PHYSICAL REVIEW D VOLUME 18, NUMBER 9 1 NOVEMBER 1978

Test of the soft-pion theorem in inclusive pion electroproduction

Susumu Koretune and Yoshiaki Masui
Department of Physics, Faculty of Science, Osaka City University, Osaka, Japan

Masatoshi Aoyama
Department of General Education, Kinki University, Higashi-Osaka, Osaka, Japan
(Received 14 March 1978)

We show that the cross section of the deep-inelastic direct pion electroproduction in the central region can be determined by the use of the PCAC (partial conservation of axial-vector current) hypothesis and the light-cone current algebra in conjunction with the scaling in x_F for each value of x_B, and that our result is consistent with the cross section of the inclusive reaction $\gamma_v p \to \pi^- X$.

The soft-pion theorem was first applied to the inclusive reaction in the neutral-pion case by N. Sakai and M. Yamada.[1] One of the present authors derived sum rules,[2] which showed that the high-energy inclusive reactions could not exist independently of the low-energy exclusive reactions, by the use of current algebra based on lightlike quantization.[3] From these theoretical approaches, it was pointed out that the PCAC (partial conservation of axial-vector current) hypothesis is applicable to the inclusive pion-production reaction in the region where the Feynman scaling variable x_F is small.[4] However, no attempt was made to compare theoretical results with experimental measurements.

In this paper, it is shown that both the inclusive soft-pion theorem and the light-cone current algebra determine the differential cross section of the processes

$$e + N \to e + \pi_s^{\pm,0} + \text{anything}, \tag{1}$$

where $\pi_s^{\pm,0}$ means soft $\pi^{\pm,0}$ and N is the initial nucleon. It is also shown, in the case of π_s^-, that the theoretical estimates, defining the condition regarded as the soft pion, are consistent with the measurements of the inclusive pion electroproduction reaction carried out by the Harvard-Cornell group.[5]

Now we consider the inclusive reaction

$$\gamma_v(k) + p(p) \to \pi_s^+(q) + \text{ anything}. \tag{2}$$

According to the usual technique,[6] we take the $q^\mu \to 0$ limit by the use of the PCAC definition

$$\partial_\mu J^{5\mu}_{1-i2} = \sqrt{2}\, m_\pi^{\,2} f_\pi \phi_{\pi^+}\,. \tag{3}$$

Then we get the soft-pion theorem as shown in Fig. 1. In the deep-inelastic region, some of the contributions to the hadronic tensor are negligible as pointed out in Refs. 1 and 2. Then $W_+^{\mu\nu}$ is given by[7,8]

$$W_+^{\mu\nu} \equiv -(g^{\mu\nu} - k^\mu k^\nu/k^2) W_1^+(\nu, k^2) + \frac{1}{M^2}(p^\mu - \nu k^\mu/k^2)(p^\nu - \nu k^\nu/k^2) W_2^+(\nu, k^2)\,, \tag{4}$$

$$= \frac{1}{4\pi M}(A + B + C + D)\,, \tag{5}$$

where + means the case of π_s^+, $\nu \equiv p \cdot k$, M is the proton mass, and[9]

$$A = \frac{1}{8 f_\pi^{\,2} (p^+)^2} \int d^4x \exp(ik\cdot x) \langle p | J_a^{5+}(0) | p\rangle \langle p | [J_b^\mu(x), J_d^\nu(0)] | p\rangle \langle p | J_c^{5+}(0) | p\rangle\,, \tag{6}$$

$$B = \frac{1}{2 f_\pi^{\,2}} g_A^{\,2}(0) (\langle n\rangle_p + \langle n\rangle_{\bar n}) \int d^4x \exp(ik\cdot x) \langle p | [J_b^\mu(x), J_d^\nu(0)] | p\rangle\,, \tag{7}$$

$$C = \frac{1}{2 f_\pi^{\,2}} f_{abe} f_{cdf} \int d^4x \exp(ik\cdot x) \langle p | [J_e^{5\mu}(x), J_f^{5\nu}(0)] | p\rangle\,, \tag{8}$$

$$D = -\frac{1}{4 f_\pi^{\,2} p^+} \Big\{ i f_{abe} \int d^4x \exp(ik\cdot x) \langle p | [J_e^{5\mu}(x), J_d^\nu(0)] | p\rangle \langle p | J_c^{5+}(0) | p\rangle - i f_{cde} \int d^4x \exp(ik\cdot x) \langle p | J_a^{5+}(0) | p\rangle \langle p | [J_b^\mu(x), J_e^{5\nu}(0)] | p\rangle \Big\}\,, \tag{9}$$

where $a \equiv 1 + i2$, $c \equiv 1 - i2$, $b \equiv d \equiv 3 + 8/\sqrt{3}$, $g_A(0)$ is the nucleon axial-vector coupling constant, $\langle n\rangle_p$ ($\langle n\rangle_{\bar n}$)

denotes the multiplicity of the proton (antineutron), and the spectral condition is used to obtain the commutator. A, B, C, and D correspond to Fig. 2(a), 2(b), 2(c), and 2(d), respectively. Since in the Bjorken limit the dominant contribution to Eqs. (6)–(9) comes from the $x^2 \simeq 0$ region, we obtain, using the light-cone current algebra,[10]

$$\begin{aligned} F_1^+(x_B) = \frac{1}{4f_\pi^2}\Big\{ & \Big[S_3(x_B) + \frac{\sqrt{3}}{3}\, i[2A_0(x_B)+A_8(x_B)]\Big] + \frac{\sqrt{3}}{9}\, g_A{}^2(0) i[2\sqrt{2}A_0(x_B) - \sqrt{3}A_3(x_B) + A_8(x_B)] \\ & + \frac{\sqrt{3}}{9}\, g_A{}^2(0)(\langle n\rangle_p + \langle n\rangle_{\bar{n}}) i[2\sqrt{2}A_0(x_B) + \sqrt{3}A_3(x_B) + A_8(x_B)] \\ & - 2g_A(0)\Big[\Big(S_3{}^5(x_B) + i\frac{\partial \bar{S}_3{}^5(x_B)}{\partial x_B}\Big) - \frac{i}{3}\Big(A_3{}^5(x_B) + i\frac{\partial \bar{A}_3{}^5(x_B)}{\partial x_B}\Big)\Big]\Big\}\ , \end{aligned} \tag{10}$$

$$F_2^+(x_B) = 2x_B F_1^+(x_B)\ , \tag{11}$$

where

$$MW_1^+(\nu, k^2) \to F_1^+(x_B) \quad \text{and} \quad \nu W_2^+(\nu, k^2)/M \to F_2^+(x_B) \tag{12}$$

as $p \cdot k$ and $k^2 \to \infty$ with $x_B = -k^2/2\nu$ fixed. The proton matrix elements of the bilocal current are defined as

$$\begin{aligned} &\langle p\,|\tfrac{1}{2}[\bar{q}(x)\gamma^\mu \tfrac{1}{2}\lambda_a q(0) + \bar{q}(0)\gamma^\mu \tfrac{1}{2}\lambda_a q(x)]\,|p\rangle \equiv p^\mu s_a(p\cdot x, x^2) + x^\mu \bar{s}_a(p\cdot x, x^2)\ , \\ &\langle p\,|\tfrac{1}{2}[\bar{q}(x)\gamma^\mu\gamma^5 \tfrac{1}{2}\lambda_a q(0) + \bar{q}(0)\gamma^\mu\gamma^5 \tfrac{1}{2}\lambda_a q(x)]\,|p\rangle \equiv \hat{s}^\mu s_a{}^5(p\cdot x, x^2) + p^\mu (x\cdot\hat{s})\bar{s}_a{}^5(p\cdot x, x^2) + x^\mu (x\cdot\hat{s})\bar{\bar{s}}_a{}^5(p\cdot x, x^2)\ , \end{aligned} \tag{13}$$

$$\begin{aligned} S_a(\eta) &\equiv (1/2\pi)\int d(p\cdot x)\exp(-i\eta p\cdot x) s_a(p\cdot x, x^2)\,|_{x^2\simeq 0}\ , \\ \bar{S}_a(\eta) &\equiv (1/2\pi)\int d(p\cdot x)\exp(-i\eta p\cdot x) \bar{s}_a(p\cdot x, x^2)\,|_{x^2\simeq 0}\ , \end{aligned} \tag{14}$$

$\hat{s}^\mu$ is the spin vector, and a similar definition is understood for the antisymmetric part. Thus we get[10]

$$\begin{aligned} F^+(p,q,k) &\equiv \frac{1}{\sigma_T}\, q^0 \frac{d\sigma^+}{d^3q} \\ &= \frac{1}{64\pi^3 f_\pi^2}\Big\{\frac{F_1^{\bar{\nu}p}(x_B)}{F_1^{ep}(x_B)} + 2g_A{}^2(0)\Big[\frac{F_1^{en}(x_B)}{F_1^{ep}(x_B)} + \langle n\rangle_p + \langle n\rangle_{\bar{n}}\Big] \\ &\qquad - \frac{2g_A(0)}{F_1^{ep}(x_B)}\Big[\Big(S_3{}^5(x_B) + i\frac{\partial \bar{S}_3{}^5(x_B)}{\partial x_B}\Big) - \frac{1}{3}i\Big(A_3{}^5(x_B) + i\frac{\partial \bar{A}_3{}^5(x_B)}{\partial x_B}\Big)\Big]\Big\}\ . \end{aligned} \tag{15}$$

The superscripts ep, en, and $\bar{\nu}p$ denote $F_1(x_B)$ in the total inclusive reaction in each process defined similarly as Eq. (12). The part proportional to $g_A(0)$, in Eq. (15), will be measured through the spin-dependent experiment in the total inclusive reactions $eN \to eX$ and $\nu N \to \mu X$. Since the measurements of these quantities are difficult, Eq. (15) is considered as another source of information on the nucleon matrix elements of the axial-vector bilocals. The same discussion is applied to π_s^- and π_s^0, and we get

$$F^-(p,q,k) = \frac{1}{64\pi^3 f_\pi^2}\Big[\frac{F_1^{\nu p}(x_B)}{F_1^{ep}(x_B)} + 2g_A{}^2(0)(\langle n\rangle_n + \langle n\rangle_{\bar{p}})\Big]\ , \tag{16}$$

$$F^0(p,q,k) = \frac{1}{64\pi^3 f_\pi^2}\,(1 + \langle n\rangle_N + \langle n\rangle_{\bar{N}})\ , \tag{17}$$

where $\langle n\rangle_N \equiv \langle n\rangle_p + \langle n\rangle_n$. Note that Figs. 2(a) and 2(d) do not contribute to F^-, since insertion of J_{1+i2}^{5+} into the initial-proton line is forbidden by isospin, and that Figs. 2(c) and 2(d) do not contribute to F^0, since the null-plane commutator be-

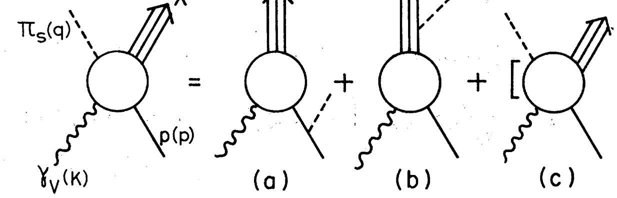

FIG. 1. The proper part of the axial-vector current (a) attaches to the initial nucleon, (b) attaches to the final nucleon and anti-nucleon (see Ref. 7), (c) shows the contribution from the equal-null-plane commutator.

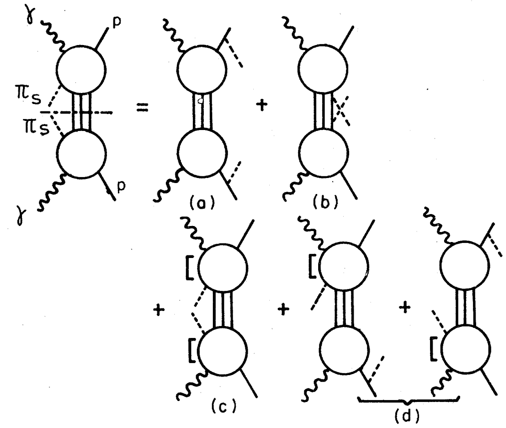

FIG. 2. The graphical expression of Eq. (5). (a), (b), and (c) is the contribution due to Figs. 1(a), 1(b) and 1(c), respectively, and 1(d) is crossing terms of Figs. 1(a) and 1(c).

tween J_3^{5+} and $J^{\mu}_{3+8/\sqrt{3}}$ vanishes. Thus F^- and F^0 are independent of axial bilocals. The application to other processes is straightforward, for example, to the inclusive reactions induced by the weak hadronic current or the inclusive reactions $e^+e^- \to \pi_s^{\pm,0}X$.[11] We will discuss these cases in a forthcoming paper; in the remaining part of this paper we discuss the validity of the soft-pion theorem for the inclusive reactions given by (1).

In the exclusive reaction, the soft-pion momenta is restricted to be less than a certain value. But in the inclusive reaction it was pointed[6] out that the assumption of PCAC in the c.m. frame in conjunction with the Feyman scaling made it possible to regard the pion in the x_F region with $q_{\parallel} < O(\sqrt{p\cdot k})$ and $q_{\perp} \simeq 0$ as the soft pion.[1] Here we assume the scaling in x_F for each value of x_B. Then we select the data which satisfy the conditions below from among the Harvard-Cornell experiment[5] of the reaction $\gamma_v p \to \pi_s^- X$, since only data on the negative pion can be easily compared with our formula. The conditions are the following:

(1) $\vec{q}_{\perp}^{\,2} < 0.02$ GeV^2.

(2) The data in the small-x_F region where the fluctuation of F^- is regarded to be small.[12]

By the use of Eq. (16) and the parton distribution functions obtained by McElhaney and Tuan,[13] F^- is given by

$$F^- = \frac{1.37(1-x_B)^{0.1}}{6.96(1+2.3x_B)+1.11(1-x_B)^{0.1}} + 0.0816\,, \qquad (18)$$

where $g_A(0) = 1.22$, f_π is determined as $0.61m_\pi$ through the Goldberger-Treiman relation,[14] and $\langle n\rangle_n + \langle n\rangle_{\bar{p}}$ is assumed to be 0.4. In Eq. (18), we neglect the sea-quark contribution since $x_B > 0.2$ in our analysis. Our theoretical results and the data selected by the conditions above are shown

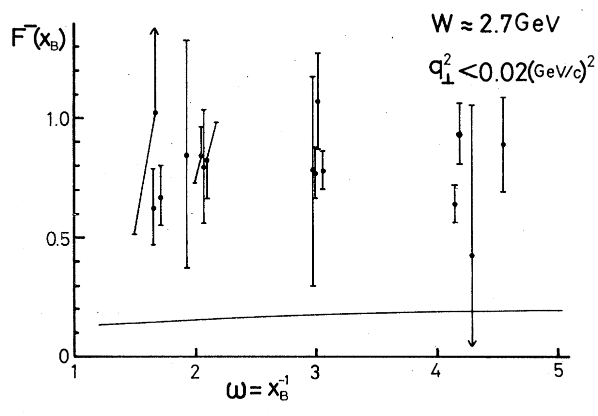

FIG. 3. The π^- invariant structure functions for $W \approx 2.7$ GeV and $q_{\perp}^2 < 0.02$ (GeV/c)^2. The solid line is the theoretical curve.

in Fig. 3.[15] The theoretical value is as the same order as the experimental one, but there is some difference between them. PCAC in the exclusive reaction, however, has validity near the pion threshold and has been used in the resonance region to express the nonresonant background.[16] Since the cross section of the inclusive reaction is the sum over the individual exclusive ones, PCAC should be applied even to the inclusive reaction according to the above sense. Then, the soft-pion theorem in inclusive reactions should not be used for resonance-decay pions. Recently, in high-energy hadronic interactions, it has been emphasized that one should distinguish between direct pions and indirect ones,[17,18] and the experimental analysis indicates that resonance-decay pions (indirect pions) are the dominant source of pion production.[18] We do not know any data concerning this point for the $\gamma_v p$ interaction, but we can expect this interaction to be similar in this respect to hadronic interactions. We may, therefore, reasonably conclude that the theoretical value is consistent with the experimental ones.[19]

In summary, we have shown that the soft pion at high energy can be regarded as the direct pion in the central region. We have also shown that the longitudinal-transverse ratio is equal to zero since the Callan-Gross relation is automatically satisfied, and that information about the nucleon matrix elements of the bilocal current is given by the experiments of direct π_s in the current-induced inclusive reaction.

ACKNOWLEDGMENTS

We would like to thank Professor T. Nakano for valuable guidance, and Dr. S. Naito, Dr. A. Niegawa, and Dr. T. Oka for useful discussions. We are also grateful to Dr. T. Odaka for interesting discussions about the direct pions.

[1]N. Sakai and M. Yamada, Phys. Lett. 37B, 505 (1971); N. Sakai, Nucl. Phys. B39, 119 (1972).

[2]S. Koretune, Phys. Rev. D 15, 132 (1977); 17, 361 (1978); Prog. Theor. Phys. 59, 1989 (1978).

[3]J. M. Cornwall and R. Jackiw, Phys. Rev. D 4, 367 (1971); H. Fritzsch and M. Gell-Mann, in *Proceedings of the XVI International Conference on High Energy Physics, Chicago-Batavia, Ill., 1972*, edited by J. D. Jackson and A. Roberts (NAL, Batavia, Ill., 1973), Vol. 2, p. 135.

[4]$x_F = p_{\parallel}^*/p_{\max}^*$, where an asterisk denotes variables in the c.m. frame.

[5]C. J. Bebek *et al.*, Phys. Rev. D 16, 1986 (1977), and references cited therein.

[6]S. L. Adler and R. F. Dashen, *Current Algebra and Application to Particle Physics* (Benjamin, New York, 1968).

[7]Here, we assume that production of hyperons is negligible.

[8]H. D. I. Abarbanel and D. J. Gross, Phys. Rev. D 5, 699 (1972); T. P. Cheng and A. Zee, *ibid.* 6, 885 (1972).

[9]The light-cone variables are defined as $p^{\pm} = 1/\sqrt{2}\,(p^0 \pm p^3)$, $g^{+-} = g^{-+} = 1$, and $g^{ij} = -\delta^{ij}$. Note, if we use the equal-time commutator, there appears the velocity factor $v = |\vec{p}\,|/p^0$ in Eqs. (6)–(9) (see Ref. 1). Thus the null-plane commutator chooses only the one point, $v = 1$.

[10]H. Fritzsch and M. Gell-Mann, in *Proceedings of Tel-Aviv Conference on Duality and Symmetry in Hadron Physics*, edited by E. Gotsman (Weizmann Science Press of Israel, Jerusalem, 1971), p. 317.

[11]A. Pais and S. B. Treiman, Phys. Rev. Lett. 25, 975 (1970).

[12]This condition is consistent with the restriction, $q_{\parallel} < m_{\pi}$, in the brick-wall frame.

[13]R. McElhaney and S. F. Tuan, Phys. Rev. D 8, 2267 (1973).

[14]M. L. Goldberger and S. B. Treiman, Phys. Rev. 110, 1178 (1958).

[15]The data only at $W = \sqrt{(p+k)^2} \approx 2.7$ GeV is selected to avoid the effect of the energy dependence in $\langle n \rangle$.

[16]S. L. Adler, in *Proceedings of the Sixth Hawaii Topical Conference in Particle Physics, University of Hawaii, 1975*, edited by P. N. Dobson, Jr., S. Pakvasa, V. Z. Peterson, and S. F. Tuan (University Press of Hawaii, Honolulu, 1976), pp. 42–72 and p. 148, and references cited therein.

[17]V. V. Anisovich and V. M. Shekhter, Nucl. Phys. B55, 455 (1973); J. D. Bjorken and G. R. Farrar, Phys. Rev. D 9, 1449 (1974).

[18]Aachen-Berlin-Bonn-CERN-Cracow-London-Vienna-Warsaw Collaboration, H. Kirk *et al.*, Nucl. Phys. B128, 397 (1977); Aachen-Berlin-Bonn-CERN-Cracow-London-Vienna-Warsaw Collaboration, H. Grässler *et al.*, *ibid.* B132, 1 (1978).

[19]According to recent data concerning $\bar{\nu} p \to \mu^+ \pi^{\pm} X$ [M. Derrick *et al.*, Phys. Rev. D 17, 1 (1978)], our theoretical estimate agrees with the ratio of direct pions in this paper.