Neutrino-antineutrino oscillations

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We show that observable neutrino-antineutrino oscillations require not only the nonconservation of lepton number and fermion number and a nonzero mass for the neutrino but also the presence of some right-handed leptonic charged current, and we discuss, very briefly, the prospects for an experimental search.

In the present note we consider the possibility of neutrino-antineutrino oscillations, i.e., of $\bar{\nu}_e \leftrightarrow \nu_e$ or $\bar{\nu}_\mu \leftrightarrow \nu_\mu$ or $\bar{\nu}_\tau \leftrightarrow \nu_\tau$ or $\cdots$ oscillations; such oscillations (as $[A,Z] \rightarrow [A,Z+2]+e^-+e^-$ or $K^+ \rightarrow \pi^- + \mu^+ + \mu^+$) violate not only lepton-number (l) conservation but also fermion-number (f) conservation. In contrast, $\nu_e \leftrightarrow \nu_\mu$ or $\nu_\mu \leftrightarrow \nu_\tau$ or $\nu_\tau \leftrightarrow \nu_e$ or $\cdots$ oscillations[1] (as $\mu^\pm \rightarrow e^\pm + \gamma$ or $\tau^\pm \rightarrow \mu^\pm + \gamma$ or $\tau^\pm \rightarrow e^\pm + \gamma$ or $\cdots$) violate the conservation of electronic lepton number (l_e), muonic lepton number (l_μ), tauonic lepton number (l_τ), etc., in such a way as to conserve $l = l_e + l_\mu + l_\tau + \cdots$ and f. For the sake of definiteness, and with the possibility of nuclear-reactor experiments in mind, we shall focus our attention on the case of $\bar{\nu}_e \leftrightarrow \nu_e$ oscillations.

To parametrize the situation as simply and as economically as possible we suppose that the $\bar{\nu}_e \leftrightarrow \nu_e$ and $\bar{\nu}_\mu \leftrightarrow \nu_\mu$ oscillations as well as the $\nu_e \leftrightarrow \nu_\mu$ and $\bar{\nu}_e \leftrightarrow \bar{\nu}_\mu$ oscillations take place between mutually orthogonal neutrino states $|\bar{\nu}_e\rangle, |\nu_e\rangle, |\bar{\nu}_\mu\rangle, |\nu_\mu|$ which can be expressed in terms of the one-particle helicity (h) eigenstates of a Dirac neutrino-antineutrino field, ψ_ν, via[2]

$$\begin{aligned}
&|\bar{\nu}_e\rangle = \alpha^*|\bar{\nu}_+\rangle + \beta^*|\bar{\nu}_-\rangle ,\\
&|\nu_e\rangle = \alpha|\nu_-\rangle + \beta|\nu_+\rangle ,\\
&|\bar{\nu}_\mu\rangle = \alpha^*|\nu_+\rangle - \beta^*|\nu_-\rangle ,\\
&|\nu_\mu\rangle = \alpha|\bar{\nu}_-\rangle - \beta|\bar{\nu}_+\rangle ,\\
&|\alpha|^2 + |\beta|^2 = 1 ,\\
&\langle h_{\bar{\nu}_e}\rangle = -\langle h_{\nu_e}\rangle = \langle h_{\bar{\nu}_\mu}\rangle = -\langle h_{\nu_\mu}\rangle \equiv \langle h_\nu\rangle ,\\
&\langle h_\nu\rangle = |\alpha|^2 - |\beta|^2 ,
\end{aligned} \tag{1}$$

with

$$\begin{aligned}
&m_{\bar{\nu}_e} = m_{\nu_e} = m_{\bar{\nu}_\mu} = m_{\nu_\mu} \equiv m_\nu ,\\
&m_\nu = \langle \nu_\pm | H | \nu_\pm \rangle_{\vec{p}_\nu = 0} = \langle \bar{\nu}_\pm | H | \bar{\nu}_\pm \rangle_{\vec{p}_\nu = 0} ,\\
&m_{\bar{\nu}_e} = \langle \bar{\nu}_e | H | \bar{\nu}_e \rangle_{\vec{p}_\nu = 0} , \text{ etc.},
\end{aligned} \tag{2}$$

where $H = H_{\text{strong}} + H_{\text{em}} + H_{\text{weak}} + \cdots$ is the world Hamiltonian. Also,

$$\langle h_\nu \rangle = \frac{1-\epsilon^2}{1+\epsilon^2} \left[\frac{|\vec{p}_\nu|}{(|\vec{p}_\nu|^2 + m_\nu^2)^{1/2}} \right], \tag{3}$$

where ϵ specifies the relative amount of right-handed leptonic charged current entering into H_{weak}, i.e.,[3]

$$\begin{aligned}
\mathcal{J}_\lambda^{\text{leptonic charged current}} &= \psi_e^\dagger \gamma_4 \gamma_\lambda \left[\frac{(1+\gamma_5) + \epsilon(1-\gamma_5)}{(1+\epsilon^2)^{1/2}} \right] \psi_\nu \\
&\quad + \psi_\mu^\dagger \gamma_4 \gamma_\lambda \left[\frac{(1+\gamma_5) - \epsilon(1-\gamma_5)}{(1+\epsilon^2)^{1/2}} \right] \psi_{\bar{\nu}} ,\\
\psi_{\bar{\nu}} = \mathcal{C}\tilde{\psi}_\nu^\dagger , &\quad \psi_\nu = \mathcal{C}\tilde{\psi}_{\bar{\nu}}^\dagger .
\end{aligned} \tag{4}$$

In a similar way, we can suppose that

$$\begin{aligned}
&|\bar{\nu}_\tau\rangle = \alpha'^*|\bar{\nu}'_+\rangle + \beta'^*|\bar{\nu}'_-\rangle ,\\
&|\nu_\tau\rangle = \alpha'|\nu'_-\rangle + \beta'|\nu'_+\rangle ,\\
&|\bar{\nu}_\sigma\rangle = \alpha'^*|\nu'_+\rangle - \beta'^*|\nu'_-\rangle ,\\
&|\nu_\sigma\rangle = \alpha'|\bar{\nu}'_-\rangle - \beta'|\bar{\nu}'_+\rangle ,\\
&|\alpha'|^2 + |\beta'|^2 = 1 ,
\end{aligned} \tag{5}$$

where σ is a charged lepton with $m_\sigma > m_\tau$, ν_σ is its associated neutrino (assuming such particles exist), and $|\nu'_\pm\rangle, |\bar{\nu}'_\pm\rangle$ are one-particle helicity eigenstates of another Dirac neutrino-antineutrino field, $\psi_{\nu'}$. It is to be noted that Eqs. (1) and (2) completely segregate $\bar{\nu}_e, \nu_e, \bar{\nu}_\mu, \nu_\mu$ from $\bar{\nu}_\tau, \nu_\tau, \bar{\nu}_\sigma, \nu_\sigma$ but this restriction can be easily removed by postulation of a more complicated relationship between $|\bar{\nu}_e\rangle, |\nu_e\rangle, |\bar{\nu}_\mu\rangle, |\nu_\mu\rangle, |\bar{\nu}_\tau\rangle, |\nu_\tau\rangle, |\bar{\nu}_\sigma\rangle, |\nu_\sigma\rangle$ and $|\bar{\nu}_+\rangle, |\bar{\nu}_-\rangle, |\nu_+\rangle, |\nu_-\rangle, |\bar{\nu}'_+\rangle, |\bar{\nu}'_-\rangle, |\nu'_+\rangle, |\nu'_-\rangle$. It is also to be noted that the states $\alpha^*|\nu_+\rangle - \beta^*|\nu_-\rangle$ and $\alpha|\bar{\nu}_-\rangle - \beta|\bar{\nu}_+\rangle$, which are identified with $|\bar{\nu}_\mu\rangle$ and $|\nu_\mu\rangle$ in Eq. (1), may not have anything to do with the muon and so should be labeled $|\bar{\nu}_\xi\rangle$ and $|\nu_\xi\rangle$ with the question of the participation of ν_ξ together with an appropriate charged lepton ξ in H_{weak} left completely open; in this case we would have, in addition to Eqs. (1)–(3),

$$
\begin{aligned}
&|\bar{\nu}_\mu\rangle = \alpha'^*|\bar{\nu}'_+\rangle + \beta'^*|\bar{\nu}'_-\rangle ,\\
&|\nu_\mu\rangle = \alpha'|\nu'_-\rangle + \beta'|\nu'_+\rangle ,\\
&|\bar{\nu}_\eta\rangle = \alpha'^*|\nu'_+\rangle - \beta'^*|\nu'_-\rangle ,\\
&|\nu_\eta\rangle = \alpha'|\bar{\nu}'_-\rangle - \beta'|\bar{\nu}'_+\rangle ,\\
&|\alpha'|^2 + |\beta'|^2 = 1 , \qquad (6)\\
&\langle h_{\bar{\nu}_\mu}\rangle = -\langle h_{\nu_\mu}\rangle \equiv \langle h_{\nu'}\rangle = |\alpha'|^2 - |\beta'|^2\\
&\qquad = \frac{1-(\epsilon')^2}{1+(\epsilon')^2}\,\frac{|\vec{p}_{\nu'}|}{(|\vec{p}_{\nu'}|^2 + m_{\nu'}{}^2)^{1/2}} ,\\
&m_{\bar{\nu}_\mu} = m_{\nu_\mu} = m_{\nu'} = \langle \nu'_\pm | H | \nu'_\pm\rangle_{\vec{p}_{\nu'}=0}\\
&\qquad = \langle \bar{\nu}'_\pm | H | \bar{\nu}'_\pm\rangle_{\vec{p}_{\nu'}=0} ,
\end{aligned}
$$

with an analogous set of equations for $|\bar{\nu}_\tau\rangle, |\nu_\tau\rangle, |\bar{\nu}_\zeta\rangle, |\nu_\zeta\rangle$.

We proceed to calculate the oscillational $\bar{\nu}_e$-survival amplitude $\langle \bar{\nu}_e | e^{-iHt}\bar{\nu}_e\rangle$ and the oscillational $\bar{\nu}_e$-transformation amplitudes $\langle \nu_e | e^{-iHt}\bar{\nu}_e\rangle$, $\langle \bar{\nu}_\mu | e^{-iHt}\bar{\nu}_e\rangle$, and $\langle \nu_\mu | e^{-iHt}\bar{\nu}_e\rangle$. Using Eq. (1), $\langle \bar{\nu}_- | e^{-iHt}\bar{\nu}_-\rangle = \langle \bar{\nu}_+ | e^{-iHt}\bar{\nu}_+\rangle$ [Eq. (2)], $\langle \nu_- | e^{-iHt}\bar{\nu}_-\rangle = \langle \nu_+ | e^{-iHt}\bar{\nu}_+\rangle$ (*CPT* invariance), and $\langle \bar{\nu}_\mp | e^{-iHt}\bar{\nu}_\pm\rangle = \langle \nu_\mp | e^{-iHt}\bar{\nu}_\pm\rangle = 0$ (angular momentum conservation) we get

$$
\begin{aligned}
&\langle \bar{\nu}_e | e^{-iHt}\bar{\nu}_e\rangle = \langle \bar{\nu}_+ | e^{-iHt}\bar{\nu}_+\rangle ,\\
&\langle \nu_e | e^{-iHt}\bar{\nu}_e\rangle = \frac{\alpha^*\beta^*}{|\alpha||\beta|}(1-\langle h_\nu\rangle^2)^{1/2}\langle \nu_+ | e^{-iHt}\bar{\nu}_+\rangle , \qquad (7)\\
&\langle \bar{\nu}_\mu | e^{-iHt}\bar{\nu}_e\rangle = \langle h_\nu\rangle\langle \nu_+ | e^{-iHt}\bar{\nu}_+\rangle ,\\
&\langle \nu_\mu | e^{-iHt}\bar{\nu}_e\rangle = 0 ,
\end{aligned}
$$

with $\langle \bar{\nu}_+ | e^{-iHt}\bar{\nu}_+\rangle$ and $\langle \nu_+ | e^{-iHt}\bar{\nu}_+\rangle$ immediately calculable once the relation between $|\bar{\nu}_\pm\rangle$, $|\nu_\pm\rangle$, and the one-neutrino mass eigenstates is specified.

To specify this relationship we assume[4]

$$
\begin{aligned}
&H|\nu_{1;\pm}\rangle = E_{\nu_1}|\nu_{1;\pm}\rangle , \quad H|\nu_{2;\pm}\rangle = E_{\nu_2}|\nu_{2;\pm}\rangle , \qquad (8)\\
&|\nu_{1;\pm}\rangle = \frac{1}{\sqrt{2}}(|\nu_\pm\rangle - |\bar{\nu}_\pm\rangle) , \quad |\nu_{2;\pm}\rangle = \frac{1}{\sqrt{2}}(|\nu_\pm\rangle + |\bar{\nu}_\pm\rangle) ,
\end{aligned}
$$

so that

$$
\begin{aligned}
E_{\nu_1} &= (|\vec{p}_\nu|^2 + m_{\nu_1}{}^2)^{1/2} = (E_\nu{}^2 - m_\nu{}^2 + m_{\nu_1}{}^2)^{1/2}\\
&\cong E_\nu + \frac{m_{\nu_1}{}^2 - m_\nu{}^2}{2E_\nu} , \qquad (9)\\
E_{\nu_2} &= (|\vec{p}_\nu|^2 + m_{\nu_2}{}^2)^{1/2} = (E_\nu{}^2 - m_\nu{}^2 + m_{\nu_2}{}^2)^{1/2}\\
&\cong E_\nu + \frac{m_{\nu_2}{}^2 - m_\nu{}^2}{2E_\nu} ,
\end{aligned}
$$

with

$$
\begin{aligned}
&m_{\nu_1} = m_\nu - m_{\nu\bar{\nu}} , \quad m_{\nu_2} = m_\nu + m_{\nu\bar{\nu}} ,\\
&m_{\nu\bar{\nu}} \equiv \mathrm{Re}\langle \nu_\pm | H | \bar{\nu}_\pm\rangle_{\vec{p}_\nu=0} . \qquad (10)
\end{aligned}
$$

Thus, substituting Eqs. (8)–(10) into Eq. (7), we obtain the oscillational $\bar{\nu}_e$-survival probability, and the oscillational $\bar{\nu}_e$-transformation probabilities for a flight path R (flight time t),

$$
\begin{aligned}
P(\bar{\nu}_e;R|\bar{\nu}_e,0) &= |\langle \bar{\nu}_e | e^{-iHt}\bar{\nu}_e\rangle|^2\\
&= |\langle \bar{\nu}_+ | e^{-iHt}\bar{\nu}_+\rangle|^2\\
&= \left[\cos\left(\frac{R(m_{\nu_2}{}^2 - m_{\nu_1}{}^2)}{4E_\nu}\right)\right]^2\\
&= \left[\cos\left(\frac{Rm_\nu m_{\nu\bar{\nu}}}{E_\nu}\right)\right]^2 ,\\
P(\nu_e;R|\bar{\nu}_e;0) &= |\langle \nu_e | e^{-iHt}\bar{\nu}_e\rangle|^2\\
&= (1-\langle h_\nu\rangle^2)|\langle \nu_+ | e^{-iHt}\bar{\nu}_+\rangle|^2\\
&= (1-\langle h_\nu\rangle^2)\left[\sin\left(\frac{R(m_{\nu_2}{}^2 - m_{\nu_1}{}^2)}{4E_\nu}\right)\right]^2\\
&= (1-\langle h_\nu\rangle^2)\left[\sin\left(\frac{Rm_\nu m_{\nu\bar{\nu}}}{E_\nu}\right)\right]^2 , \qquad (11)\\
P(\bar{\nu}_\mu;R|\bar{\nu}_e;0) &= |\langle \bar{\nu}_\mu | e^{-iHt}\bar{\nu}_e\rangle|^2\\
&= \langle h_\nu\rangle^2|\langle \nu_+ | e^{-iHt}\bar{\nu}_+\rangle|^2\\
&= \langle h_\nu\rangle^2\left[\sin\left(\frac{R(m_{\nu_2}{}^2 - m_{\nu_1}{}^2)}{4E_\nu}\right)\right]^2\\
&= \langle h_\nu\rangle^2\left[\sin\left(\frac{Rm_\nu m_{\nu\bar{\nu}}}{E_\nu}\right)\right]^2 ,\\
P(\nu_\mu;R|\bar{\nu}_e;0) &= |\langle \nu_\mu | e^{-iHt}\bar{\nu}_e\rangle|^2 = 0 .
\end{aligned}
$$

Equations (11) and (3) show that $P(\nu_e;R|\bar{\nu}_e;0)$ is small for all R since

$$
1-\langle h_\nu\rangle^2 = \frac{4\epsilon^2}{(1+\epsilon^2)^2} + \left(\frac{1-\epsilon^2}{1+\epsilon^2}\right)^2 \frac{m_\nu^2}{|\vec{p}_\nu|^2 + m_\nu{}^2} < 0.04 , \qquad (12)
$$

where the numerical upper bound is obtained on the basis of measurements of the longitudinal polarization of electrons

$$
\left(=\langle h_e\rangle = \frac{1-\epsilon^2}{1+\epsilon^2}\,\frac{|\vec{p}_e|}{(|\vec{p}_e|^2 + m_e{}^2)^{1/2}}\right)
$$

emitted in nuclear β decay. We also mention that the "inhibition factor," $\mathcal{I}_{ee}$, for $[A,Z] \to [A,Z+2] + e^- + e^-$ may be calculated from Eqs. (1) and (8) and turns out to be approximately equal to $P(\nu_e;R|\bar{\nu}_e;0)$ [Eqs. (11) and (12)] with $R \cong 1/E \cong$ radius of $[A,Z]$, i.e., $<10^{-34}$ for $(m_\nu m_{\nu\bar{\nu}})^{1/2} < 0.2$ eV [see Eq. (13) just below]—this is to be compared with the experimental upper bound[5] on g_{ee}, $\{g_{ee}\}_{\text{exper}} < 10^{-9}$.

To discuss the implications of Eqs. (11) and (12) on performed and proposed neutrino-oscillation experiments, we first note that a study of the positron energy spectrum from $\bar{\nu}_e$ (reactor) $+p \to e^+ + n$ has set a lower bound on $P(\bar{\nu}_e; R = 11.1\text{ m} | \bar{\nu}_e; 0)$ of about 0.9; using $\langle 1/E_\nu^2\rangle_{\text{aver}} \cong 1/(3\text{ MeV})^2$ this yields

$$(m_\nu m_{\nu\bar{\nu}})^{1/2} < 0.2\text{ eV}.^6 \tag{13}$$

Further, any actual observation of $\bar{\nu}_e \leftrightarrow \nu_e$ oscillations with monoenergetic $\bar{\nu}_e$ beams (or energy-selective ν_e detectors) would determine $1 - \langle h_\nu\rangle^2$ and $m_\nu m_{\nu\bar{\nu}}$ separately [from the value of $P(\nu_e; R = (\pi/2)\, E_\nu/m_\nu m_{\nu\bar{\nu}} | \bar{\nu}_e; 0)$]; unfortunately, sufficiently intense monoenergetic $\bar{\nu}_e$ beams are not easily available so that consideration must be given to $\bar{\nu}_e \leftrightarrow \nu_e$ oscillation experiments with the nonmonoenergetic (but extremely pure) $\bar{\nu}_e$ beams emerging from nuclear reactors. In this case, the effective cross section for a $\bar{\nu}_e$ (reactor)-induced reaction [e.g., $\bar{\nu}_e$ (reactor) $+ {}^{37}\text{Cl} \to e^- + {}^{37}\text{A}$, or $\bar{\nu}_e$ (reactor) $+ {}^7\text{Li} \to e^- + {}^7\text{Be}$, or $\bar{\nu}$(reactor) $+ {}^{71}\text{Ga} \to e^- + {}^{71}\text{Ge}$)] at an energy-nonselective detector distant R from the reactor is

$$\begin{aligned}\sigma_{\text{eff}}(R; \bar{\nu}_e) &= \frac{\int \sigma(E_\nu; \nu_e) P(\nu_e; R | \bar{\nu}_e; 0)\phi(E_\nu; \bar{\nu}_e) dE_\nu}{\int \phi(E_\nu; \bar{\nu}_e) dE_\nu} \\ &\equiv \langle\sigma\rangle_\phi \langle P\rangle_{\sigma\phi}, \\ \langle\sigma\rangle_\phi &\equiv \frac{\int \sigma(E_\nu; \nu_e)\phi(E_\nu; \bar{\nu}_e) dE_\nu}{\int \phi(E_\nu; \bar{\nu}_e) dE_\nu}, \\ \langle P\rangle_{\sigma\phi} &\equiv \frac{\int P(\nu_e; R | \bar{\nu}_e; 0)\sigma(E_\nu; \nu_e)\phi(E_\nu; \bar{\nu}_e) dE_\nu}{\int \sigma(E_\nu; \nu_e)\phi(E_\nu; \bar{\nu}_e) dE_\nu},\end{aligned} \tag{14}$$

where $\sigma(E_\nu; \nu_e)$ is the cross section for the ν_e-induced reaction at energy E_ν, $\phi(E_\nu; \bar{\nu}_e) dE_\nu$ is the flux of $\bar{\nu}_e$ with energy between E_ν and $E_\nu + dE_\nu$ emerging from the reactor, and $P(\nu_e; R | \bar{\nu}_e; 0)$ is given by Eqs. (11)–(13). Since in all practical situations $\sigma(E_\nu; \nu_e)\phi(E_\nu; \bar{\nu}_e)$ has a reasonably sharp maximum at $E_\nu = E_\nu^* \gg m_\nu$ we can write

$$\begin{aligned}\langle P\rangle_{\sigma\phi} &= \left\langle (1 - \langle h_\nu\rangle^2)\left[\sin\left(\frac{R m_\nu m_{\nu\bar{\nu}}}{E_\nu}\right)\right]^2\right\rangle_{\sigma\phi} \\ &\cong \begin{cases}\left[\dfrac{4\epsilon^2}{(1+\epsilon^2)^2}\right](R m_\nu m_{\nu\bar{\nu}})^2 \left\langle\dfrac{1}{E_\nu^2}\right\rangle_{\sigma\phi}, & \left(\dfrac{R m_\nu m_{\nu\bar{\nu}}}{E_\nu^*}\right) \ll 1, \\ \left[\dfrac{4\epsilon^2}{(1+\epsilon^2)^2}\right]\dfrac{1}{2}, & \left(\dfrac{R m_\nu m_{\nu\bar{\nu}}}{E_\nu^*}\right) \gg 1.\end{cases}\end{aligned} \tag{15}$$

Thus, observation of $\sigma_{\text{eff}}(R; \bar{\nu}_e)$ as a function of R, together with a calculation of $\langle\sigma\rangle_\phi$ and $1/\langle E_\nu^2\rangle_{\sigma\phi}$, will yield both $m_\nu m_{\nu\bar{\nu}}$ and $4\epsilon^2/(1+\epsilon^2)^2 \cong 1 - \langle h_\nu\rangle^2$. We also note that if $\epsilon = 0$ (no right-handed leptonic charged current), $\langle(1 - \langle h_\nu\rangle^2)\rangle_{\sigma\phi} = \langle m_\nu^2/(E_\nu)^2\rangle_{\sigma\phi} \lesssim 10^{-10}$ and $\langle P\rangle_{\sigma\phi}$ will be immeasurably small.

We proceed to give estimates of $\langle\sigma\rangle_\phi$ and $\langle 1/E_\nu^2\rangle_{\sigma\phi}$ for $\nu_e + {}^{37}\text{Cl} \to e^- + {}^{37}\text{A}$ and $\nu_e + {}^7\text{Li} \to e^- + {}^7\text{Be}$. Using all available nuclear physics data to calculate $\sigma(E_\nu; \nu_e)$,[7] and the expression for $\phi(E_\nu, \bar{\nu}_e)$ given by Avignone,[8] we obtain from Eq. (14)[7]

$\nu_e + {}^{37}\text{Cl} \to e^- + {}^{37}\text{A}$:

$$\langle\sigma\rangle_\phi = 1.3\times 10^{-44}\text{ cm}^2, \quad \left\langle\frac{1}{E_\nu^2}\right\rangle_{\sigma\phi} = \frac{1}{(3.5\text{ MeV})^2}, \tag{16}$$

$\nu_e + {}^7\text{Li} \to e^- + {}^7\text{Be}$:

$$\langle\sigma\rangle_\phi = 2.9\times 10^{-43}\text{ cm}^2, \quad \left\langle\frac{1}{E_\nu^2}\right\rangle_{\sigma\phi} = \frac{1}{(2.9\text{ MeV})^2}. \tag{17}$$

Thus, assuming $R m_\nu m_{\nu\bar{\nu}}/E_\nu^* \ll 1$ and combining Eqs. (16) and (17) with Eqs. (14), (15), (12), and (13)

$\bar{\nu}_e$ (reactor) $+ {}^{37}\text{Cl} \to e^- + {}^{37}\text{A}$:

$$\begin{aligned}\sigma_{\text{eff}}(R; \bar{\nu}_e) &\cong (1.3\times 10^{-44}\text{ cm}^2)\left\{\frac{4\epsilon^2}{(1+\epsilon^2)^2}\left(\frac{R m_\nu m_{\nu\bar{\nu}}}{3.5\text{ MeV}}\right)^2\right\} \\ &< (2\times 10^{-46}\text{ cm}^2)\left(\frac{R}{10\text{ m}}\right)^2,\end{aligned} \tag{18}$$

$\bar{\nu}_e$ (reactor) $+ {}^7\text{Li} \to e^- + {}^7\text{Be}$:

$$\begin{aligned}\sigma_{\text{eff}}(R; \bar{\nu}_e) &\cong (2.9\times 10^{-43}\text{ cm}^2)\left\{\frac{4\epsilon^2}{(1+\epsilon^2)^2}\left(\frac{R m_\nu m_{\nu\bar{\nu}}}{2.9\text{ MeV}}\right)^2\right\} \\ &< (6\times 10^{-45}\text{ cm}^2)\left(\frac{R}{10\text{ m}}\right)^2.\end{aligned} \tag{19}$$

which is to be compared with an experimental upper bound obtained by Davis,[9]

$\bar{\nu}_e$ (reactor) $+ {}^{37}\text{Cl} \to e^- + {}^{37}\text{A}$:

$$\sigma_{\text{eff}}(R = 10.7\text{ m}; \bar{\nu}_e) < 2.5\times 10^{-46}\text{ cm}^2. \tag{20}$$

We therefore see that a significant improvement of this upper bound would yield an upper bound on

$$(1 - \langle h_\nu\rangle^2)(m_\nu m_{\nu\bar{\nu}})^2 \cong \left[\frac{4\epsilon^2}{(1+\epsilon^2)^2}\right](m_\nu m_{\nu\bar{\nu}})^2$$

that is significantly smaller than that available from Eqs. (12) and (13). We also see that in view of the relatively large value of $\langle\sigma\rangle_\phi$ and, consequently of $\sigma_{\text{eff}}(R; \bar{\nu}_e)/R^2$, for $\bar{\nu}_e$ (reactor) $+ {}^7\text{Li} \to e^- + {}^7\text{Be}$ [Eqs. (17) and (19)], a search for this process appears to be especially attractive.[10] Such a search, if a negative result emerges, should yield a much smaller upper bound on

$$(1 - \langle h_\nu\rangle^2)(m_\nu m_{\nu\bar{\nu}})^2 \cong \left[\frac{4\epsilon^2}{(1+\epsilon^2)^2}\right](m_\nu m_{\nu\bar{\nu}})^2$$

than is now available and, if a positive result is found, would provide a finite value for $(m_\nu m_{\nu\bar{\nu}})^{1/2}$ and for $1-\langle h_\nu\rangle^2 \cong 4\epsilon^2/(1+\epsilon^2)^2$ and so establish

(i) the nonconservation of lepton number and fermion number,

(ii) a nonzero mass for the neutrino (ν_e), and

(iii) the presence of some right-handed leptonic (e, ν_e) charged current in H_{weak}.

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[1]B. Pontecorvo, Zh. Eksp. Teor. Fiz. 53, 1717 (1967) [Sov. Phys.—JETP 26, 984 (1968)]; V. Gribov and B. Pontecorvo, Phys. Lett. 28B, 495 (1969); J. N. Bahcall and S. Frautschi, *ibid.* 29B, 263 (1969); H. Fritzsch and P. Minkowski, *ibid.* 62B, 72 (1976); S. Nussinov, *ibid.* 63B, 201 (1976); A. K. Mann and H. Primakoff, Phys. Rev. D 15, 655 (1977); L. Wolfenstein, *ibid.* 17, 2369 (1978).

[2]H. Primakoff and S. P. Rosen, Phys. Rev. D 5, 1784 (1972).

[3]Taking α and β to have a general form appropriate to Eq. (4) and using Eqs. (1) and (3) yields

$$\alpha = \frac{1}{(1+\epsilon^2)^{1/2}}\left(\frac{1+v_\nu}{2}\right)^{1/2} + i\,\frac{\epsilon}{(1+\epsilon^2)^{1/2}}\left(\frac{1-v_\nu}{2}\right)^{1/2},$$

$$\beta = \frac{\epsilon}{(1+\epsilon^2)^{1/2}}\left(\frac{1+v_\nu}{2}\right)^{1/2} + i\,\frac{1}{(1+\epsilon^2)^{1/2}}\left(\frac{1-v_\nu}{2}\right)^{1/2},$$

where

$$v_\nu = |\vec{p}_\nu|/(|\vec{p}_\nu|^2 + m_\nu^2)^{1/2}.$$

[4]This prodecure is analogous to that used in the theory of $K^0 \longleftrightarrow \bar{K}^0$ oscillations where $|K_S^0\rangle \cong |K_1^0\rangle = (1/\sqrt{2})(|K^0\rangle - |\bar{K}^0\rangle)$, $|K_L^0\rangle \cong |K_2^0\rangle = (1/\sqrt{2})(|K^0\rangle + |\bar{K}^0\rangle)$.

[5]See the recent comprehensive analysis by D. Bryman and C. Picciotto, Rev. Mod. Phys. 50, 11 (1978). Our $\{g_{ee}\}_{\text{exper}}$ corresponds to the $\{\eta^2\}_{\text{exper}}$ of this reference.

[6]F. Reines, B. Lee Memorial International Conference, 1977 (unpublished) and private communication.

[7]J. N. Bahcall, Rev. Mod. Phys. (to be published). These cross sections have also been calculated by G. V. Domogatskii, Yad. Fiz. 22, 1267 (1975) [Sov. J. Nucl. Phys. 22, 657 (1975)] who obtains values of 2.4 (^{7}Li) and (^{37}Cl) smaller than ours. These differences are apparentlydue in part to the fact that he gives four-component-neutrino cross sections (which are to begin with a factor of two smaller than the two-component cross sections given here).

[8]F. T. Avignone, Phys. Rev. D 2, 2609 (1970); F. T. Avignone and Z. D. Greenwood, *ibid.* 17, 154 (1978).

[9]R. J. Davis, Jr., Phys. Rev. 97, 766 (1955); R. J. Davis, Jr., in *Proceedings of the First UNESCO International Conference, Paris, 1967* (Pergamon, London, 1958), Vol. 1, p. 728; R. J. Davis, Jr. and D. S. Harmer, Bull. Am. Phys. Soc. 4, 217 (1959). Equations (20) and (16) also show that the ν_e impurity in the $\bar{\nu}_e$ beam from the reactor could not have exceeded $(2.5\times10^{-46})/(1.3\times10^{-44}) = 2\%$. In fact, such an impurity [which provides an R-independent background to $\sigma_{\text{eff}}(R;\bar{\nu}_e)$] is expected to be much less than 1%.

[10]Another promising candidate is $\bar{\nu}_e$ (reactor) $+\,^{71}\text{Ga} \to e^- + ^{71}\text{Ge}$. In connection with further possible searches for neutrino-antineutrino oscillations, we note that failure to observe ν_μ + nucleon $\to \mu^+$ + anything at a given level to precision can be used to set an upper bound on

$$P(\bar{\nu}_\mu; R|\nu_\mu; 0) = P(\nu_e; R|\bar{\nu}_e; 0) = (1-\langle h_\nu\rangle^2)\left[\sin\left(\frac{R\,m_\nu m_{\nu\bar{\nu}}}{E_\nu}\right)\right]^2 \cong \left(\frac{4\epsilon^2}{(1+\epsilon^2)^2}\right)\left(\frac{R\,m_\nu m_{\nu\bar{\nu}}}{E_\nu}\right)^2.$$

Thus, with the results of Holder *et al.* [Phys. Lett. 74B, 277 (1978)] which correspond to $\langle P(\bar{\nu}_\mu; R = 610\text{ m}\,|\,\nu_\mu; 0)\rangle_{\sigma\phi} < 1.6\times10^{-4}$ for

$$\left\langle \frac{1}{E_\nu^2}\right\rangle_{\sigma\phi} = \frac{1}{(148\times10^3\ \text{MeV})^2},$$

we get

$$(m_\nu m_{\nu\bar{\nu}})^{1/2} < (1.6\ \text{eV})\left\{4\times10^{-2}\bigg/\left[\frac{4\epsilon^2}{(1+\epsilon^2)^2}\right]\right\}^{1/4}.$$

In view of Eq. (12) [and Eqs. (1)–(4)] this upper bound on $(m_\nu m_{\nu\bar{\nu}})^{1/2}$ is at least 8 times as large as the upper bound in Eq. (13). However, this upper bound is extracted from a consideration of $\nu_\mu \longleftrightarrow \bar{\nu}_\mu$ oscillations and, if these are governed by Eq. (6) rather than Eq. (1), it is really an upper bound on $(m_{\nu'} m_{\nu'\bar{\nu}'})^{1/2}$, i.e.,

$$(m_{\nu'} m_{\nu'\bar{\nu}'})^{1/2} < (1.6\ \text{eV})\left\{\frac{6\times10^{-2}}{4(\epsilon')^2/[1+(\epsilon')^2]^2}\right\}^{1/4},$$

where $4(\epsilon')^2/[1+(\epsilon')^2]^2 < 0.06$ on the basis of measurements of the angular asymmetry of electrons emitted in muon decay.