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Test of the Veneziano-type πNN form factor

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We have investigated Dominguez's Veneziano-type πNN form factor by attempting to use it to fit $d\sigma/dt$ data for $np \to pn$ and $\bar{p}p \to \bar{n}n$ at 8 and 23.5 GeV/c in the interval $0 < -t < 0.1$ GeV^2. With $n = 5/2$ as proposed by Dominguez it is not possible to fit the data. A fit can be obtained for other values of n.

I. INTRODUCTION

Dominguez[1] has proposed, in analogy to the parmetrization of the electromagnetic form factor[2] by

$$G(t)_{E,M} = \frac{C\Gamma(1-\alpha_\rho(t))}{\Gamma(\frac{7}{2}-\alpha_\rho(t))},$$

a Veneziano-type expression for the πNN form factor. For the divergence of the axial-vector current, he then writes

$$(m_p + m_n)G_A(t) + t\,G_p(t) = f_\pi \mu^2 F_D(t) \tag{1}$$

with

$$F_D(t) = \frac{\sqrt{2}\,G\Gamma(n)\Gamma(-\alpha_\pi(t))}{\Gamma(n-\alpha_\pi(t))}, \tag{2}$$

where, with $\alpha_\pi(t) = t - \mu^2$,

$$\lim_{t\to\mu^2} (\mu^2 - t)F_D(t) = \sqrt{2}\,G.$$

From Eq. (1) Dominguez assumed that $F_D(t)$ has the same asymptotic behavior for large negative t as $G_A(t)$, which in turn appears to scale like $G(t)_{E,M}$ (see Ref. 3) (an assumption we consider further in Sec. IV). Dominguez chose n to be equal to $\frac{5}{2}$. (n must be a half-integer so that $F_D(t)$ is built up from an infinite number of resonances—an integer would only give a finite number of resonance poles.) Thus Dominguez's final form for $F_D(t)$ is

$$F_D(t) = \frac{\sqrt{2}\,G\Gamma(\frac{5}{2})\Gamma(\mu^2 - t)}{\Gamma(\frac{5}{2}+\mu^2 - t)}, \tag{3}$$

which gives as a value for the corrections to the Goldberger-Treiman relation

$$\Delta_{\rm GTR} = 1 - \frac{\Gamma(\frac{5}{2})\Gamma(1+\mu^2)}{\Gamma(\frac{5}{2}+\mu^2)} \sim 0.03, \tag{4}$$

which is not unreasonable when compared to other theoretical estimates of $\Delta_{\rm GTR}$.[4] To provide a further experimental test of the reasonableness of the form factor of Eq. (3) we have used it to compute the average of the differential cross section for $np \to pn$ and $\bar{p}p \to \bar{n}n$.

Bongardt, Pilkuhn, and Schlaile[5] pointed out that these reactions provide a test of the πNN form factor. They show that for $0 < -t < 0.15$ GeV^2

$$\frac{1}{2}\frac{d\sigma}{dt}(np) + \frac{1}{2}\frac{d\sigma}{dt}(\bar{p}p) = \frac{\pi}{2m^2 P_{\rm lab}^2}\left(\frac{G^2}{4\pi}\right)^2 \left[\left(C_\pi + \frac{tF^2}{\mu^2 - t}\right)^2 + \left(\frac{tF^2}{\mu^2 - t}\right)^2 + D^2\right], \tag{5}$$

where C_π is the real part of the negative-G-parity, helicity-0, slowly varying background amplitude and is parametrized as $C_\pi = Ae^{bt}$ to account for the exponential dependence of the differential cross section on t. The data are averaged to eliminate the interference between the ρ- and A_2-exchange terms, which produces about 40% difference in the cross section above $-t = 0.03$ GeV^2. D^2 represents all other non-π-exchange terms and was found to be less than 10% for $0 < -t < 0.1$ GeV^2.[5]

Further, Bizard and Diu[8] have done a phenomenological analysis of np charge-exchange (CEX) data for 1 GeV/$c < p_{\rm lab} < 25$ GeV/c, using an equation which isolates the pion contribution but is essentially model independent. Their expression for small negative t looks like Eq. (5), with F^2 replaced by an exponential form factor constrained to have the same slope as C_π. [This type of form factor would give $\Delta_{\rm GTR} = 0$ and $F(\mu^2) = e^{a\mu^2/2} = 1.06$ for $a = 6$, the value found by Bizard and Diu.] Their results indicate that the data in this range is not inconsistent with a negligible value for the incoherent background (D^2) and further that the nonzero amplitudes remain remarkably energy independent, despite the expected competition between pion exchange with its associated background, and the ρ, A_2 exchanges which are expected to dominate at higher energies. (See however, Ref. 16, in which Bouquet and Diu show that at Fermilab energies the growth of energy-dependent terms in pn CEX scattering is too rapid for compatibility with ρ and A_2 exchange dominance.) In the spirit of this analysis, we set $D^2=0$. A is not expected to be very different from 1, and b must not be too large (less than 6, say[8]) otherwise we no longer have a slowly varying background.

The πNN form factor F is related to $F_D(t)$ by

$$\frac{\sqrt{2}\,GF}{\mu^2-t}=F_D(t)$$

since $F_D(t)$ already includes the pion pole. With Dominguez' parametrization of $F_D(t)$, this gives

$$F=\frac{\Gamma(\frac{5}{2})\Gamma(1+\mu^2-t)}{\Gamma(\frac{5}{2}+\mu^2-t)}. \qquad (6)$$

The data on $\frac{1}{2}[d\sigma/dt(np)+d\sigma/dt(\bar{p}p)]$, along with Eq. (5), provides a constraint on the otherwise unknown behavior of the form factor.

II. RESULTS

Fits have been made to experimental data at $P_{\rm lab}=8$ GeV/c[6,7] and 23.5 GeV/c.[9,10] The data have been smoothed and scaled to allow averaging and the value at $t=0$ has been obtained by extrapolation. In each case, the values for the constants A and b in C_π have been chosen to give the best fit. As can be seen from Fig. 1(a) and 1(b), Dominguez's expression, with the choice of $n=\frac{5}{2}$, does not satisfactorily account for the behavior of the data. We therefore turn to the possibility of varying n, which is the other parameter in Dominguez's form factor.

We now take as our form factor, for any general half-integer n,

$$F_n(t)=\frac{\Gamma(n)\Gamma(1+\mu^2-t)}{\Gamma(n+\mu^2-t)}. \qquad (7)$$

Figure 2(a) shows the best fits to the 8-GeV/c data for $n=\frac{5}{2}$, $\frac{7}{2}$, and $\frac{9}{2}$. There is quite a change between the $n=\frac{5}{2}$ (χ^2 of 6.2 per point) and the $n=\frac{7}{2}$ curve (χ^2 of 0.48 per point), but a lesser change between the $n=\frac{7}{2}$ and the $n=\frac{9}{2}$ curve (χ^2 of 0.17 per point), and in fact higher values of n also fit the curve to a similar degree of reliability. In particular we note that $n=\frac{13}{2}$, which cannot be distinguished from $n=\frac{9}{2}$ in Fig 2(a) gives a χ^2 per point of 0.14.

The behavior can be understood as follows: it can be shown using Stirling's formula, that for large n and $0<-t<0.15$,

$$F_n(t)\sim\frac{1}{n^{(\mu^2-t)}}\Gamma(1+\mu^2-t), \quad \text{as } n\to\infty. \qquad (8)$$

(This approximation is good to within about 2% for n as low as $\frac{7}{2}$ in this range of t.)

In other words, for n larger than say $\frac{7}{2}$, $F_n(t)$ is a very slowly varying function of n. The first reasonable fit occurs at $n=\frac{7}{2}$ for the 8-GeV/c data.

III. THE GOLDBERGER-TREIMAN RELATION

With the form factor written as in (7), then

$$\Delta_{\rm GTR}=1-F_n(t=0)$$
$$=1-\frac{\Gamma(n)\Gamma(1+\mu^2)}{\Gamma(n+\mu^2)}. \qquad (9)$$

For $n=\frac{7}{2}$, $\Delta_{\rm GTR}=0.033$. As n increases, the value of $\Delta_{\rm GTR}$ increases and at $n=\frac{27}{2}$, $\Delta_{\rm GTR}^{\rm theor}=0.061$ compared to the experimental value of $\Delta_{\rm GTR}=0.06\pm0.01$.[11]

Figure 2(b) shows best fits to the 23.5-GeV/c data for $n=\frac{11}{2}$, $\frac{13}{2}$, and $\frac{27}{2}$. Surprisingly, in this data there is a transition from an "undesirable"

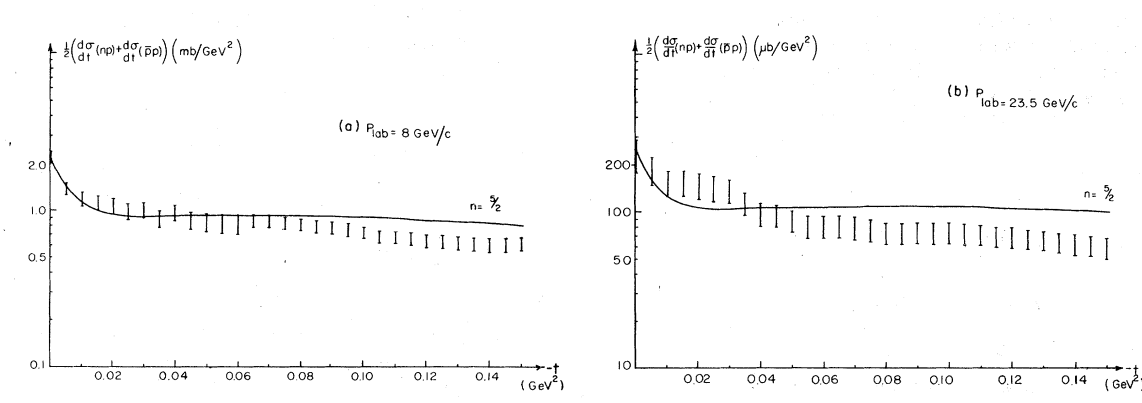

FIG. 1. (a) Plot of Eq. (5) with $n=\frac{5}{2}$, $C_\pi=0.97$ exp $(4.0t)$. The smoothed and averaged data at $P_{\rm lab}=8$ GeV/c are from Refs. 6 and 7. (b) Plot of Eq. (5) with $n=\frac{5}{2}$, $C_\pi=0.95$ exp $(3.9t)$, against smoothed and averaged data (Refs. 9 and 10) at $P_{\rm lab}=23.5$ GeV/c.

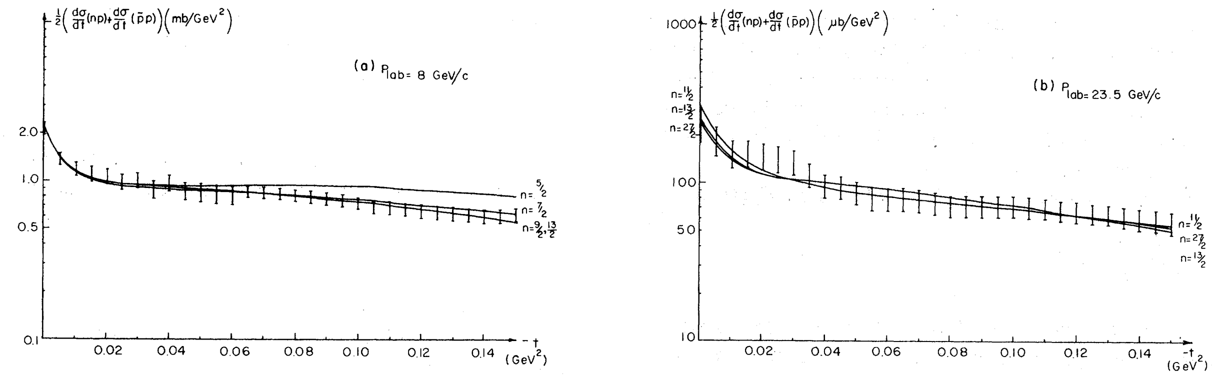

FIG. 2(a) Plots of $n=\frac{5}{2}$, $C_\pi=0.97\exp(4.0t)$; $n=\frac{7}{2}$, $C_\pi=0.98\exp(4.5t)$; $n=\frac{9}{2}$, $C_\pi=0.97\exp(3.2t)$; $n=\frac{13}{2}$, $C_\pi=0.96\exp(2.2t)$ against the $P_{\rm lab}=8$ GeV/c data (Refs. 6 and 7). (b) Plots of $n=\frac{11}{2}$, $C_\pi=1.08\exp(7.7t)$; $n=\frac{13}{2}$, $C_\pi=0.99\exp(4.1t)$; $n=\frac{27}{2}$, $C_\pi=0.95\exp(2.9t)$; against the $P_{\rm lab}=23.5$ GeV/c data (Refs. 9 and 10).

value of A and b (a large b leading to a rapidly varying background, and $A>1$ leading to a large value for $d\sigma/dt$ at $t=0$) to "desirable" ones at $n=\frac{13}{2}$ as shown in Table I. This value, $n=\frac{13}{2}$, corresponds to $\Delta_{\rm GTR}=0.046$ roughly within the limits of the theoretical value for $\Delta_{\rm GTR}$. This transition behavior is not seen in the 8-GeV/c data.

As in this case we are trying to fit the region of small $-t$ (i.e., the peak), then the first reasonable fit at 23.5 GeV/c is for $n=\frac{13}{2}$. To summarize these results, a good fit of Eq. (5) to the data at 8 and 23.5 GeV/c can be obtained for $n=\frac{13}{2}$. The values of A, b, and χ^2 per point are shown in Table II.

It is clear from our analysis that a Dominguez-type form factor with $n=\frac{5}{2}$ does not give a sufficiently rapid variation of the form factor with $-t$. Higher values of n are required, and there is an added bonus that the required n values give a much better value for the Goldberger-Treiman discrepancy.

TABLE I. Best fits for various n, at $p_L=2.35$ GeV/c. [Apparent decrease in χ^2 for increasing n does not continue indefinitely—eventually ($n\sim 770.5$, $\chi^2\sim 7.18$) there is a turn around. As $n\to\infty$, $F\to 0$, $\chi^2\to 18.1$.]

n	A	b	χ^2 (31 points)
$\frac{5}{2}$	0.95	3.9	250.2
$\frac{7}{2}$	1.01	5.4	64.7
$\frac{9}{2}$	1.05	6.6	16.4
$\frac{11}{2}$	1.08	7.7	10.8
$\frac{13}{2}$	0.99	4.1	14.7
$\frac{15}{2}$	0.98	3.6	15.0
$\frac{17}{2}$	0.97	3.3	14.6
$\frac{19}{2}$	0.96	3.0	14.2

The absence of $\bar{p}p\to\bar{n}n$ data makes it difficult to test the form factor at other energies. (A fit was attempted at $p_{\rm lab}=0.7$ GeV/c[12,13] but results were inconclusive, as is to be expected at such a low energy. n of about $\frac{25}{2}$ was required for $\chi^2=0.65$ per point.) For completeness, we mention that a contradiction has been found[15] between measurements of $d\sigma/dt$ in $\bar{p}p\to\bar{n}n$ at 40 GeV/c and expected theoretical behavior. The type of form factor discussed cannot reproduce the data without the presence of a very rapidly varying background ($C_\pi \simeq 2.0e^{63t}$). Inclusion of ρ and A_2 contributions,[15] while affecting the final curve, still do not improve the fit. It is interesting to note however, that no such anomalous behavior is seen in either the Serpukhov[9] or the Fermilab data[17] for $np\to pn$, the crossed reaction.

IV. DISCUSSION

Dominguez originally chose $G_A(t)$ to scale like $G_{E,M}(t)$,[3] which according to Di Vecchia and Drago[2] scales like

$$G_{E,M}(t)\propto t^{-5/2}, \quad \text{as } |t|\to\infty .$$

TABLE II. Fitted parameters for the $n=\frac{13}{2}$ form factor.

Beam momentum	A	b	χ^2 per point
8 GeV/c	0.96	2.2	0.14
23.5 GeV/c	0.99	4.1	0.47

However, Frampton[14] has shown that it is possible to obtain equally good fits with

$$\frac{G_M(t)}{G_A(t)} \propto t^{-1/2}, \quad \text{as } |t| \to \infty$$

and phenomenologically he finds that

$$G_A(t) \propto t^{-(1.75 \pm 0.2)}, \quad \text{as } |t| \to \infty$$

$$G_M(t) \propto t^{-(2.25 \pm 0.05)}, \quad \text{as } |t| \to \infty$$

which is not inconsistent with the asymptotic behavior found for $G_M(t)$ by Di Vecchia and Drago. Thus $G_A(t)$ has the same[3] or possibly a slower[14] asymptotic behavior than $G_M(t)$.

Since for a general value of n one can write

$$F_D(t) = \frac{\sqrt{2}\, G\Gamma(n)\Gamma(\mu^2 - t)}{\Gamma(n + \mu^2 - t)},$$
$$F_D(t) \propto t^{-n}, \quad \text{as } |t| \to \infty \tag{10}$$

and our results indicate that $n > \frac{5}{2}$, one can conclude that $F_D(t)$ has a faster asymptotic behavior than $G_M(t)$, and moreover a faster asymptotic behavior than $G_A(t)$, in direct contradiction with Dominguez' original assumption that $F_D(t)$ scales like $G_A(t)$.

[It is interesting to note here that in their phenomenological analysis of np CEX scattering, Bizard and Diu use as a form factor $F^2 = e^{at}$ where a is found to be approximately energy independent and equal to 6 in the range $1.4 < p_{\rm lab} < 25$ GeV/c. This type of form factor (necessitated by the data) has an even faster asymptotic behavior for large negative t than $F_n(t)$.]

There are two possible explanations of this. Either there is a cancellation at large t between the G_A and G_p terms of Eq. (1), which leads to F_D approaching zero more rapidly than G_A, or the PCAC (partial conservation of axial-vector current) connection between the pion form factor F, and the form factor F_D of the divergence of the axial current does not hold at large t.

Further evidence for this separation of the asymptotic behavior of F_D and G_A comes from the form factor used to fit the 8-GeV/c data by Bongardt, Pilkuhn, and Schlaile.[5] Their form factor gives the reasonable value $\Delta_{\rm GTR} = 0.05$, but does not scale like G_A.

To conclude, we reiterate that the Veneziano-type πNN form factor of Dominguez can fit the available data on the average differential cross section for np and $\bar{p}p$ scattering, and at the same time give a reasonable value for the Goldberger-Treiman discrepancy. However, at large $-t$, the asymptotic behavior implied for the πNN form factor is not that of the axial-vector form factor—the decay is more rapid.

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