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Note on the phenomenology of nonleptonic processes

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We propose that the nonleptonic weak Hamiltonian consists of a dynamically enhanced current × current octet belonging to the $\underline{20}''$ representation of SU(4), and a new nonconventional octet belonging to the $\underline{15}$ representation of SU(4). We show that the phenomenology of the nonleptonic processes can be correctly reproduced through such a Hamiltonian.

The conventional current × current theory of weak interactions[1] has proved to be successful in describing the leptonic and semileptonic weak processes. The theory has further gained in respectability since it has been presented as the lowest-order phenomenology arising out of an SU(2) × U(1) gauge theory of leptons,[2] which can be extended to hadrons by adding an extra degree of freedom (charm[3]), and which is renormalizable.[4] However, the use of the conventional weak-interaction theory for describing nonleptonic processes has always been beset with problems. The theory has conspicuously failed in describing the following phenomena observed in nonleptonic processes[6]:

(a) *Octet dominance*. The available data on the nonleptonic processes satisfy the so-called $\Delta I=\frac{1}{2}$ rule within a few percent.[5] This implies that the nonleptonic Hamiltonian is dominated by the octet of SU(3) — while the conventional theory assigns equal strengths to the $\underline{8}$ and $\underline{27}$ of SU(3).

On fixing the weak coupling constant from the leptonic and semileptonic decay amplitudes and then using the conventional theory to calculate nonleptonic decay amplitudes, one finds that the strength of the $\underline{27}$ ($\Delta I=\frac{3}{2}$) part of the decay amplitudes is correctly predicted, but the octet ($\Delta I=\frac{1}{2}$) part is underestimated by an order of magnitude.[7] In fact the octet part of the nonleptonic Hamiltonian of the current × current theory needs to be enhanced by a factor of about 15–20.

(b) *$K\to 2\pi$ decay and the asymmetry in $\Sigma^+\to p\gamma$.* The conventional weak-interaction theory forbids the decay $K\to 2\pi$ in the SU(3) limit. Experimentally, $K\to 2\pi$ decay is one of the fastest known decays. A related problem is the explanation of the asymmetry parameter for the decay $\Sigma^+\to p\gamma$. In the SU(3) limit it is zero, if one accepts the conventional theory. Experimentally the value of the asymmetry parameter is $-1.03^{+0.52}_{-0.42}$,[8] consistent with the maximal asymmetry value of −1.

(c) *Inconsistency in the s- and p-wave baryon decay amplitudes*. Within the framework of the conventional theory it has not been possible to fit the s- and p-wave baryon decay amplitudes simultaneously in any consistent manner[9] — even if the octet dominance is assumed.

There is a long history of ad hoc attempts at solving these problems of the conventional weak-interaction theory when applied to the nonleptonic processes.[6] It is only in the recent past that a satisfactory picture of the weak processes has started emerging. In this note we want to present the status of the nonleptonic weak Hamiltonian as it stands today, and to explore its phenomenological implications.

The problem (a), i.e., the problem of octet dominance, has long been suspected to be a manifestation of the renormalization caused by the strong-interaction dynamics. Now this conjecture has been given a concrete shape. It has been rigorously proved that in the asymptotically free gauge theories of strong interactions, unified suitably with weak and electromagnetic gauge models, the octet of the current × current theory indeed gets enhanced.[10] Numerical estimates of the enhancement factor in models with no nonconventional right-left transitions are of the order of 10. This seemingly solves the problem (a), though the typical enhancement factors obtained are below the required enhancement factor of about 15–20.

The enhancement of the conventional current × current octet leaves the problems (b) and (c) untouched. The solution of these problems seems to require the addition of a new nonconventional SU(3)-octet piece to the weak Hamiltonian for nonleptonic decay. Several more or less successful attempts to generate new nonconventional pieces through (i) Higgs-scalar-meson exchange,[11] (ii) introduction of right-handed currents,[12] and (iii) specific spontaneous symmetry breakdown[13] have been made in the recent past. In all of these attempts it has been assumed that the new nonconventional octet dominates the nonleptonic Hamiltonian to the complete exclusion of the current × current part. This assumption is apparently bad if the octet part of the conventional theory indeed gets enhanced by as large a factor as ~10, which has been shown to be possible.[10] Significantly, all these attempts have failed in comprehensively explaining the phenomenology of the nonleptonic processes.

In the background of the developments sketched above, we believe that the effective Hamiltonian for the nonleptonic decays must contain both the conventional current × current theory octet and a nonconventional octet. The phenomenological weak Hamiltonian for the nonleptonic processes thus consists of the following:

(1) The conventional current × current Hamiltonian with the strangeness-changing $\Delta I = \frac{1}{2}$ piece enhanced by a factor of about 10. The SU(3) chiral structure of the enhanced octet is $(\underline{1},\underline{8}) + (\underline{8},\underline{1})$.[10] At the SU(4) level, since the $\underline{15}$ representation of SU(4) does not make any contribution to the current × current Hamiltonian,[14] the enhanced octet must be a part of the $\underline{20}''$ representation of SU(4). Under exact SU(4) invariance the d/f ratio for the SU(3) octet in the $\underline{20}''$ of SU(4) is known to be −1.[15] Therefore, we expect the d/f ratio for the conventional octet piece to lie near −1.

The strength of this octet should, in principle, be determinable from leptonic and semileptonic decay amplitudes. However, since the enhancement factor is model-dependent, the strength becomes arbitrary and is to be determined phenomenologically from the nonleptonic processes. In most models the enhancement factor is of the order of 10. Hence, the contribution of this octet to the nonleptonic processes is expected to be large.

(2) A nonconventional, explicitly octet piece. Phenomenologically this piece is equivalent to a "tadpole" model.[16] The typical chiral SU(3) × SU(3) structure of this piece is $(\underline{3},\underline{3}^*) + (\underline{3}^*,\underline{3})$, and the octet is the SU(3) subgroup of a $\underline{15}$-dimensional representation of SU(4).[12,13] From the equivalence with "tadpole" models the d/f ratio for this octet is expected to be $-\frac{1}{3}$.

This new octet contributes only to the $\Delta S = 1$, $\Delta I = \frac{1}{2}$ part of the Hamiltonian. Its strength is not normalized with respect to the $\Delta I = \frac{3}{2}$ part or with respect to the leptonic decays. Therefore, this strength is also to be fixed phenomenologically.

Having decided upon the structure of the nonleptonic Hamiltonian, we proceed to test it for the various nonleptonic processes.

PARITY-CONSERVING BARYONIC DECAYS

Since the nonconventional octet in the nonleptonic Hamiltonian is phenomenologically equivalent to a tadpole Hamiltonian, this octet cannot contribute to the parity-conserving processes. This is because a tadpole term in the Hamiltonian can be transformed away if the SU(3) symmetry is exact for the strong-interaction part of the Hamiltonian.[16] In fact, this transformation goes through even when the SU(3) symmetry is systematically broken.[17] Hence, the only term contributing to the parity-conserving baryon decays is the enhanced conventional octet belonging to the $\underline{20}''$ of SU(4). In this situation the decay amplitudes arise from Born terms alone.[6] The expressions for these decay amplitudes are

$$B(\Lambda^0_-) = (\tfrac{2}{3})^{1/2} g f_{20''}(m_\Lambda + m_N)\left[\frac{3 + (d/f)_{20''}}{2m_N(m_\Lambda - m_N)} + \frac{2D[(d/f)_{20''} - 1)]}{(m_\Sigma + m_\Lambda)(m_\Sigma - m_N)}\right],$$

$$B(\Xi^-_-) = -(\tfrac{2}{3})^{1/2} g f_{20''}(m_\Xi + m_\Lambda)\left[\frac{2D[(d/f)_{20''} + 1]}{(m_\Lambda + m_\Sigma)(m_\Xi - m_\Sigma)} - \frac{(D-F)[(d/f)_{20''} - 3]}{2m_\Xi(m_\Xi - m_\Lambda)}\right],$$

$$B(\Sigma^+_+) = g f_{20''}(m_\Sigma + m_N)\left[\frac{(d/f)_{20''} - 1}{m_N(m_N - m_\Sigma)} + \frac{F[(d/f)_{20''} - 1]}{m_\Sigma(m_\Sigma - m_N)} - \frac{2D[(d/f)_{20''} + 3]}{3(m_\Lambda + m_\Sigma)(m_\Lambda - m_N)}\right],$$

$$B(\Sigma^-_-) = -g f_{20''}(m_\Sigma + m_N)\left[\frac{F[(d/f)_{20''} - 1]}{m_\Sigma(m_\Sigma - m_N)} + \frac{2D[(d/f)_{20''} + 3]}{3(m_\Lambda + m_\Sigma)(m_\Lambda - m_N)}\right].$$

Here g is the strong coupling constant $g_{NN\pi}$, and strong $D/F = 1.76$ with $D + F = 1$. We find a good fit can be obtained (Table I) with $(d/f)_{20''} = -0.84$. This is remarkably close to the SU(4)-invariant value of −1.

PARITY-VIOLATING BARYONIC DECAYS

Both the conventional and nonconventional octets contribute to these decays. The resultant expressions for the decay amplitudes are

$$A(\Lambda^0_-) = \lambda \frac{f_{20''}}{\sqrt{3} f_\pi}[(d/f)_{20''} + 3] + \frac{f_{15}}{\sqrt{3} f_\pi}[(d/f)_{15} + 3],$$

TABLE I. Parity-conserving baryonic decay amplitudes. Here $(d/f)_{20''} = -0.84$ and $g f_{20''} = 24.65 \times 10^2$ MeV. The definitions of the amplitudes and the experimental values are those of Ref. 5, and the sign convention is that of Marshak *et al.* (Ref. 6).

Process	Calculated amplitude	Experimental value
$B(\Lambda^0_-)$	10.17	10.17 ± 0.24
$B(\Xi^-_-)$	6.68	6.73 ± 0.41
$B(\Sigma^+_+)$	17.47	19.05 ± 0.16
$B(\Sigma^-_-)$	0.05	−0.65 ± 0.08
$B(\Sigma^+_0)$	12.32	12.04 ± 0.59

TABLE II. Parity-violating baryonic decay amplitudes. We use $(d/f)_{20''}=-0.84$, $f_{20''}/f\pi=1.4$, and $\lambda=0.17$. The values in column 3 are those of Ref. 18 normalized properly to conform with our definitions of the amplitudes. Correspondingly, $f_{15}/f\pi=0.76$. We use the definitions of the amplitudes and the experimental values as given in Ref. 5, and the sign convention of Marshak *et al.* (Ref. 6).

Process	20″ contribution	15 contribution	Total	Experimental
$A(\Lambda_-^0)$	0.30	1.18	1.48	1.48 ± 0.01
$A(\Xi_-^-)$	−0.53	−1.35	−1.88	-2.04 ± 0.02
$A(\Sigma_+^+)$	0	0	0	0.06 ± 0.02
$A(\Sigma_-^-)$	0.64	1.35	1.98	1.93 ± 0.01
$A(\Sigma_0^+)$	−0.44	−0.96	−1.40	-1.48 ± 0.05

$$A(\Xi_-^-)=\lambda\frac{f_{20''}}{\sqrt{3}f\pi}\left[(d/f)_{20''}-3\right]+\frac{f_{15}}{\sqrt{3}f\pi}\left[(d/f)_{15}-3\right],$$

$$A(\Sigma_+^+)=0,$$

$$A(\Sigma_-^-)=-\lambda\frac{\sqrt{2}f_{20''}}{f\pi}\left[(d/f)_{20''}-1\right]-\frac{\sqrt{2}f_{15}}{f\pi}\left[(d/f)_{15}-1\right].$$

The contribution of the nonconventional octet, belonging to the 15 representation of SU(4) and with $(d/f)_{15}=-\frac{1}{3}$, is equivalent to the K^*-pole contribution.[6] This contribution can be completely determined from the $K\to2\pi$ decay width, because only the nonconventional octet contributes to the $K\to2\pi$ decay. Many authors have determined the K^*-pole contribution to the parity-violating baryonic decays, assuming that the $K\to2\pi$ decay proceeds through the tadpole term.[13,18] We use the values obtained by J. J. Sakurai[18] (column 3, Table II) which are in very good agreement with a recent calculation in a chiral-symmetric gauge model.[13]

The current × current contribution is also completely determinable once $f_{20''}$ has been fixed from the parity-conserving decays. However, we find that to obtain a good fit this contribution must be considerably lowered. Therefore we have multiplied this contribution with an arbitrary phenomenological factor λ. Excellent fit with experiment (Table II) is obtained for $\lambda=0.17$. We note that this factor just about cancels the enhancement in the current × current octet. It could perhaps mean that the equal-time commutator contribution arises from the original unenhanced octet and that the enhancement is valid only for the pole terms.

ASYMMETRY PARAMETER FOR $\Sigma^+\to p\gamma$

With the new structure of the nonleptonic Hamiltonian the asymmetry parameter for the weak radiative decay $\Sigma^+\to p\gamma$ is no longer zero. Now the parity-conserving decay arises through the usual current × current octet and the parity-violating decay from the nonconventional octet. Since we have already determined the strengths of the two octets, the asymmetry parameter in the pole model can be determined. In the SU(3) limit, we obtain $\alpha(\Sigma^+\to p\gamma)\approx-0.5$, in good agreement with the experimental value $\alpha=-1.03^{+0.52}_{-0.42}$.[8]

We conclude that the present understanding of the nonleptonic Hamiltonian as consisting of an enhanced current × current octet and a nonconventional tadpole-type octet is phenomenologically correct.

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