is important to understand the experimental results for the tensor force in more detail. From Table II, we note that the experimental value

$$\langle B\rangle_{I=1/2}\simeq 7 \text{ MeV} \tag{7.3}$$

is negligible compared to Eq. (7.2) and

$$\langle B\rangle_{I=0,s\bar{s}}\simeq 50 \text{ MeV}. \tag{7.4}$$

The large value (7.4) for the $I=0$, $s\bar{s}$ tensor force is a consequence of the spacing $f'(1514)$, $E(1420)$, and $S^*(993)$, with the fact that $E(1420)$ is closer to f' than S^* being dominant in deducing (7.4). We see that the tensor force is negligible in the $I=1$ and $I=\frac{1}{2}$ multiplets, and substantial in the $I=0$ multiplets.

Let us consider the predictions of Eq. (4.5) for the tensor force, which are presented in Table III. (The entries in Table IV are identical, since scalar exchange makes no contribution to the tensor interaction.) Note that within the context of our model (or similar models) there is not mechanism available for reducing $\langle B\rangle$ for $I=1$ or $I=\frac{1}{2}$ mesons towards the experimental values, since there are no cancellations possible within a valence-quark model with Coulomb + scalar exchange. (This is in contrast to the situation with the spin-orbit force.) We summarize the status of our model for the tensor force as follows:

$$\langle B\rangle_{I=1}=61\pm 10\text{ MeV},\quad \text{expt: } 2.6\text{ MeV},\tag{7.5}$$

$$\langle B\rangle_{I=1/2}=37\pm 7\text{ MeV},\quad \text{expt: } 6.9\text{ MeV},\tag{7.6}$$

$$\langle B\rangle_{I=0,u\bar{u}}=61\pm 10\text{ MeV},\quad \text{expt: } 82\text{ MeV},\tag{7.7}$$

and

$$\langle B\rangle_{I=0,s\bar{s}}=36\pm 8\text{ MeV},\quad \text{expt: } 52\text{ MeV}.\tag{7.8}$$

We conclude that our predictions are an order of magnitude too large for the $I=1$ and $I=\frac{1}{2}$ states, and qualitatively correct for the $I=0$ states.

Consideration of the ratio

$$R_2=\langle B\rangle_{I=1}/\langle B\rangle_{I=1/2}\tag{7.9}$$

reinforces our conclusion that the $I=1$ and $I=\frac{1}{2}$ tensor forces do not behave as expected in a valence-quark model, since the experimental value is

$$(R_2)_{\text{exp}}\simeq 0.4,\tag{7.10}$$

while we found

$$(R_2)_{\text{model}}\simeq 1.6.\tag{7.11}$$

The prediction (7.11) can be understood as a kinematical effect of the constituent-quark mass ratio, since R_2 should vary approximately as

$$\frac{m_s m_u}{(m_u)^2}=\frac{m_s}{m_u}\simeq 1.6\tag{7.12}$$

according to (4.5). The discrepancy between (7.10) and (7.11) is significant since, *if* $R_2>1$ as expected in a $q\bar{q}$ model, then one must have the $I=\frac{1}{2}$ tensor force smaller than the $I=1$ tensor force. This in turn implies that $K^*(1426)$, Q_B, Q_A, and κ would have to obey an equal-spacing rule at least as accurate as the $I=1$ sequence, which is not the case. Hence, Eq. (7.10) tells us that the tensor force for the P-wave $I=1$ and $I=\frac{1}{2}$ states cannot be understood in terms of a pure valence-quark model.

On the other hand, not only does the $q\bar{q}$ model give the correct order of magnitude for $\langle B\rangle_{I=0}$, but it also gives an understanding of the ratio

$$R_3=\langle B\rangle_{I=0,\bar{u}u}/\langle B\rangle_{I=0,\bar{s}s}.\tag{7.13}$$

The experimental value is

$$(R_3)_{\text{exp}}\simeq 1.6,\tag{7.14}$$

while our model predicts

$$(R_3)_{\text{model}}\simeq 1.7.\tag{7.15}$$

The prediction of the model is predominately a kinematical effect of the quark mass ratio

$$\left(\frac{m_s}{m_u}\right)^2\simeq 2.6,\tag{7.16}$$

with some suppression of R_3 due to the variation of the matrix element $\alpha_s\langle r^{-3}\rangle$ between the $I=0$ $\bar{u}u$ multiplet and $\bar{s}s$ multiplet. Thus, the $I=0$ tensor forces are compatible with our model.

A comparison of our model with that of other workers is presented in Table VI. The systematics presented in this Table, and analyzed above, leads to the following conclusions: The $q\bar{q}$ model

(1) correctly predicts the $I=0$ tensor forces of the ordinary mesons, but

(2) badly overestimates the magnitude of the $I=1$ and $I=\frac{1}{2}$ tensor forces. These matrix elements should be considered abnormally small compared to the energy scales set by the $I=0$ tensor force, the model, and the P-wave spin-orbit force. Therefore, a tensor force of ~ 50–80 MeV is to be regarded as the normal magnitude to be expected in ordinary P-wave mesons. Much smaller values require more elaborate explanations outside the scope of the valence-quark model.

TABLE VI. P-wave matrix elements of tensor force $\langle B\rangle$ in MeV. Same as Table V, but for the tensor force.

Multiplet	This paper	Barbieri *et al.* (Ref. 5)	DGG (Ref. 5)	Experiment
$I=1$	61 ± 10	20 ($\alpha_s=0.32$)	2.6 (input)	2.6
$I=0$ ($u\bar{u}+d\bar{d}$)	61 ± 10	20 ($\alpha_s=0.32$)	2.6	82
$I=\frac{1}{2}$	37 ± 7	13.6 ($\alpha_s=0.30$)	1.4	6.9
$I=0$ ($s\bar{s}$)	36 ± 8	12 ($\alpha_s=0.27$)	0.5	52
$C=1$ ($c\bar{u}$)	7.2 ± 2	4.9 ($\alpha_s=0.25$)	...	...
($c\bar{s}$)	7.0 ± 1.4	4.9 ($\alpha_s=0.25$)	...	...

B. Induced tensor force

Faced with an inability of the valence-quark model to explain the $I=1$ and $I=\frac{1}{2}$ tensor force, we look for a mechanism outside the scope of this class of models which can reduce the tensor force without seriously affecting the successful predictions for the spin-spin, spin-orbit, or $I=0$ tensor forces. A clue comes from the extensive work of the Cornell group,[30] who showed that charmed meson decays of charmonium induces a tensor force which mixes $^3S_1 \leftrightarrow {}^3D_1$. In the case of the ψ system, the atomic (potential model) contribution to the tensor force is too small to explain the experimentally observed mixing, so that for ψ' the induced tensor force gives almost the entire effect. The Cornell workers also showed[30] that the virtual charmed-particle decays were not significant for the $n=1$, charmonium P states, aside from a renormalization of the strength of the linear potential.

Guided by this pioneering work, we seek a *qualitative* picture of tensor forces in the ordinary mesons with contributions from *both* atomic $(q\bar{q})$ configurations and induced tensor forces owing to couplings to open decay channels. Unfortunately, the Cornell model[30] is not directly applicable to our problem, since their calculations do not make provision for direct $q\bar{q}$ spin-dependent forces. In our case this effect is important, so that a quantitative calculation must allow for the interference of $q\bar{q}$ and induced tensor forces of comparable magnitude. Such a model is not yet available, so that we must be content with offering a qualitative explanation of the cancellation of the $I=1$ and $I=\frac{1}{2}$ tensor forces by induced effects.

Since we speculate that real (but not virtual) decays make an important contribution to the tensor force of the ordinary P-wave mesons, we begin with a summary of the dominant decay modes in Table VII. Note that, for kinematical reasons, there are no decays: P state $\to V+V$, where V represents a vector-meson state. Further, the dominant modes are all two-body final states, *except* for the decays of the $I=0$, $^3P_1$ states

$$D \to (\pi\pi)_{I=2} + \rho, \quad (L \geq 1) \tag{7.17}$$

and

$$E \to \eta + (\pi\pi)_{I=0}, \quad (L \geq 1). \tag{7.18}$$

[We have indicated the required isospin of the $(\pi\pi)$ system, and its angular momentum L relative to the third particle.] These decays of the $I=0$, $^3P_1$ states stand in sharp contrast to the principle decays of the $I=1$ and $I=\frac{1}{2}$, $^3P_1$ and $^1P_1$ states, for which

$$^3P_1 \to V + M \quad (L=0 \text{ dominant}) \tag{7.19}$$

and

$$^1P_1 \to V + M \quad (L=0 \text{ dominant}) \tag{7.20}$$

are dominant, where M denotes a pseudoscalar meson. Similarly, for the $J=2$ levels, only the $I=1$ and $I=\frac{1}{2}$ multiplets exhibit the decay mode

$$^3P_2 \to V + M \quad (L=2). \tag{7.21}$$

In view of the systematics summarized[16] in Table VII and Eqs. (7.17)–(7.21), we are able to formulate a set of hypothesis for the contributions of the observed decay modes to the spin-dependent forces of the ordinary P-wave mesons. We *assume to first approximation* that for the ordinary P-wave mesons

(1) virtual decay modes are not relevant to our problem,

(2) two-body decay modes are most important,

TABLE VII. Dominant decay modes of the ordinary P-wave mesons, from Ref. 16.

Multiplet	$^3P_2$	$^1P_1$	$^3P_1$	$^3P_0$
$I=1$	$A_2 \to \rho\pi$ (71%) $\to \eta\pi$ (15%)	$B \to \omega\pi$ (100%) D wave/S wave $\sim\frac{1}{4}$	$A_1 \to \rho\pi$ (100%)	$\delta \to \eta\pi$
$I=0$ $(u\bar{u}+d\bar{d})$	$f \to \pi\pi$ (81%)	...	$D \to \pi\pi\rho$ (dominant)	$\epsilon \to \pi\pi$
$I=\frac{1}{2}$	$K^*(1420) \to K\pi$ (56%) $\to K^*\pi$ (31%)	$\begin{Bmatrix} Q_A \\ Q_B \end{Bmatrix}$ ⟶	$\begin{Bmatrix} K^*\pi \\ \rho\ K \end{Bmatrix}$	$\kappa \to K\pi$
$I=0$ $(s\bar{s})$	$f' \to K\bar{K}$		$E(1470)$ $\to \eta\pi\pi$ (60%) $\to K^*\bar{K}+\bar{K}^*K$ (20%)	$S^* \to K\bar{K}$ $\to (\pi\pi)$

(3) because of statistical factors, we need only retain $V+M$ final states, and may neglect $M+M$ in consideration of induced spin dependence,

(4) $V+M$ rescattering effects are attractive, i.e., lower energies of the relevant states

(5) the $L=0$, $V+M$ final states are of major importance, with higher partial waves of lesser importance,

(6) effects which give more or less uniform energy shifts to all P states need not be discussed, as their contribution is absorbed in a renormalized value for the centroid of the P states.

The decay modes which survive these criteria are listed in Table VIII.

To proceed further, we assume that the minor mode

$$E \to K^*K + \bar{K}^*K, \tag{7.22}$$

which is only 20% of the decay fraction, need not be considered in a qualitative discussion, although it may have a small quantitative contribution. Omitting (7.22), only the $I=1$ and $I=\frac{1}{2}$, $J=2$, and $J=1$ decays survive the selection criteria. Hence our hypotheses successfully segregates the $I=1$ and $I=\frac{1}{2}$ multiplets from the $I=0$ multiplets. A demonstration that our assumptions are supported by detailed dynamical calculations requires an investigation which is beyond the scope of this paper.

The essence of the above discussion is that the processes

$I=1,\frac{1}{2}$

$$^3P_1 \to V+M \to {}^3P_1, \quad (L=0) \tag{7.23}$$

$$^1P_1 \to V+M \to {}^1P_1, \tag{7.24}$$

and

$$^3P_2 \to V+M \to {}^3P_2, \quad (L=2) \tag{7.25}$$

are the only ones that need be considered in a *qualitative* discussion of induced spin-dependent effects. The combined effect of (7.23)–(7.25) will be to lower *both* $J=2$ and $J=1$ states relative to $J=0$, but *for* $I=1$ *and* $I=\frac{1}{2}$ *only*. Further, because (7.23) and (7.24) should be more important than (7.25) [hypothesis (5) above], there will also be a lowering of $J=1$ relative to $J=2$. The net effect of (7.23)–(7.25) should then be a small decrease of $E(^3P_2)$ and a larger decrease of $E(^3P_1)$, both measured relative to $E(^3P_0)$.

This energy shifts can be related to induced spin-dependent forces by means of the relations

$$3E(^3P_1) - E(^3P_2) - 2E(^3P_0) = \tfrac{72}{5}\langle B\rangle \tag{7.26}$$

and

$$5E(^3P_2) - 3E(^3P_1) - 2E(^3P_0) = 12\langle A\rangle \tag{7.27}$$

obtained from (2.2)–(2.4). We now understand the importance of process (7.23), since the large decrease of $E(^3P_1)$ implies a significant *decrease* in $\langle B\rangle$, as can be seen from (7.26). Further, if $E(^3P_2)$ decreases by a lesser amount, then $[5E(^3P_2) - 3E(^3P_1)]$ is *not* expected to change significantly. From (7.27) this means that there is little or no induced spin-orbit force. Finally, because (7.23) and (7.24) should be of comparable strength, we do not expect an induced spin-spin force.

In summary, we have proposed a mechanism which will induce a tensor force in the $I=1$ and $I=\frac{1}{2}$ multiplets that tends to cancel the large tensor force of the valence-quark model, and which will leave the predictions of the valence-quark model for all $I=0$ spin-dependent forces, and $I=1$ and $I=\frac{1}{2}$ spin-orbit and spin-spin forces substantially unchanged.

In view of the important contribution of the processes (7.23) and (7.24) to the tensor force, one should regard the $J=1$ states of the $I=1$ and $I=\frac{1}{2}$ multiplets as having significant non-valence-quark configurations. That is, one should think

TABLE VIII. Decay modes of the ordinary P-wave mesons which satisfy the criteria of Sec. VII B.

Multiplet	$^3P_2$	$^1P_1$	$^3P_1$	$^3P_0$
$I=1$	$A_2 \to \rho\pi$ ($L=2$) (71%)	$B \to \omega\pi$ (100%) ($L=0$ dominant)	$A_1 \to \rho\pi$ ($L=0$ dominant)	δ: none
$I=0$ ($u\bar{u}+d\bar{d}$)	f: none	$\cdots$	D: none	ϵ: none
$I=\frac{1}{2}$	$K^*(1420) \to K^*\pi$ (31%) $L=2$	$\begin{Bmatrix} Q_A \\ Q_B \end{Bmatrix}$ ($L=0$ dominant) $\longrightarrow$	$\begin{Bmatrix} K^*\pi \\ \rho\ \pi \end{Bmatrix}$	κ: none
$I=0$ ($s\bar{s}$)	f': none	$\cdots$	$E \to (K^*\bar{K}+\bar{K}^*K)$ (20%)	S^*: none

of these mesons as

$$A_1 = (q\bar{q}) + (\rho\pi) = (q\bar{q}) + (q\bar{q})(q\bar{q}), \quad (7.28)$$

$$B = (q\bar{q}) + (\omega\pi) = (q\bar{q}) + (q\bar{q})(q\bar{q}), \quad (7.29)$$

and

$$Q_{A_1}Q_B = (q\bar{q}) + \begin{Bmatrix} K^*\pi \\ \rho K \end{Bmatrix} = (q\bar{q}) + (q\bar{q})(q\bar{q}) \quad (7.30)$$

configurations. There should be come configuration mixing in the $J=2$, $I=1$, and $I=\frac{1}{2}$ states, but to a much lesser extent than that of the $J=1$ states.[31]

We recognize that our picture of induced tensor forces is speculative; however, it has a fair degree of consistency, and also supplements the atomic $q\bar{q}$ model in a rather satisfactory way. Although the details of our analysis of these induced effects may require some adjustments, we believe that the qualitative features have a good chance of being correct.

VIII. SPIN-ORBIT MIXING

Mesons which are not eigenstates of charge conjugation are subject to the spin-orbit mixing force

$$D(r)(\vec{S}_1 - \vec{S}_2)\cdot\vec{L}, \quad (8.1)$$

which connects the $^3P_1$ and $^1P_1$ levels of the P-wave mesons. Our prediction, obtained from (4.5) gives

$$D(r) = \frac{(m_2{}^2 - m_1{}^2)}{4m_1{}^2 m_2{}^2}\left(\frac{4}{3}\frac{\alpha_s}{r^3} - \frac{a}{r}\right) \quad (8.2)$$

when $f=0$. The only measurement of this effect to date is that of the Q_A, Q_B sector in the $I=\frac{1}{2}$ multiplet. Further opportunities to observe spin-orbit mixing will occur in P-wave charmed mesons and mesons of the type $(b\bar{u})$ and $(b\bar{s})$.

The experimental value of $\langle D\rangle$, determined from Q_A, Q_B mixing, is given in Table II, while our prediction is in Table III. Predictions for P-wave charmed mesons are also included in that table. Comparison of our results with that of other workers[18] is given in Table IX. The sign of the experimental result is taken from the analysis of Barbieri, Gatto, and Kunszt (BGK).[18] The sign of $\langle D\rangle$ for the Coulomb + scalar-exchange model is opposite to that of the pure Coulomb calculation of BGK.[18] We claim that $\langle D\rangle$ has contributions from induced effects due to couplings to open decay channels, for much the same reason as the $I=1$ and $I=\frac{1}{2}$ tensor force. Therefore, final judgment should be reserved as to the degree of success of the Coulomb + scalar-exchange valence-quark model in predicting the *sign* of $\langle D\rangle$ until a detailed calculation of induced effects is available.

In Sec. VII B we discussed a possible mechanism for induced tensor forces in the $I=1$ and $I=\frac{1}{2}$ mesons. It is plausible that the couplings

$$\begin{Bmatrix} Q_A \\ Q_B \end{Bmatrix} \to \begin{Bmatrix} K^*\pi \\ \rho K \end{Bmatrix} \to \begin{Bmatrix} Q_A \\ Q_B \end{Bmatrix} \quad (8.3)$$

may induce spin-orbit mixings as well. Unfortunately, we have been unable to give even a qualitative discussion of the sign of the effect. Therefore, it is not clear whether (8.3) will be effective in reducing the magnitude of $\langle D\rangle$ predicted by the valence-quark model.

IX. CHARMONIUM

A great deal of attention has been given to the spin-dependent level structure of charmonium.[1,3-7,9-12] Analyses of this system have made it evident that pure vector exchange does not seem to work in the ψ system.[1,6,7] This situation prompted Henriques, Kellet, and Moorhouse[9] to investigate the Coulomb + scalar-exchange model for charmonium, with a resulting improvement

TABLE IX. Same as Table V, but for the spin-orbit mixing force. The sign convention is that of Ref. 18.

Multiplet	This paper Coulomb + scalar	Pure Coulomb	Barbieri *et al.* (Ref. 18) pure Coulomb	Experiment
$I=\frac{1}{2}$ $(\bar{s}u)$	−44 ± 6	40 ± 6	23.3 ($\alpha_s=0.30$)	−12.2
$C=1$ D mesons	−103 ±10	38 ±10	34.2 ($\alpha_s=0.25$)	⋯
$C=1$ F mesons $(\bar{c}s)$	−40.4 ± 4.2	20.8 ± 4.2	15.4 ($\alpha_s=0.25$)	⋯

in the P-state predictions of the model. This conclusion was subsequently confirmed by other workers.

Here we discuss P-wave charmonium level structure[32] from the point of view of this paper. If we assume $f=0$, as in the ordinary mesons, we obtain the predictions presented in Table III. For comparison, observe the predictions of pure Coulomb exchange in Table IV. Both models underestimate the energy scale of the spin-orbit and tensor forces. (One could increase these predictions by reducing the charmed quark mass, which we reject.) Instead we consider the possibility that the charmed quark-gluon anomalous moment $\kappa_c \neq 0$, *and* that $f_c \neq 0$. In this case Eq. (4.5) becomes[21]

$$\begin{aligned} H = H_0 + (ar+b) &+ \text{spin-independent corrections} \\ &+ \frac{1}{2m_c^2}\left\{[4f_c(1+\kappa_c)-1]\left(\frac{a}{r}\right)+\frac{4\alpha_s}{r^3}\right\}\vec{L}\cdot\vec{S} \\ &+ \frac{1}{12m_c^2}\left\{[f_c(1+\kappa_c)^2]\left(\frac{a}{r}\right)+\frac{4\alpha_s}{r^3}\right\}S_{12} \\ &+ \frac{1}{6m_c^2}\left\{[2f_c(1+\kappa_c)^2]\left(\frac{a}{r}\right)+4\pi\left(\frac{4}{3}\alpha_s\right)\delta^3(\vec{r})\right\}\sigma_1\cdot\sigma_2, \end{aligned} \tag{9.1}$$

when restricted to the $(c\bar{c})$ system. Note the appearance of two free parameters f_c and κ_c.

It is an interesting exercise to fit the two parameters to the two ^3P energy differences, since the χ states seem to be well founded.[32] The prescription for α_s formulated in Sec. V gives

$$\alpha_s = 0.45 \tag{9.2}$$

for the low-lying charmonium levels. Our fit to the charmonium P states gives

$$f_c = 0.059 \tag{9.3}$$

and

$$(1+\kappa_c) = 6.26\,, \tag{9.4}$$

which means that the charmed-quark–colored-gluon anomalous magnetic moment is [cf (4.1) and (4.2)]

$$\sqrt{f_c}\ \kappa_c = 1.27\,. \tag{9.5}$$

This value is close to that obtained in an earlier attempt[6] to explain ψ-$\eta_c(2850)$ splitting. However, here we only use the χ states as input. Our results, (9.3)–(9.5), are close to the values found by other workers[11] who have studied charmonium by means of (9.1), but used somewhat different methods to evaluate matrix elements.

The implication of

$$f_c \simeq 0.06$$

is that the Coulomb + long-range scalar potential is essentially flavor independent, as expected from theoretical considerations. Combining (9.3) and (9.4), we obtain

$$f_c(1+\kappa_c) = 0.37\,, \tag{9.6}$$

which is in contrast with

$$f(1+\kappa_u) \simeq f(1+\kappa_s) \simeq 0 \tag{9.7}$$

as found in Sec. V. Thus the quark-gluon anomalous moment is flavor dependent, indicating a qualitative change in the quark-gluon vertex as one replaces a light quark with a heavy quark.

Based on our parameters (9.3) and (9.4) we now *predict*

$$n=1:\ E(^3S_1) - E(^1S_0) = 204\ \text{MeV}, \tag{9.8}$$

$$n=2:\ E(^3S_1) - E(^1S_0) = 170\ \text{MeV}, \tag{9.9}$$

and

$$E(^1P_1) = 3417\ \text{MeV}\,. \tag{9.10}$$

These predictions are in qualitative agreement with

$$E(\psi) - E(\eta_c(2830)) = 265\ \text{MeV} \tag{9.11}$$

and

$$E(\psi') - E(\chi(3455)) = 229\ \text{MeV}\,. \tag{9.12}$$

In our mind, the results of this section heighten the mystery surrounding the nature of $\eta_c(2830)$ and $\chi(3455)$.[33]

X. LARGE QUARK MASSES

A. Inverted multiplets

In this section we show that Coulomb + scalar exchange predicts inverted multiplets[13] for the charmed D and F mesons, and mesons of the type $(b\bar{u})$ and $(b\bar{s})$. Consider Eq. (4.5), and isolate the spin-orbit forces in the limit $m_2 \gg m_1$. One finds

$$\begin{aligned} H_{\text{so}} \xrightarrow[m_2 \gg m_1]{} &\frac{1}{4m_1^2}\left\{\frac{4}{3}\frac{\alpha_s}{r^3} - [1-2f(1+\kappa_1)]\frac{a}{r}\right\} \\ &\times[\vec{L}\cdot\vec{S} + (\vec{S}_1 - \vec{S}_2)\cdot\vec{L}]\,. \end{aligned} \tag{10.1}$$

In the same limit, the P-wave matrix elements (4.7) become

$$\langle r^{-1}\rangle_{P\ \text{wave}} \simeq 0.8(am_1)^{1/3} \tag{10.2}$$

and

$$\langle r^{-3}\rangle_{P\ \text{wave}} \simeq 0.453(am_1)\,. \tag{10.3}$$

For a u- or d-type light quark one obtains the result

$$H_{so} \xrightarrow[m_2 >> m_u]{} (93.6)\{\alpha_s - 1.66[1 - 2f(1+\kappa_u)]\} \times [\vec{L}\cdot\vec{S} + (\vec{S}_1 - \vec{S}_2)\cdot\vec{L}]\ \text{MeV} \quad (10.4)$$

with the parameters (4.8). If the light quark is an s quark, one has

$$H_{so} \xrightarrow[m_2 >> m_s]{} (63.6)\{\alpha_s - 1.23[1 - 2f(1+\kappa_s)]\} \times [\vec{L}\cdot\vec{S} + (\vec{S}_1 - \vec{S}_2)\cdot\vec{L}]\ \text{MeV}. \quad (10.5)$$

Since $m_2 \gg m_s$ by assumption,

$$\alpha_s(M^2) \simeq \alpha_s(m_2^2) < 1 \quad (10.6)$$

is valid. Further

$$f(1+\kappa_u) \simeq 0, \quad f(1+\kappa_s) \simeq 0, \quad (10.7)$$

as has been shown in the analysis of the $D^* - D$ and F^*-F splittings in Sec. V. One thus observes that the long-range scalar exchange dominates (10.4) and (10.5) leading to a *negative* spin-orbit force, i.e., inverted multiplets in the charmed mesons, and mesons with $(b\bar{u})$ and $(b\bar{s})$ valence quarks. It is this feature of the spin-orbit interaction which leads to our prediction of $\langle A\rangle < 0$ for the P-wave charmed mesons, as presented in Tables III, V, and IX.

We now present our predictions for the P-wave spin-orbit forces for $(b\bar{u})$ and $(b\bar{s})$, assuming $f(1+\kappa_1) = 0$. Estimating that $m_b \simeq 5$ GeV, and using (10.6) and (5.15)–(5.17) to fix $\alpha_s(m_b) \simeq 0.35$, we obtain

$$H_{so} \simeq -123[\vec{L}\cdot\vec{S} + (\vec{S}_1 - \vec{S}_2)\cdot\vec{L}]\ \text{MeV}, \quad \text{for } (\bar{b}u) \text{ mesons}, \quad (10.8)$$

and

$$H_{so} \simeq -56[\vec{L}\cdot\vec{S} + (\vec{S}_1 - \vec{S}_2)\cdot\vec{L}]\ \text{MeV}, \quad \text{for } (\bar{b}s) \text{ mesons}. \quad (10.9)$$

From Eq. (4.5) and Table III we also have

$$H_{so} \simeq -[80\vec{L}\cdot\vec{S} + 103(\vec{S}_1 - \vec{S}_2)\cdot\vec{L}]\ \text{MeV}, \quad \text{for } (\bar{c}u) \text{ mesons} \quad (10.10)$$

and

$$H_{so} \simeq -[20\vec{L}\cdot\vec{S} + 40(\vec{S}_1 - \vec{S}_2)\cdot\vec{L}]\ \text{MeV}, \quad \text{for } (\bar{c}s) \text{ mesons}. \quad (10.11)$$

[We have included a finite quark mass m_c in (10.10) and (10.11).] There are also possible contributions from induced effects to the spin-orbit mixing term as discussed in Sec. VIII. However, there is no evidence for induced effects contributing to the $\vec{L}\cdot\vec{S}$ term in ordinary mesons or in charmonium, so that we do not expect coupling to decay channels to affect the $\vec{L}\cdot\vec{S}$ force here either.

The prediction of a *negative* spin-orbit force for $(\bar{c}u)$, $(\bar{c}s)$, $(\bar{b}u)$, and $(\bar{b}s)$ mesons is a rather clean consequence of the Coulomb + effective long-range scalar-exchange kernel, since it rests on a rather simple and physical picture. When $m_2 \gg m_1$, the light quark is on the average "far" from the heavy quark, since the "size" of the meson is approximately governed by $(m_1)^{-1/3}$ according to (10.2). That is, the light quark sets the average distance scale of the meson. However, at large distances, the light quark only interacts with the (effective) confining scalar field, which, as is well known, leads to an inverted multiplet. That is, one predicts the sequence

$$E(^3P_0) > E(^3P_1) > E(^3P_2) \quad (10.12)$$

for the $(\bar{c}u)$, $(\bar{c}s)$, $(\bar{b}u)$, and $(\bar{b}s)$ mesons. Equivalent to the above qualitative discussion is the fact that

$$\langle r^{-3}\rangle / \langle r^{-1}\rangle \sim (m_1)^{2/3} \text{ for } m_2 \gg m_1, \quad (10.13)$$

and that $\alpha_s \to 0$ in this limit, so that the contribution of the Coulomb exchange is unimportant for $m_2 \gg m_1$.

It should be emphasized that (10.12) is *not* a property of charmonium or the new $(\bar{b}b)$ Υ states, since $\langle r^{-1}\rangle \sim (m_2)^{1/2}$ and $\langle r^{-3}\rangle/\langle r^{-1}\rangle \sim (m_2)^{1/3}$ for $m_1 = m_2$. Therefore, in the ψ and Υ systems, the Coulomb exchange is not negligible, and in fact becomes more important as $m_1 = m_2$ gets large. Therefore, the *prediction* of *inverted multiplets depends crucially on:*

(a) $m_2 \gg m_1$

and

(b) a long-range effective scalar exchange.

Observation of these inverted multiplets will provide dramatic confirmation of our views.

We now emphasize that there already exists some experimental evidence for the effects discussed in this section. As presented earlier in this paper, our prediction for the spin-orbit forces of the P-wave $I = \frac{1}{2}$ mesons is

$$(H_{so})_{\text{this paper}} \simeq [40\vec{L}\cdot\vec{S} - 45(\vec{S}_1 - \vec{S}_2)\cdot\vec{L}]\ \text{MeV}, \quad (10.14)$$

which neglects possible induced effects in the spin-orbit mixing term [cf. Sec. VIII]. This should be compared with the experimental result

$$(H_{so})_{\exp} \simeq [49\vec{L}\cdot\vec{S} - 12(\vec{S}_1 - \vec{S}_2)\cdot\vec{L}]\ \text{MeV}. \quad (10.15)$$

In the Coulomb + scalar-exchange $(\bar{q}q)$ model, we found

$$\langle A\rangle_{I=1} \simeq \langle A\rangle_{I=0,\bar{u}u} \simeq \langle A\rangle_{I=0,\bar{s}s} \quad (10.16)$$

and

$$\langle A\rangle_{I=1/2} \simeq \tfrac{1}{2}\langle A\rangle_{I=1}, \quad (10.17)$$

where (10.17) is the result of a partial enhancement of the scalar exchange relative to Coulomb exchange owing to the kinematical effects of the quark-mass ratio $(m_s/m_u) \simeq 1.6$. This mass ratio is not large enough to permit the scalar exchange to overwhelm the Coulomb term; nevertheless, the effect is in the direction of (10.5). Recall, that the *experimental* values of the spin-orbit forces are

$$\langle A\rangle_{I=1} \simeq \langle A\rangle_{I=0,\bar{u}u} \simeq \langle A\rangle_{I=0,\bar{s}s} \simeq 100 \pm 10 \text{ MeV} \qquad (10.18)$$

and

$$\langle A\rangle_{I=1/2} \simeq 50 \text{ MeV}. \qquad (10.19)$$

Therefore, the experimental results (10.18) and (10.19) appear to confirm the picture of a kinematical enhancement of scalar exchange. The predicted inverted multiplets of the charmed mesons is merely a logical continuation of the physics already seen in the $I=\frac{1}{2}$ multiplet.

B. Self-conjugate mesons

In Sec. X we analyzed the spin-orbit force for mesons for which $m_2 \gg m_1$. In this subsection we consider self-conjugate mesons with heavy quarks. At this time there are only two systems which are relevant, the $(c\bar{c})$ and $(b\bar{b})$ mesons, although other examples might be relevant in the future. [Possible $(b\bar{c})$ mesons should be discussed as in Sec. X A, as they probably will exhibit some of the features of the $I=\frac{1}{2}$ states.] In Sec. IX we considered some problems of charmonium level spacings. Here we attempt to generalize those lessons extracted from the known mesons, so as to extend our understanding to the $(b\bar{b})$ mesons, and beyond.

We do not discuss the *spin-independent* structure of the upsilon system; the reader is referred to the work of others[1,34-36] in this regard. Here we wish to emphasize the contribution of Pignon and Piketty,[31] who argue that a Coulomb + linear (scalar) confining potential *can* explain the approximate equal spacing of the $^3S_1$ level spacings of bottomium *if* α_s has a magnitude which is compatible with the prescription formulated in Eqs. (5.15)–(5.17), ff. Therefore, there may not be a need for a logarithmic contribution to the intermediate range of the spin-independent potential,[36] contrary to earlier expectations. This question deserves further study in light of our findings.

We now turn to questions of the spin dependence of new mesons. First we summarize those features of our work required for extension to higher-mass mesons. It was concluded that

(1) the fraction f, of long-range vector exchange appears to be very nearly flavor independent. We found that

(a) $f \simeq 0$ for the ordinary mesons, and the charmed D and F mesons, and

(b) $f \simeq 0.06$ for charmonium.

(2) The long-range quark–colored-gluon anomalous moment appears to be strongly flavor dependent, as one passes from light to heavy quarks. Our study of S-wave hyperfine splittings of both ordinary and charmed mesons (cf. Sec. V), and the P-wave spectroscopy of ordinary mesons and charmonium indicate that

$$\begin{aligned} &\sqrt{f}\,(1+\kappa_u) \simeq 0,\\ &\sqrt{f}\,(1+\kappa_s) \simeq 0,\\ &\sqrt{f_c}\,(1+\kappa_c) \simeq 1.5, \end{aligned} \qquad (10.20)$$

where $\sqrt{f}\,(1+\kappa)$ is the total (long-distance) quark-gluon chromomagnetic moment.

The following generalities serve as an introduction to the questions of spin dependence of the Υ system and other heavy mesons for which $m_1 \simeq m_2 = m \gg m_s$.

(1) Equation (4.6) indicates that the average size of such mesons decreases as $(m)^{-1/3}$.

(2) Thus, Coulomb exchange, measured relative to the long-range scalar exchange, should be somewhat more important for the low-lying states of the $(\bar{b}b)$ mesons than for charmonium.

(3) If one includes an intermediate-ranged logarithmic potential,[36] one must decide whether this exchange has a vector, scalar, or mixed character. As remarked above, it is not obvious that such an exchange is needed, or what its Lorentz properties would be if present.

(4) Since f has changed from $f=0$ to $f_c \simeq 0.06$ charmonium, the fraction of long-ranged vector exchange *may* be increasing with increased mass. We speculate that this is so, which, if correct, implies that $f_b > f_c$ for the $(\bar{b}b)$ mesons.

(5) Combining (2) with (4), for sufficiently large quark mass m, the self-conjugate mesons will have a large fraction of the long-ranged exchange vector in character. In the extreme, $m \gtrsim 50$ GeV, say, the spectrum will be similar to that of positronium, with the appropriate scale changes, since the Coulomb exchange will be dominant for a large number of low-lying states. The existence of mesons with quark masses, $m \gtrsim 50$ GeV would also be interesting because the absence of flavor-changing neutral currents in light hadrons seems to require such super-heavy hadrons to be quasi-stable.[37]

One of the difficulties in making quantitative predictions for the spin structure of the Υ levels is the lack of knowledge of the long-range b quark-gluon anomalous moment. Even though f_b is probably small, Eq. (10.20) suggests that $\sqrt{f_b}\,(1+\kappa_b)$ is likely to be large. Unfortunately, there is no phenomenological or theoretical guidance for the extrapolation of the quark chromomagnetic moment from the charmed quark to the b quark. Therefore, one must leave f_b and $(1+\kappa_b)$ as free parameters in the $\bar{b}b$ Hamiltonian. As remarked earlier, the work of Pignon and Picketty[35] suggests that the Coulomb + scalar linear potential model, without a logarithmic potential, may be adequate for the Υ mesons *if* $\alpha_s(M_\Upsilon^2)$ is as large as that given by our prescription Eqs. (5.15)–(5.17) ff. We assume this to be the case, so that Eq. (9.2) will be applicable with appropriate notational changes. We shall not give a complete discussion of all the multiplet splittings of the Υ system, since they are expected to be smaller than charmonium and difficult to measure. However, the S-wave hyperfine splitting is interesting because of the large value of the wave function at the origin expected.

The S-wave hyperfine splitting is given by Eq. (5.3), which for the $\bar{b}b$ mesons is

$$\Delta E_\Upsilon = \frac{32\pi}{9m_b^2}\alpha_s|\psi(0)|^2 + \frac{4}{3}\frac{f_b(1+\kappa_b)^2}{m_b^2}a\langle r^{-1}\rangle_{S\text{ wave}} . \tag{10.21}$$

We adopt the parameters

$$m_b \simeq 5 \text{ GeV}$$

and (10.22)

$$\alpha_s(M_\Upsilon^2) \simeq 0.27 ,$$

as well as (4.6)–(4.8) and (5.9). [The value of α_s comes from (5.15)–(5.17).] One can estimate $|\psi(0)|^2$ as in Refs. 12 and 27, and reported in Eqs. (5.5)–(5.8). Extrapolating the straight lines of Fig. 13 of Jackson,[27] we find

$$|\psi(0)|^2 = (296 \pm 60)\times 10^{-3}\text{ GeV}^3 \quad (\Upsilon \text{ mesons}) , \tag{10.23}$$

which should be compared with (39×10^{-3}) GeV^3 for charmonium. As a consequence of (10.21)–(10.23), we obtain

$$\Delta E = [35.5 \pm 7 + 8f_b(1+\kappa_b)^2] \text{ MeV} \tag{10.24}$$

for the ground-state ${}^3S_1$-${}^1S_0$ hyperfine splitting of the Υ system. If $f_b(1+\kappa_b)^2 \gtrsim f_c(1+\kappa_c)^2 \simeq 2.3$, then $\Delta E \gtrsim 55$ MeV is possible, which could be observable. Therefore, observation of Υ-system hyperfine splittings will be extremely useful in clarifying the question of anomalous quark-gluon interactions.

Two DESY groups[37a] have recently observed $\Upsilon \to e^+e^-$ directly. They find $M_\Upsilon = 9.46 \pm 0.01$ GeV and $\Gamma(\Upsilon \to e^+e^-) = 1.3 \pm 0.4$ keV. From Eq. (5.4) this corresponds to

$$|\psi(0)|^2 = (391 \pm 120)\times 10^{-3}\text{ GeV}^3 , \tag{10.25}$$

which is compatible with our estimate (10.23).

XI. SUMMARY

We now summarize a number of the principal findings of our analysis, and present two additional pieces of experimental evidence for scalar confinement.

(1) Analysis of the $I=0$ multiplets leads to the prediction of masses of the missing ${}^1P_1$ states. (See Table I and Sec. II for details.)

(2) A valence-quark model with Coulomb + long-range scalar exchange gives an excellent overall account of the spin structure of ordinary mesons, *if* $\alpha_s(M^2)$ is sufficiently large. A detailed prescription for α_s specified in Sec. V leads to many qualitative and quantitative successes.

(3) This model predicts inverted ${}^3P$ multiplets for the D and F charmed mesons and mesons of the type $(b\bar{u})$ and $(b\bar{s})$. The prediction is a consequence of a kinematical enhancement of the long-range scalar exchange relative to the short-range Coulomb exchange.

(4) The details of the spin-orbit and spin-spin forces in ordinary mesons are very well understood within the context of the model. Similarly, the systematics of the $I=0$ tensor force is predicted correctly.

(5) The valence-quark model *cannot* account for the absence of a significant tensor force in the $I=1$ and $I=\frac{1}{2}$ multiplets. As a consequence we suggest that there is an important contribution to the tensor force induced by couplings to open decay channels. (See Sec. VII B for a detailed description.) We argue that the $I=1$ and $I=\frac{1}{2}$, $J=1$ mesons may have significant contributions from $(q\bar{q})(q\bar{q})$ configurations, in addition to the usual valence $(q\bar{q})$ states.

(6) The sign, but not the magnitude, of the spin-orbit mixing of $I=\frac{1}{2}$, P-wave mesons is correctly predicted by the valence-quark model with scalar confinement. However, induced effects due to coupling to open decay channels are expected to be relevant as well.

(7) Analysis of the P-wave spectrum of charmonium indicates that the fraction (f) of long-range vector exchange is nearly flavor independent. However, the quark-gluon anomalous moment changes significantly from light to heavy quarks. Discovery of the Υ (${}^3S_1$-${}^1S_0$) hyperfine splitting will give further information on a possible heavy quark-gluon anomalous moment.

(8) A fit to the charmonium P states then

predicts

$$E(\psi)-E(\eta_c)=204 \text{ MeV}$$

and

$$E(\psi')-E(\eta_c')=170 \text{ MeV}$$

in qualitative agreement with observation.

There are two additional pieces of evidence for scalar confinement that should be brought to the reader's attention. The first involves relativistic potential models for spin-averaged meson spectra. Some time ago, Kang and Schnitzer[2] constructed a relativistic linear-potential model, assuming that the quark-confining potential transformed as a Lorentz four-vector. In that work the *first* radial recurrence of $\rho(770)$ was $\rho(1570)$. It is very difficult to avoid this result *if* (i) the confining potential is flavor independent, and (ii) the potential transforms as a Lorentz four-vector. On the other hand, Gunion and Li[3] obtain $\rho(1250)$ as the first radial excitation of $\rho(770)$ from a relativistic potential model with scalar confinement. In the scalar-exchange model it is very difficult to accommodate $\rho(1570)$ as the first radial excitation of the ρ. This conclusion has recently been confirmed by Bradley.[38] We can offer a qualitative explanation of this result. These two models differ in that the vector confining potential is velocity dependent, while the scalar potential is velocity independent. Hence, for a *given* kinetic energy, one expects excitation energies which are higher in the vector model than in the scalar model. Since there is increasing evidence[39] for $\rho(1250)$ as the first radial excitation of ρ, this situation also appears to favor scalar confinement.

Finally one should note that baryon spectroscopy has undergone a development which parallels that of meson spectroscopy.[5,40,41] The evidence from the baryons, particularly the 70^- supermultiplet, also favors scalar confinement.[40] The reader is referred to the literature for details.

Note added. The model presented in this paper apparently provides another success for the nonrelativistic constituent quark model. There are numerous applications of the nonrelativistic, constituent quark model for confined, light quarks, where one would expect relativistic corrections to be important. Yet it appears as if these corrections may be ignored for reasons which are not well understood. This situation is common to problems in both meson and baryon spectroscopy.[40]

One can speculate why certain *specific* nonrelativistic predictions work so well. For example, the calculations of Sec. V only demonstrate that the ratio

$$R_4=\left[\frac{\Gamma(V\to l^+l^-)}{\Delta E}\right]\left(\frac{2\alpha_s}{\alpha^2 e_q{}^2}\right)\left(\frac{M_v{}^2}{m_1 m_2}\right)=O(1)\,, \quad (11.1)$$

obtained from (5.3) and (5.4), is of order one for light-quark systems, even when higher-order corrections are considered. Thus, the unknown relativistic corrections to (5.3) and (5.4) are comparable in magnitude, and are less important in the ratio R_4 than the experimental uncertainties inherent in (11.1). The fact that R_4 is order one does not test absolute energy scales without further input.

Since (5.4) gives a reasonable estimate of the *absolute* decay rates of vector mesons, one must go beyond (11.1) to understand this. A detailed consideration of the corrections to (5.4) shows that[42]

$$\Gamma(V\to l^+l^-)\cong(\text{usual factors})|\psi_0(0)|^2 \times\left[1-\frac{8\alpha_s}{3\pi}-\frac{1}{6}\langle(v/c)^2\rangle+\cdots\right]^2 \quad (11.2)$$

to lowest order in α_s and the quark (velocity)^2. The α_s term is the short-distance gluon effect discussed in Ref. 5, Eq. (11), while the $(v/c)^2$ correction[42] is the leading long-distance effect due to the quark confinement which cannot be absorbed in the nonrelativistic wave function $\psi_0(0)$. Therefore, comparing (5.4) with (11.2), one observes that (5.4) should not be used as input to fix $|\psi_0(0)|^2$ in potential models.[24] Rather, potential models should give wave functions which are larger than the *effective* $|\psi_0(0)|^2$ defined by (5.4) if the potential model is to be compatible with (11.2).

One can also show[42] that, to leading order in α_s and $(v/c)^2$, one can write (11.2) in the alternate form[43]

$$\Gamma(V\to l^+l^-)\simeq(\text{usual factors})|\psi_0(r\simeq 1/m)|^2\,. \quad (11.3)$$

One may speculate that (11.3) is also approximately valid for light quarks as well, which would explain why (5.4) gives a reasonable estimate of the absolute leptonic decay rates of the vector mesons, even for light quarks. If both (11.1) and (11.3) are correct, then one understands why the absolute value of the ρ-π mass difference is correctly predicted in Sec. V, even though light quarks are involved. It may well be that other successes of the nonrelativistic constituent quark model may be explained by similar considerations.

XII. CONCLUSIONS

The overall conclusion of this paper is that meson spectroscopy provides extensive evidence to support the view that the effective interaction between $q\bar{q}$ pairs is a long-range scalar confining force, together with a short-range Coulomb-type vector-gluon exchange, governed by QCD. The valence-quark model presented here, based on this effective interaction, achieves a number of qualitative and quantitative successes in understanding meson spectroscopy. However, the $I=1$ and $I=\frac{1}{2}$ tensor force cannot be understood within the framework of a valence-quark model; some sort of "induced" tensor force is required. A qualitative explanation for this effect is offered.

We have stressed a qualitative understanding of many of the detailed conclusions of the paper. This is best illustrated by the discussion in Sec. III, which argues for a long-range scalar exchange, without undue dependence on detailed numerical results, but rather on systematic trends in the data. We believe that these qualitative conclusions will survive changes in the data, and improvements in calculational technique.

ACKNOWLEDGMENTS

We wish to thank Fred Gilman and Enrico Poggio for conversations on problems of spectroscopy, and for reading the manuscript. This research was supported in part by the U. S. Department of Energy under Contract No. E(11-1)3230.

---

[1]For recent reviews see J. D. Jackson, in *Proceedings of the 1977 European Conference on Particle Physics, Budapest*, edited by L. Jenik and I. Montvay (CRIP, Budapest, 1978); in *Proceedings of the International Symposium on Lepton and Photon Interactions at High Energies, Hamburg, 1977*, edited by F. Gutbrod (DESY, Hamburg, 1977).

[2]H. J. Schnitzer, Brandeis University Report, 1974 (unpublished); J. S. Kang and H. J. Schnitzer, Phys. Rev. D 12, 841 (1975); 12, 2791 (1975); B. J. Harrington, S. Y. Park, and A. Yildiz, Phys. Rev. Lett. 34, 168 (1975); E. Eichten *et al.*, *ibid.* 34, 369 (1975); J. F. Gunion and R. S. Willey, Phys. Rev. D 12, 174 (1975); M. Machacek and Y. Tomozawa, Ann. Phys. (N.Y.) 110, 407 (1978).

[3]H. J. Schnitzer, Phys. Rev. D 13, 74 (1976).

[4]H. J. Schnitzer, Phys. Rev. Lett. 35, 1540 (1975).

[5]A. De Rújula, H. Georgi, and S. L. Glashow, Phys. Rev. D 12, 147 (1975); J. Pumplin, W. Repko, and A. Sato, Phys. Rev. Lett. 35, 1538 (1975); R. Barbieri, R. Gatto, R. Kogerler, and Z. Kunszt, Phys. Lett. 57B, 455 (1975); Nucl. Phys. B105, 125 (1976); P. M. Fishbane, D. Horn, and S. Meshkov (unpublished); W. Celmaster, H. Georgi, and M. Machacek, Phys. Rev. D 17, 879 (1978); 17, 886 (1978).

[6]H. J. Schnitzer, Phys. Lett. 65B, 239 (1976).

[7]X. Y. Pham and J. M. Richards, Phys. Lett. 70B, 370 (1977).

[8]For other motivation see W. A. Bardeen *et al.*, Phys. Rev. D 11, 1094 (1975) and R. Friedberg and T. D. Lee, *ibid.* 16, 1096 (1977).

[9]A. B. Henriques, B. H. Kellett, and R. G. Moorhouse, Phys. Lett. 64B, 85 (1976).

[10]D. Gromes, Nucl. Phys. B131, 80 (1977); D. Pignon and C. A. Picketty, Phys. Lett. 74B, 108 (1978); A. Khare, *ibid.* 73B, 296 (1978).

[11]Lai-Him Chan, Phys. Lett. 71B, 422 (1977); C. E. Carlson and F. Gross, Phys. Lett. 74B, 404 (1978).

[12]H. J. Schnitzer, Phys. Lett. 69B, 477 (1977).

[13]H. J. Schnitzer, Phys. Lett. 76B, 461 (1978).

[14]R. K. Carnegie *et al.*, Nucl. Phys. B127, 509 (1977); M. Bowler, J. Phys. G 3, 775 (1977); D. W. G. S. Leith, in Proceedings of the 5th International Conference on Experimental Meson Spectroscopy, Boston, 1977 (unpublished); F. J. Gilman, SLAC Report No. 2054, 1977 (unpublished).

[15]Pluto Collaboration, Phys. Lett. 73B, 99 (1978).

[16]Particle Data Group, Rev. Mod. Phys. 48, S1 (1976).

[17]W. Wetzel *et al.*, Nucl. Phys. B115, 208 (1976); N. N. Cason *et al.*, Phys. Rev. Lett. 36, 1485 (1976); A. J. Pawlicki *et al.*, *ibid.* 37, 1666 (1976); Phys. Rev. D 15, 3196 (1977).

[18]R. Barbieri, R. Gatto, and Z. Kunszt, Phys. Lett. 66B, 349 (1977).

[19]H. Georgi (unpublished).

[20]De Rújula *et al.*, Ref. 5; Barbieri *et al.*, Ref. 5.

[21]The spin-orbit force given in Ref. 6 has been corrected in Refs. 7, 11, and Gromes, Ref. 10, and W. Celmaster and F. S. Henyey, Phys. Rev. D 17, 3268 (1978).

[22]D. Gromes, Ref. 10.

[23]Kang and Schnitzer, Ref. 2; E. Eichten and K. Gottfried, Phys. Lett. 66B, 286 (1977); W. Celmaster, H. Georgi, and M. Machachek, Phys. Rev. D 17, 879 (1978). Within the context of QCD, see T. Appelquist, M. Dine, and I. J. Muzinich, Phys. Lett. 69B, 231 (1977); Phys. Rev. D 17, 2074 (1978); E. Poggio, Phys. Lett. 71B, 381 (1977); W. Fischler, Nucl. Phys. B129, 157 (1977).

[24]It is expected that a potential model will overestimate $|\psi(0)|^2$. The reason for this is explained in the "Note added" to Sec. XI. For example, see Table II of J. S. Kang and H. J. Schnitzer, Phys. Rev. D 12, 841 (1975). Therefore we do not use model calculations to find $|\psi(0)|^2$. One should not use the experimental value of $|\psi(0)|^2$ as input to fix the parameters of a potential-model calculation for the same reason.

[25]H. J. Schnitzer and S. Weinberg, Phys. Rev. 164, 1828 (1967); S. L. Glashow and S. Weinberg, Phys. Rev. Lett. 20, 224 (1967); I. S. Gerstein and H. J. Schnitzer, Phys. Rev. 175, 1876 (1968).

[26]V. A. Novikov *et al.*, Phys. Rev. Lett. 38, 626 (1977).

[27]J. D. Jackson, in Proceedings of the 1976 SLAC Summer Institute on Particle Physics, SLAC Report No. 198, p. 147 (unpublished).

[28]K. Lane and S. Weinberg, Phys. Rev. Lett. 37, 717 (1976). See also A. De Rújula, H. Georgi, and S. L. Glashow, *ibid.* 37, 398 (1976); S. Ono, *ibid.* 37, 655 (1976); W. Celmaster, *ibid.* 37, 1042 (1976); H. Fritzsch, Phys. Lett. 63B, 419 (1976); L.-H. Chan, Phys.

Rev. D 15, 2478 (1976); J. Sanchez-Guillen, Lett. Nuovo Cimento 18, 218 (1977); D. C. Peaslee, Phys. Rev. D 15, 3495 (1977).

[29]A. De Rújula, H. Georgi, and H. D. Politzer, Ann. Phys. (N.Y.) 103, 315 (1977); E. C. Poggio, H. R. Quinn, and S. Weinberg, Phys. Rev. D 13, 1958 (1976); R. Shankar, *ibid.* 15, 755 (1977). See also R. Barbieri and R. Gatto, Phys. Lett. 66B, 181 (1977).

[30]K. Gottfried, Ref. 1, and references therein. In particular see E. Eichten *et al.*, Phys. Rev. Lett. 36, 500 (1976); and K. Lane and E. Eichten, *ibid.* 37, 477 (1976).

[31]For discussion of $(q\bar{q})$ $(q\bar{q})$ configurations in other contexts see Ref. 30; M. Bander *et al.*, Phys. Rev Lett. 36, 695 (1976); C. Rosenzweig, *ibid.* 36, 697 (1976); A. De Rújula and R.L. Jaffe, MIT Report No. CTP 658, 1977 (unpublished); R. L. Jaffe, Phys. Rev. D 15, 267 (1977); 15, 281 (1977); S. Nussinov and D. Sidhu, Nuovo Cimento 44, 230 (1978).

[32]M. S. Chanowitz and F. J. Gilman, Phys. Lett. 63B, 178 (1976).

[33]See Ref. 1 for a detailed discussion of the difficulties surrounding these states. See also G. Feldman and T. Fulton, Phys. Lett. 77B, 89 (1978).

[34]E. Eichten and K. Gottfried, Phys. Lett. 66B, 286 (1977); W. Celmaster, H. Georgi, and M. Machacek, Phys. Rev. D 17, 879 (1978).

[35]D. Pignon and C. A. Picketty, Ref. 10.

[36]M. Machacek and Y. Tomozawa, Ref. 10; W. Celmaster, *et al.*, Ref. 34; D. Lichtenberg and J. Wilk, Indiana Univ. report (unpublished). See also E. Poggio and H. Schnitzer, Brandeis Univ. reports, July 1978 and Sept. 1978 (unpublished).

[37]E. Poggio and H. J. Schnitzer, Phys. Rev. D 15, 1973 (1977); R. N. Cahn, Phys. Rev. Lett. 40, 80 (1978).

[37(a)]PLUTO Collaboration, Phys. Lett. 76B, 243 (1978); DASP Collaboration, *ibid.* 76B, 246 (1978).

[38]A. Bradley, University of Durham report, 1977 (unpublished).

[39]S. Baralucci *et al.*, DESY Report No. DESY 77/59, 1977 (unpublished).

[40]D. Gromes, Nucl. Phys. B130, 18 (1977); N. Isgur and G. Karl, Phys. Lett. 72B, 109 (1977); *ibid.* 74B, 353 (1978); and University of Toronto report (unpublished).

[41]A full strength spin-orbit force fails badly in the baryon 70^- multiplet. Therefore the work of D. Gromes and I. O. Stamatescu, Nucl. Phys. B112, 213 (1976); and W. Celmaster, Phys. Rev. D 15, 1391 (1977) is superseded by Ref. 40.

[42]E. Poggio and H. J. Schnitzer (in preparation).

[43]See also A. Bradley, Phys. Lett. 77B, 422 (1978); J. F. Gunion and L. F. Li, Phys. Rev. D 13, 82 (1976).