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$\phi\phi$ decay as a parity and signature test

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Chang and Nelson have recently constructed an analog of Yang's parity test for a spin-zero particle which decays into $\phi\phi$. We show that a simple generalization of this test allows one to determine the parity of a system of any spin J which decays into $\phi\phi$. In addition, with certain exceptions, one can determine $(-1)^J$.

Recently, Chang and Nelson[1] have constructed an analog of Yang's[2] parity test that enables one to determine the parity of a spin-zero particle which decays into two ϕ mesons. The test is based on the dependence of the decay distribution on the azimuthal angle χ between the two $\varphi \rightarrow K^+K^-$ decay planes. They show that the distribution is given by

$$1+\beta\cos 2\chi\,,$$

where

$$\beta=-1, \quad \text{pseudoscalar particle,}$$

$$0\leq\beta\leq 1, \quad \text{scalar particle.}$$

In this note, we extend their result to a particle of spin J and show that measurements of this type can be very useful in determining the parity η and the signature $(-1)^J$ of the particle. In addition to the dependence on χ, we will also make some use of the dependence on θ, the polar angle of the K^+ momentum in the rest system of the φ with respect to the helicity axis. (See Fig. 1 for the definition of the angles.) The main advantage of this test is that it is independent of the polarization state of the particle.[3]

The matrix element for the decay of the particle into $\phi\phi$ is defined by

$$\langle\Theta,\Phi,\lambda_1,\lambda_2,\,|JM\rangle=D^{*}_{m\lambda}(\Phi,\Theta,-\Phi)a_{\lambda_1\lambda_2}\,, \tag{1}$$

where λ_1, λ_2 denote the helicities of the two ϕ's and $\lambda=\lambda_1-\lambda_2$. The angles are defined in the particle's rest system in the usual way; see Fig. 2.

According to Jacob and Wick,[4] parity conservation in the decay implies that

$$a_{-\lambda_1,-\lambda_2}=\eta(-1)^J a_{\lambda_1\lambda_2}\,. \tag{2}$$

Likewise, we learn from that paper that the identity of the two ϕ's requires in addition that

$$a_{\lambda_1\lambda_2}=(-1)^J a_{\lambda_2\lambda_1}\,. \tag{3}$$

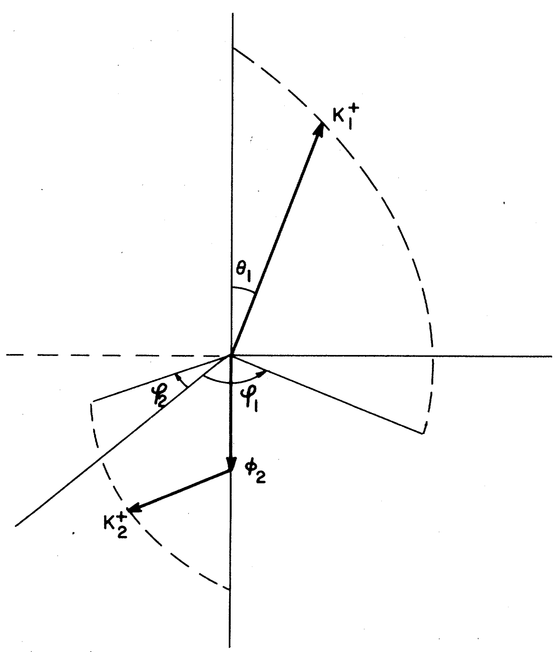

FIG. 1. The angles describing the ϕ decay, in the ϕ_1 rest system, ϕ_2 moving in the negative z direction.

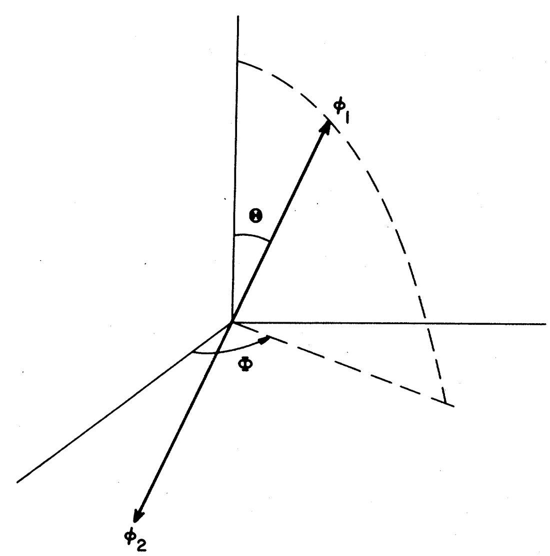

FIG. 2. The angles describing the $\phi_1\phi_2$ distribution in their center-of-mass system.

These are the relations on which our results are based. The point is that for certain values of η, $(-1)^J$, λ_1, and λ_2, (2) and (3) are incompatible and force the corresponding $a_{\lambda_1\lambda_2}$ to vanish. In Table I, we list the nonvanishing amplitudes for the various values of $(-1)^J$ and η. We see that for $\eta=(-1)^J=-1$ there is only a single nonvanishing amplitude, for $\eta=-(-1)^J$ there are two independent amplitudes, and only for $(-1)^J=\eta=+1$ are all four amplitudes present. One difficulty always encountered in this sort of problem is that some amplitudes may vanish even though (2) and (3) allow them to be nonzero. We shall see that this difficulty can be overcome to a large extent, but not completely.

The decay matrix element for the ϕ decay into K^+K^- is given simply by

$$\langle \theta_1\varphi_1|1\lambda_1\rangle = C D^{1*}_{\lambda_1 0}(\varphi_1,\theta_1,-\varphi_1), \qquad (4)$$

where C is a constant, irrelevant to our problem. Thus, if the initial density matrix of the particle decaying into $\phi\phi$ is $\rho_{MM'}$, the general decay distribution is given by

$$I(\Theta,\Phi,\theta_1,\varphi_1,\theta_2,\varphi_2)=\sum_{\substack{M_1M'\\ \lambda_1,\lambda_2,\lambda',\lambda_2'}} \rho_{MM'}D^{J*}_{M\lambda}(\Phi,\Theta,-\Phi)D^{J}_{M'\lambda'}(\Phi,\Theta,\Phi)a_{\lambda_1\lambda_2}a^*_{\lambda_1'\lambda_2'}D^{1*}_{\lambda_1 0}(\varphi_1,\theta_1,-\varphi_1)D^{1*}_{\lambda_2 0}(\varphi_2,\theta_2,-\varphi_2)$$
$$\times D^{1}_{\lambda_1' 0}(\varphi_1,\theta_1,\varphi_1)D^{1}_{\lambda_2' 0}(\varphi_2,\theta_2,-\varphi_2). \qquad (5)$$

It is clear that if we define

$$\varphi_2=-(\varphi_1-\chi), \qquad (6)$$

then χ is the azimuthal angle between the two decay planes. Let us keep χ fixed and integrate over all $0\le\varphi_1\le 2\pi$; that is, we keep the angle between the decay planes fixed and sum over all orientations of the decay planes about the axis defined by the ϕ momentum. Notice from (5) that the dependence on φ_1 is of the form

$$\exp[i\varphi_1(\lambda_1-\lambda_2-\lambda_1'+\lambda_2')],$$

so the integral over all φ_1 leaves only those terms in (5) with

$$\lambda_1-\lambda_2=\lambda_1'-\lambda_2'$$

or

$$\Lambda=\lambda_1-\lambda_1'=\lambda_2-\lambda_2'. \qquad (7)$$

Now, with $\lambda=\lambda'$ we can use the orthogonality of the $D^J_{M\lambda}(\Phi,\Theta,-\Phi)$ to integrate the distribution over all ϕ directions. This forces $M=M'$ and the sum on M in (5) can be done yielding $\mathrm{Tr}\,\rho$ as a factor. Hence, the remaining distribution are independent of the initial polarization state. Finally, the Clebsch-Gordan series[5] for combining products of D's can be used to write the remaining distribution as

$$I(\theta_1,\theta_2,\chi)=\mathrm{Tr}\,\rho\sum_{\substack{\lambda_1\lambda_2\\ \lambda_1'\lambda_2'\\ \Lambda\\ j,j'}} a_{\lambda_1\lambda_2}a^*_{\lambda_1'\lambda_2'}e^{i\Lambda\chi}(-1)^{\lambda_1-\lambda_2}(2j+1)(2j'+1)\begin{pmatrix}1&1&j\\0&0&0\end{pmatrix}\begin{pmatrix}1&1&j'\\0&0&0\end{pmatrix}\begin{pmatrix}1&1&j\\\lambda_1&-\lambda_1'&-\Lambda\end{pmatrix}\begin{pmatrix}1&1&j'\\\lambda_2&-\lambda_2'&-\Lambda\end{pmatrix}$$
$$\times d^j_{\Lambda 0}(\theta_1)d^{j'}_{\Lambda 0}(\theta_2). \qquad (8)$$

This same distribution is obtained if the initial particle is unpolarized, independently of the integral just done; in that case, the distribution is isotropic in Φ, Θ, and φ_1.

We now wish to consider separately the χ and θ_1 distribution given by (8). The general form for the χ distribution is

$$F(\chi)=1+\alpha\cos\chi+\beta\cos 2\chi, \qquad (9)$$

$\cos\chi$ coming from the terms in (8) with $\Lambda=\pm1$ and $\cos 2\chi$ coming from those with $\Lambda=\pm2$. The value of α and β depends on the θ_1 and θ_2 acceptances integrated over in going from (8) to (9). If the full π acceptance (or any acceptance symmetric about 90°) is integrated over, $\alpha=0$. This result depends only on the assumption that the K^+K^- comes from a ϕ decay and not on any assumption regarding the source of the ϕ's. This comes about because

$$\int_0^\pi d(\cos\theta)\,d^j_{10}(\theta)=0 \quad \text{for } j \text{ even},$$

and the only way that an odd j could occur in (8) is if the K^+K^- system occurs in both even and odd partial waves. $[\begin{pmatrix}j_1&j_2&j\\0&0&0\end{pmatrix}=0$ unless $(-1)^{j_1+j_2+j}=+1.]$ Hence a nonzero α in such circumstance is a direct indication that there is interference

TABLE I. The decay helicity amplitudes which are permitted to be nonzero for the various possible values of parity η and signature $(-1)^J$.

η	$(-1)^J$	Nonvanishing amplitudes
-1	-1	$a_{+0}=a_{-0}=-a_{0+}=-a_{0-}$
$+1$	-1	$a_{+0}=-a_{-0}=-a_{0+}=a_{0-}$ $a_{+-}=-a_{-+}$
-1	$+1$	$a_{+0}=a_{0+}=-a_{-0}=-a_{0-}$ $a_{++}=-a_{--}$
$+1$	$+1$	$a_{+0}=a_{0+}=a_{-0}=a_{0-}$ $a_{+-}=a_{-+}$ $a_{++}=a_{--}$ a_{00}

between the φ decay and some even angular momentum background, α and β can be worked out for an arbitrary acceptance but we present results only for an acceptance of $0 \le \theta_i \le \frac{1}{2}\pi$. With this assumed acceptance, we can directly work out the expressions for α and β, using Eqs. (2) and (3) to express the result in terms of four independent amplitudes

$$\beta = \frac{2\eta\,|a_{++}|^2}{2|a_{++}|^2 + |a_{00}|^2 + 4|a_{0+}|^2 + 2|a_{+-}|^2}\,, \tag{10}$$

$$\alpha = \frac{2\mathrm{Re}(a_{++}a_{00}^*) - 2\eta\,|a_{0+}|^2}{2|a_{++}|^2 + |a_{00}|^2 + 4|a_{0+}|^2 + 2|a_{+-}|^2}\,. \tag{11}$$

(β is the same for π or $\frac{1}{2}\pi$ acceptance. α vanishes for π acceptance.)

We see immediately from this and Table I that if $\beta \ne 0$, then $(-1)^J = +1$; otherwise, $a_{++}=0$. At the same time, the sign of β gives the parity of the state. Thus, if $\beta \ne 0$, all the information obtainable by these measurements is fixed and the remaining dependences are not needed, but of course can be used for a check.

If $\beta = 0$, then from (11) we see that η is again determined. In fact, we see from Table I that if $a_{++}=0$, $\eta=-1$ requires that $\alpha=\frac{1}{2}$ for either value of $(-1)^J$. $\eta=+1$ simply requires that $-\frac{1}{2} \le \alpha \le 0$.

To obtain information about $(-1)^J$ when $\beta=0$, we go to our third parameter ζ. If we integrate the distribution (8) over χ and θ_2, we get a θ_1 distribution of the form

$$G(\theta_1) = 1 + \zeta P_2(\cos\theta_1)\,, \tag{12}$$

when

$$\zeta = 2\,\frac{|a_{00}|^2 - |a_{++}|^2 - |a_{+-}|^2 + |a_{+0}|^2}{2|a_{++}|^2 + |a_{00}|^2 + 2|a_{+-}|^2 + 4|a_{+0}|^2} \tag{13}$$

In general, $-1 \le \zeta \le 2$; however, if $a_{00}=0$, then $\zeta \le \frac{1}{2}$, so if $\zeta > \frac{1}{2}$ we uniquely have $(-1)^J = \eta = +1$. If $\alpha = \frac{1}{2}$ and $\beta = 0$ so $\eta = -1$, we have uniquely $\zeta = \frac{1}{2}$; i.e.,

$$G(\theta_1) = \tfrac{3}{4}(1+\cos^2\theta)$$

independent of $(-1)^J$, so for $\eta=-1$ and $\beta=0$ we cannot determine $(-1)^J$ by this method. For $\eta=+1$ we can do better because, if $\beta=0$, then

$$\zeta + 3\alpha = -1 + \frac{4|a_{00}|^2}{|a_{00}|^2 + 2|a_{+-}|^2 + 4|a_{+0}|^2}$$

by combining (11) and (12). Hence, if $\zeta + 3\alpha \ne -1$ we must have $a_{00} \ne 0$ and $(-1)^J = +1$. If $\zeta + 3\alpha = -1$ we cannot determine $(-1)^J$ in this way.

This seems to be the maximum amount of information that can be determined in this way. The results are summarized in Table II. We might mention several cross checks that can be made, which must be satisfied if the φ's are in a state of definite η and $(-1)^J$: (i) $\beta < 0$ requires $\alpha = \frac{1}{2}(1+\beta)$ because $\eta=-1$; (ii) if $\beta=0$ then if $\alpha>0$, α must have the value $\frac{1}{2}$; (iii) if $\beta=0$ then $\zeta - 3|\alpha| \ge -1$.

Finally, although one cannot in general determine the particle's spin in this way, it may be possible to rule out $J=0$ or 1 because then there are fewer amplitudes $a_{\lambda_1\lambda_2}$ and constraints are obtained. Thus, if $J=0$, as in the case considered by Chang and Nelson, only a_{00} and $a_{++}=\eta a_{--}$ are

TABLE II. Allowed states for various values or ranges of the parameters α, β, and ζ.

Value of parameters	Allowed states
$\beta > 0$	$(-1)^J = \eta = +1$
$\beta < 0$	$(-1)^J = +1,\ \ \eta = -1$
$\zeta > \frac{1}{2}$	$(-1)^J = \eta = +1$
$\beta = 0$ and $\alpha = \frac{1}{2}$	$(-1)^J = \pm 1,\ \ \eta = -1$
$\beta = 0$ and $\alpha \le 0$ $\zeta > 3\lvert\alpha\rvert - 1$	$(-1)^J = \eta = +1$
$\beta = 0$ and $\alpha \le 0$ and $\zeta = 3\lvert\alpha\rvert - 1$	$(-1)^J = \pm 1,\ \ \eta = +1$

nonzero. For $\eta=-1$, $a_{00}=0$ and we have

$$\beta=-1,\quad \alpha=0,\quad \zeta=-1, \qquad \text{for } J=0,\ \eta=-1.$$

For $J=0$ and $\eta=+1$, the constraints are weaker but it is easy to combine (10), (11), and (13) to show that

$$0\leqslant\beta\leqslant1,\quad \zeta=2-3\beta,\quad \alpha\leqslant[2\beta(1-\beta)]^{1/2} \qquad \text{for } J=0,\ \eta=+1.$$

For $J=1$ only $a_{\lambda_1\lambda_2}$ with $|\lambda_1-\lambda_2|=1$ are allowed; from Table I, we see that this leaves only one amplitude and so the distributions are unique

$$\beta=0,\quad \alpha=-\eta/2,\quad \zeta=\tfrac{1}{2}, \qquad \text{for } J=1.$$

These results obviously do not depend on any special properties of the ϕ and will be valid for any suitable identical vector-meson system.

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