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Arguments concerning an SU(3)-scalar term in the electromagnetic current operator and the value for $\Gamma(\rho\to\pi\gamma)$

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It is shown that arguments given by Edwards and Kamal, against the explanation of the experimental value $\Gamma(\rho\to\pi\gamma) = 35\pm 10$ keV by an SU(3)-scalar term in the electromagnetic current operator, are wrong. Sensitive experimental criteria for such an SU(3)-scalar term are listed and the very recently obtained experimental value of the branching ratio $(\eta'\to\rho\gamma)/(\eta'\to\omega\gamma) = 9.9\pm 2$ is discussed.

In a recent Letter,[1] Edwards and Kamal studied the vector- and pseudoscalar-meson radiative decays, and in particular the problem posed by the experimental value[2] $\Gamma(\rho\to\pi\gamma)=35\pm10$ keV. They also attempted to raise a theoretical objection to our suggestion[3] that the experimental ratio $\Gamma(\rho\to\pi\gamma)/\Gamma(\omega\to\pi\gamma)$ could be explained by the introduction of an SU(3)-scalar term V_μ^s in the electromagnetic current operator

$$V_\mu^{\mathrm{el}} = V_\mu^{\pi^0} + \frac{1}{\sqrt{3}} V_\mu^\eta + V_\mu^s, \tag{1}$$

where $V_\mu^{\pi^0} + (1/\sqrt{3})V_\mu^\eta$ is the usual Gell-Mann–Nishijima term.

The purpose of this paper is to point out that the objections of Edwards and Kamal are based upon a mistake or at least upon some unmentioned additional assumptions connecting charges with magnetic moments, which, however, are unfounded. Furthermore, in the main part of this paper, we provide criteria on the existence of an SU(3)-scalar term in the meson magnetic transition moments and compare them with the experimental data.

The possible existence of an SU(3)-scalar term has been discussed before in connection with other processes.[4,5]

Since a discussion of the ratio $\Gamma(\rho\to\pi\gamma)/\Gamma(\omega\to\pi\gamma)$ does not require SU(4), it is advantageous to restrict ourselves to SU(3). [A detailed discussion in terms of SU(4) is given in the Appendix.] The connection between the SU(3)-scalar term V_μ^s and the expressions in Refs. 1 and 3 is $V_\mu^s=\Pi[-(\frac{2}{3})^{1/2}V_\mu^\chi + V_\mu^\sigma]\Pi$ where Π is the projection operator onto the space of old hadrons. If one were to require (as is automatically done in naive quark models, for example) that all predictions of SU(3) and the Gell-Mann–Nishijima formula $V_\mu^{\mathrm{el}} = V_\mu^\pi + (1/\sqrt{3})V_\mu^\eta$ should be retained then V_μ^s would have to vanish, as already mentioned in Ref. 3. However, this condition is not necessary. All that is required of V_μ^s is that it does not contribute to the charges, because the charges must be given by the Gell-Mann–Nishijima formula. V_μ^s can, however, contribute to the magnetic moments of baryons and occur in the magnetic transition $V\to P\gamma$. Thus any argument in terms of charges is of no relevance for the existence of an SU(3)-scalar term in the magnetic transitions. The argument of Edwards and Kamal following Eq. (5) of Ref. 1 is, however, in terms of charges and therefore irrelevant.

The only way to make the arguments of Ref. 1 applicable to the problem is to postulate relations between electric and magnetic matrix elements such as those which exist for the electron and are often postulated for the quarks. Even for the diagonal matrix elements of V_μ^{el} between the baryon states there is no *a priori* relation between the charge and the magnetic moment terms; the former is entirely given by the F-type Clebsch-Gordan coefficients and the latter definitely containing also D-type contributions and possibly SU(3)-scalar contributions.[6]

After seeing that the argument of Ref. 1 is invalid, one might ask whether there are other theoretical relations which make the explanation of the experimental ratio of $\Gamma(\rho\to\pi\gamma)/\Gamma(\omega\to\pi\gamma)$ by an SU(3)-scalar term V_μ^s impossible. One suggestion of such a theoretical relation is a connection between the transition magnetic moments for $V\to P\gamma$ and the baryon magnetic moments. The possibility of an SU(3)-scalar term for the baryon magnetic moments has been discussed in Refs. 5 and 6 where it has been shown that, though the experimental data give a slight preference for (1) with a V_μ^s, the fit to the present experimental magnetic-moment values is in neither case really acceptable. But, whatever the experimental situation for the baryon magnetic moments, they do not provide information on a possible SU(3)-scalar term in the magnetic transitions of vector mesons. The contribution of V_μ^s to the baryon magnetic moment is f_2^s given by

$$\langle B | V_\mu^s | B\rangle \sim \bar{u}(f_2^s i\sigma_{\mu\nu} q^\nu)u. \tag{2}$$

(f_1^s has to be zero because the charges are given by the Gell-Mann–Nishijima formula.)

The scalar term $\mathcal{S}$ that occurs in the radiative transition is given by

$$\langle P\,|\,V_\mu^s\,|\,V\rangle \sim \mathcal{S}\epsilon_{\mu\nu\rho\sigma}p_P^\nu p_V^\rho \epsilon_V^\sigma\,. \tag{3}$$

f_2^s is a reduced matrix element in the $\frac{1}{2}^+$-baryon subspace, and $\mathcal{S}$ is a reduced matrix element in the meson subspace (the direct sum of the pseudoscalar-meson subspace and vector-meson subspace) of the same operator V_μ^s. However—unless one postulates an additional assumption which relates these matrix elements—there is no relation between these two reduced matrix elements. An operator can certainly have zero matrix elements in one subspace (e.g., $f_2^s=0$) and nonzero matrix elements in another subspace (e.g., $\mathcal{S}\neq 0$). Only from $V_\mu^s=0$ can one conclude $f_2^s=0$ and $\mathcal{S}=0$. From $f_2^s=0$ one *cannot* conclude $V_\mu^s=0$ and therewith $\mathcal{S}=0$.

Although there are no valid theoretical arguments that relate the value of $\Gamma(\rho\to\pi\gamma)/\Gamma(\omega\to\pi\gamma)$ to experimental data that were known when the investigation in Ref. 3 was undertaken, some experimental results[7,8] have recently been published which are related to $\Gamma(\rho\to\pi\gamma)/\Gamma(\omega\to\pi\gamma)$ by theoretical arguments and which may cast some doubt upon the experimental value of $\Gamma(\rho\to\pi\gamma)$. We will devote the second part of this communication to the discussion of these connections.

As the fact that SU(3) is not a symmetry group has probably a non-negligible effect upon the matrix elements, in the investigation of the form of the current operator (1) one should only use the ratios of decay rates in which the effect of a possibly mass-dependent symmetry-breaking correction factor (suppression factor) cancels as in the ratio $\Gamma(\rho\to\pi\gamma)/\Gamma(\omega\to\pi\gamma)$. Because $m_\rho\approx m_\omega$, this ratio is given entirely by the ratio of the SU(3) matrix elements. All SU(3) matrix elements $\langle P\,|\,V^{\mathrm{el}}\,|\,V\rangle$ can be expressed in terms of two arbitrary parameters[3] for which one conveniently chooses

$$d=\langle\pi^0\,|\,V^{\mathrm{el}}\,|\,\omega\rangle,\qquad S=\langle\pi\,|\,V^{\mathrm{el}}\,|\,\rho\rangle. \tag{5}$$

Whereas the absence of the SU(3)-scalar term, $V_\mu^s=0$, in (1) requires

$$(S/d)^2=(-\tfrac{1}{3})^2, \tag{6}$$

the experimental value of $\Gamma(\rho\to\pi\gamma)/\Gamma(\omega\to\pi\gamma)$ requires

$$|S/d\,|^2=\frac{1}{25.1}\pm 29\%=\frac{1}{25.1\pm 7.3}\,. \tag{7}$$

The predictions that one obtains from these values for other branching ratios are the following:

$$\frac{\Gamma(K^{+*}\to K^+\gamma)}{\Gamma(K^{0*}\to K^0\gamma)}=\tfrac{1}{4}\ \text{ for (6), i.e., with no scalar term} \tag{6a}$$

and

$$\frac{\Gamma(K^{+*}\to K^+\gamma)}{\Gamma(K^{0*}\to K^0\gamma)}\approx \tfrac{1}{16}\text{ or }\tfrac{1}{36}\ \text{ for (7), i.e., with scalar term determined from }\Gamma(\rho\to\pi\gamma)\,, \tag{7a}$$

$$\frac{\Gamma(\rho\to\eta\gamma)}{\Gamma(\omega\to\eta\gamma)}=\left|\frac{d}{S}\right|^2=\frac{9}{(8.05)}\ \text{ for (6), i.e., with no scalar term} \tag{6b}$$

and

$$\frac{\Gamma(\rho\to\eta\gamma)}{\Gamma(\omega\to\eta\gamma)}=\left|\frac{d}{S}\right|^2=\frac{25.1\pm 7.3}{(22.5\pm 6.5)}\ \text{ for (7), i.e., with scalar term determined from }\Gamma(\rho\to\pi\gamma). \tag{7b}$$

The numbers in parentheses are the predictions that are obtained if one takes the ρ-ω mass difference in the phase-space factor $m_V^3[1-(m_P/m_V)^2]^3$ into account.

Whereas $\Gamma(K^{+*}\to K^+\gamma)$ is not known, recent experimental results[7] for $\Gamma(\rho\to\eta\gamma)$ and $\Gamma(\omega\to\eta\gamma)$ give

$$\frac{\Gamma(\rho\to\eta\gamma)}{\Gamma(\omega\to\eta\gamma)}=\frac{50\pm 13}{3.0^{+2.5}_{-1.8}}\,, \tag{8}$$

which, however, is still too inaccurate to discriminate between (6) and (7). Thus the present data on the radiative decays of vector mesons are still too inaccurate for a check of the presence of an SU(3)-scalar term V_μ^s of the magnitude required by the experimental value for $\Gamma(\rho\to\pi\gamma)$.

The only published experimental value accurate enough to discriminate between (6) and (7) comes from the radiative decays of pseudoscalar mesons.[8] From one experiment of the branching ratio $(\eta'\to\omega\gamma)/(\eta'\to\eta\pi^+\pi^-)$ and the table values of other branching ratios Ref. 8 one obtains

$$\Gamma(\eta'\to\rho\gamma)/\Gamma(\eta'\to\omega\gamma)=9.9\pm 2. \tag{9}$$

As $|\langle V\,|\,V_\mu^{\mathrm{el}}\,|P\rangle|=|\langle P\,|\,V_\mu^{\mathrm{el}}\,|\,V\rangle|$ one can calculate the branching ratio (9) in terms of S and d.[9] The additional complication due to the singlet-octet mixing of the physical η'_{ph},

$$|\eta'_{\mathrm{ph}}\rangle=\cos\theta_P\,|\,\sigma\rangle+\sin\theta_P\,|\,\eta\rangle, \tag{10}$$

is irrelevant because for the usually accepted, though perhaps questionable, assumptions[10] one obtains

$$\langle\sigma\,|\,V^{\mathrm{el}}\,|\,\rho\rangle=(\tfrac{2}{3})^{1/2}d,\quad \langle\sigma\,|\,V^{\mathrm{el}}\,|\,\omega\rangle=(\tfrac{2}{3})^{1/2}S \tag{11}$$

and

$$\langle\eta\,|\,V^{\mathrm{el}}\,|\,\rho\rangle=-\frac{1}{\sqrt{3}}\,d,\quad \langle\eta\,|\,V^{\mathrm{el}}\,|\,\omega\rangle=-\frac{1}{\sqrt{3}}\,S, \tag{12}$$

which for any mixing angle leads to[10]

$$\frac{\langle \eta' | V^{el} | \rho \rangle}{\langle \eta' | V^{el} | \omega \rangle} = d/S. \tag{13}$$

Thus the prediction that one obtains for the branching ratio (9) is

$$\frac{\Gamma(\eta' \to \rho\gamma)}{\Gamma(\eta' \to \omega\gamma)} = \frac{9}{(10.4)} \text{ from (6), with no scalar term } V^s_\mu \tag{6c}$$

and

$$\frac{\Gamma(\eta' \to \rho\gamma)}{\Gamma(\eta' \to \omega\gamma)} = \frac{25.1 \pm 7.3}{(29.0 \pm 8.4)} \text{ from (7), with a scalar term determined from } \Gamma(\rho \to \pi\gamma). \tag{7c}$$

Again, the value in parentheses are the predictions with the corrected phase-space factor.

The experimental value (9) clearly favors (6) and is in contradiction to the value obtained from $\Gamma(\rho \to \pi\gamma)$.[10,11]

Summarizing, we have demonstrated that the theoretical arguments given in Ref. 1 against the explanation of the experimental value $\Gamma(\rho \to \pi\gamma) = 35 \pm 10$ keV by a magnetic SU(3)-scalar term in the electromagnetic current operator are wrong, because they are given in terms of charges which have in general no relation to the magnetic transition moments. We have then explained that the baryon magnetic moments cannot be related by theoretical arguments to the transition magnetic moment of $\rho \to \pi\gamma$. In the second part of the paper we have listed sensitive experimental criteria for such an SU(3)-scalar term in the electromagnetic current operator and have compared them with the available experimental data. We found that the two experimental values $\Gamma(\rho \to \pi\gamma) = 35 \pm 10$ keV [assuming the correctness of the well-established value for $\Gamma(\omega \to \pi\gamma)$] and $\Gamma(\eta' \to \rho\gamma)/\Gamma(\eta' \to \omega\gamma) = 9.9 \pm 2$ are incompatible under the above assumptions.[10,11] The former definitely requires an additional term in the electromagnetic current operator whereas the latter can be explained under the old assumptions. The preference one may like to give to the last value comes from the familiarity with the Gell-Mann–Nishijima formula for the electromagnetic current operator without scalar term. Though this value is under sufficiently general assumptions[10] not in conflict with an SU(3)-scalar term, the only evidence for such an additional term comes from the disputed experimental value of $\Gamma(\rho \to \pi\gamma)$.

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APPENDIX

The discussions in Refs. 1 and 3 were given in the framework of SU(4) where the electromagnetic current operator was written

$$V^{el}_\mu = V^{\pi^0}_\mu + \frac{1}{\sqrt{3}} V^{\eta}_\mu + (\tfrac{2}{3})^{1/2} V^{\chi}_\mu + V^{\sigma}_\mu; \tag{1'}$$

$V^{\pi^0}_\mu$, V^{η}_μ, and V^{χ}_μ are components of a 15-plet operator whereas V^{σ}_μ is an SU(4)-singlet operator. Edwards and Kamal argued that there must be a relationship between the reduced matrix elements of the 15-plet operator and the singlet operator and they calculate such a relationship from the *charges* of the multiplets. However, this relationship holds only for the reduced matrix elements that are connected with the charges, i.e., for the charge form factors (factor of γ_μ in the baryon matrix element of V^{el}_μ) of baryons or the reduced matrix elements between pseudoscalar mesons and pseudoscalar mesons $\langle 0^-, \{15\} \| V^{(15)} \| \{15\}, 0^- \rangle_F$ $\langle 0^-, \{15\} \| V^{(1)} \| \{15\} 0^- \rangle$, etc. The relationship derived from the charges does *not* hold for the reduced matrix elements connected with the magnetic moments such as the f_2 form factors (factors of $\sigma_{\mu\nu}$) for baryons or the reduced matrix elements between pseudoscalar mesons and vector mesons, $\langle 0^-, \{15\} \| V^{(15)} \| \{15\}, 1^- \rangle_D = \sqrt{3}\, d$ and $\langle 0^- \{\ \} \| V^{(1)} \| \{\ \}, 1^- \rangle = S$. The mistake of Edwards and Kamal was to apply the relationship derived from the charges to the magnetic moments.

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[9]S is not the matrix element of SU(3)-scalar V^s but of the SU(4)-scalar V^σ.

[10]A derivation of this result is given in A. Bohm and R. B. Teese, University of Texas report (unpublished). The assumptions that are used in this derivation are

$$\langle \text{vector meson } \{8\} \| V^8 \| \text{ pseudoscalar } m. \ \{1\}\rangle = \langle \text{pseudoscalar } \{8\} \| V^8 \| \text{vector } \{1\}\rangle = \text{real}$$

and

$$\langle \text{pseudoscalar } \{1\} \| V^s \| \text{ vector } \{1\}\rangle = \langle \text{pseudoscalar } \{8\} \| V^s \| \text{ vector } \{8\}\rangle ,$$

which can certainly be contested. Without these assumptions there are enough free parameters to make (9) compatible with (7).

[11]There exist, however, some preferences to take the $E(1420)$ rather than the $\eta'(958)=X(958)$ as the η' for the pseudoscalar-meson 16-plet, in which case the value of $\Gamma(\eta'\to\rho\gamma)/\Gamma(\eta'\to\omega\gamma)$ may have no bearing on our problem. S. Oneda *et al.*, University of Maryland, Tech. Rep. No. 76-116, 1976 (unpublished).