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On muonic double-β decays of pseudoscalar mesons

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The rate for the decay of any pseudoscalar meson $H^{\pm}$ into two identical muons $\mu^{\pm}$ and another pseudoscalar meson $h^{\mp}$ is calculated in the tree and box-diagram approximation. This double-β decay is assumed to be induced by a Majorana lepton N_{μ} of mass m_{σ}. The rate for K^- decay at rest is 3.3×10^{-16} sec^{-1} for $m_{\sigma}=2$ GeV/c^2, and it is 4 sec^{-1} for the charmed meson F^- decay. Detailed dependence of the decay rate on m_{σ} is given.

I. INTRODUCTION

Recent high-precision experiments[1] searching for $\mu^- \to e^-\gamma$ and μ^- +nucleus $\to e^-$ +nucleus demonstrate that muonic quantum number does not convert to electronic quantum number.[2] Thus, the mixing between muon and electron, if it exists at all, must be very small. The question of violating electron quantum number by two units has been examined in nuclear *neutrinoless* double-β decay

$$(A, Z) \to (A, Z+2) + 2e^- . \tag{1}$$

The accumulated experimental evidence[3] indicates that this mode is highly suppressed. For gauge-theory models of weak and electromagnetic interactions where the gauge group is SU(2) $\times$ U(1) and the leptons are arranged in doublets such that the left-handed (L) and right-handed (R) doublets exist symmetrically,

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \text{ and } \begin{pmatrix} N_e \\ e^- \end{pmatrix}_R , \tag{2}$$

where N_e is a heavy Majorana lepton,[4] reaction (1) will proceed as a second-order weak process. It has been shown[5,6] that current data[3] imply the mass of N_e is greater than 10^5 GeV. Consequently, the right-handed electron must couple predominantly to a four-component Dirac particle.

The question remains whether or not the right-handed muon can form a doublet with a massive Majorana lepton, N_{μ}, giving the following structure[6]:

$$\begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \text{ and } \begin{pmatrix} N_\mu \\ \mu^- \end{pmatrix}_R . \tag{3}$$

A clear experimental signature can be obtained by searching for rare muon double-β decay of a charged hadron $H^{\pm}$ into another hadron $h^{\mp}$ or into the doublet ($e^{\pm}\nu_e$) plus two *identical* muons, $\mu^{\pm}$:

$$H^{\pm} \to h^{\mp} + \mu^{\pm} + \mu^{\pm} . \tag{4}$$

One such example will be the kaon decay mode:

$$K^{\mp} \to \pi^{\pm} + \mu^{\mp} + \mu^{\mp} , \tag{5}$$

which is kinematically allowed. However, we have found no experimental information on this reaction.[7]

In this paper we will study the reaction (4) in detail. We shall assume that the mass of N_{μ}, denoted by m_{σ}, is greater than the mass, m_H, of the parent hadron, H, which we will assume to be a 0^- meson. The decay (4) will then proceed through the tree diagram Fig. 1(a) and the box diagram Fig. 1(b). The specific lepton-number nonconservation is transmitted through the propagation of the virtual Majorana lepton N_{μ}. The cross in Fig. 1 denotes a mass-insertion term $m_{\sigma}\bar{N}_{\mu}N_{\mu}^c$ where N_{μ}^c is the charge-conjugate field of N_{μ}. This mass insertion acts as a source or a sink of N_{μ}. The amplitude squared, A^2, for the process given by the tree graph Fig. 1(a) will then in general be for $m_{\sigma} \gg m_H$

$$A^2 \sim G_F{}^4 \frac{m_H{}^7}{m_{\sigma}{}^2} f_H{}^2 f_h{}^2 , \tag{6}$$

where G_F is the Fermi coupling constant f_H, and f_h are respectively the form factors of the axial-vector part of the charged weak current for the mesons H and h. The factor $m_{\sigma}{}^2$ comes from the mass insertion and the two fermion propagators appearing in the Feynman diagram, and the mass m_H, supplies the correct dimension. In the case of kaon decay at rest [Eq. (5)] and for $m_{\sigma} \approx 3$ GeV/c^2,

$$A^2 \sim 10^{-5}\ \text{sec}^{-1} \tag{7}$$

with a Cabibbo factor $\sin^2\theta_C$ included. However, a phase-space factor cuts it down to[8] a rate R $\sim 10^{-8}$ sec^{-1}. An estimate using the box diagram Fig. 1(b) gives the same order of magnitude for the case of reaction (6). For details see Sec. II.

On the other hand, the purely lepton mode

$$K^- \to \mu^-\mu^- e^+ \nu_e \tag{8}$$

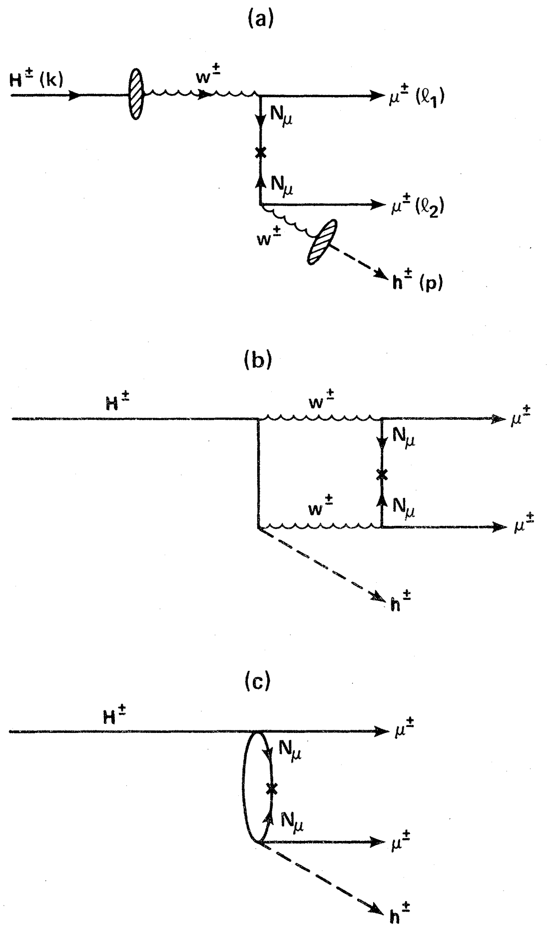

FIG. 1. Feynman diagrams for the neutrinoless muonic double-β decay of pseudoscalar mesons. The cross denotes a mass insertion term $m_\sigma \overline{N}_{\mu R} N^c_{\mu R}$ which acts as a source and sink for Majorana fermion lines. The tree graphs is given in (a) and the box graph is represented in (b) while (c) is the box graph in the limit of large W-boson mass.

does not proceed via the box diagram and it is less suppressed kinematically. However, it is a four-body decay and its actual rate is lower than the three-body mode for kaon decays giving

$$R \sim 2\times 10^{-10}\ \mathrm{sec}^{-1}. \tag{9}$$

In general the estimates using the tree diagram have an enhancement factor when the parent meson and the Majorana lepton are almost degenerate in mass. This comes from the propagator in the virtual-lepton line in Fig. 1(a). A factor-of-100 increase over the rate as given in Eq. (7) for $m_\sigma \approx 500\ \mathrm{MeV}/c^2$ is obtained. This feature coupled with the larger phase space available makes mesons with heavy-quark content such as the D mesons more favorable for detecting the effects of N_μ, if it exists. Otherwise, one can set a limit on its mass. However, the charmed-meson decays pose more background problems than the kaons.

Next we discuss the case when $m_H > m_\sigma > m_h$. The virtual line in Fig. 1(a) becomes real, and in principle one can observe the decay of N_μ via the sequence

$$H^+ \to \mu^+ N_\mu \tag{10a}$$

$$\searrow \mu^+ + h^- \tag{10b}$$

or

$$\mu^+ + e^- \overline{\nu}_e . \tag{10c}$$

Since no such striking signature is observed in kaon decays one can immediately rule out the possibility that m_σ lies between the mass of the kaon and the mass of the electron, m_e. Similarly one can search for double-β decays in charmed mesons that are produced in electron-positron annihilation.[9] One such mode will be

$$D^+ \to \mu^+ N_\mu \tag{11a}$$

$$\searrow \mu^+ \pi^- \tag{11b}$$

and m_σ will be given by the invariant mass of the $\mu^+\pi^-$ pair. The main experimental difficulty is to detect the neutral N_μ which will be rather long-lived and will behave very much like a neutron. Nonobservation of such a mode will immediately set $m_\sigma > m_D$.

The final case when $m_\sigma < m_e$ corresponds to having a Majorana muonic neutrino of very light mass.[10] One can test this possibility by doing neutrino oscillation experiments,[11] but this is beyond the scope of this study.

In Sec. II we spell out the assumptions and the details of the calculation. Cases when the box diagram dominates over the tree diagram or vice versa are given. In Sec. III we discuss our results of treating the three-body phase space exactly via numerical methods. Finally, in Sec. IV we will discuss our results and their implications.

II. MODELS AND CALCULATIONS

We will consider SU(2)×U(1) gauge models in which both the left-handed and right-handed electron and muon and their neutrino are arranged in doublets as given in Eqs. (2) and (3). The neutrinos ν_{eL} and $\nu_{\mu L}$ are assumed to be massless. The neutral Dirac lepton N_e is taken to have a mass greater than that of the charmed quark so that it is assumed not to have been produced at present accelerator energies. It does not enter into our calculations. The weak-interaction Lagrangian involving the Majorana fermion field N_μ is assumed to have the form[12]

$$\mathcal{L} = \bar{N}_\mu i\gamma^\rho(1+\gamma_5)\partial_\rho N_\mu - m_\sigma \bar{N}_\mu(1+\gamma_5)N_\mu^c$$
$$-f^2(W_\rho^\dagger j_l^\rho + W_\rho^\dagger J_h^\rho + \text{H.c.})\,, \tag{12}$$

where $N_\mu^c = C\bar{N}_\mu^\dagger$ (C denotes the charge-conjugation operator) and f is the gauge coupling. The intermediate-vector-boson field is denoted by W^ν. The leptonic weak charged current j_ρ^l is given by

$$j_l^\rho = \bar{\mu}\gamma^\rho(1+\gamma_5)N_\mu + \bar{\mu}\gamma^\rho(1-\gamma_5)\nu_\mu$$
$$+\bar{e}\gamma^\rho(1-\gamma_5)\nu_e + \bar{e}\gamma^\rho(1+\gamma_5)N_e + \cdots\,, \tag{13}$$

where we have neglected terms involving other possible heavy leptons. Such a current will not have an axial-vector part in the neutral current. For the hadronic charged weak current we adopt the usual Glashow-Iliopoulos-Maiani (GIM) form

$$J_h^\lambda = \cos\theta_C[\bar{u}\gamma^\lambda(1-\gamma_5)d + \bar{c}\gamma^\lambda(1-\gamma_5)s]$$
$$+\sin\theta_C[\bar{u}\gamma^\lambda(1-\gamma_5)s + \bar{c}\gamma^\lambda(1-\gamma_5)d] + \cdots\,, \tag{14}$$

where we have used u, d, s, and c to represent the up, down, strange, and charmed quarks, respectively, and θ_C is the Cabibbo angle. Here we have neglected possible right-handed currents, since they involve much heavier quarks and thus are irrelevant for us. As we shall see below, our calculation is general enough that including heavier quarks follows straightforwardly. Since we have assumed universal weak coupling, the gauge coupling constant will eventually be replaced by the Fermi coupling constant G_F via the relation

$$\frac{G_F}{\sqrt{2}} = \frac{f^2}{M_W{}^2}\,, \tag{15}$$

where $M_W{}^2$ is the mass of the intermediate boson, and $G_F m_p{}^2 = 1.023\times10^{-5}$ (m_p is mass of the proton). The Majorana fermion N_μ is described by a two-component Weyl field with definite helicities specified by the projection operators $\frac{1}{2}(1+\gamma_5)$ for antiparticles. The mass term in Eq. (12) is not diagonalized. This can be easily done by the Pauli-Gürsey transformation.[13]

For clarity we will give our calculations for 0^- decays only. In particular, we study the following decays:

$$K^- \to \mu^-\mu^-\pi^+\,, \tag{16a}$$

$$D^- \to \mu^-\mu^-\pi^+\,, \tag{16b}$$

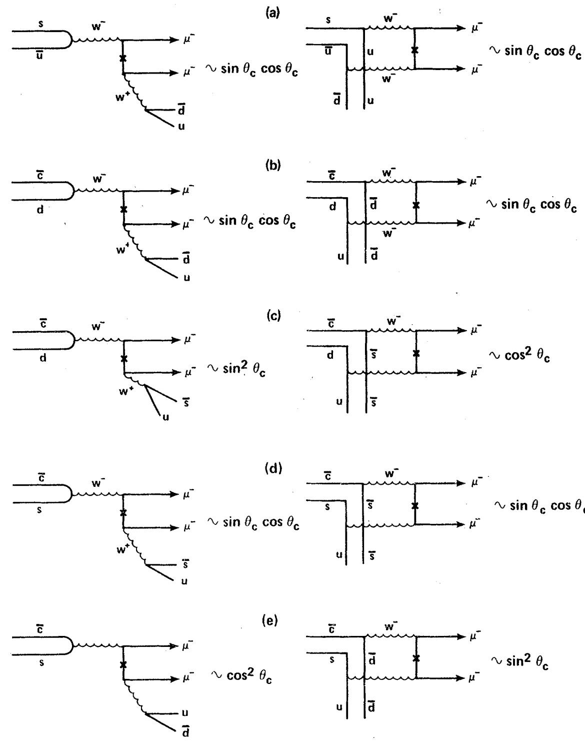

FIG. 2. Quark diagrams for the decays (a) $K^-\to\mu^-\mu^-\pi^+$, (b) $D^-\to\mu^-\mu^-\pi^+$, (c) $D^-\to\mu^-\mu^-K^+$, (d) $F^-\to\mu^-\mu^-K^+$, and (e) $F^-\to\mu^-\mu^-\pi^+$ with Cabibbo factors given for each case. The diagrams on the left-hand side and the right-hand side for each case correspond respectively to the tree and the box graphs of Fig. 1.

$$D^- \to \mu^-\mu^-K^+\,, \tag{16c}$$

$$F^- \to \mu^-\mu^-\pi^+\,, \tag{16d}$$

and

$$F^- \to \mu^-\mu^-K^+\,. \tag{16e}$$

We display in Fig. 2(a)–2(e) the quark diagrams corresponding to the reactions (16a)–(16e). Using the GIM current, we see that (16d) dominates over (16e) for F^- meson decay, and (16c) is not Cabibbo suppressed in the box diagram, whereas for the kaon the decay, (16a) is Cabibbo suppressed. Hence, for the charmed mesons the decays to study are (16c) and (16d).

For definiteness we will give the calculation for the reaction $H^-\to l^-l^-h^+$. The transition amplitude is given by perturbation theory as

$$\langle l^-l^-h^+|H^-\rangle = -f^4\int d^4x\int d^4x'\int d^4y\int d^4y'\, i\Delta_{\mu\nu}(x-x')\, i\Delta_{\rho\sigma}(y-y')\langle h^+l^-l^-|J_h^\rho(y)j_l^\sigma(y')j_l^\nu(x')J_h^\mu(x)|H^-\rangle\,, \tag{17}$$

where the W-boson propagator is given by

$$\Delta_{\mu\nu}(x-x')=\int\frac{d^4q}{(2\pi)^4}e^{iq\cdot x}\frac{g_{\mu\nu}-q_\mu q_\nu/M_W^2}{q^2-M_W^2}\,. \tag{18}$$

Since the leptonic current has no strong interaction, knowledge of the two-current correlation function

$$M^{\rho\mu}=\langle h^+|J_h^\rho J_h^\mu|H^-\rangle \tag{19}$$

will determine the transition rate completely. The completeness condition dictates that

$$M^{\rho\mu}=\sum_{\text{all }x}\langle h^+|J_h^\rho|x\rangle\langle x|J_h^\mu|H^-\rangle$$

$$=\langle h^+|J_h^\rho|0\rangle\langle 0|J_h^\mu|H^-\rangle+\langle h^+|J_h^\rho|\delta^0\rangle\langle\delta^0|J_h^\mu|H^-\rangle+\cdots\,. \tag{20}$$

In Eq. (20), the first term denotes the vacuum intermediate state insertion, the second term gives the single-particle state insertions, which in our cases are neutral meson states, and the dots represent higher-multiplicity particle states. The vacuum intermediate state results in the tree diagram [Fig. 1(a)] and the single-particle intermediate state gives the box diagram [Fig. 1(b)]. We shall also assume that the higher-multiplicity intermediate states do not make a significant contribution. Thus, the decay amplitude is then the sum of the tree-graph amplitude T_1 and the box graph amplitude T_2. As we have seen before, owing to the structure of the GIM current the charmed-meson decays have either T_1 or T_2 as the dominant decay mode. On the other hand, for the kaon decay in Fig. 2(a) both amplitudes are proportional to $\sin\theta_C\cos\theta_C$.

Now we return to the general case. The tree-graph amplitude T_1 is given explicitly by

$$T_1=i\left(\frac{f^2}{M_W^2}\right)^2 m_\sigma\langle h^-|J_h^\mu|0\rangle\left\{\frac{\bar u(l_1)\gamma_\mu(1+\gamma_5)(\not k-\not l_1+m_\sigma)(1+\gamma_5)(\not k-\not l_1-m_\sigma)(1+\gamma_5)\gamma_\nu\bar{\bar u}(l_2)}{[(k-l_1)^2-m_\sigma^2]^2}-(l_1-l_2)\right\}\langle 0|J_h^\nu|H^-\rangle, \tag{21}$$

where the kinematics is defined in Fig. 1(a), and we have taken the large-M_W^2 limit. Using the principle of partially conserved axial-vector current and then squaring the amplitude we obtain the rate R_1 for H decaying at rest as given below:

$$R_1=\frac{1}{2}\frac{1}{2m_H}\frac{1}{(2\pi)^5}\int\frac{d^3l_1}{2E_1}\int\frac{d^3l_2}{2E_2}\int\frac{d^3p}{2E}\delta^4(k-l_1-l_2-p)|T_1|^2$$

$$=\frac{G_F^4m_\sigma^6f_H^2f_h^2}{4\pi^5m_H}\int\frac{d^3l_1}{2E_1}\int\frac{d^3l_2}{2E_2}\int\frac{d^3p_h}{2E_h}\delta^4(k-l_1-l_2-p)$$

$$\times\left(\frac{1}{(\Delta M^2-2k\cdot l_1)^4}\{2(l_2\cdot p)[2(l_1\cdot k)(k\cdot p)-(l_1\cdot p)m_H^2]-m_h^2[2(l_1\cdot k)(l_2\cdot k)-(l_1\cdot l_2)m_H^2]\}\right.$$

$$+\frac{1}{(\Delta M^2-2k\cdot l_2)^4}\{2(l_1\cdot p)[2(l_2\cdot k)(k\cdot p)-(l_2\cdot p)m_H^2]-m_h^2[2(l_2\cdot k)(l_1\cdot k)-(l_1\cdot l_2)m_h^2]\}$$

$$+\frac{2}{(\Delta M^2-2k\cdot l_1)^2(\Delta M^2-2k\cdot l_2)^2}\{2(k\cdot p)[(k\cdot p)(l_1\cdot l_2)-(l_1\cdot p)(l_2\cdot k)-(l_2\cdot p)(l_1\cdot k)]$$

$$\left.+m_H^2[2(l_1\cdot p)(l_2\cdot p)-m_h^2(l_1\cdot l_2)]+2m_h^2(l_1\cdot k)(l_2\cdot k)\}\right)\times(\text{Cabibbo factor})\,, \tag{22}$$

where

$$\Delta M^2=m_H^2+m_\mu^2-m_\sigma^2\,. \tag{23}$$

The quantities f_H and f_h are given by

$$\langle 0|J_\mu^+|H\rangle=if_Hk_\mu \tag{24a}$$

and

$$\langle h|J_\mu|0\rangle=if_hp_\mu\,. \tag{24b}$$

For the case of K^- decay, f_H and f_h become f_K, the weak kaon form factor and f_π, the pion decay constant respectively. The appropriate Cabibbo factor that multiplies Eq. (18) can be read from Fig. 2.

In the limit $m_\sigma \gg m_H$, Eq. (18) becomes

$$R_1 \xrightarrow[m_\sigma \gg m_H]{} \frac{G_F{}^4 f_H{}^2 f_\pi{}^2}{\pi^2 m_\sigma{}^2 m_H} \int \frac{d^3 l_1}{2E_1} \int \frac{d^3 l_2}{2E_2} \int \frac{d^3 p}{2E} \delta^4(k-p-l_1-l_2)(l_1 \cdot l_2)(k \cdot p)^2 \tag{25}$$

and hence falls as $m_\sigma{}^{-2}$.[14]

Next we consider the box-diagram amplitude, T_2. Explicitly, we have

$$T_2 = 8f^4 m_\sigma{}^3 \int \frac{d^4 q}{(2\pi)^4} [\bar{u}(l_2)(\not{k} - \not{q})(1+\gamma_5)\not{k} C \bar{\tilde{u}}(l_1) I(l_1, q) - (l_1 \leftrightarrow l_2)] \tag{26a}$$

and

$$I(l_1, q) = 1/\{(q^2 - M_W{}^2)[(k-q-p)^2 - M_W{}^2][(q-l_1)^2 - m_\sigma{}^2]^2[(k-q)^2 - m_\delta{}^2]\}. \tag{26b}$$

The four-momentum carried by the W boson is given by q^ρ. The mass of the intermediate-state meson is denoted by m_δ. Since we are only interested in an estimate of the contribution to the decay rate due to the box diagram we shall make the following assumptions: (i) The couplings of the W boson to the pseudoscalar particles are given by $f\epsilon^\mu K_\mu$ where ϵ^μ is the polarization vector of the W boson, K_μ is the four-momentum of the decaying meson, and f is a form factor. (ii) The form factors f involved in K decay will just be the $K\pi W$ and $\pi\pi W$ vertices and the former can be extracted from K_{l3} decay. On the other hand, at present no reliable information can be obtained for charmed-meson decays of Eqs. (16b–16e). For simplicity we shall assume that all form factors are constants. Furthermore, we have used the 't Hooft–Feynman gauge for the W-boson propagators in Eq. (26a). The loop integral in Eq. (26) is convergent and one can combine the denominators in Eq. (26b) using Feynman parameters (see Appendix). Thus, the rate calculated from the box diagram (with $M_W{}^2 \to \infty$) is

$$\begin{aligned} R_2 = \frac{G_F{}^4 m_\sigma{}^6}{256 m_H \pi^9} f_H{}^2 f_h{}^2 \int \frac{d^3 l_1}{2E_1} \int \frac{d^3 l_2}{2E_2} \int \frac{d^3 p}{2E} \delta^4(k - l_1 - l_2 - p) \\ \times \Bigg\{ \frac{J^2(l_1)}{m_\mu{}^2 + m_H^2 - 2l_1 \cdot k} [(m_H{}^2 - 2k \cdot l_1)^2 (l_1 \cdot l_2) + 2m_\mu{}^2 (l_2 \cdot k)(m_H{}^2 - k \cdot l_1) \\ - m_\mu{}^2 m_H{}^2 (l_1 \cdot l_2)] \\ + (l_1 \leftrightarrow l_2) + \frac{2J(l_1)J(l_2) m_H{}^2 (l_1 \cdot l_2)}{(m_\mu^2 + m_H{}^2 - 2l_1 \cdot k)(m_\mu{}^2 + m_H{}^2 - 2l_2 \cdot k)} (m_H{}^2 + m_\mu{}^2 - l_1 \cdot k - l_2 \cdot k) \Bigg\}. \end{aligned} \tag{27}$$

and $J(l)$ is given in the Appendix. In the limit of large m_σ, the controlling mass of the loop integral is just m_σ as expected and gives

$$R_2 \sim \frac{G_F{}^4 m_\sigma^2 f_H{}^2 f_h{}^2}{\pi^9} m_H{}^3. \tag{28}$$

Thus, R_2 increases as m_σ^2 for m_σ large but still smaller than $M_W{}^2$. The final three-body phase-space is done numerically. For the case where m_σ is large and neglecting m_μ the phase-space integrals can be evaluated analytically and serves as a check.

III. NUMERICAL RESULTS

As we have seen in Sec. II, the amplitudes corresponding to Figs. 1(a) and 1(b) should be added, since they are respectively the vacuum and single-particle intermediate state insertion. However, for $D^- \to K^+ \mu^- \mu^-$ the box-diagram amplitude prevails, being $\sim \cos^2\theta_C$, whereas for $F^- \to \pi^+ \mu^- \mu^-$ the tree-diagram amplitude dominates ($\sim \cos^2\theta_C$). The expressions given by Eqs. (22) and (27) are adequate estimates[15] for the decays of Eqs. (16d) and (16c).

The mass of the $D^\pm$ meson[16] is taken to be 1.87 GeV/c^2 and the $F^\pm$ meson[17] is assumed to have a mass of 2.03 GeV/c^2. The form factors f_D and f_F

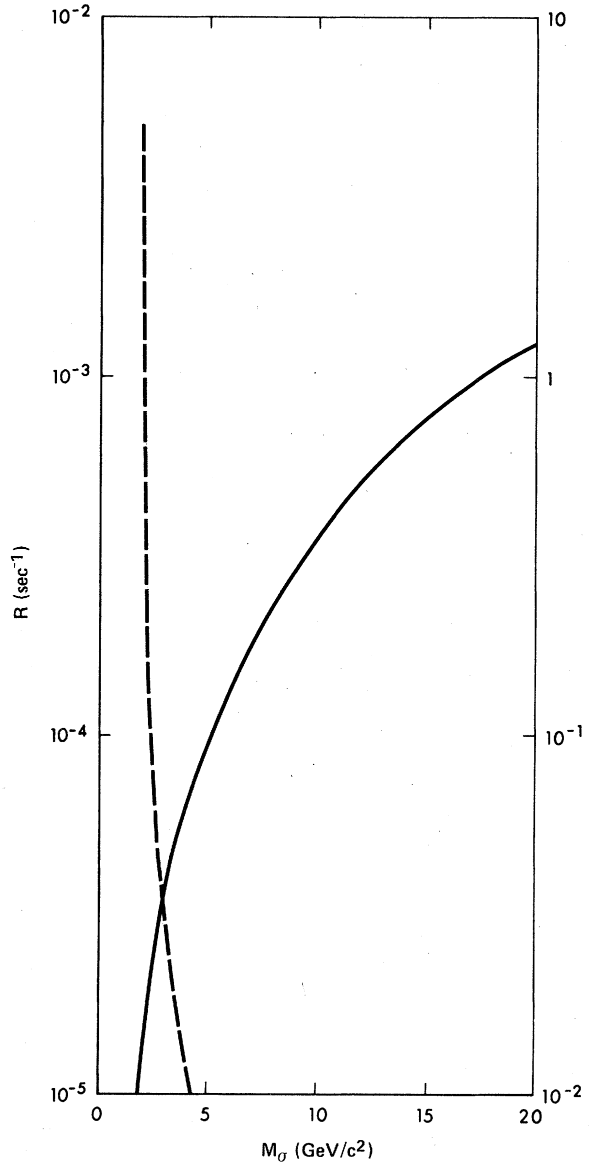

FIG. 3. The rates for $D^- \to K^+\mu^-\mu^-$ and $F^- \to \mu^-\mu^-\pi^+$ decays at rest as a function of M_σ. The solid line stands for D^- decay with scale on the right-hand side and the dotted line gives the F^- decay with scale given on the left-hand side.

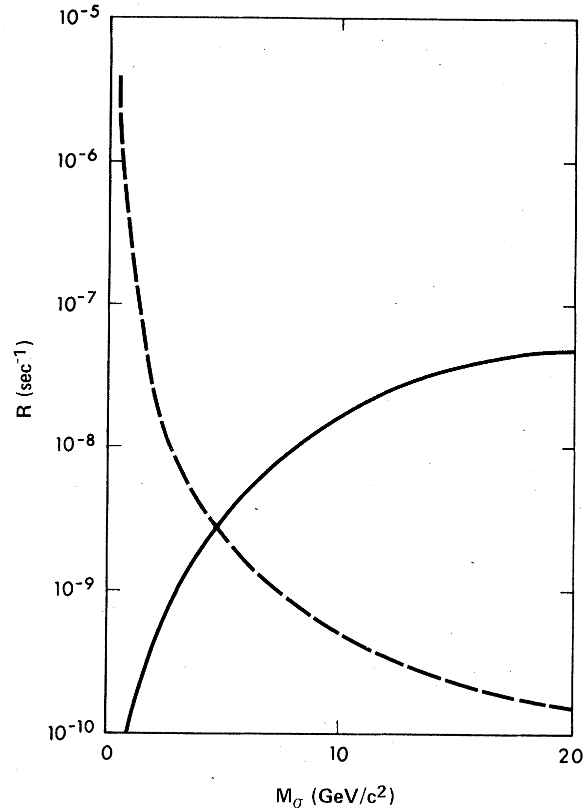

FIG. 4. The rate calculated by the tree diagram for $K^- \to \mu^-\mu^-\pi^+$ is given by the solid line and the contribution from the box diagram is denoted by the dashed line.

are assumed to the same as f_K, the K_{l3} form factors for the kaon. This would be the case for an SU(4)-invariant theory and it is adequate for our purposes.

In Fig. 3 we show the behavior of the decay rates as a function of M_σ for D^- and F^- decays. The largest rate we obtain is the F^- decay with $M_\sigma = 2$ GeV/c^2 and $M_F = 2.03$ GeV/c^2 is $R = 4$ sec^{-1} and falls rapidly with M_σ. The D^- decay rises rapidly with M_σ and goes to an $M_\sigma^{\,2}$ variation for large M_σ. When $M_\sigma > 25$ GeV our assumption of $M_W^{\,2} \to \infty$ will no longer be good, and the estimate for the rate will no longer be reliable. Since charmed mesons are expected to have lifetimes of the order of 10^{-14} sec, the above estimates correspond to branching ratios of the order of 10^{-15} for F mesons and 10^{-18} for D mesons.

We illustrate the corresponding results for kaon double-β decay in Fig. 4. Since we know the lifetime of the K mesons, we can give the branching ratio (B) into two identical muons, for example, for $m_\sigma = 500$ MeV/c^2

$$B(K^- \to \mu^-\mu^-\pi)/(K^- \to \text{all}) = 4.9\times 10^{-14};$$

for $m_\sigma = 2$ GeV/c^2,

$$B = 3.3\times 10^{-16};$$

and for $m_\sigma = 20$ GeV/c^2, we have

$$B = 6\times 10^{-16}.$$

IV. CONCLUSIONS

Although the lepton-number-violating decays of pseudoscalar mesons we have considered have spectacular signatures in the form of two identical muons, the rates are all discouragingly small. The decay mode has the added virtue that it contains no missing neutral particles in the decay product. Thus, for the D and F mesons which are produced in e^+e^- annihilation machines the candidates for the decays (16b) and (16c) will have the four-momenta of the decay products reconstructed to the mass of the parent meson enabling one to distinguish them from background events. Thus, in principle no new techniques other than that used in detecting charged decay, say, $K^+\pi^-$, of the D^0 meson are required.[16] Moreover, one needs a formidably high number of produced D^+ or D^- mesons to observe the decay.

Although the rate of F-meson decays is more favorable, this has to be weighed against the less copious production of the $F^\pm$ compared to $D^\pm$ mesons thus making the experiment just as difficult.

The availability of high-intensity separated $K^\pm$ meson beams makes reaction (5) the only currently possible reaction for the search for muonic neutrinoless double-β decay. The present experimental status has reached 10^{-8} in branching ratio for rare decays. Figure 4 shows that in order to even set an interesting limit on m_σ one requires measurements of rates of the order 10^{-8} sec^{-1}. Thus long running time on high-intensity slow $K^\pm$ beams is required.

The calculation we have done rests on a set of assumptions based on gauge theories. Among them is the assumption of universality of weak-interaction couplings, i.e., all weak interactions couple with strength G_F, and that the doubly charged exchange currents involved here are mediated by two W-boson exchanges. Any experimental signature above that we have calculated will indicate a breakdown of one or more of the above assumptions, and will have profound implications in weak interactions.

We have seen that the experiments are very difficult. Since the decays concern the very important question of lepton-number conservation and the existence of Majorana fermions or a doubly charged current, they warrant careful experimental studies.

ACKNOWLEDGMENTS

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APPENDIX

We derive here the integral that is used in calculating the box-diagram amplitude, T_2. We define ($l = l_1$ or l_2)

$$I = f^4 \int \frac{d^4q}{(2\pi)^4} \frac{(\not{k} - \not{q})}{(q^2 - M_W^2)[(k-q-p)^2 - M_W^2][(q-l)^2 - m_\sigma^2]^2[(k-q)^2 - m_\delta^2]}$$

$$\xrightarrow[M_W^2 \to \infty]{} \frac{f^4}{M_W^4} \frac{2}{(2\pi)^4} \int d^4q \int_0^1 dx \, \frac{x[\not{q} + (\not{l} - \not{k})x]}{(q^2 - D^2)^3}, \tag{A1}$$

where

$$D^2 = (m_\mu^2 - 2l\cdot k + m_K^2)x^2 - (m_\mu^2 + m_K^2 - 2l\cdot k + m_\delta^2 - m_\sigma^2)x + m_\delta^2 \tag{A2}$$

and the usual Feynman parametrization of the denominators in (A1) has been used. Standard techniques reduce (A2) to

$$I = \frac{f^4}{M_W^4}\left(-\frac{i}{16\pi^2}\right)\frac{\not{k} - \not{l}}{s^2}\left[1 + \frac{s^2 + m_\delta^2 - m_\sigma^2}{2s^2} \ln\frac{m_\sigma^2}{m_\delta^2} + \frac{(s^2 + m_\delta^2 - m_\sigma^2)^2 - 2m_\sigma^2 s^2}{2s^2} J\right], \tag{A3}$$

where

$$s^2 \equiv m_H^2 + m_\mu^2 - 2k\cdot l, \tag{A4}$$

$$\Delta \equiv (s^2 + m_\delta^2 - m_\sigma^2)^2 - 4s^2 m_\delta^2, \tag{A5}$$

and

$$\bar{J} = \frac{1}{\sqrt{\Delta}} \ln \frac{(m_\sigma^2 - m_\delta^2 + s^2 - \sqrt{\Delta})(m_\sigma^2 - m_\delta^2 - s^2 + \sqrt{\Delta})}{(m_\sigma^2 - m_\delta^2 + s^2 + \sqrt{\Delta})(m_\sigma^2 - m_\delta^2 - s^2 - \sqrt{\Delta})} \text{ for } \Delta > 0$$

$$= -\frac{4s^2}{(s^2 + m_\delta^2 - m_\sigma^2)(s^2 - m_\delta^2 + m_\sigma^2)} \text{ for } \Delta = 0$$

$$= \frac{1}{\sqrt{-\Delta}} \left(\tan^{-1} \frac{s^2 - m_\delta^2 + m_\sigma^2}{\sqrt{-\Delta}} + \tan^{-1} \frac{s^2 + m_\delta^2 - m_\sigma^2}{\sqrt{-\Delta}} \right) \text{ for } \Delta < 0 . \qquad \text{(A6)}$$

The quantity $J(l)$ appearing in Eq. (27) is given by the square bracket in Eq. (A3).

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[1]P. Depommier *et al.*, Phys. Rev. Lett. 39, 1113 (1977); H. P. Povel *et al.*, Phys. Lett. 72B, 183 (1977).

[2]In this paper we assume the usual scheme of assigning a muon number $L_\mu = 1$ to μ^- and an electron number $L_e = 1$ to the electron. For a discussion of other schemes see S. Frankel, in *Muon Physics*, edited by V. Hughes and C. S. Wu (Academic, New York, 1975), Vol. II, p. 83.

[3]For the most recent data see R. J. Cleveland, W. R. Leo, C. S. Wu, L. R. Kasday, P. K. Gollon, and J. D. Ullman, Phys. Rev. Lett. 35, 737 (1975).

[4]For a review on the properties of Majorana spinors see R. E. Marshak, Riazuddin, and C. P. Ryan, *Theory of Weak Interactions in Particle Physics* (Wiley-Interscience, New York, 1969), p. 66.

[5]A. Halprin, P. Minkowski, H. Primakoff, and S. P. Rosen, Phys. Rev. D 13, 2567 (1976).

[6]This type of structure exists in vectorlike gauge theories; see H. Fritzsch, M. Gell-Mann, and P. Minkowski, Phys. Lett. 59B, 256 (1976). The left-right-symmetric theories using the $SU(2)_L \times SU(2)_R \times U(1)$ gauge group also have similar doublets. The argument given in Ref. 5 also applies to these models. Currently the viability of $SU(2) \times U(1)$ vectorlike models based on SU(2) weak doublets only is placed in serious doubt by experimental data on hadronic neutral currents in high-energy ν_μ hadron scatterings. For a recent review see H. Fritzsch, in *Proceedings of the International Neutrino Conference, Aachen, 1976*, edited by H. Faissner, H. Reithler, and P. Zerwas, (Vieweg, Braunschweig, West Germany, 1977), p. 575. However, experimentally the question of vectorlike leptonic currents is still open.

[7]There exists a stringent limit on $K^\pm \to \pi^\mp e^\pm e^\pm$. See C. Y. Chang *et al.*, Phys. Rev. Lett. 20, 510 (1968). However, this gives a theoretical constraint on N_e which is consistent with the one from nuclear neutrinoless double-β decay.

[8]A similar value for the double-β decay of the kaon is obtained in Ref. 5 using a different method.

[9]G. Goldhaber *et al.*, Phys. Rev. Lett. 37, 255 (1976).

[10]The mass limit on $\nu_{\mu L}$ is only $\lesssim 6$ MeV/c^2. Hence, this case has to be considered in conjunction with the neutrino mass problem. See T. P. Cheng, Phys. Rev. D 14, 1367 (1976).

[11]A. K. Mann and H. Primakoff, Phys. Rev. D 15, 655 (1977).

[12]We use the metric notation of J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields* (McGraw-Hill, New York, 1965). Then $N_{\mu R} = \frac{1}{2}(1+\gamma_5)N_\mu$.

[13]W. Pauli, Nuovo Cimento 6, 204 (1957); F. Gürsey, *ibid.* 7, 411 (1958).

[14]The same behavior is also found in Ref. 5.

[15]The amplitude, T_2, due to the loop integral, is in general smaller than T_1. However, the $\sin^2\theta_C$ suppression in Fig. 2(c) makes the tree-diagram contribution negligible for $M_\sigma \gg M_D{}^2$ For $M_\sigma = 1.9$ GeV/c^2, $R_1 = 8 \times 10^{-5}$ sec^{-1} and for $M_\sigma = 4$ GeV/c^2 we have $R_1 = 7 \times 10^{-7}$ sec^{-1}. An upper limit for the R when T_1 and T_2 are comparable can be obtained from R_1 and R_2 by using the Schwarz inequality.

[16]I. Peruzzi *et al.*, Phys. Rev. Lett. 39, 1301 (1977).

[17]W. Braunschweig *et al.*, Phys. Lett. 70B, 132 (1977). The mass of $F^\pm$ is taken from this experiment.