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$X^0(958)\to\eta\pi\pi$ decay and ηX^0 mixing

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Within the framework of a broken-SU(6) $\times$ O(3) quark model of Lorentz-invariant hadron-coupling structures, the three-body decay process $X^0(958)\to\eta\pi\pi$ is investigated with a view to assessing the value of the ηX^0 mixing angle. The model assumes that the amplitude for this process is dominated by a $\delta(970)\pi$ intermediate state so that $X^0\to\delta\pi\to\eta\pi\pi$. The results obtained seem to favor a quadratic mass formula for pseudoscalar mesons with an ηX^0 mixing angle of $-10°$, in agreement with recent analyses.

Ever since its discovery[1] the $X^0(958)$ meson has been a subject of considerable interest. Although its existence has been confirmed, its properties are not quite clear. It is generally believed that the spin-parity assignment (J^P) of X^0 is either 0^- or 2^-, the evidence for which comes from the Dalitz-plot analyses of the $X^0\to\eta\pi\pi$ and $X^0\to\pi^+\pi^-\gamma$ decays and from the observation of the $X^0\to\gamma\gamma$ mode.[2] These analyses favor the pseudoscalar assignment but cannot rule out spin 2. The indication of anisotropy in the decay of the very forward produced X^0 has not yet been established, thus favoring spin 0 strongly. Support for the pseudoscalar assignment also comes from the theoretical considerations, for instance, of Parashar[3] and of Graham and Ng.[4] Furthermore, the X^0 meson is known to have substantial mixing with its pseudoscalar partner η. It is the purpose of the present investigation to study the effect of the ηX^0 mixing on the decay width of the three-body process $X^0\to\eta\pi\pi$.

There have been many theoretical attempts[5] to study this rare decay, for it is expected to be an important decay channel for testing the various ηX^0 mixing schemes. In what follows, we compute the decay width for $X^0\to\eta\pi\pi$ within the general theoretical framework of Mitra's version of the quark model of relativistic hadron couplings,[6] which not only has intrinsic appeal but also provides a very convenient language for their description. The amplitude for this process is assumed to be dominated by a $\delta\pi$ intermediate state,[7] so that the three-body decay is reduced essentially to a two-step process involving merely three-point vertices such as $X^0\to\delta\pi\to(\eta\pi)\pi$. Here δ is an isovector scalar ($I=1$, $J^P=0^+$) resonance with a mass ~976 MeV and total width ~50 MeV. The relevant vertices involved in the calculation of the decay width for $X^0\to\delta\eta\to\eta\pi\pi$ are $X^0\delta\pi$ and $\delta\eta\pi$, the appropriate coupling structures for which are constructed essentially in a phenomenological sense by making extensive use of the broken SU(6)$\times$O(3) group. The chief ingredient in the present scheme is provided by a single-quark transition with the emission of a pseudoscalar meson (P) regarded as a radiation quantum. In the case of a vertex involving two P mesons, the heavier of the two is regarded as a radiation quantum on the basis of purely phenomenological considerations.

Now we apply the general prescriptions of the model to the calculation of the decay width for $X^0\to\eta\pi\pi$ proceeding as a two-step process $X^0\to\delta\pi$ followed by $\delta\to\eta\pi$. In the context of the present model X^0 is taken to belong to the pseudoscalar nonet (0^-) with $L=0$, whereas δ is assigned to a scalar nonet (0^+) with $L=1$ supermultiplet. The couplings governing the $X^0\delta\pi$ and $\delta\eta\pi$ vertices can be written down explicitly in the form

$$C_1 g_1(2m_\delta/m_{X^0}^2)(m_{X^0}/m_\pi)^{1/2}(-m_{X^0}^2/\sqrt{3}) \quad (X^0\delta\pi), \tag{1}$$

$$C_2 g_1(2m_\delta/m_\eta^2)(m_\eta/m_\pi)^{1/2}(-m_\eta^2/\sqrt{3}) \quad (\delta\eta\pi), \tag{2}$$

where g_1 is a dimensionless coupling constant governing the entire supermultiplet transition from $L=1$ to $L=0$. The value of g_1 has already been fixed from the known width of $A_1\to\rho\pi$ and is given by $g_1^2/4\pi=0.08$; whereas C_1 and C_2 are the SU(3) coefficients appropriate for the $X^0\delta\pi$ and $\delta\eta\pi$ vertices, respectively, which are evaluated with the help of quark-model prescriptions by taking the ηX^0 mixing angle (θ) into account. For this purpose, we take the bare states to be $\eta_8=(\frac{1}{6})^{1/2}(u\bar{u}+d\bar{d}-2s\bar{s})$ and $\eta_1=(\frac{1}{3})^{1/2}(u\bar{u}+d\bar{d}+s\bar{s})$ with the physical states given by $\eta=\eta_8\cos\theta+\eta_1\sin\theta$ and $X^0=-\eta_8\sin\theta+\eta_1\cos\theta$. The coefficients C_1 and C_2 work out as

$$C_1=(2/\sqrt{3})(-\sin\theta+\sqrt{2}\cos\theta),$$

$$C_2=(2/\sqrt{3})(\cos\theta+\sqrt{2}\sin\theta).$$

The invariant T-matrix element squared for $X^0\to\eta\pi\pi$, dominated by the δ-meson pole, can now be easily written down. Using the covariant phase-space calculations for the three-particle final state,[8] the resulting expression for the decay width $\Gamma(X^0 \to \eta\pi\pi)$ is given by

$$\Gamma(X^0 \to \eta\pi\pi) = \tfrac{1}{3}(\tfrac{2}{9})^2 (g_1{}^2/4\pi)^2 (m_\delta{}^4\, m_\eta / m_{X^0}{}^2 m_\pi{}^2) \times (\sin 2\theta + 2\sqrt{2}\cos 2\theta)^2 I(X^0 \to \eta\pi\pi)\,, \quad (3)$$

where $I(X^0 \to \eta\pi\pi)$ is a phase-space integral given by the expression

$$I(X^0 \to \eta\pi\pi) = \int_{(m_\eta + m_\pi)^2}^{(m_{X^0} - m_\pi)^2} \left(\frac{ds}{s}\right) \times \frac{\lambda^{1/2}(s, m_\pi{}^2, m_\eta{}^2)\lambda^{1/2}(m_{X^0}{}^2, s, m_\pi{}^2)}{(s - m_\delta{}^2)^2 + m_\delta{}^2\Gamma_\delta{}^2}\,. \quad (4)$$

Here s is a Mandelstam-type variable defined in terms of the four-momenta of the particles participating in the interaction.[8] The function $\lambda(m_1, m_2, m_3)$ appearing in (4) is defined as

$$\lambda(m_1, m_2, m_3) = m_1{}^2 + m_2{}^2 + m_3{}^2 - 2(m_1 m_2 + m_2 m_3 + m_3 m_1)\,. \quad (5)$$

From the experimental point of view, the value of the ηX^0 mixing angle (θ) is $\pm 10°$ for the quadratic mass formula and $\pm 24°$ for the linear mass formula for pseudoscalar mesons. We use these four values of θ to compute the decay rate $\Gamma(X^0 \to \eta\pi\pi)$ from expressions (3) and (4). The results are shown in Table I.

For the sake of completeness, we also list the widths of the other radiative decay modes of X^0, viz., $X^0 \to (\rho\gamma, \gamma\gamma, \omega\gamma, \pi\gamma\gamma, \pi\pi\gamma)$, calculated in the present model for various values of the mixing angle θ.

In order to shed light on the mixing of the ηX^0 complex, we must compare our results in Table I with the experimental data. At present the available experimental evidence on the absolute decay rates of X^0 is not sufficiently definitive to warrant a serious test of ηX^0 mixing; for there exist only experimental upper bounds on these rates, and any model giving a value below that bound is obviously allowed. Furthermore, given the nature of the bound here, a violation of the order of 10–15% is not very critical. The upper bound cannot, therefore, be used to advantage in the present context to sort out the ηX^0 mixing problem. The experimentally significant result, however, is the accurately known ratios $B(X^0 \to \eta\pi\pi)/B(X^0 \to \rho\gamma) \approx 2.2$ and $B(X^0 \to \rho\gamma)/B(X^0 \to \gamma\gamma) \approx 15$. Comparing the predictions of the present model with these ratios, we find the ratio $\eta\pi\pi/\rho\gamma = 2.1$ for $\theta = -10°$, in excellent agreement with the experimental value of 2.2; whereas the theoretical value of the ratio $\rho\gamma/\gamma\gamma = 17.7$ obtained for $\theta = -10°$ compares reasonably well with the experimental value of 15. We are, therefore, led to conclude that if the underlying assumption of the dominance of the decay process $X^0 \to \eta\pi\pi$ by the scalar isovector $\delta(970)$ resonance is valid, the ηX^0 mixing angle should necessarily be $-10°$, thereby supporting the quadratic version of the mass formula for pseudoscalar mesons.

This conclusion is in agreement with the recent analysis of Isgur,[10] who establishes from symmetry-breaking considerations at the quark level that the pseudoscalar mixing angle of $-10°$ is an attribute of the near degeneracy in the masses of the strange and the nonstrange pseudoscalar mesons, while the mixing angle for more massive nonets, on the other hand, is expected to be very nearly ideal. Recently there have been many attempts to estimate the mixing of the ηX^0 complex within the framework of various symmetry schemes. For instance, Edwards and Kamal[11] have studied this problem on the basis of SU(3) and nonet symmetry-breaking schemes and found general agreement with experiment with a mixing angle of $-10°$. A similar analysis was carried out by Boal, Graham, and Moffat[12] who obtain solutions for both $\theta = -10°$ and $-24°$. The calculations

TABLE I. The decay widths of $X^0(958)$ for various values of the ηX^0 mixing angle ($\theta = \pm 10°$ and $\theta = \pm 24°$). Experimental values are taken from Ref. 9.

Decay mode of X^0	$\theta = +10°$	$\theta = -10°$	$\theta = +24°$	$\theta = -24°$	Experimental value
$\eta\pi\pi$ (MeV)	0.81	0.52	0.75	0.15	< 0.68
$\pi\pi\gamma$ (keV)	135	202	64	238	<205
$\pi\gamma\gamma$ (eV)	3.0	5.9	1.9	7.0	···
$\rho\gamma$ (keV)	149	246	78	280	< 304
$\omega\gamma$ (keV)	28.2	46	14.7	60	<50
$\gamma\gamma$ (keV)	6.8	13.9	3.5	13.1	<20
$\eta\pi\pi/\rho\gamma$	5.4	2.1	9.6	0.54	2.2
$\rho\gamma/\gamma\gamma$	22.0	17.7	22.3	21.4	15

of O'Donnell[13] lend support to the quadratic version of the mass formula but suffer from inaccurate normalization of the decay rates. Our result for the ratio $\pi\pi\gamma/\gamma\gamma = 14.5$ (with $\theta = -10°$) compares well with the value 14–16 predicted by Kotlewski, Lee, Suzuki, and Thaler;[14] but unfortunately there is no experimental confirmation of this ratio at present. Using an effective Lagrangian approach, Torgerson[15] obtains agreement with the experimental data with $\theta = -10°$. The ratio $\rho\gamma/\gamma\gamma$ of the present calculation compares well with that obtained by many others.[11-13,15] The work of Thews[16] and of Chase and Vaughn[17] treats the ηX^0 mixing problem on the basis of the quark-loop model, while Barnes[18] has applied recoil effect corrections to the magnetic dipole decays using pseudoscalar mixing angle of $-10°$.

The quark model with broken $SU(6) \times O(3)$ used here has the intrinsic merit of being mathematically simple and provides a consistent description of both mesons and baryons in a unified framework. The chief advantage of this scheme is that the whole supermultiplet transition is governed by a single dimensionless coupling constant (as opposed to several parameters used in other approaches).

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