Cabibbo angle in a six-quark gauge model

T. Kitazoe* and K. Tanaka
Department of Physics, The Ohio State University, Columbus, Ohio 43210
(Received 14 June 1978)

General structures of the mass matrices resulting from a quark–Higgs-boson Yukawa interaction are discussed. We assume that there are two sets of Higgs bosons in a six-quark $SU(2)_L \times SU(2)_R \times U(1)$ gauge model and require that the interaction be invariant under a general discrete symmetry. We find three physically acceptable mass matrices including those of Fritzsch and Hagiwara *et al*. Special attention is paid to the phase of the vacuum expectation value of the Higgs boson, which is related to *CP* violation. A mass relation among six quarks, $m_t = m_b(m_u m_c/m_d m_s)^{1/2}$, is obtained for the latter models. No new mass matrices are found when there are three sets of Higgs bosons.

I. INTRODUCTION

There is recent interest in computing the Cabibbo angle in terms of quark mass ratios in six-quark models[1-3] in the framework of a gauge group $SU(2)_L \times SU(2)_R \times U(1)$. In order to reduce the parameters, specific discrete symmetries have been used. In a previous article,[3] an extended Cabibbo current in a six-quark gauge model is obtained in terms of Eulerian angles which are functions of the quark mass ratios and phases in the quark mass matrices. The model predicted *CP* violation for $|\Delta S| = 1$ nonleptonic decays, a long-lived heavy quark and a dominant b-quark decay mode $b \to u$. It is interesting to examine whether other such cases may exist in six-quark models. We therefore consider the general type of discrete symmetry $Q_i \to \eta^i Q_i$, $\phi_i \to \chi^i \phi_i$ ($|\eta^i| = |\chi^i| = 1$) together with the symmetry $Q_i \leftrightarrow Q_j$, $\phi_i \leftrightarrow \phi_j$ (if needed) and pursue the question of how many physically interesting mass matrices exist, where Q_i and ϕ_i represent the quark and Higgs field, respectively. We assume left-right symmetry and require that the Cabibbo (Eulerian) angle[1] be expressible in terms of quark mass ratios and phases. The maximum number of Higgs bosons which interact with quarks should be three due to the latter requirement, unless there are some relations among vacuum expectation values of Higgs bosons. The one-Higgs-boson case leads to trivial mass matrices. The two-Higgs-boson case is the most interesting and is discussed in detail. The three-Higgs-boson case which is given in the Appendix leads to either mass matrices which are included in the two-Higgs-boson case or to one which is physically unacceptable.

Interesting results are obtained in the two-Higgs-boson case. A mass relation exists between the upper quark (u, c, t) and the lower quark (d, s, b). There are three physically acceptable mass matrices, i.e., the F model,[2] the O model,[3] and a new model. It is shown that when m_u, $m_c \ll m_t$ and $m_d, m_s, \ll m_b$, the Glashow-Iliopoulos-Maiani (GIM) mechanism[4] is well satisfied in the u, c, d, s sector.

The plan of the paper is as follows: In Sec. II we discuss the discrete symmetry that leads to various Higgs-boson–quark couplings, in Sec. III the resulting mass matrices, and in Sec. IV the extended Cabibbo current for the Fritzsch model and a new model. Finally, remarks are made in Sec. V and a new quark mass relation is presented.

II. HIGGS-BOSON-QUARK COUPLING AND DISCRETE SYMMETRY

We propose that a quark mass matrix is generated by a quark–Higgs-boson Yukawa interaction for the case of two Higgs bosons,

$$H = \sum_{\substack{i,j=1,2,3\\ n=1,2}} g_{ij}^{n} \overline{Q}_{iR} \phi_n Q_{jL} + \text{H.c.}, \qquad (1)$$

where the Higgs scalars ϕ_n transforms as $(\frac{1}{2}, \frac{1}{2}, 0)$. Interactions of the form $\overline{Q}_{iR}\tilde{\phi}_n Q_{jL}$, where $\tilde{\phi}_n = \tau_2 \phi_n^* \tau_2$, are eliminated by requiring invariance under the discrete transformation $Q_{iL} \to Q_{iL}$, $Q_{iR} \to -iQ_{iR}$, $\phi_n \to -i\phi_n$. The $Q_{iL,R}$ represent the quark doublets, $Q_{1L,R} = (u_0, d_0)_{L,R}$, $Q_{2L,R} = (c_0, s_0)_{L,R}$, and $Q_{3L,R} = (t_0, b_0)_{L,R}$.

The coupling constant g_{ij}^n in (1) that are taken to be real are suppressed and the discrete transformation

$$\overline{Q}_{1R}, \overline{Q}_{2R}, \overline{Q}_{3R}, Q_{1L}, Q_{2L}, Q_{3L} \to \eta^a \overline{Q}_{1R}, \eta^b \overline{Q}_{2R}, \eta^c \overline{Q}_{3R}, \eta^d Q_{1L}, \eta^e Q_{2L}, \eta^f Q_{3L} \qquad (2)$$

is considered, where a, b, c, d, e, and f are arbitrary numbers and $|\eta^i| = 1$. Then the quark part $\overline{Q}_{iR} Q_{jL}$ is transformed to

$$(\overline{Q}_{1R}, \overline{Q}_{2R}, \overline{Q}_{3R})\begin{bmatrix} \eta^{a+d} & \eta^{a+e} & \eta^{a+f} \\ \eta^{b+d} & \eta^{b+e} & \eta^{b+f} \\ \eta^{c+d} & \eta^{c+e} & \eta^{c+f} \end{bmatrix} \begin{bmatrix} Q_{1L} \\ Q_{2L} \\ Q_{3L} \end{bmatrix}. \quad (3)$$

We impose left-right symmetry, extract an arbitrary phase η^{a+d}, put $\eta=\eta^{b-a}$ and $\xi=\eta^{c-a}$ ($|\eta|=|\xi|=1$), and obtain

$$\begin{bmatrix} 1 & \eta & \xi \\ \eta & \eta^2 & \xi\eta \\ \xi & \xi\eta & \xi^2 \end{bmatrix}. \quad (4)$$

There are six independent phases ($1, \eta, \xi, \eta^2, \xi^2, \eta\xi$) in (4). When the Higgs fields ϕ_1 and ϕ_2 transform so as to cancel out the phases above, then one obtains a Hamiltonian invariant under a discrete transformation. When some of the phases are equal, then there are some interesting couplings. The general conditions that any two of them have the same value are given as (a) $\eta=\xi^2$, (b) $\eta\xi=1$, (c) $\eta^2=1$, and (d) $\eta=\xi$. It is noted that the alternative cases $\xi=\eta^2$ and $\xi^2=1$ to (a) and (c), respectively, lead to the same result when the Q_2 and Q_3 quarks are exchanged.

Let us first consider the case (a) $\eta=\xi^2$ in which case (4) becomes

$$\begin{bmatrix} 1 & \xi^2 & \xi \\ \xi^2 & \xi^4 & \xi^3 \\ \xi & \xi^3 & \xi^2 \end{bmatrix}. \quad (5)$$

We find from (5) that the Higgs fields can couple to $\overline{Q}_{iR}$ and Q_{jL} in the following ways in matrix form[5]:

$$A=\begin{bmatrix} \phi_1 & \phi_1 & \phi_2 \\ \phi_1 & \phi_1 & \phi_2 \\ \phi_2 & \phi_2 & \phi_1 \end{bmatrix} \quad (\xi^2=1), \quad (6)$$

$$O=\begin{bmatrix} 0 & \phi_1 & \phi_2 \\ \phi_1 & \phi_2 & 0 \\ \phi_2 & 0 & \phi_1 \end{bmatrix} \quad (\xi^3=1), \quad (7)$$

$$F=\begin{bmatrix} 0 & \phi_1 & 0 \\ \phi_1 & 0 & \phi_2 \\ 0 & \phi_2 & \phi_1 \end{bmatrix}, \quad B=\begin{bmatrix} 0 & 0 & \phi_1 \\ 0 & 0 & \phi_2 \\ \phi_1 & \phi_2 & 0 \end{bmatrix} \quad (\xi^4=1). \quad (8)$$

The notation $\xi^n=1$ (following the Higgs coupling) means $\xi=\exp(2\pi i/n)$. Trivial cases in which the u. c, d, s sector decouples from the t, b sector are neglected. The matrices which coincide with A, O, F, and B by exchange of Q_i and Q_j quarks are also neglected because they are equivalent.

The second case (b) $\eta\xi=1$ leads to the same matrices as in (a) because if we set $\eta=\xi^{-1}$ in (4), multiplying by ξ^2 and transforming (Q_1, Q_2, Q_3) to (Q_3, Q_1, Q_2), we obtain (5). There are two additional matrices in case (c),

$$C=\begin{bmatrix} \phi_1 & \phi_1 & \phi_2 \\ \phi_1 & \phi_1 & \phi_2 \\ \phi_2 & \phi_2 & 0 \end{bmatrix}, \quad D=\begin{bmatrix} 0 & 0 & \phi_2 \\ 0 & 0 & \phi_2 \\ \phi_2 & \phi_2 & \phi_1 \end{bmatrix} \quad (\eta=1). \quad (9)$$

The case (d) does not lead to any new matrix.

We now discuss the quark mass matrices for the Higgs coupling given above and show that the physically acceptable models are F, O and a new model D. We incorporate the consequences of left-right symmetry, $g^{\eta}_{ij}=g^{\eta}_{ji}$. The condition that the Eulerian angles in weak currents be expressible in terms of quark mass ratios and phases requires that the number of parameters in mass matrices should at most be three up to phases. Therefore, in order to reduce the number of parameters, we sometimes assume invariance of the interaction under the exchange of some quark fields $Q_i \leftrightarrow Q_j$ which imposes corresponding equalities among the couplings g^{η}_{ij}. We can perform the exchange due to the fact that the strong interaction (mediated by gluons), electromagnetic, and weak interactions are invariant under the exchange.

III. MASS MATRICES

Let us take up model A of (6) that includes six coupling constants. We introduce a (discrete) symmetry $(Q_1, Q_2, Q_3)\to(Q_2, Q_1, Q_3)$ which yields $g^1_{11}=g^1_{22}$ and $g^2_{13}=g^2_{23}$. The resulting mass matrix, when the neutral scalar Higgs fields develop vacuum expectation values, can be written as

$$m^0_A=\begin{bmatrix} \beta & \alpha & \gamma \\ \alpha & \beta & \gamma \\ \gamma & \gamma & b \end{bmatrix}. \quad (10)$$

The phase of γ and the common phase of α, β, and b are suppressed. Upon exchange (redefinition) of $(Q_1, Q_2, Q_3)\to(Q_3, Q_2, Q_1)$, one obtains

$$m^{0\prime}_A=\begin{bmatrix} b & \gamma & \gamma \\ \gamma & \beta & \alpha \\ \gamma & \alpha & \beta \end{bmatrix}. \quad (11)$$

A constant unitary transformation leads to

$$U^\dagger m_A'^0 U = \begin{bmatrix} b & \sqrt{2}\gamma & 0 \\ \sqrt{2}\gamma & \beta+\alpha & 0 \\ 0 & 0 & \beta-\alpha \end{bmatrix},$$

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}. \qquad (12)$$

In view of the fact that a weak current is invariant under a constant unitary transformation, the Q_3 quark is decoupled in the resulting weak current from Q_1 and Q_2. In model C of (9), after introducing $(Q_1, Q_2, Q_3) \leftrightarrow (Q_2, Q_1, Q_3)$ symmetry and $(Q_1, Q_2, Q_3) \rightarrow (Q_3, Q_2, Q_1)$ exchange, we obtain

$$m_C^0 = \begin{bmatrix} 0 & \gamma & \gamma \\ \gamma & \beta & \alpha \\ \gamma & \alpha & \beta \end{bmatrix},$$

which upon comparison with (11) indicates that this model also belongs to a decoupled case. The model B of (8) gives zero and two degenerate eigenvalues, so it is not interesting.

We now have the nontrivial quark mass matrices for models O, F, and D, given in (7), (8), and (9). After imposing the symmetry $(Q_1, Q_2, Q_3)_{L,R} \rightarrow (Q_1, Q_3, Q_2)_{L,R}$ and $(\phi_1, \phi_2) \rightarrow (\phi_2, \phi_1)$ for the O model, the resulting mass matrices can be written as

$$m_F^0 = \begin{bmatrix} 0 & a & 0 \\ a & 0 & be^{i\delta} \\ 0 & be^{i\delta} & c \end{bmatrix}. \qquad (13)$$

$$m_O^0 = \begin{bmatrix} 0 & \epsilon e^{i\delta} & \beta\epsilon/b \\ \epsilon e^{i\delta} & \beta & 0 \\ \beta\epsilon/b & 0 & be^{i\delta} \end{bmatrix}. \qquad (14)$$

$$m_D^0 = \begin{bmatrix} 0 & 0 & \gamma e^{i\delta} \\ 0 & 0 & \beta e^{i\delta} \\ \gamma e^{i\delta} & \beta e^{i\delta} & \alpha \end{bmatrix}, \qquad (15)$$

where the subscripts of the parameters in the matrix elements are suppressed. The quark mass terms take the form $\sum_{i=1,2} \bar\psi_{iR}^0 e^{i\rho_i} m_i^0 \psi_{iL}^0 + \text{H.c.}$, and $\psi_{1L,R}^0 = (u_0, c_0, t_0)_{L,R}$, and $\psi_{2L,R}^0 = (d_0, s_0, b_0)_{L,R}$. The matrix m_F^0 of (13) differs from Fritzsch's Hermitian mass matrix due to the fact that here the expectation values of the Higgs scalar are complex, whereas there it is the coupling constants that are complex. The mass matrix m_O^0 of (14) is discussed in Ref. 3, so no further remarks are made. The mass matrix m_D^0 is a new case that is discussed in Sec. IV.

IV. EXTENDED CABIBBO CURRENT

We now consider the quark mass matrix and the extended Cabibbo current for the Fritzsch model (13) and the new model (15). The mass matrix m_F^0 is diagonalized by the transformation $KU_0^\dagger P_0 m_F^0 P_0 U_0 = m$, where the subscript i is suppressed and[1]

$$U_0 = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta\cos\varphi & \cos\theta\cos\varphi & -\sin\varphi \\ \sin\theta\sin\varphi & \cos\theta\sin\varphi & \cos\varphi \end{bmatrix},$$

$$P_0 = \begin{bmatrix} e^{i\delta} & 0 & 0 \\ 0 & e^{-i\delta} & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad K = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \qquad (16)$$

Put $V^\dagger = KU_0^\dagger P_0$, $U = P_0 U_0$, then the mass eigenstates are expressible in terms of the original states by a biunitary transformation as $V^\dagger m_{F i}^0 U = m_i$.

Let us define $\psi_{i,L} = P_i^\dagger U_i^\dagger \psi_{iL}^0$ and $\psi_{i,R} = e^{-i\rho_i} P_i^\dagger V_i^\dagger \psi_{iR}^0$ and choose P_i so that the 2×2 sector of the extended left-handed Cabibbo current $J_{\mu L}$ is real.[6] The extended Cabibbo current $J_{\mu L,R}$ can be written as

$$J_{\mu L,R} = \bar\psi_{1L,R}^0 \gamma_\mu \psi_{2L,R}^0 = \bar\psi_{1L,R} \Gamma_{L,R}^C \gamma_\mu \psi_{2L,R},$$

$$\Gamma_L^C = P_1^\dagger U_1^\dagger U_2 P_2 = \begin{bmatrix} \cos\theta_C & -\sin\theta_C & -\varphi_C \sin\theta_1 e^{i(\xi-\eta)} \\ \sin\theta_C & \cos\theta_C & -\varphi_C\cos\theta_1 \\ \varphi_C \sin\theta_2 e^{i\eta} & \varphi_C\cos\theta_2 e^{-i(\xi+\delta)} & 1 \end{bmatrix}, \qquad (17)$$

where

$$\begin{aligned}
&\cos\varphi_C e^{i\xi_\varphi}=\cos\varphi_1\cos\varphi_2 e^{-i\delta'}+\sin\varphi_1\sin\varphi_2\,,\\
&\sin\varphi_C e^{i\eta_\varphi}=\cos\varphi_1\sin\varphi_2-e^{-i\delta'}\sin\varphi_1\cos\varphi_2\,,\\
&\cos\theta_C e^{i\xi}=\cos\theta_1\cos\theta_2 e^{-i\delta}+\sin\theta_1\sin\theta_2\,,\\
&\sin\theta_C e^{i\eta}=\cos\theta_1\sin\theta_2-e^{-i\delta}\sin\theta_1\cos\theta_2\,,\\
&\delta'=\delta_2-\delta_1\,,\quad \delta=\delta'-\xi\,,\\
&\tan\theta_1=(m_u/m_c)^{1/2}\,,\quad \tan\theta_2=(m_d/m_s)^{1/2}\,,\\
&\sin\varphi_1=-[(m_c-m_u)/m_t]^{1/2}\,,\quad \sin\varphi_2=-[(m_s-m_d)/m_b]^{1/2}\,.
\end{aligned}\tag{18}$$

Equation (17) is written in the limit $\sin\varphi_C\sim\varphi_C$. The right-handed Cabibbo current is then given below to the zeroth order in φ_C since the right-handed current interaction is suppressed by the mass ratio of the vector bosons $(M_{WL}/M_{WR})^2\ll 1$,

$$\Gamma_R^C e^{-i(\rho_1-\rho_2)}=P_1^\dagger V_1^\dagger V_2P_2=(P_1^\dagger)^2K\Gamma_L^{c*}K(P_2)^2$$

$$=\begin{bmatrix}\cos\theta_C e^{2i(\xi-\xi_\varphi)} & \sin\theta_C e^{-2i(\delta+\eta+\xi_\varphi)} & 0\\ -\sin\theta_C e^{2i(\eta-\xi_\varphi)} & \cos\theta_C e^{-2i(\delta+\xi+\xi_\varphi)} & 0\\ 0 & 0 & e^{2i(\delta+2\xi_\varphi)}\end{bmatrix}.\tag{19}$$

The mass matrix m_D^0 is diagonalized similarly by the transformation $KU_0^\dagger P_0 m_D^0 P_0 U_0=V^\dagger m_D^0 U=m$, where the subscript i is suppressed and

$$U_0=\begin{bmatrix}\cos\theta & \sin\theta\cos\varphi & \sin\theta\sin\varphi\\ -\sin\theta & \cos\theta\cos\varphi & \cos\theta\sin\varphi\\ 0 & -\sin\varphi & \cos\varphi\end{bmatrix},$$

$$P_0=\begin{bmatrix}e^{-i\delta} & 0 & 0\\ 0 & e^{-i\delta} & 0\\ 0 & 0 & 1\end{bmatrix},\quad K=\begin{bmatrix}1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 1\end{bmatrix},\tag{20}$$

where

$$\begin{aligned}
&\tan^2\varphi_1=m_c/m_t\,,\quad \tan^2\varphi_2=m_s/m_b\,,\\
&[\alpha_1^2+4(\beta_1{}^2+\gamma_1{}^2)]^{1/2}=m_c+m_t\,,\\
&[\alpha_2{}^2+4(\beta_2{}^2+\gamma_2{}^2)]^{1/2}=m_s+m_b\,,\\
&\alpha_1=m_t-m_c\,,\quad \alpha_2=m_b-m_s\,,\\
&\tan\theta_1=\gamma_1/\beta_1\,,\quad \tan\theta_2=\gamma_2/\beta_2\,.
\end{aligned}\tag{21}$$

This is a model that gives $m_d=m_u=0$ and θ_i is not expressible in terms of quark mass ratios as one cannot determine γ_i and β_i separately.

The left-handed Cabibbo current $J_{\mu L}$ can be written as $J_{\mu L}=\bar\psi_{1L}\Gamma_L^C\gamma_\mu\psi_{2L}$,

$$\Gamma_L^C=e^{i\delta}U_1^\dagger U_2$$

$$=\begin{bmatrix}\cos\theta_C & \sin\theta_C & \varphi_2\sin\theta_C\\ -\sin\theta_C & \cos\theta_C & \varphi_2\cos\theta_C-\varphi_1 e^{i\delta}\\ -\varphi_1\sin\theta_C & \varphi_1\cos\theta_C-\varphi_2 e^{i\delta} & e^{i\delta}\end{bmatrix},\tag{22}$$

where

$$\theta_C=\theta_2-\theta_1\,,\quad \delta=\delta_2-\delta_1\,.$$

Equation (22) is written in the limit $\sin\varphi_i\sim\varphi_i$. The right-handed current is again given to the zeroth order in φ_i,

$$\begin{aligned}
\Gamma_R^C e^{-i(\rho_1-\rho_2)}&=V_1^\dagger V_2 e^{i\delta}\\
&=e^{2i\delta}K\Gamma_L^{*C}K\\
&=e^{2i\delta}\begin{bmatrix}\cos\theta_C & -\sin\theta_C & 0\\ \sin\theta_C & \cos\theta_C & 0\\ 0 & 0 & e^{-i\delta}\end{bmatrix}.
\end{aligned}\tag{23}$$

V. REMARKS

We finally compare the three models discussed above. One notes from (17) and (19) that CP invariance is violated for $|\Delta S|=1$ nonleptonic decays in the F model[2] as in the O model[3] due to the phase difference $(\xi+\eta+\delta)$. The CP violation is due solely to spontaneous symmetry breakdown. By comparison of (22) and (23), one notes that the new model satisfies CP invariance in $|\Delta S|=1$ nonleptonic decays.

One notes from (7) that the $g_{12}{}^1\bar Q_{1R}Q_{2L}$, $g_{12}{}^1\bar Q_{2R}Q_{1L}$, and $g_{33}{}^1\bar Q_{3R}Q_{3L}$ terms couple to the same Higgs boson ϕ_1. Therefore, one finds from (7) and (14) that

$$|g_{12}{}^1\langle\phi_1\rangle_i|=\epsilon_i\,,\quad |g_{33}{}^1\langle\phi_1\rangle_i|=b_i\tag{24}$$

where $\langle\phi_1\rangle_i$ expresses the vacuum expectation value of ϕ_1. The parameters are found in Ref. 3 to be $\epsilon_1=(m_u m_c)^{1/2}$, $b_1=m_t$, $\epsilon_2=(m_d m_s)^{1/2}$, and b_2

TABLE I. The order of magnitude of the weak-current couplings. The letters u,c,t,d,s,b, label the quarks and F, O, and D label the models.

	$\vert g_{td}\vert$	$\vert g_{ts}\vert$	$\vert g_{ub}\vert$	$\vert g_{cb}\vert$	$\vert g_{tb}\vert$
F	$\left(\frac{m_u^2 m_c}{m_b^2 m_s}\right)^{1/4}$	$\left(\frac{m_c m_s}{m_b^2}\right)^{1/4}$	$\left(\frac{m_u^2 m_s}{m_b^2 m_c}\right)^{1/4}$	$\left(\frac{m_c m_s}{m_b^2}\right)^{1/4}$	~ 1
O	$\left(\frac{m_s^2 m_u m_c}{m_b^4}\right)^{1/2}$	$\left(\frac{m_u^2 m_c m_s}{m_b^4}\right)^{1/2}$	$\left(\frac{m_s^2 m_u m_c}{m_b^4}\right)^{1/2}$	$\frac{m_s m_u}{m_b^2}$	~ 1
D	$\sin\theta_C\left(\frac{m_c}{m_t}\right)^{1/2}$	$\left(\frac{m_c}{m_t}\right)^{1/2}$	$\sin\theta_C\left(\frac{m_s}{m_b}\right)^{1/2}$	$\left(\frac{m_c}{m_t}\right)^{1/2}$	~ 1

$=m_b$ so that the equality $\epsilon_1/b_1=\epsilon_2/b_2$ leads to

$$m_t=\left(\frac{m_u}{m_d}\,\frac{m_c}{m_s}\right)^{1/2} m_b\approx 3m_b\,. \tag{25}$$

This relation holds approximately in the Fritzsch model.

The strength of the weak current in the u, c, d, s sector is similar, and the GIM mechanism is satisfied for the F, D, and O models. There is, however, a marked difference in the strength of the couplings between the u, c, d, s and t, b sectors. When the weak current is written for simplicity as

$$J_\mu=g_{ub}\bar{u}b+g_{cb}\bar{c}b+g_{td}\bar{t}d+g_{ts}\bar{t}s+g_{tb}\bar{t}b\,, \tag{26}$$

the orders of magnitude of g_{ij} in terms of quark mass ratios are given for the various models in Table I.

The ratio of the t-quark to u-quark production in neutrino reactions

$$R_t=\sigma_\nu(d\to t)/\sigma_\nu(d\to u)=|g_{td}|^2\,, \tag{27}$$

is severely suppressed, $R_t\sim 10^{-6}$ in the O model, while it is reasonable $R_t\sim 10^{-2}$–10^{-3} in the F and D models, provided the masses $m_u{:}m_d{:}m_s{:}m_c{:}m_b=1{:}1{:}15{:}150{:}500$ and Eq. (25) are used.[2,3] The b quark produced in high-energy reactions has a rather long lifetime $\tau(b\to u\mu\bar{\nu})\sim 10^{-8}$ sec in the O model, while it has a short lifetime in the F and D models. The dominant decay mode of the b quark is u for the O model and c for the F and D models.

It is not possible to explain the trimuon events in terms of the cascade decay of the t quark in O due to the smallness of R_t, but it is possible in F and D. On the other hand, a rough estimate indicates that the production ratio

$$\sigma(\nu+N\to\mu^-\mu^+\mu^++\dots)/\sigma(\nu+N\to\mu^-\mu^+\mu^-+\dots)$$

is rather large for the F and D models provided the t-quark cascade process is a dominant process for trimuon events. Therefore, it seems difficult to understand the trimuon events by the t-quark cascade via the weak current that is a consequency of a discrete symmetry.

A new model does not appear in the case of three Higgs bosons as shown in the Appendix. Therefore, the two-Higgs-boson case is sufficient to express the Eulerian angles in terms of quark mass ratios. It is remarkable that the F, D, and O models are the only possible ones that resulted from the discrete symmetry considered.

ACKNOWLEDGMENTS

We thank T. Hagiwara and B. Mainland for discussions. This work was supported in part by the U. S. Department of Energy under Contract No. EY-76-C-02-1545. *000.

APPENDIX

We study here the three-Higgs-boson case. Since there are already three parameters $\langle\phi_1\rangle$, $\langle\phi_2\rangle$, and $\langle\phi_3\rangle$, no additional parameters can be introduced. Therefore, one boson, say ϕ_1, cannot couple to quark fields in the form $g_{ii}\bar{Q}_{iR}Q_{iL}\phi_1+g_{jk}\bar{Q}_{jR}Q_{kL}\phi_1$ $(j\neq k)$ because the off-diagonal coupling g_{jk} cannot be related to the diagonal coupling g_{ii} by any symmetry consideration.

In the case in which phases $(1,\eta,\xi,\eta^2,\xi^2,\xi\eta)$ in (4) are all different, we have a quark–Higgs-boson interaction

$$\bar{Q}_R\begin{bmatrix}0 & \phi_1 & \phi_2\\ \phi_1 & 0 & \phi_3\\ \phi_2 & \phi_3 & 0\end{bmatrix}Q_L\,, \tag{A1}$$

in addition to the interactions which give mass matrices essentially identical to those of the F

and D model (the only difference is the phase). In the case (a) $\eta=\xi^2$, we have the new matrix

$$\begin{bmatrix} \phi_3 & 0 & \phi_1 \\ 0 & \phi_3 & \phi_2 \\ \phi_1 & \phi_2 & 0 \end{bmatrix} \quad (\xi^4=1)\,. \tag{A2}$$

In the case (c) $\eta^2=1$, we have

$$\begin{bmatrix} \phi_1 & \phi_2 & \phi_3 \\ \phi_2 & \phi_1 & 0 \\ \phi_3 & 0 & 0 \end{bmatrix} \quad (\eta=-1)\,. \tag{A3}$$

Case (d) $\eta=\xi$ does not lead to a new matrix.

The mass matrix from (A1) is given as

$$\begin{bmatrix} 0 & \alpha & \beta \\ \alpha & 0 & \gamma \\ \beta & \gamma & 0 \end{bmatrix}, \tag{A4}$$

where the phases are suppressed. The matrix (A4) is diagonalized after eliminating the phases in the elements. The eigenvalue equation $\lambda^3 - (|\alpha|^2+|\beta|^2+|\gamma|^2)\lambda - 2|\alpha||\beta||\gamma|=0$ leads to $\pm m_d \pm m_s \pm m_b=0$ and $\pm m_u \pm m_c \pm m_t=0$. Any combination of signs in these equations is not satisfied by the usual quark masses.

After introducing the symmetry $(Q_1, Q_2, Q_3) \to (Q_2, Q_1, Q_3)$ in (A2), the mass matrix is given as

$$D'=\begin{bmatrix} -\alpha & 0 & \gamma \\ 0 & -\alpha & \beta \\ \gamma & \beta & 0 \end{bmatrix}. \tag{A5}$$

This has the same structure as the D model in Eq. (15), except for the phases in the matrix elements, because $D'=D-\alpha I$ (I is a unit matrix). Therefore, the unitary transformation and the weak current in the D' model is almost similar to those in the D model. The u, d quark masses are nonvanishing in the D' model. Equation (A3) leads to a mass matrix which includes four parameters and phases. One cannot reduce the parameters to three.

---

*Permanent address: Kobe University, Kobe, Japan.

[1]For previous work on the Cabibbo angle see, for example, R. Gatto, in *Springer Tracts in Modern Physics*, (Springer, Berlin, Heidelberg, New York, 1970), Vol. 53. Recent papers on the u, d, s, c sector are H. Fritzsch, Phys. Lett. 70B, 436 (1977); F. Wilczek and A. Zee, Phys. Lett. 70B, 418 (1977); S. Weinberg, Festschrift in honor of I. I. Rabi, 1977 (to be published); A. De Rújula, H. Georgi, and S. L. Glashow, Ann. Phys. (N.Y.) (to be published); S. Pakvasa and H. Sugawara, Phys. Lett. 73B, 61 (1978); R. N. Mohapatra and G. Sanjanović, Phys. Lett. 73B, 176 (1978).

[2]H. Fritzsch, Phys. Lett. 73B, 317 (1978).

[3]T. Hagiwara, T. Kitazoe, G. B. Mainland, and K. Tanaka, Phys. Lett. 76B, 602 (1978).

[4]S. L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D 2, 1285 (1970).

[5]For the O case, for example, $(\phi_1\phi_2)$ are transformed as $(\xi\phi_1, \xi^2\phi_2)$ with $\xi^3=1$. Then, the quark–Higgs-boson interaction that is invariant under this transformation is expressible as

$$\bar{Q}_R \begin{bmatrix} 0 & g_{12}{}^1\phi_1 & g_{13}{}^2\phi_2 \\ g_{12}{}^1\phi_1 & g_{22}{}^2\phi_2 & 0 \\ g_{13}{}^2\phi_2 & 0 & g_{33}{}^1\phi_1 \end{bmatrix} Q_L \,.$$

[6]The diagonal matrices P_1 and P_2 are chosen to be

$$P_1=\begin{bmatrix} e^{-i(\xi-\eta+\eta_\varphi-\xi_\varphi)} & 0 & 0 \\ 0 & e^{-i(\eta_\varphi-\xi_\varphi)} & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$P_2=\begin{bmatrix} e^{i(\eta-\eta_\varphi)} & 0 & 0 \\ 0 & e^{-i(\delta+\eta_\varphi+\xi)} & 0 \\ 0 & 0 & e^{i(\delta+2\xi_\varphi)} \end{bmatrix}.$$