Testing unified gauge theories with polarized-electron–nucleon elastic scattering

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The parity-violating left-right asymmetry in longitudinally-polarized-electron elastic scattering off unpolarized nucleons is studied in the contest of unified gauge theories of the weak and electromagnetic interactions. Predictions of a large variety of gauge models of present interest are given. It is shown that measurements at *low* electron beam energies, typically 150 to 800 MeV, and over a wide range of scattering angles, are of particular importance for distinguishing between various SU(2) ⊗ U(1) models and between models based on larger gauge groups, such as $SU(2)_L \otimes SU(2)_R \otimes U(1)$ and SU(3) ⊗ U(1) theories.

I. INTRODUCTION

Until now only neutrino-nucleon scattering and atomic-physics experiments have been performed to establish the existence of neutral weak currents predicted by unified gauge theories of weak and electromagnetic interactions.[1] Specifically, most of these theories suggested so far predict very different neutral (Z^0) couplings to leptonic and hadronic vertices, which can hardly be tested in detail with conventional (unpolarized) scattering experiments. However, the recent precision measurements of the optical rotation in Bi atoms[2] due to parity violation of weak neutral currents could constitute an ideal test for the parity structure of leptonic and hadronic neutral currents.[3,4] Unfortunately, the theoretical assumptions needed to interpret the atomic-physics experiments are not quite certain, depending on models for the atomic wave function and for many-electron effects.[4,5]

To get further information on neutral-current parity-violating asymmetries, there are (longitudinally)-polarized-electron–proton deep-inelastic scattering experiments under construction at SLAC,[6] and for elastic scattering at Mainz[7] and MIT-Yale.[8] These experiments constitute a rather clean test for models of unified gauge theories since the theoretical calculations of the asymmetries do not depend on additional model assumptions about the nuclear or atomic wave functions. Especially in the elastic scattering experiments[7,8] the asymmetries will be measured to a level of 10^{-5} to 10^{-7}, whereas the theoretically predicted values[9] are in the range of about 10^{-5} for electron laboratory energies of $E \simeq 300$ MeV. When this kind of experiment was originally proposed,[9] the parity-violating asymmetry, originating from an interference of the weak Z^0-exchange and the usual QED γ-exchange amplitudes, was exclusively studied within the framework of the standard Weingerg-Salam model.[10] Recently, Cahn and Gilman[11] gave predictions for a wider class of gauge theories concentrating on only high-energy experiments ($E \simeq 20$ GeV, appropriate for SLAC[6]) where $-q^2/2ME \ll 1$ with M being the nucleon mass—an approximation which is certainly not adequate for the Mainz[7] and MIT-Yale[8] experiments. Further, by this approximation only those terms in the asymmetry survive which consist of the neutral axial-vector current of the electron and of the vector current of the nucleons, whereas the contribution from the electronic vector current and the nucleonic axial-vector current is neglected. This leads, for example, to a zero prediction for the SU(3)⊗SU(1) model of Lee and Weinberg,[12] whereas an exact calculation at lower energies gives a value greater than for the standard Weinberg-Salam model. Thus, sticking to measurements of parity-violating left-right asymmetries at high energies only, one gives up the possibility of testing and differentiating between a variety of gauge theories with a vanishing neutral axial-vector coupling of electrons, such as in SU(2)⊗U(1) with $I^e_{3R} = -\frac{1}{2}$ or in the Lee-Weinberg SU(3)⊗U(1) model.

In Sec. II we present the general theoretical framework leading to the exact expressions for the parity-violating asymmetry for both proton and neutron targets as a function of the vector and axial-vector parts of the weak-neutral currents of the electrons and of the dominant u and d quarks. In doing so we have to express the weak-neutral nucleonic form factors through the fundamental neutral-weak couplings of electrons and quarks, fixed by the given gauge theory, and through the known electromagnetic and charged weak form factors. In Sec. III we generally classify and discuss the details of most unified gauge theories suggested so far, and give numerical predictions of only those models which are in agreement with present elastic and inelastic neutrino experiments. These quantitative predictions are presented for various beam energies (E = 150, 300, 600 MeV) and scat-

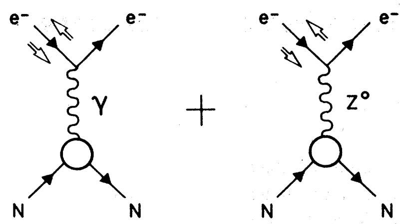

FIG. 1. Lowest-order Feynman diagrams contributing to longitudinally-polarized-electron elastic scattering off unpolarized nucleons.

tering angles, and for various values of the Weinberg angle or its equivalents. Finally, our conclusions are summarized in Section IV.

II. THEORETICAL FRAMEWORK

A. Left-right asymmetry

The most general expressions for the fundamental weak-neutral current of leptons and quarks can be written as

$$j_r^{0\mu}=g_V^r\bar{r}\gamma^\mu r+g_A^r\bar{r}\gamma^\mu\gamma_5 r\,, \tag{1}$$

with $r=e,u,d,\ldots$, and where the vector and axial-vector couplings g_V^r and g_A^r are uniquely determined by the gauge model under consideration, as we shall see below. Assuming time-reversal invariance and only first-class currents, the electromagnetic and weak nucleon form factors are defined by ($N=p$ or n)

$$\begin{aligned}J_N^{\mathrm{em},\mu}&\equiv\langle N(p')\,|j^{\mathrm{em},\mu}\,|N(p)\rangle\\&=\bar{N}(p')[G_M^N\gamma^\mu-F_2^N(p+p')^\mu]N(p)\end{aligned} \tag{2}$$

for the electromagnetic current, and for the weak-neutral current

$$\begin{aligned}J_N^{0\mu}&\equiv\langle N(p')\,|j^{0\mu}\,|N(p)\rangle\\&=\bar{N}(p')[g_V^N\gamma^\mu-g_A^N\gamma^\mu\gamma_5\\&\qquad-f_V^N(p+p')^\mu-h_A^N(p-p')^\mu\gamma_5]N(p)\,,\end{aligned} \tag{3}$$

and similarly for the three-component of the weak current

$$\begin{aligned}J_N^{3\mu}&\equiv\langle N(p')\,|j^{3\mu}\,|N(p)\rangle\\&=\bar{N}(p')[g_{3V}^N\gamma^\mu-g_{3A}^N\gamma^\mu\gamma_5\\&\qquad-f_{3V}^N(p+p')^\mu-h_{3A}^N(p-p')^\mu\gamma_5]N(p)\,.\end{aligned} \tag{4}$$

The nucleonic charged weak current is defined by

$$\begin{aligned}J^{c\mu}&\equiv\langle n(p')\,|j^{c\mu}\,|p(p)\rangle\\&=\bar{n}(p')[g_{cV}\gamma^\mu-g_{cA}\gamma^\mu\gamma_5\\&\qquad-f_{cV}(p+p')^\mu-h_{cA}(p-p')^\mu\gamma_5]p(p)\,.\end{aligned} \tag{5}$$

The form factors in Eqs. (2)–(5) are functions of $q^2=(p-p')^2<0$.

It is now straightforward to calculate the amplitudes for the diagrams depicted in Fig. 1 for longitudinally left-handed (L) or right-handed (R) polarized incoming electrons and unpolarized nucleon targets. Denoting the weak neutral coupling constant by $\tilde{g}$ one obtains, to leading order,

$$|\mathfrak{M}_R^N|^2+|\mathfrak{M}_L^N|^2=\frac{4\pi^2\alpha^2}{m^2M^2q^4}\{[(G_M^N-2MF_2^N)^2-q^2(F_2^N)^2][8M^2E^2+2q^2(M^2+2ME)]+q^4(G_M^N)^2\} \tag{6}$$

and

$$\begin{aligned}|\mathfrak{M}_R^N|^2-|\mathfrak{M}_L^N|^2=\frac{2\pi\alpha\tilde{g}^2}{m^2M^2M_Z^2q^2}\,&\{g_A^e g_V^N[(G_M^N-2MF_2^N)(8M^2E^2+4MEq^2+2M^2q^2)+q^4G_M^N]\\&-g_A^e f_V^N[2MG_M^N-(4M^2-q^2)F_2^N][8M^2E^2+4MEq^2+2M^2q^2]+g_V^e g_A^N G_M^N(4MEq^2+q^4)\}\,,\end{aligned} \tag{7}$$

where m is the lepton mass, M_Z denotes the mass of the exchanged neutral vector boson, and E is the laboratory beam energy of the incoming lepton. The calculation was made in the limit of $m^2\ll M^2-q^2$ but it differs quantitatively from the exact results[13] ($m\neq 0$) only by about 30% if, instead of electrons, a muon beam is used at $E\simeq 300$ MeV and for small scattering angles. Furthermore, in the Z^0 propagator we have neglected terms of order q^2/M_Z^2. To lowest order, Eq. (6) is just twice the square of the usual Rosenbluth amplitude. The parity-violating left-right asymmetry is then defined by

$$A_N\equiv\frac{d\sigma_R^N-d\sigma_L^N}{d\sigma_R^N+d\sigma_L^N}=\frac{|\mathfrak{M}_R^N|^2-|\mathfrak{M}_L^N|^2}{|\mathfrak{M}_R^N|^2+|\mathfrak{M}_L^N|^2}\,. \tag{8}$$

For models with more than one neutral current[14,15] one simply has to sum in Eq. (7) over their individual contributions.

B. Relations between form factors

Since common gauge models specify only the fundamental vertices in Eq. (1), the left-right asymmetry can be uniquely calculated once we know the nucleonic weak neutral form factors defined in Eq. (3) and needed in Eq. (7). These experimentally unknown neutral form factors can be related to the fundamental couplings $g^{r}_{V,A}$ and to the known electromagnetic and charged weak form factors of Eqs. (2) and (5), respectively, in the following way. The electromagnetic current of the proton is, neglecting the small contributions of s and c quarks,

$$J_p^{\mathrm{em},\mu} = \langle p \,|\, \tfrac{2}{3}\bar{u}\gamma^\mu u - \tfrac{1}{3}\bar{d}\gamma^\mu d \,|\, p\rangle \tag{9}$$

and with an isospin rotation one obtains for the neutron

$$J_n^{\mathrm{em},\mu} = \langle p \,|\, \tfrac{2}{3}\bar{d}\gamma^\mu d - \tfrac{1}{3}\bar{u}\gamma^\mu u \,|\, p\rangle , \tag{10}$$

or

$$\begin{aligned} \langle p\,|\bar{u}\gamma^\mu u\,|p\rangle &= 2J_p^{\mathrm{em},\mu} + J_n^{\mathrm{em},\mu} ,\\ \langle p\,|\bar{d}\gamma^\mu d\,|p\rangle &= J_p^{\mathrm{em},\mu} + 2J_n^{\mathrm{em},\mu} , \end{aligned} \tag{11}$$

with $J_N^{\mathrm{em},\mu}$ given by Eq. (2). Furthermore, using the SU(6) quark representation of proton and neutron wave functions, the ratios of isoscalar and isovector weak vector and axial-vector currents are given by[16]

$$\begin{aligned} \frac{\langle p\,|\bar{u}\gamma^\mu u + \bar{d}\gamma^\mu d\,|p\rangle}{\langle p\,|\bar{u}\gamma^\mu u - \bar{d}\gamma^\mu d\,|p\rangle} &= 3 \\ \frac{\langle p\,|\bar{u}\gamma^\mu\gamma_5 u + \bar{d}\gamma^\mu\gamma_5 d\,|p\rangle}{\langle p\,|\bar{u}\gamma^\mu\gamma_5 u - \bar{d}\gamma^\mu\gamma_5 d\,|p\rangle} &= \frac{3}{5} \end{aligned} \tag{12}$$

which imply

$$\langle p\,|\bar{u}\gamma^\mu u\,|p\rangle = 2\langle p\,|\bar{d}\gamma^\mu d\,|p\rangle \tag{13}$$

$$\langle p\,|\bar{u}\gamma^\mu\gamma_5 u\,|p\rangle = -4\langle p\,|\bar{d}\gamma^\mu\gamma_5 d\,|p\rangle . \tag{14}$$

Needless to say that in general these relations will hold only approximately.[16]

For a given gauge theory the most general expression for $j^{3\mu}$ in Eq. (4) reads

$$\begin{aligned} j^{3\mu} = {} & (I_{3R}^u + I_{3L}^u)\bar{u}\gamma^\mu u + (I_{3R}^u - I_{3L}^u)\bar{u}\gamma^\mu\gamma_5 u \\ & + (I_{3R}^d + I_{3L}^d)\bar{d}\gamma^\mu d + (I_{3R}^d - I_{3L}^d)\bar{d}\gamma^\mu\gamma_5 d , \end{aligned} \tag{15}$$

which, together with $J_p^{3\mu} = \langle p\,|j^{3\mu}\,|p\rangle$ and Eqs. (11) and (14), yields

$$\begin{aligned} \langle p\,|\bar{u}\gamma^\mu\gamma_5 u\,|p\rangle = {} & \frac{4}{4I_{3R}^u - 4I_{3L}^u - I_{3R}^d + I_{3L}^d} \\ & \times [J_p^{3\mu} - (2I_{3R}^u + 2I_{3L}^u + I_{3R}^d + I_{3L}^d)J_p^{\mathrm{em},\mu} \\ & - (I_{3R}^u + I_{3L}^u + 2I_{3R}^d + 2I_{3L}^d)J_n^{\mathrm{em},\mu}] . \end{aligned} \tag{16}$$

This, using Eq. (14), gives $\langle p\,|\bar{d}\gamma^\mu\gamma_5 d\,|p\rangle$. Extending the conserved-vector-current (CVC) hypothesis to the neutral currents where there are both isovector and isoscalar terms, one can express[16] the form factors of the three-component of the weak current in Eq. (4) through the known electromagnetic and charged weak form factors in Eqs. (2) and (5): By applying Eq. (12) to the isovector and isoscalar parts of (15) one obtains

$$\begin{aligned} g_{3V}^p = c_1[& \tfrac{1}{2}(I_{3R}^u + I_{3L}^u - I_{3R}^d - I_{3L}^d)(G_M^p - G_M^n) \\ & + \tfrac{3}{2}(I_{3R}^u + I_{3L}^u + I_{3R}^d + I_{3L}^d)(G_M^p + G_M^n)] \end{aligned} \tag{17}$$

and the same for f_{3V}^p with F_2 instead of G_M, and

$$\begin{aligned} g_{3A}^p = c_2[& \tfrac{1}{2}(I_{3R}^u - I_{3L}^u - I_{3R}^d + I_{3L}^d) \\ & + \tfrac{3}{10}(I_{3R}^u - I_{3L}^u + I_{3R}^d - I_{3L}^d)]g_{cA} \end{aligned} \tag{18}$$

and the same for h_{3A}^p with h_{cA} instead of g_{cA}. To fix the normalization constants $c_{1,2}$ let us consider the standard SU(2)⊗U(1) model[10] where $I_{3R}^u = I_{3R}^d = 0$, $I_{3L}^u = +\frac{1}{2}$, and $I_{3L}^d = -\frac{1}{2}$, for which the weak isospin is identical to the nuclear isospin. Therefore, in this model one gets by an isospin rotation

$$\begin{aligned} & g_{3V}^p = \tfrac{1}{2}g_{cV} , \quad f_{3V}^p = \tfrac{1}{2}f_{cV} , \\ & g_{3A}^p = \tfrac{1}{2}g_{cA} , \quad h_{3A}^p = \tfrac{1}{2}h_{cA} , \end{aligned} \tag{19}$$

and, by the usual CVC theorem

$$g_{cV} = G_M^p - G_M^n , \quad f_{cV} = F_2^p - F_2^n . \tag{20}$$

Since the general Eqs. (17) and (18) must hold for any model under consideration, we have to choose, according to Eqs. (19) and (20), the normalization constants to be $c_1 = +1$ and $c_2 = -1$. Therefore, Eqs. (17) and (18) yield the general relations

$$\begin{aligned} g_{3V}^p = {} & (2I_{3R}^u + 2I_{3L}^u + I_{3R}^d + I_{3L}^d)G_M^p \\ & + (I_{3R}^u + I_{3L}^u + 2I_{3R}^d + 2I_{3L}^d)G_M^n \end{aligned} \tag{21}$$

and similarly for f_{3V}^p with F_2 instead of G_M, and

$$g_{3A}^p = \tfrac{1}{5}(-4I_{3R}^u + 4I_{3L}^u + I_{3R}^d - I_{3L}^d)g_{cA} \tag{22}$$

and similarly for h_{3A}^p with h_{cA} instead of g_{cA}. Inserting these expressions into Eq. (16), using Eqs. (2) and (4), one obtains

$$\begin{aligned} \langle p\,|\bar{u}\gamma^\mu\gamma_5 u\,|p\rangle &= \tfrac{4}{5}\bar{p}(p')[g_{cA}\gamma^\mu\gamma_5 + h_{cA}(p-p')^\mu\gamma_5]p(p) \\ \langle p\,|\bar{d}\gamma^\mu\gamma_5 d\,|p\rangle &= -\tfrac{1}{5}\bar{p}(p')[g_{cA}\gamma^\mu\gamma_5 + h_{cA}(p-p')^\mu\gamma_5]p(p) . \end{aligned} \tag{23}$$

Thus, Eqs. (11) and (23) allow us to express the neutral weak current in Eq. (3) completely through the electromagnetic and charged weak form factors

$$\begin{aligned} J_p^{0\mu} = {} & \langle p(p')\,|g_V^u\bar{u}\gamma^\mu u + g_A^u\bar{u}\gamma^\mu\gamma_5 u + g_V^d\bar{d}\gamma_\mu d + g_A^d\bar{d}\gamma^\mu\gamma_5 d\,|p(p)\rangle \\ = {} & \bar{p}(p')\{[(2g_V^u + g_V^d)G_M^p + (g_V^u + 2g_V^d)G_M^n]\gamma^\mu - [(2g_V^u + g_V^d)F_2^p + (g_V^u + 2g_V^d)F_2^n](p+p')^\mu \\ & - (-\tfrac{4}{5}g_A^u + \tfrac{1}{5}g_A^d)g_{cA}\gamma^\mu\gamma_5 - (-\tfrac{4}{5}g_A^u + \tfrac{1}{5}g_A^d)h_{cA}(p-p')^\mu\gamma_5\}p(p) , \end{aligned} \tag{24}$$

which, when compared with Eq. (3), yields the final relations for the weak neutral form factors

$$
\begin{aligned}
g_V^p &= (2g_V^u + g_V^d)G_M^p + (g_V^u + 2g_V^d)G_M^n\,,\\
f_V^p &= (2g_V^u + g_V^d)F_2^p + (g_V^u + 2g_V^d)F_2^n\,,\\
g_A^p &= (-\tfrac{4}{5}g_A^u + \tfrac{1}{5}g_A^d)g_{cA}\,,\\
h_A^p &= (-\tfrac{4}{5}g_A^u + \tfrac{1}{5}g_A^d)h_{cA}\,.
\end{aligned}
\tag{25}
$$

By analogy, similar expressions follow for the neutral weak form factors of neutrons[13]

$$
\begin{aligned}
g_V^n &= (g_V^u + 2g_V^d)G_M^p + (2g_V^u + g_V^d)G_M^n\,,\\
f_V^n &= (g_V^u + 2g_V^d)F_2^p + (2g_V^u + g_V^d)F_2^n\,,\\
g_A^n &= (\tfrac{1}{5}g_A^u - \tfrac{4}{5}g_A^d)g_{cA}\,,\\
h_A^n &= (\tfrac{1}{5}g_A^u - \tfrac{4}{5}g_A^d)h_{cA}\,,
\end{aligned}
\tag{26}
$$

which follow from Eq. (25) by interchanging the quark labels u with d. For a given process $eN \to eN$ and a given gauge model, Eqs. (25) and (26) uniquely determine Eq. (7), i.e. the asymmetry in (8), using

$$
\begin{aligned}
\frac{G_M^p(q^2)}{1+\mu_p} \simeq \frac{G_M^n(q^2)}{\mu_n} \simeq \frac{F_2^p(q^2)}{\mu_p/2M} \simeq \frac{F_2^n(q^2)}{\mu_n/2M}\\
\simeq \frac{g_{cA}(q^2)}{1.2} \simeq \left(1 - \frac{q^2}{0.71\ \mathrm{GeV}^2}\right)^{-2},
\end{aligned}
\tag{27}
$$

with $\mu_p = 1.79$ and $\mu_n = -1.91$.

III. GAUGE THEORIES AND NUMERICAL RESULTS

A. Unified gauge models

For our actual calculations we shall use the following seven general classes of unified gauge theories which yield nontrivial results for the left-right asymmetry A_N: Models A–E all have a SU(2) $\otimes$U(1) gauge structure with $I_{3L}^e = -\frac{1}{2}$, $I_{3L}^u = +\frac{1}{2}$, $I_{3L}^d = -\frac{1}{2}$, and

(A) $I_{3R}^e = 0$, $I_{3R}^u = 0$, $I_{3R}^d = 0$ (the standard model[10]);

(B) $I_{3R}^e = 0$, $I_{3R}^u = 0$, $I_{3R}^d = +\frac{1}{2}$ (Refs. 16 and 17);

(C) $I_{3R}^e = -\frac{1}{2}$, $I_{3R}^u = 0$, $I_{3R}^d = +\frac{1}{2}$ (Refs. 16 and 17);

(D) $I_{3R}^e = -\frac{1}{2}$, $I_{3R}^u = 0$, $I_{3R}^d = 0$ (the hybrid model[18]);

(E) $I_{3R}^e = -\frac{1}{2}$, $I_{3R}^u = +\frac{1}{2}$, $I_{3R}^d = 0$ (Refs. 19–24);

(F) $SU(2)_L \otimes SU(2)_R \otimes U(1)$ ambidextrous model[18,25];

(G) SU(3)$\otimes$U(1) model of Lee and Weinberg[12];

(H) $SU(2)_L \otimes SU(2)_R \otimes U(1)$ models of the kind suggested in Refs. 15, 26, and 27 have no parity violation in eN interactions and give trivially $A_N = 0$.

The values of the fundamental couplings $g_{V,A}^r$ and the weak neutral coupling constant $\tilde{g}$ for these models are summarized in Table I. For the SU(2) $\otimes$U(1) models A–E the general expressions for these couplings are

$$
g_V^r = I_{3R}^r + I_{3L}^r - 2Q_r \sin^2\theta_W\,,\quad g_A^r = I_{3R}^r - I_{3L}^r\,,
\tag{28}
$$

with $Q_e = -1$, $Q_u = +\frac{2}{3}$, $Q_d = -\frac{1}{3}, \ldots$. Note that mod-

TABLE I. Values for the fundamental weak neutral coupling constants ($G_F = 1.01 \times 10^{-5} M^{-2}$).

Model	$\tilde{g}^2/M_Z{}^2$	g_V^e	g_A^e	g_V^u	g_A^u	g_V^d	g_A^d	Field	A_N
A	$\sqrt{2}G_F$	$-\frac{1}{2}+2\sin^2\theta_W$	$\frac{1}{2}$	$\frac{1}{2}-\frac{4}{3}\sin^2\theta_W$	$-\frac{1}{2}$	$-\frac{1}{2}+\frac{2}{3}\sin^2\theta_W$	$\frac{1}{2}$	Z^0	Finite
B	$\sqrt{2}G_F$	$-\frac{1}{2}+2\sin^2\theta_W$	$\frac{1}{2}$	$\frac{1}{2}-\frac{4}{3}\sin^2\theta_W$	$-\frac{1}{2}$	$\frac{2}{3}\sin^2\theta_W$	1	Z^0	Finite
C	$\sqrt{2}G_F$	$-1+2\sin^2\theta_W$	0	$\frac{1}{2}-\frac{4}{3}\sin^2\theta_W$	$-\frac{1}{2}$	$\frac{2}{3}\sin^2\theta_W$	1	Z^0	Finite
D	$\sqrt{2}G_F$	$-1+2\sin^2\theta_W$	0	$\frac{1}{2}-\frac{4}{3}\sin^2\theta_W$	$-\frac{1}{2}$	$-\frac{1}{2}+\frac{2}{3}\sin^2\theta_W$	$\frac{1}{2}$	Z^0	Finite
E	$\sqrt{2}G_F$	$-1+2\sin^2\theta_W$	0	$1-\frac{4}{3}\sin^2\theta_W$	0	$-\frac{1}{2}+\frac{2}{3}\sin^2\theta_W$	$\frac{1}{2}$	Z^0	Finite
F	$\frac{G_F}{\sqrt{2}}\cos^2\beta$	$-\frac{1}{2}+2\sin^2\theta_W$	$\frac{1}{2}$	$\frac{1}{2}-\frac{4}{3}\sin^2\theta_W$	$-\frac{1}{2}$	$-\frac{1}{2}+\frac{2}{3}\sin^2\theta_W$	$\frac{1}{2}$	W^0	Finite
	$\frac{G_F}{\sqrt{2}}\tan^2\alpha\cos^2\beta$	$-1+4\sin^2\theta_W$	0	$1-\frac{8}{3}\sin^2\theta_W$	0	$-1+\frac{4}{3}\sin^2\theta_W$	0	V^0	0
G	$G_F\frac{1+l}{2\sqrt{2}}$	$-1+3w$	0	$1-2w$	-1	$-1+w$	0	Z^0	Finite
	$\sqrt{2}G_F\frac{1+l}{l}$	0	$\frac{1}{2}$	0	0	0	$\frac{1}{2}$	Y^0	0
H	$\frac{G_F}{\sqrt{2}}\frac{1}{1+\epsilon}$	0	1	0	-1	0	1	Z_1	0
	$\frac{G_F}{\sqrt{2}}\frac{1}{1-\epsilon}$	$1-2\sin^2\theta_W$	0	$-\frac{1}{3}+\frac{4}{3}\sin^2\theta_W$	0	$-\frac{1}{3}-\frac{2}{3}\sin^2\theta_W$	0	Z_2	0

els C–E and G correspond to $g_A^e=0$, still predicting a nontrivial asymmetry $A_N \neq 0$, and are thus in agreement with the small parity violation found in the atomic Bi experiments[2] which are sensitive only to the interference of g_A^e and the hadronic weak neutral vector current. The contributions of these models to our left-right asymmetry A_N is then entirely due to the leptonic vector current (g_V^e) term in Eq. (7). Thus, at least for this class of gauge theories, a measurement of A_N would yield a clean test for the structure of the electronic vector and hadronic axial-vector weak neutral current. The essential difference of the $SU(2)_L \otimes SU(2)_R \otimes U(1)$ models of type F and H is, that one of the two neutral weak vector bosons in H has only vector couplings and the other has only axia-vector couplings, giving no parity violation in the interactions between electrons and nucleons. Model F, on the other hand, has one neutral gauge field with purely vector couplings, and one is just like the Z^0 in the standard $SU(2)\otimes U(1)$ model, the contribution of which to parity violation is, however, reduced by a factor $\cos^2\beta$, with β being a mixing angle.[25]

Besides the types of $SU(2)\otimes U(1)$ models listed above there exist also various other isosinglet and isodoublet classifications, but most of them appear to be in conflict[28] with (anti)neutrino-electron and (anti)neutrino-hadron scattering experiments. Hagiwara and Tagasugi[28] have done a comprehensive study of all these possible representations for $SU(2)\otimes U(1)$ models and came to the conclusion that only leptonic parts with $I_{3L}^e=-\frac{1}{2}$ and $I_{3R}^e=0$ or $-\frac{1}{2}$ are allowed by the experimental values of g_V^e and g_A^e from neutrino reactions, provided multiplets of order higher than 3 are not taken into account. Considering $SU(2)\otimes U(1)$ models with the standard left-handed doublets (and perhaps additional new ones) then, because of $\sigma(\bar\nu N \to \bar\nu X)/\sigma(\nu N \to \nu X) < 1$ and $\sigma(\bar\nu p \to \bar\nu p)/\sigma(\nu p \to \nu p) < 1$, only models with $(I_{3R}^u, I_{3R}^d) = (0,0)$, $(\frac{1}{2},0)$, $(0,\frac{1}{2})$, $(-\frac{1}{2},0)$, and $(0,-\frac{1}{2})$ are allowed. Furthermore, the measured ratio $\sigma(\bar\nu p \to \bar\nu p)/\sigma(\nu p \to \nu p) = 0.4 \pm 0.2$ eliminates models with $(I_{3R}^u, I_{3R}^d) = (-\frac{1}{2},0)$, and the choice $(0,-\frac{1}{2})$ can be excluded because it predicts too flat a q^2 dependence for $d\sigma(\nu p\to\nu p)/dq^2$ and $d\sigma(\bar\nu p \to \bar\nu p)/dq^2$ in disagreement[28] with experiment. Therefore, $SU(2)\otimes U(1)$ models with only $(I_{3R}^u, I_{3R}^d) = (0,0)$, $(\frac{1}{2},0)$, and $(0,\frac{1}{2})$ survive. In our calculations we did not use the model corresponding to $I_{3R}^e=0$, $I_{3R}^u=+\frac{1}{2}$, and $I_{3R}^d=0$ since it predicts a high-y anomaly for antineutrino scattering as well as a large atomic parity violation, in disagreement with experiment.

B. Numerical results

In Figs. 2–4 we show the values of the left-right asymmetry A_N for both proton and neutron targets as predicted by the general types of gauge models A–G for electron beam energies $E=150$, 300, and 600 MeV and as a function of the lab scattering angle θ. Here we have used a Weinberg angle $\sin^2\theta_W=0.3$ or its equivalents, corresponding to the best-fitted results obtained from presently known neutrino experiments.[1,29] For the $SU(3)\otimes U(1)$ model we have taken[12] $l=w=0.2$. Recall that the predictions of model H are identically 0. As can be seen from Figs. 2–4 the asymmetries $A_{p,n}$ increase by typically a factor of 2 by doubling the energy. Similarly, the asymmetries for neutron targets are larger[9] in magnitude by a factor of 2 to 7 than those for proton targets, depending on the model under consideration. Note especially the large asymmetries resulting from the hybrid model D and from the $SU(3)\otimes U(1)$ model G which, for proton targets, are about 4 to 10 times larger than the results of the standard $SU(2)\otimes U(1)$ model A. This clearly shows the importance of low-energy ($E \lesssim 1$ GeV) elastic scattering experiments[7,8] for discriminating between different types of gauge theories, since at high energies[6] such that $-q^2/$

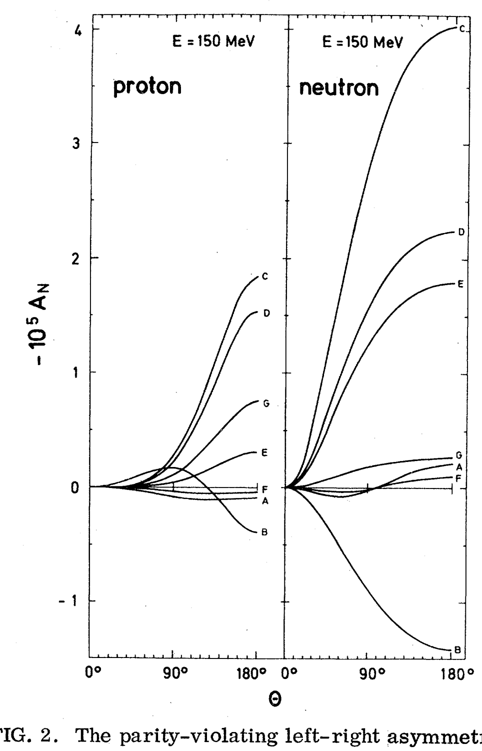

FIG. 2. The parity-violating left-right asymmetry $A_{N=p,n}$ of Eq. (8) for unpolarized proton and neutron targets and for an electron beam energy $E=150$ MeV, as a function of the laboratory scattering angle. The predictions for the $SU(2)\otimes U(1)$ models A–E correspond to $\sin^2\theta_W=0.3$, and for the ambidextrous model F we have used in addition $\cos^2\beta=0.95$. The $SU(3)\otimes U(1)$ model G has been calculated with $l=w=0.2$.

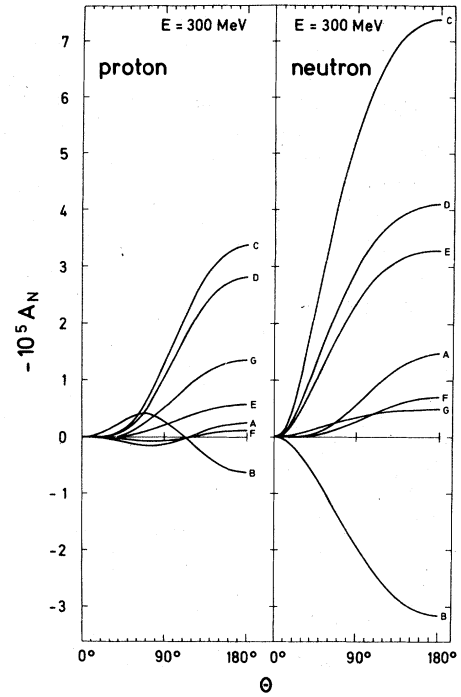

FIG. 3. The asymmetries A_p and A_n at $E = 300$ MeV. Notation and input parameters are as in Fig. 2.

$2ME \ll 1$, A_N becomes negligibly small[11] for C–E and G.

Most of the asymmetries in Figs. 2–4 show a very pronounced angular dependence. Thus, detailed measurements of A_N in the forward as well as backward hemisphere for proton as well as neutron (deuteron) targets should enable us to distinguish rather firmly between different unified gauge theories. In addition, this kind of experiment should provide us with a rather clean information on the right-handed isospin structure of the electron because of the very different angular dependence predicted by models A and D, or by models B and C. On the other hand, scattering longitudinally polarized electrons off proton targets only, it should be possible, as can be seen from Fig. 3, for example, to distinguish at least the predictions of models B–E and G from those corresponding to A and F which show a rather similar structure. In addition, measurements of the energy dependence of A_N, say, by going from 150 MeV to 600–800 MeV, would greatly facilitate one to discriminate between different gauge theories.

In case one does not accept the simplest standard Higgs-doublet structure of SU(2)⊗U(1) models, i.e., $\kappa \equiv M_W^2/M_Z^2 \cos^2\theta_W = 1$, then all our predictions

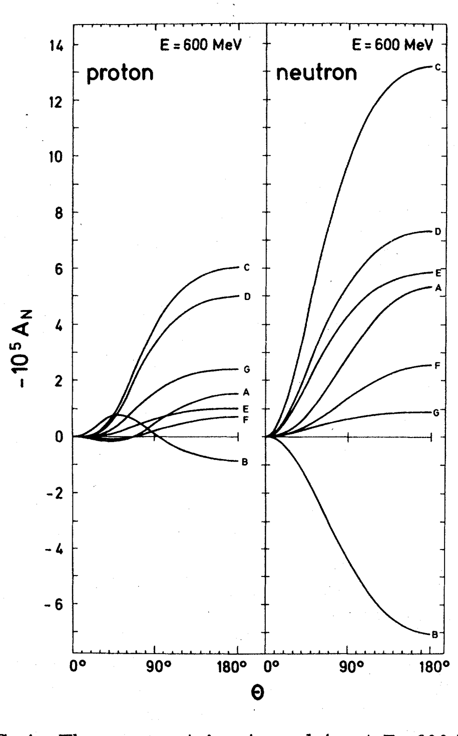

FIG. 4. The asymmetries A_p and A_n at $E = 600$ MeV. Notation and input parameters are as in Fig. 2.

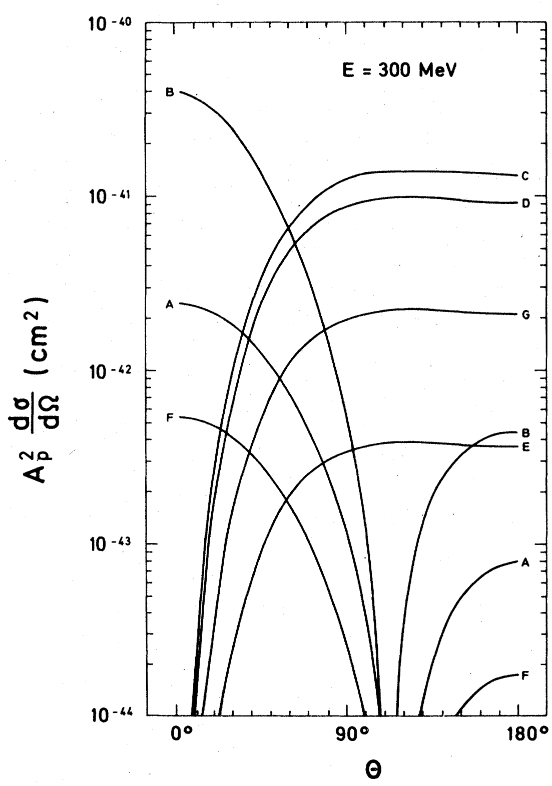

FIG. 5. $A_p{}^2 d\sigma/d\Omega$ distributions at $E = 300$ MeV. Notation and input parameters are as in Fig. 2.

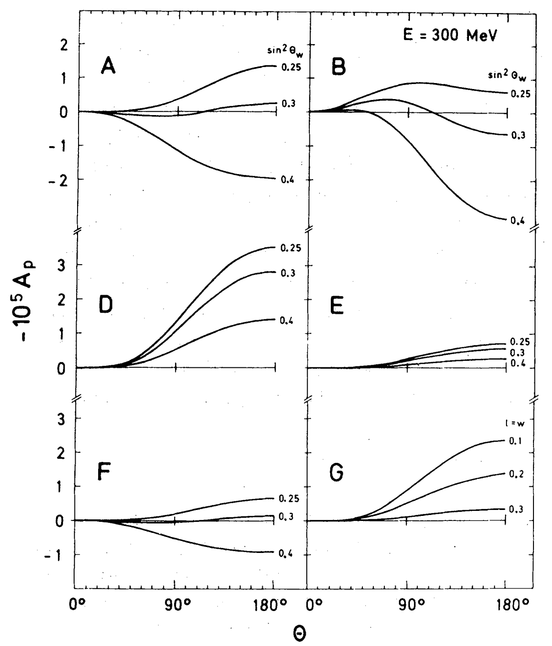

FIG. 6. Dependence of A_p on a variation of the input parameters (extreme choices of mixing angles) as denoted on the plots, at $E = 300$ MeV.

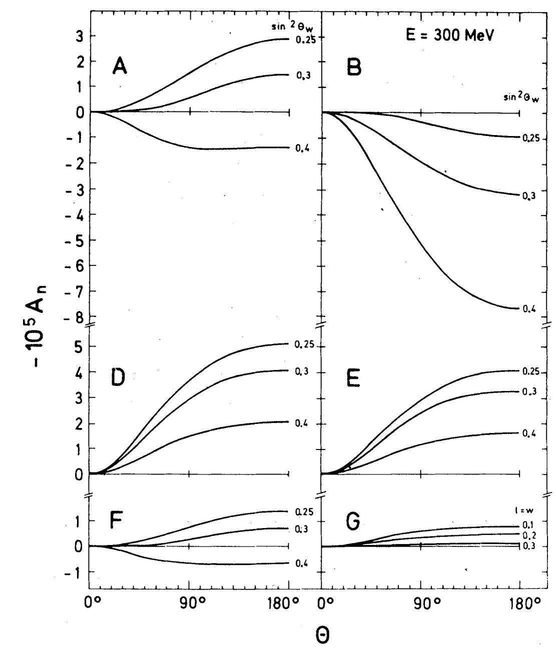

FIG. 7. Dependence of A_n on a variation of the input parameters (extreme choices of mixing angles) as denoted on the plots, at $E = 300$ MeV.

have to be simply multiplied by $\kappa^2 \neq 1$.

Of immediate interest are the predictions in Fig. 3 at $E = 300$ MeV, the typical energy range of the Mainz[7] and Yale-MIT[8] experiments. Here, $|A_p|$ reaches values of $(0.25 - 1) \times 10^{-5}$ at $\theta \simeq 90°$, and becomes as large as 3×10^{-5} in the backward hemisphere at $\theta \simeq 150°$, whereas $|A_n| \simeq (0.25 - 5) \times 10^{-5}$ for $\theta \simeq 90°$ and increases to 7×10^{-5} at $\theta \simeq 150°$ which is well within the expected experimental accuracy.[7,8] Regarding the right choice of energy and scattering angles,[9] Figs. 2–4 might be misleading since they do not contain the rapid decrease of the (Rosenbluth) cross section at high q^2. We therefore show in Fig. 5, as an example, the statistically relevant quantity $A_p^2 d\sigma/d\Omega$. At least at large scattering angles the gain in asymmetry is partly balanced by the decrease of the cross section.

In order to show how the model predictions for the left-right asymmetry depend on the Weinberg angle, or its equivalents, we have calculated A_p and A_n at $E = 300$ MeV in Figs. 6 and 7, respectively, for extreme high and low values of these parameters allowed by neutrino experiments.[1,29] The predictions for model C are qualitatively the same as those shown for D. As can be seen, the dependence of the asymmetry on these parameters (mixing angles) is remarkable. A measurement at *one* given scattering angle could hardly discriminate between various models, since their different predictions, as show in Fig. 3, do partly overlap with the rather large spread of A_N in Figs. 6 and 7 due to variations of $\sin^2\theta_W$. Again, measurements at *various* scattering angles will prove very important to distinguish between various gauge models.

IV. CONCLUSIONS

Measurements of parity-violating effects in longitudinally polarized electron-nucleon scattering, arising from the interference between neutral weak and electromagnetic amplitudes, are of particular importance for discriminating between a large class of different models of unified gauge theories, and for testing the nature of the electronic and hadronic vector and axial-vector couplings because this process involves the neutral-current couplings in a clean and well-defined manner. Having generally classified most of the $SU(2) \otimes U(1)$, $SU(2)_L \otimes SU(2)_R \otimes U(1)$, and $SU(3) \otimes U(1)$ models which show agreement with present neutrino experiments, we calculated these parity-violating left-right asymmetries at various low electron energies ($E = 150$, 300, and 600 MeV) and laboratory scattering angles for seven rather general classes of gauge models. In contrast to high-energy measurements, such that $-q^2/2ME \ll 1$, where the $SU(2) \otimes U(1)$ models with $I_{3R}^e = -\frac{1}{2}$ (like the hybrid model[18]) and the $SU(3) \otimes U(1)$ model of Lee and Weinberg,[12] for example, predict practically vanishing asymmetries, low-energy measurements ($E \lesssim 1$ GeV) are an important tool to discriminate between different gauge theories.

Of course it would be very interesting to establish experimentally whether there is any finite parity-violating effect in elastic eN scattering at all; but this would eliminate only a small fraction of principally distinct gauge models suggested so far. To discriminate, however, also between models which all have a parity-violating eN interaction in common, one needs measurements over a rather wide range of the scattering angle (forward/backward hemisphere). Additional knowledge of the energy dependence of the left-right asymmetry (say, $150 \lesssim E \lesssim 800$ MeV) would be helpful in better delineating the differences coming from ambiguities of the model parameters (i.e., mixing angles such as $\sin^2\theta_W$) and from different models. At electron beam energies of $E \simeq 300$ MeV, typically in the range of the Mainz[7] and Yale-MIT[8] experiments, we expect the parity violation to be about 2×10^{-5} for (unpolarized) proton targets and 4×10^{-5} for neutron targets. Both asymmetries, however, show a strong dependence on the scattering angle (see, e.g., Fig. 3), and increase by more than a factor of two by going to $E = 600$ MeV. These predicted values of parity violation lie well within the experimentally expected[7,8] level of accuracy.

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