Mesons with b quarks: Spectroscopy and electromagnetic properties

David H. Boal
Theoretical Physics Institute, Department of Physics, University of Alberta, Edmonton, Alberta, Canada
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We treat the recently discovered Υ particle at 9.41 GeV as evidence for a fifth quark flavor, the b quark. An SU(5) mass formula is then used to predict the masses of other $b\bar{b}$ states. The masses of the b-quark analogs of the D and F mesons (B_N, B_s, B_c, etc.) are predicted, as are their production cross sections in e^+e^- annihilation. At best, the $B_N^* \bar{B}_N^*$ combination will be seen at 12 GeV with a cross section of ~ 0.3 nb. As with the charm domain, the $B_N^*-B_N$ mass difference is about one pion mass. The predicted electromagnetic mass splitting is only about 2 MeV for $B_N^+-B_N^0$, so that the strong-interaction decay widths of the B^*'s will be of the same order as the electromagnetic decay widths.

I. INTRODUCTION

Recent experiments[1] involving the reaction

$$p + (\mathrm{Cu, Pt}) \to \mu^+ + \mu^- + \text{anything}$$

show structures in the differential cross section which have been interpreted as new particles with mass 9.41 and 10.06 GeV. The width of these states is less than the experimental resolution, suggesting that they may be new narrow resonances much like the ψ family. If we are to believe the high-y anomaly[2,3] of neutrino physics then a new quark with charge $-\frac{1}{3}$ is required.[4] Most analyses put the effective mass of this quark at 4–5 GeV, which means the Υ particle is handily interpreted as a $(b\bar{b})$ state, where b represents the new quark.

In this paper, we will assume that this interpretation of the Υ is correct. The SU(4) mass operator, which was used in calculating the masses of charmed mesons, is extended to SU(5) in Sec. II. The masses of the $J^P = 0^-$, 1^-, and 2^+ meson $\underline{24}$-plets of SU(5) are calculated using a linear mass formula. Assuming that the charge of the b quark is $-\frac{1}{3}$, the electromagnetic mass splitting of the pseudoscalar $(b\bar{n})$ states (where n denotes the u and d quarks) is calculated in Sec. III. In Sec. IV, the cross sections for the production in e^+e^- annihilation of pairs of mesons with b quarks are predicted. Our conclusions are summarized in the last section.

II. MASSES

Following the approach taken in SU(4), we write the SU(5) mass operator in the form[5,6]

$$\hat{M} = T_0 + aT_8 + bT_{15} + cT_{24}, \tag{1}$$

where the T_i's ($i = 8$, 15, and 24) are members of the same $\underline{24}$ representation in SU(5), and T_0 is an SU(5) singlet. The parameters a, b, and c are measures of the SU(3), SU(4), and SU(5) breaking, respectively. This operator gives rise to the following mass matrix for the mesons of the $\underline{24} \oplus \underline{1}$ representations of SU(5):

$$M_{ij} = \overline{M}\delta_{ij} + A(d_{i8j} + xd_{i15j} + yd_{i24j}), \tag{2}$$

$$M_{0i} = B(\delta_{8i} + x\delta_{15i} + y\delta_{24i}), \tag{3}$$

$$M_{00} = M_0, \tag{4}$$

where $\overline{M}$ and M_0 are the average $\underline{24}$ and $\underline{1}$ masses, respectively and $i,j = 1, \ldots 24$. Here, A and B are the products of reduced matrix elements and the SU(3)-symmetry-breaking parameter. Thus x and y are the ratio of SU(4):SU(3) and SU(5):SU(3) symmetry breaking, respectively.

In our previous application[6] of this operator to the SU(4) domain, we found that the observed masses of the D and D^* particles (and now, the preliminary[7] results of the F and F^* masses at 2.03 ± 0.06 and 2.14 ± 0.06 GeV, respectively) favored a linear mass formula. However, the mixing angles determined from an SU(4) invariant analysis[6] of the ψ decays favored the quadratic formula. Since there now appears to be SU(4) breaking in the coupling constants which determine the mixing angles,[8] it remains to be seen whether they still favor the quadratic formula. To motivate the choice adopted here, we look to the expressions obtained for the masses in the ideal mixing case.

With ideal mixing of the $\underline{24} \oplus \underline{1}$ representations, the ω, ϕ, ψ, and Υ particles (to use the vector states as an example) have quark content $(1/\sqrt{2})(u\bar{u} + d\bar{d})$, $s\bar{s}$, $c\bar{c}$, and $b\bar{b}$. In terms of the SU(5) basis states of the $\underline{24} \oplus \underline{1}$ representations, this means

$$|\omega\rangle = \frac{1}{\sqrt{3}}|8\rangle + \frac{1}{\sqrt{6}}|15\rangle + \frac{1}{\sqrt{10}}|24\rangle + (\tfrac{2}{5})^{1/2}|0\rangle, \tag{5}$$

$$|\phi\rangle = -(\tfrac{2}{3})^{1/2}|8\rangle + \frac{1}{2\sqrt{3}}|15\rangle + \frac{1}{2\sqrt{5}}|24\rangle + \frac{1}{\sqrt{5}}|0\rangle, \tag{6}$$

$$|\psi\rangle = -\frac{\sqrt{3}}{2}|15\rangle + \frac{1}{2\sqrt{5}}|24\rangle + \frac{1}{\sqrt{5}}|0\rangle, \tag{7}$$

$$|\Upsilon> = -\frac{2}{\sqrt{5}}|24> + \frac{1}{\sqrt{5}}|0> . \tag{8}$$

When the 4×4 block of the mass matrix responsible for $\underline{24}\oplus\underline{1}$ mixing is diagonalized by these eigenvectors, we find $\overline{M} = M_0$ and $A = (\frac{5}{2})^{1/2}B$. The masses are given in terms of the four remaining parameters by (we denote the mass of the particle simply by its name)

$$\rho = \omega = \overline{M} + A\left(\frac{1}{\sqrt{3}} + \frac{x}{\sqrt{6}} + \frac{y}{\sqrt{10}}\right), \tag{9}$$

$$\phi = \overline{M} + A\left(-\frac{2}{\sqrt{3}} + \frac{x}{\sqrt{6}} + \frac{y}{\sqrt{10}}\right), \tag{10}$$

$$\psi = \overline{M} + A\left(-\frac{3x}{\sqrt{6}} + \frac{y}{\sqrt{10}}\right), \tag{11}$$

$$\Upsilon = \overline{M} - \frac{4yA}{\sqrt{10}}. \tag{12}$$

The remainder of the masses obey the following sum rules:

$$\begin{aligned} &K^* = \frac{\omega+\phi}{2}, \quad D^* = \frac{\omega+\psi}{2}, \quad F^* = \frac{\phi+\psi}{2} \\ &B_N^* = \frac{\omega+\Upsilon}{2}, \quad B_s^* = \frac{\phi+\Upsilon}{2}, \quad B_c^* = \frac{\psi+\Upsilon}{2} \end{aligned} \tag{13}$$

where $B_{N,s,c}$ denote mesons with one b quark and one n, s, or c quark, respectively. For a linear mass formula, the results of Eq. (13) are the same as those found for a simple quark model where the binding energy per quark flavor is constant from meson to meson. That is, denoting the effective mass of the ith quark by m_i, the linear formula is equivalent to

$$(m_i + m_j)_a = \frac{(2m_i)_b + (2m_j)_c}{2}, \tag{14}$$

while the quadratic is equivalent to

$$(m_i + m_j)_a{}^2 = \frac{(2m_i)_b{}^2 + (2m_j)_c{}^2}{2}, \tag{15}$$

where the labels a, b, and c refer to particular meson states. With the exception of the pseudoscalar mesons (where relativistic effects are important for the π), Eq. (14) applies quite well. Thus we choose to work with the linear mass formula.

The parameters are determined for the vectors by fitting the ρ, K^*, D^*, F^*, ω, ϕ, ψ, and Υ masses. By assuming that the ratio of SU(5) to SU(4) breaking is a constant (i.e., y/x = constant), the tensor parameters can be determined from the A_2, $K^*(1420)$, f, f', and $\chi(3551)$ masses. We should add that in these fits, all masses were weighted equally (except the F^*, which was given only $\frac{1}{12}$ as much weight). The results are given in Table I. Table II shows the quark content of the mixed states.

TABLE I. Masses of $J^P = 0^-$, 1^-, and 2^+ mesons. The observed masses are indicated in parentheses. For the vectors, $\overline{M} = 3.144$ GeV, $M_0 = 2.504$ GeV, $A = -0.149$ GeV, $B = -0.080$ GeV, $x = 9.83$, and $y = 35.97$. For the tensors, $\overline{M} = 3.484$ GeV, $M_0 = 3.362$ GeV, $A = -0.128$ GeV, $B = -0.080$ GeV, $x = 10.47$, and $y = 38.31$. All masses are in GeV. Data are from Particle Data Group, Rev. Mod. Phys. 48, S1 (1976) and W. Tanenbaum *et al.*, Phys. Rev. D 17, 1731 (1978).

$J^P=0^-$		$J^P=1^-$		$J^P=2^+$	
η_b	9.15	Υ	9.41 (9.41)	χ_b	9.65
B_c	6.44	B_c^*	6.58	B_c^{**}	6.64
B_s	5.33	B_s^*	5.47	B_s^{**}	5.61
B_N	5.202	B_N^*	5.340	B_N^{**}	5.503
η_c	(2.83)	ψ	3.10 (3.098)	χ	3.551 (3.551)
F	(2.03)	F^*	2.14 (2.13)	F^{**}	2.55
D	(1.866)	D^*	2.006 (2.007)	D^{**}	2.44
		ϕ	1.017 (1.020)	f'	1.516 (1.516)
		K^*	0.899 (0.894)	K^{**}	1.421 (1.421)
		ω	0.784 (0.783)	A_2	1.310 (1.31)
		ρ	0.770 (0.773)	f	1.271 (1.271)

For the pseudoscalar mesons, our calculations are not as straightforward. We know that the π and η cannot be fitted properly when the simple quark model is applied to the pseudoscalars,[9] so it comes as no surprise that the linear mass formula also has difficulty. To complete the analysis, we would have to include symmetry breaking from $I = Y = C = \mathcal{B} = 0$ members of higher representations in the $\underline{24}\otimes\underline{24}$ decomposition ($\mathcal{B}$ refers to the b-quark quantum number). To avoid the problems associated with the low-mass pseudoscalars, we make use of some sum rules involving mesons with charm. For example, we can eliminate all the parameters except the SU(5):SU(4) symmetry-breaking ratio (which we assume is a constant) by taking the following combination of vector mesons:

TABLE II. Quark content of the vector and tensor mesons (rounded off at three decimal places).

Particle	$\frac{1}{\sqrt{2}}(u\bar{u}+d\bar{d})$	$s\bar{s}$	$c\bar{c}$	$b\bar{b}$
ω	0.998	0.008	0.042	0.045
ϕ	−0.011	0.999	0.037	0.033
ψ	−0.044	−0.039	0.997	0.049
Υ	−0.043	−0.032	−0.053	0.997
f	0.994	0.107	0.014	0.006
f'	−0.107	0.994	0.009	0.003
χ	−0.013	−0.011	1.000	0.005
χ_b	−0.005	−0.004	−0.006	1.000

$$\frac{B_N^* - D^*}{\psi - \frac{1}{2}(D^* + K^*)} = \frac{1}{2}\left[\left(\frac{5}{3}\right)^{1/2}\frac{y}{x} - 1\right]. \qquad (16)$$

Since the η_c appears to be ideally mixed, we can assume this quantity is a constant for both $J^P = 1^-$ and 0^-, and so determine the B_N mass in terms of the predicted B_N^* mass. Similarly the combination

$$\frac{\Upsilon - \psi}{\psi - \frac{1}{2}(D^* + K^*)} = \left(\frac{5}{3}\right)^{1/2}\frac{y}{x} - 1 \qquad (17)$$

can be used to determine the η_b mass. The results for the pseudoscalars are also given in Table I.

We can also apply the ideal-mixing mass formulas to calculate the masses of the $(b\bar{b})$ analogs of the χ states (with $J^P = 0^+$ and 1^+) as well as the radial excitations of the η_b and Υ. However, since there are fewer known masses to use as inputs, the estimates of the masses will probably not be as accurate. First, we will deal with the $\chi_b(0^+)$ and $\chi_b(1^+)$ states. Since the isoscalar states of these multiplets are incomplete, and hence their mixing undetermined, we will not make use of sum rules involving them. Similarly, because of the A_1 controversy,[10] we will not make use of the isovector states. This leaves us with the strange mesons. For ideal mixing, we find (using the vectors as an example)

$$\frac{\psi - K^*}{\Upsilon - \psi} = \frac{4 - 1/\sqrt{2}\,x}{\sqrt{15}\,y/x - 3}. \qquad (18)$$

We assume the ratio y/x is a constant, and since the $J^P = 0^+$, 1^+, and 2^+ states are linked together in the quark model, we will assign x a common value of 10.5 (from $J^P = 2^+$). It remains only to input the $\kappa(1250)$ and $Q(1350)$ (the average of the Q_1Q_2 pair is taken to eliminate mixing effects[11]) to predict that $\chi_b(0^+)$ has a mass of 9.56 GeV and $\chi_b(1^+)$ has a mass of 9.62 GeV (see Fig. 1).

Several points should be noted regarding the above masses. Firstly, the B_N^*-B_N mass difference is approximately one pion mass, so that the electromagnetic mass differences will be important in determining the decay rates. Secondly, the 2^+-0^+ mass splitting continues to decrease with quark mass. For the $(1/\sqrt{2})(u\bar{u} - d\bar{d})$, $(c\bar{c})$, and $(b\bar{b})$ states we find that this difference is 334, 137, and 90 MeV, respectively. This general trend is also what one expects from the quark model, although the simple quark model[9] gives only about 20 MeV for the splitting of the $(c\bar{c})$ states and 2 MeV for the $(b\bar{b})$ states. However, since this quark-model result is an underestimate for the χ states, it is also likely an underestimate for the χ_b states. Lastly, if we assign the B_N^* and B_N^{**} particles to the same Regge trajectory, then the slope of that trajectory will be about 0.57 GeV^{-2}

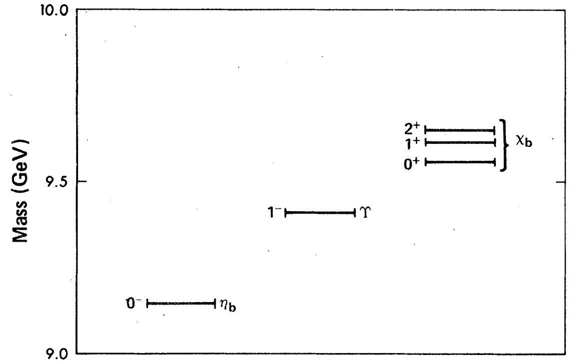

FIG. 1. Predicted mass spectrum of some of the $(b\bar{b})$ states in the region 9–11 GeV.

and it will have $\alpha(0) \simeq -15$. With such a large negative intercept, the cross section for a reaction of the sort $\pi p \to B_N B_1$ [B_1 is a (bnn) state] will be very small.

III. ELECTROMAGNETIC MASS DIFFERENCES

Up until this point, the calculations have only assumed that a new quantum number does exist; no assumptions have been made about the charge of the new quark which carries it. As we have pointed out above, the electromagnetic mass differences of the B_N and B_N^* mesons will critically determine the strong decays of the B_N^*. In order to predict these rates, we will assume that the b quark has a charge of $-\frac{1}{3}$.

In previous work,[12] we used the Cottingham formula with broken SU(4) symmetry to estimate the D^+-D^0 mass splitting. The prediction of 5.8 MeV is in good agreement with the recent experimental measurement[13] of 5.1 ± 2.8 MeV. The approach was also used for the D^* mass splitting,[14] yielding a prediction of ≈ 5 MeV, compared to the experimental result of 2.6 ± 1.8 MeV. In view of the success of this approach, we will extend it to SU(5).

We break up the electromagnetic corrections to the meson propagator in three parts: a Born term [representing the process $B_N \to B_N + \gamma(\text{virtual}) \to B_N$], a contribution from the vector meson intermediate state [$B_N \to B_N^* + \gamma(\text{virtual}) \to B_N$], and finally, a tadpole term. We deal with the last term first.

In the $K^+ - K^0$ mass splitting case, the Born and intermediate state contributions yield a positive Δm_K (we define $\Delta m_P \equiv P^+ - P^0$), and a substantial negative tadpole term is required to produce the observed $\Delta m_K = -3.99$ MeV. To within a Clebsch-Gordan coefficient, the contribution of the tadpole to the propagator (that is, the mass squared difference) will be constant, so that its contribution to Δm_P for higher mass P will decrease in magnitude. For Δm_D we found a contribution of $+1.71$

MeV, while for Δm_{B_N} we find -0.61 MeV.

The Born-term contribution to the self-mass, dm has the form[15]

$$dm_P=\frac{i\alpha}{8\pi^3 m_P}\int d^4q\,\frac{3q^2-4q\cdot p-4m^2}{q^2(q^2-2q\cdot p)}\times[F(PP,q^2)]^2, \qquad (19)$$

where p and m_P are the pseudoscalar meson's momentum and mass, respectively. In the ideal mixing case, the pseudoscalar electromagnetic form factors have the pole-dominated form

$$F(B_N^{\pm,0}B_N^{\pm,0},q^2)=\pm\frac{1}{2}\frac{m_\rho^2}{m_\rho^2-q^2}+\frac{1}{6}\frac{m_\omega^2}{m_\omega^2-q^2}+\frac{1}{3}\frac{m_\Upsilon^2}{m_\Upsilon^2-q^2}, \qquad (20)$$

$$F(B_sB_s,q^2)=-\frac{1}{3}\frac{m_\phi^2}{m_\phi^2-q^2}+\frac{1}{3}\frac{m_\Upsilon^2}{m_\Upsilon^2-q^2}, \qquad (21)$$

$$F(B_cB_c,q^2)=\frac{2}{3}\frac{m_\psi^2}{m_\psi^2-q^2}+\frac{1}{3}\frac{m_\Upsilon^2}{m_\Upsilon^2-q^2}, \qquad (22)$$

where we have given the complete set for use in Sec. IV [the $F(B^*B^*,q^2)$ have the same form]. When the expression is integrated out,[12] we find $(\Delta m_{B_N})_{\text{Born}}=+2.31$ MeV. Hence, the Born term dominates the tadpole and is of the opposite sign.

The self-mass contribution of the first intermediate state has the form

$$dm_P=g_{PV}{}^2\frac{ie^2}{(2\pi)^4m_P}\int\frac{[(q\cdot p)^2-q^2m_P{}^2][F(PV,q^2)]^2}{q^2(q^2-2q\cdot p+m_P{}^2-m_V{}^2)}, \qquad (23)$$

where the subscript V refers to the intermediate vector meson. In our previous work,[12] we found that the use of an SU(4)-invariant coupling for g_{PV} (which was determined by the radiative decay rates of $V\to P\gamma$ and $P\to V\gamma$ to be 2.59 GeV^{-1}) led to cross sections in e^+e^- annihilation which were far too high, and also to a value for Δm_D which, in retrospect, was also too high. Since g_{PV} has dimensions of inverse mass, we can introduce SU(5) breaking by writing $g_{PV}=f_{PV}/m_V$, where f_{PV} is a dimensionless SU(5) invariant coupling constant, and m_V is the mass of the intermediate vector meson, as in Eq. (23). Of course, there are several other possibilities open to us for the introduction of SU(5) breaking. One could use m_P or $(m_Vm_P)^{1/2}$ in place of m_V, but because $m_B\approx m_{B^*}$ to within 3%, the results will not change substantially. Other authors[16] have considered introducing symmetry breaking in the form factors, but such a possibility will not be pursued here. We normalize f_{PV} to the ω mass,[17] so that $f_{PV}=2.03$. Again, we dominate the form factors by poles,

$$F(B_N^{+,0}B_N^{*-,0},q^2)=\pm\frac{1}{2}\frac{m_\rho^2}{m_\rho^2-q^2}+\frac{1}{6}\frac{m_\omega^2}{m_\omega^2-q^2}-\frac{1}{3}\frac{m_\Upsilon^2}{m_\Upsilon^2-q^2}, \qquad (24)$$

$$F(B_sB_s^*,q^2)=-\frac{1}{3}\frac{m_\phi^2}{m_\phi^2-q^2}+\frac{1}{3}\frac{m_\Upsilon^2}{m_\Upsilon^2-q^2}, \qquad (25)$$

$$F(B_cB_c^*,q^2)=\frac{2}{3}\frac{m_\psi^2}{m_\psi^2-q^2}-\frac{1}{3}\frac{m_\Upsilon^2}{m_\Upsilon^2-q^2}. \qquad (26)$$

When Eq. (23) is integrated, we find the first intermediate-state contribution to $\Delta m_{B_N}=+0.30$ MeV. Thus $\Delta m_{B_N}=+2.00$ MeV in total. If $\Delta m_{B_N^*}$ decreases in the same proportion to Δm_{D^*} as Δm_B does to Δm_D, then we would predict $\Delta m_{B_N^*}\approx 1.7$ MeV if $\Delta m_{D^*}\approx 5$ MeV.

The mass splitting should therefore be considerably less than that observed for the D-D^* mesons, and the widths for strong decays correspondingly narrower. For the sake of argument, let us put the B_N^0 and $B_N^\pm$ mesons at 5.201 and 5.203 GeV, and the B_N^{*0} and $B_N^{*\pm}$ mesons at 5.339 and 5.341 GeV, respectively. We see immediately that the reaction $B_N^{*\pm}\to B_N^0\pi^\pm$ is just at threshold, while $B_N^{*0}\to B_N^+\pi^-$ is forbidden. The other two decays involving neutral pions are allowed. If we use a VPP coupling constant[6] such that $g_{VPP}{}^2/4\pi=3.27$, then a width of $\sim\frac{1}{2}$ keV is predicted for the decays $B_N^{*+,0}\to B_N^{+,0}\pi^0$. The $PV\gamma$ coupling with SU(5) breaking introduced here can be used to calculate the radiative decay $B_N^*\to B_N\gamma$. One finds $\Gamma(B_N^{*+}\to B_N^+\gamma)=0.10$ keV while $\Gamma(B_N^{*0}\to B_N^0\gamma)=0.39$ keV. Since these decay modes will be the dominant ones for B_N^*, we see that the electromagnetic width will be of the same order as the strong width. For comparison, $\Gamma(B_s^*\to B_s\gamma)=0.39$ keV while $\Gamma(B_c^*\to B_c\gamma)=0.07$ keV in the broken-SU(5) scheme. [In an SU(5)-symmetric scheme, the rates would be $\Gamma(B_N^{*+}\to B_N^+\gamma)=4.59$ keV, $\Gamma(B_N^{*0}\to B_N^0\gamma)=18.4$ keV, $\Gamma(B_s^*\to B_s\gamma)=19.2$ keV, and $\Gamma(B_c^*\to B_c\gamma)=4.83$ keV.]

IV. PRODUCTION CROSS SECTIONS

Armed with these predictions for masses and coupling constants, the obvious question to ask is, "Where will we find visible b quarks?" As we indicated before, it is unlikely that we will find them in hadronic collisions, so we must turn to the electromagnetic domain. The pole-dominated form factors which were used in calculating the electromagnetic mass differences can be used here to predict $\sigma(e^+e^-\to PP,\ PV,\text{ and }VV)$. In terms of these form factors, the cross sections can be expressed as

$$\sigma(e^+e^- \to P\overline{P}) = \frac{\pi\alpha^2}{3s}\left(1-\frac{4m_P^2}{s}\right)^{3/2}|F(PP,s)|^2, \tag{27}$$

$$\sigma(e^+e^- \to P\overline{V}) = \sigma(e^+e^- \to \overline{P}\,V)$$

$$= \frac{\pi\alpha^2}{6}\left(1-\frac{(m_P+m_V)^2}{s}\right)^{3/2}\left(1-\frac{(m_P-m_V)^2}{s}\right)^{3/2} g_{PV}^{\,2}|F(PV,s)|^2, \tag{28}$$

$$\sigma(e^+e^- \to V\overline{V}) = \frac{\pi\alpha^2}{12m_V^{\,4}s}\,(s^2+20sm_V^{\,2}+12m_V^{\,4})\left(1-\frac{4m_V^{\,2}}{s}\right)^{3/2}|F(VV,s)|^2. \tag{29}$$

The predicted cross sections are shown in Figs. 2–4 for the individual reactions with $10 \leq \sqrt{s} \leq 17$ GeV. For the $e^+e^- \to (P\overline{V} + \overline{P}\,V)$ reaction, the same symmetry breaking is used as was introduced in Sec. III. To convert the broken-symmetry results to the SU(5)-invariant results, simply multiply by $m_V^{\,2}/m_\omega^{\,2}$.

As a general rule, we observe that the $(b\overline{n})$ mesons are the most likely to be produced, followed by $(b\overline{s})$, and then $(b\overline{c})$. Within a given set of quantum numbers, the order is $B^*B^* > BB^* > BB$. The maximum production of the mesons with a single b quark will be at $\sqrt{s} \approx 12$ GeV, with $B_N^*\overline{B}_N^*$ having a cross section of about 0.3 nb and $B_N\overline{B}_N^* + \overline{B}_N B_N^*$ having a cross section of about 0.2 nb. In Fig. 5, we show the total cross section for the B and B^* particles. For comparison, the cross section for the reaction $e^+e^- \to \mu^+\mu^-$ is also shown. Since the ratio of $\sigma(e^+e^- \to B, B^*)$ to $\sigma(e^+e^- \to \mu^+\mu^-)$ is comparable to that found for the D, D^* mesons, there should be no difficulty in seeing these new mesons.

V. CONCLUSIONS

We have treated the recently observed Υ particle as a bound state of a new b quark and its antiquark. The mass spectrum of the $(b\overline{b})$ family is then predicted. Since the masses of the Υ family are so

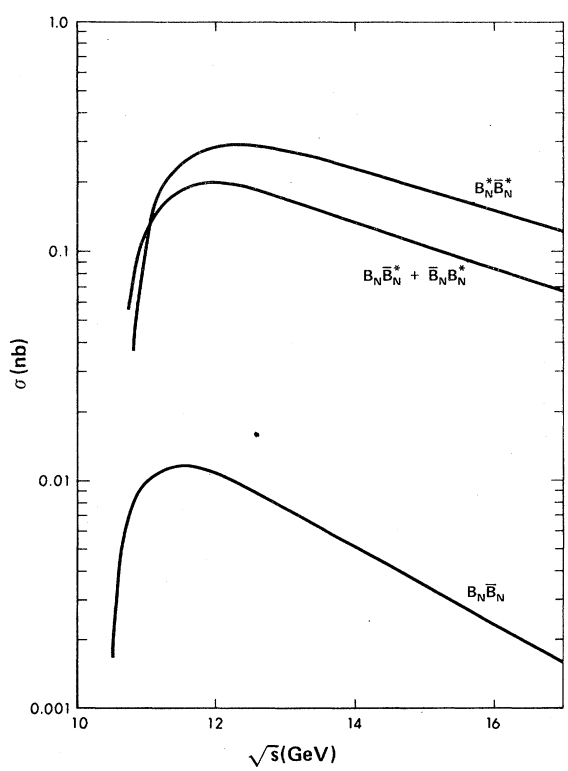

FIG. 2. Predicted cross sections for the production of $(b\overline{n})$ meson pairs in e^+e^- annihilation shown for $10 \leq \sqrt{s} \leq 17$ GeV.

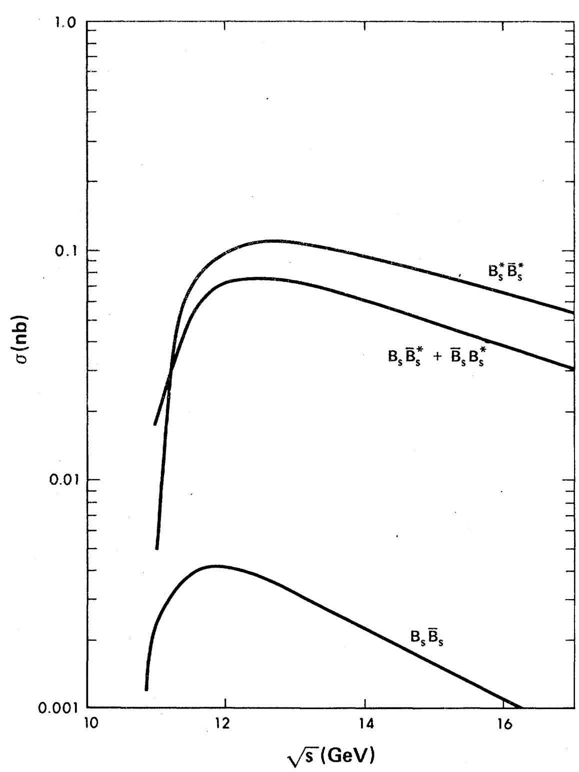

FIG. 3. Predicted cross sections for the production of $(b\overline{s})$ meson pairs in e^+e^- annihilation shown for $10 \leq \sqrt{s} \leq 17$ GeV.

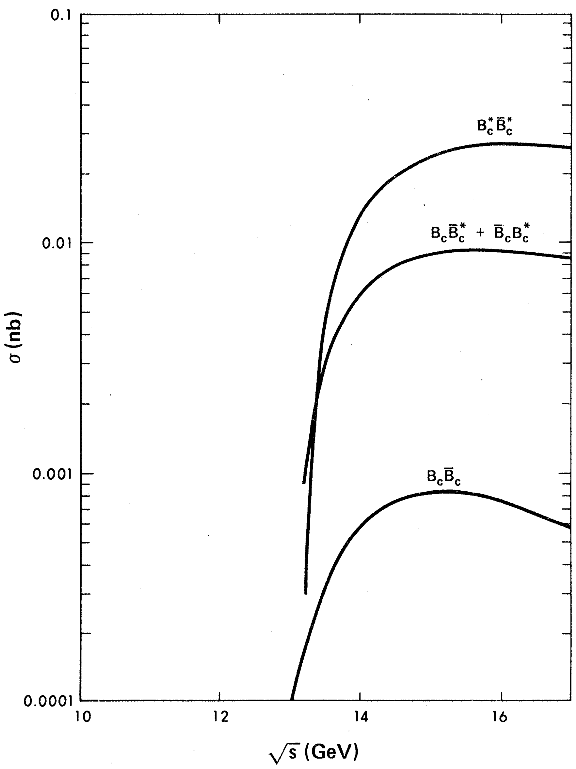

FIG. 4. Predicted cross sections for the production of $(b\bar{c})$ meson pairs in e^+e^- annihilation shown for $10 \leq \sqrt{s} \leq 17$ GeV.

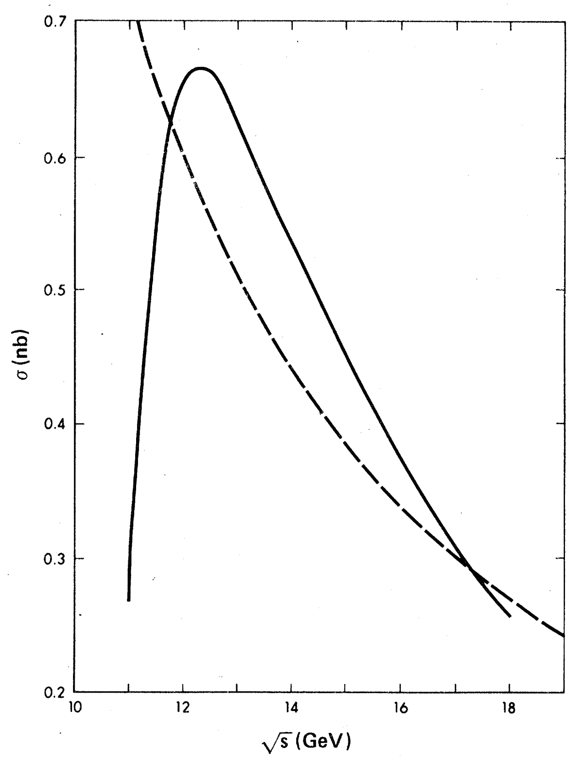

FIG. 5. Predicted cross section for ($e^+e^- \to PP$, PV, VV with b quarks) for $10 \leq \sqrt{s} \leq 17$ GeV (solid curve). For comparison, $\sigma(e^+e^- \to \mu\mu)$ is also given for the same energy range (dashed curve).

large, a 1% error in the mass formulas represent a 100 MeV change in the masses. As much as possible, then, we have tried to make use of mass formulas which involve few independent parameters and many input masses. Hence, our results (particularly for the 2^+, 1^+, and 0^+ states) may be in error by up to 100 MeV, although the relative order of the states should stay the same. The overall splitting is roughly comparable to that predicted by Eichten and Gottfried[18] or Quigg and Rosner[19] using various quark-model potentials.

The same general conclusions hold for the $B_{N,s,c}$ states, although here the error should be less. We expect the B^{**} particles will be higher in mass than predicted, from our experience with the D mesons. There, the linear mass formula predicts masses which were low by about 70 MeV, if only the ψ and η_c masses are used as input. In the predictions made here of the B and B^* masses, this is taken into account by actually using the D and D^* masses as input. However, the D^{**} has yet to be observed. If our experience with the P and V multiplets is applied to the Υ multiplet, then the D^{**}, F^{**}, and B^{**} masses quoted here are probably too low by ~ 70 MeV.

The differences in the masses of the B_i^*-B_i mesons ($i = N, s, c$) are all approximately one pion mass. The calculated electromagnetic mass splitting for B_N is only 2.0 MeV, about a third of the size of the D splitting, so that the radiative decay mode of the B_N^* should be a substantial fraction of its decay width.[20] The pole-dominated form factors for the (PP, PV, and VV) coupling to the photon were used not only in the calculation of the electromagnetic mass differences, but also in the prediction of the production cross sections in e^+e^- annihilation. For large s, the cross sections go like s^{-3} (PP), s^{-2} (PV), and s^{-1} (VV). Thus we expect $B_N^*B_N^*$ production to dominate at large s.

In the pole-dominated form factors, we only included contributions from the lowest-lying $24 \oplus 1$ representations of the vector mesons. It has been argued[21] that the coupling of higher mass $24 \oplus 1$-plets is small, perhaps of the order of 10%. Since we expect the Υ''' to have a mass of about 11 GeV, our estimates for the cross sections are probably only to be trusted from about $\sqrt{s} = 12$ GeV.

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[1]S. W. Herb *et al.*, Phys. Rev. Lett. 39, 252 (1977).
[2]B. Aubert *et al.*, Phys. Rev. Lett. 33, 984 (1974); A. Benvenuti *et al.*, *ibid* 36, 1478 (1976); 37, 189 (1976); B. C. Barish *et al.*, *ibid.* 38, 314 (1977).
[3]M. Holder *et al.*, Phys. Rev. Lett. 39, 433 (1977).
[4]R. M. Barnett, Phys. Rev. Lett. 34, 41 (1975); V. Barger, R. J. N. Phillips, and T. Weiler, Phys. Rev. D 14, 1276 (1976); L. N. Chang and J. N. Ng, *ibid.* 16, 3157 (1977).
[5]S. Borchardt, V. S. Mathur, and S. Okubo, Phys. Rev. Lett. 34, 38 (1975); V. S. Mathur, S. Okubo, and S. Borchardt, Phys. Rev. D 11, 2572 (1975).
[6]D. H. Boal and R. Torgerson, Phys. Rev. D 15, 327 (1977).
[7]R. Brandelik *et al.*, Phys. Lett. 70B, 132 (1977).
[8]H. Fritzsch and J. D. Jackson, Phys. Lett. 66B, 365 (1977).
[9]A. De Rújula, H. Georgi, and S. L. Glashow, Phys. Rev. D 12, 147 (1975).
[10]See C. Baltay, C. V. Cautis, and M. Kalelkar, Phys. Rev. Lett. 39, 591 (1977) and references therein.
[11]D. H. Boal, B. J. Edwards, A. N. Kamal, and R. Torgerson, Phys. Rev. D 14, 2998 (1976). See also later work by R. Barbieri, R. Gatto, and Z. Kunszt, Phys. Lett. 66B, 349 (1977); R. K. Carnegie *et al.*, *ibid.* 68B, 287 (1977).
[12]D. H. Boal and A. C. D. Wright, Phys. Rev. D 16, 1505 (1977). Similar results were obtained by H. Fritzsch, Phys. Lett. 63B, 419 (1976); K. Lane and S. Weinberg, Phys. Rev. Lett. 37, 717 (1976); see also S. Borchardt *et al.*, Phys. Lett. 66B, 95 (1977).
[13]G. Goldhaber, in *Proceedings of the* 1977 *European Conference on Particle Physics*, *Budapest*, edited by L. Jenik and I. Montvay (CRIP, Budapest, 1978).
[14]A. C. D. Wright, Phys. Rev. D 16, 2265 (1977).
[15]Riazuddin, Phys. Rev. 114, 1184 (1959); R. H. Socolow, *ibid.* 137, B1221 (1965).
[16]R. Thews, Phys. Rev. D 14, 3021 (1975); D. W. McKay and B.-L. Young, *ibid.* 15, 1282 (1977); N. Chase and M. T. Vaughn, Phys. Lett. 61B, 175 (1976).
[17]This number can be obtained from quark-loop anomaly arguments, see Ref. 6.
[18]E. Eichten and K. Gottfried, Phys. Lett. 66B, 286 (1977).
[19]C. Quigg and J. L. Rosner, Phys. Lett. 71B, 153 (1977).
[20]A discussion of the weak decays can be found in J. Ellis, M. K. Gaillard, D. V. Nanopoulos, and S. Rudaz, Nucl. Phys. B131, 285 (1977).
[21]P. L. Brunini, F. Rimondi, and G. Venturi, Lett. Nuovo Cimento 10, 693 (1974); McKay and Young (Ref. 17).