PHYSICAL REVIEW D VOLUME 18, NUMBER 1 1 JULY 1978

Are there an elementary length $l_0 = 0.66$ fm and an elementary time $\tau_0 = 0.66$ fm/c associated with the strong interaction?

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(Received 23 December 1977)

In an earlier paper we found an indication that particle lifetimes were quantized in units of 0.66 fm/c = 2.20×10^{-24} sec. We now find that four other kinds of data show indications of an elementary length, l_0, and time, $\tau_0 = l_0/c = 0.66$ fm/c, connected with the strong interaction. A string model and a string-lattice model are suggested as ways of interpreting the observed regularities. The existence of an elementary length and time may also be relevant to lattice gauge theories. Several experimental tests of the model are suggested.

EMPIRICAL EVIDENCE FOR AN ELEMENTARY LENGTH AND TIME, $l_0 = c\tau_0 = 0.66$ fm

It was previously reported[1] that all resonant-state lifetimes computed from their widths, using the width-lifetime relation $\Gamma\tau = \hbar$, were reasonably consistent with being integral multiples of one half the ρ-meson lifetime:

$$\tau_0 = \tfrac{1}{2}\hbar/\Gamma_\rho = 2.20 \times 10^{-24}\ \text{sec} = 0.66\ \text{fm}/c.$$

This was based on an observed statistical regularity in resonance widths, which were found to be consistent with $\Gamma = (298/n)$ MeV, where $n = 1, 2, 3, \ldots$. Unfortunately, the large values of measurement uncertainty for many resonsnce widths prevents a definitive test of the quantized-lifetime hypothesis. Moreover, problems connected with the model dependence of fits to resonance parameters (particularly for baryons) may be responsible for the poorer agreement with the quantized-lifetime hypothesis for more recent data on some of the baryon states.[2] Nevertheless, all those meson-resonance widths that are well measured continue to be in approximate agreement with the quantized-lifetime hypothesis as indicated in Table I. The worst case is the B meson which is generally produced in final states having a large background.

As seen from Table I, the B-meson lifetime, computed from its width, is now approximately two standard deviations from a value demanded by the quantized-lifetime hypothesis. Despite this serious defect, we note that the suggestion of an elementary time τ_0 was originally based strictly on an observed regularity of one particular type (particle lifetimes). It is therefore not insignificant that there now exist four other kinds of data showing further indications for an elementary time τ_0 and an elementary length $l_0 = c\tau_0 = 0.66$ fm.

There are a number of different ways one would expect elementary lengths and times to reveal themselves. The first entry in Table II is the value from the fit to resonant-state lifetimes previously mentioned. Secondly, we list a value based on our interpretation of a recent article by Tryon.[3] Using a $q\bar{q}$ potential model of mesons, Tryon adjusts the quark masses and the meson e^+e^- partial decay widths.[4] In Tryon's model, the $q\bar{q}$ potential is related to the tension in a connecting string which undergoes transverse harmonic vibrations of wavelength $\lambda_n = 2l_0/n$, where l_0 is the string

TABLE I. Full widths and lifetimes of meson resonant states, using values from the Table of Particle Properties [Particle Data Group, Rev. Mod. Phys. 48, S1(1976)]. Lifetimes are expressed in units of one half the ρ-meson lifetime, where the width of the ρ is taken to be $\Gamma_\rho = 148.8$ MeV. Only states having an uncertainty in lifetime of less than one-sixth the ρ-meson lifetime are listed.

Meson resonances	Full width (Γ) in MeV	Lifetime (τ) in units of one half the ρ lifetime $\tau = 2\Gamma_\rho/\Gamma$
$g(1680)$	180±30	$1.65^{+0.33}_{-0.21}$
$f(1270)$	180±20	$1.65^{+0.21}_{-0.16}$
$\rho(770)$	152±3	1.96±0.04
$\omega(1675)$	150±20	$1.98^{+0.31}_{-0.23}$
$B(1235)$	125±10	$2.38^{+0.21}_{-0.18}$
$K^*(1420)$	108±10	$2.76^{+0.28}_{-0.24}$
$A_2(1310)$	102±5	$2.92^{+0.15}_{-0.14}$
$K^*(892)$	49.8±1.0	5.98±0.12

TABLE II. Nine values for possible elementary units of time and length, together with the source of each value. The two quantities indicated by arrows represent times and are in units of fm/c; the other seven quantities represent lengths and are in units of fm. All values are reasonably consistent with $\tau_0 = l_0/c = 0.66$ fm/$c = 2.20 \times 10^{-24}$ sec$=\hbar/(298$ MeV/$c^2)$. The horizontal lines separate the nine values according to the different methods by which they are obtained. The suggestion that any of the listed values correspond to a fundamental length or time has not been made by any of the authors whose work is cited.[3-10]

Length in fm or time in fm/c	Source
$\rightarrow 0.66 \pm 0.01$	Minimum resonant-state lifetime, $\tau_{\min} = \frac{1}{2}\tau_\rho$, for which other resonance lifetimes appear to be consistent with $\tau = n\tau_{\min}$, $n = 1, 2, 3, \ldots$.
0.62^{a}	Minimum string length for the $q\bar{q}$-potential-model fit to the meson mass spectrum and e^+e^- partial widths (assuming entire mass of string to come from quantum fluctuations)
0.65 ± 0.02	Lower bound on rms impact parameter of coherently produced $\pi^+\pi^+\pi^-$ meson system in 12 GeV/c π^+ nucleus interactions.
0.66 ± 0.04	Lower bound on rms impact parameter of coherently produced $\pi^+\pi^0\pi^0$ meson system in 12 GeV/c π^+ nucleus interactions.
0.67 ± 0.02	Lower bound on rms impact parameter of coherently produced $K^+\pi^+\pi^-$ meson system in 10 GeV/c K^+ nucleus interactions.
$\rightarrow 0.59 \pm 0.17$	Lifetime of centrally produced hadronic matter in 28 GeV/c pp collisions.
$0.73^{+0.10}_{-0.11}$	Spatial dimension along the collision axis of centrally produced hadronic matter in 28 GeV/c pp collisions.
$0.69^{+0.007}_{-0.014}$ [b]	Radius of the charged pion from average of measurements of pion form factor.
0.62 ± 0.02	Calculated radii of $K\pm$ and $\pi\pm$ mesons.

[a] In a private communication, Tryon notes that while the uncertainty in the string tension T from which $\lambda_{\min}$ is computed is only 15%, questions about the validity of the nonrelativistic analysis make the value of $\lambda_{\min}$, and hence l_0, uncertain by a factor of the order of unity.

[b] The value quoted is from a world average of pion-form-factor measurements. Since the listed uncertainty includes statistical errors only, it is very probably an underestimate.

length, and $n = 1, 2, 3, \ldots$. In order to avoid an ultravoilet divergence, Tryon introduces a cutoff wavelength $\lambda_{\min}$ associated with high values of n, i.e., for $n \gtrsim 2l_0/\lambda_{\min}$. Tryon suggests that a nonzero string diameter may be the physical mechanism responsible for suppressing the short-wavelength modes.

We choose to interpret the appearance of the minimum wavelength $\lambda_{\min}$, demanded by Tryon's fit to the meson mass spectrum and e^+e^- partial widths, in a different way. We suggest that the appearance of $\lambda_{\min}$ is indirect evidence for a minimum allowed string length, whose length would then be given by $l_0 = \frac{1}{2}n\lambda_{\min}$, with $n = 1$. If we use the value $\lambda_{\min} = 1.24$ fm that Tryon finds from his best fit, we obtain $l_0 = 0.62$ fm, assuming the string to be classically massless.

A third area from which we find an indication of an elementary length (entries 3, 4, 5 in Table II), is in an impact-parameter analysis of high-energy interactions. Webber[5] has suggested a method making it possible to determine, for specific final states, lower bounds on the rms impact parameter b. Moreover, the inequality represented by the lower bound remains strong as long as the phase of the scattering amplitude depends weakly on the transverse particle momenta. According to Webber,[5] a realistic estimate would be that at high energy the rms impact parameter would be no more than 13% above the computed lower bound. Using Webber's method, Arnold *et al.*[6] compute lower bounds on $b_{\rm rms}$ for a number of final states produced in 12 GeV/c π^+ + nucleus and 10 GeV/c K^+ + nucleus interactions. They report values for three coherently produced meson systems: $\pi^+\pi^+\pi^-$, $\pi^+\pi^0\pi^0$, and $K^+\pi^+\pi^-$. All three values, as indicated in Table II, are quite consistent with $b_{\rm rms} = l_0 = 0.66$ fm. In view of the fact that incoherently produced meson systems, and other reactions involving free proton and deuteron targets, do not show such a regularity,[6,7] it is therefore essential to explain why an elementary length would appear only in an impact-parameter analysis in which coherent nuclear interactions are studied. For a high-energy particle interacting coherently with a heavy nucleus, we may consider the nucleus to be a large piece of hadronic matter of radius R having a relatively well-defined edge If hadrons (quarks) interact when they are a distance l_0 apart, then the effective radius of the nucleus seen by the incoming hadron is $R+l_0$, which can, in principle, be estimated experimentally using

$$\langle (R+l_0)^2\rangle^{1/2} \approx \frac{\hbar}{\langle p_\perp{}^2\rangle^{1/2}} .$$

Thus l_0 is deducible from an impact-parameter analysis of coherent nuclear interactions in a way that is not possible for incoherent interactions with single nucleons.

A fourth type of analysis, by Ezell *et al.*,[8] gives an indication of both an elementary length $l_0 = 0.66$ fm, and an elementary time $\tau_0 = 0.66$ fm/c, as shown in Table II, but the authors again make no such claim. Ezell *et al.*[8] report a high-statistics study of the reaction $pp \to pN+$ mesons at 28.5 GeV/c. They have measured for centrally produced pions, the ratio of negative-pion pairs to that of negative minus positive pion pairs as a function of the Kopylov variables. From the shape of the correlation function, they determine the spatial size and temporal duration of the centrally produced hadronic matter. They report values for the lifetime and the spatial extensions parallel and perpendicular to the collision axes that are all consistent with 0.66 fm/c and 0.66 fm. We have only listed two of the three values in Table II, since the value for the transverse dimension, $r_\perp = 1.65^{+3.56}_{-0.99}$ fm, has a very large uncertainty.

A fifth place one might expect an elementary length to appear is in the rms charge radii of particles. Since mesons consist of a $q\bar{q}$ pair, we might expect the $q\bar{q}$ separation to be l_0, or perhaps some integer multiple of l_0. The value for the pion charge radius based on a world average of many measurements[9] is in fair agreement with the value $l_0 = 0.66$ fm, as indicated in Table II. It should, however, be noted that the experimental uncertainty for $\langle r_\pi{}^2\rangle^{1/2} = 0.69^{+0.007}_{-0.014}$ fm includes statistical errors only, making it very likely an underestimate. For the $K^\pm$ radii, Chou[10] has used data for $\pi^\pm p$ and $K^\pm p$ elastic scattering to calculate the charge radii of $\pi^\pm$ and $K^\pm$ mesons. He finds that "remarkably" the charge radii and form factors of all four particles are the same, and the charge radii are found to be $\langle r_\pi{}^2\rangle^{1/2} = \langle r_K{}^2\rangle^{1/2} = 0.62 \pm 0.02$ fm, the last entry in Table II. All nine entries in Table II are seen to be reasonably consistent with $l_0 = c\tau_0 = 0.66$ fm.

One might expect elementary units of time and length to have some simple relation to other elementary constants. It is interesting to note that the mass $\mu = \hbar/\tau_0 c^2 = 298$ MeV, is quite close to the rest mass of the nonstrange quarks in Tryon's string-model fit to the meson mass spectrum[4]: $m_u = m_d = 292$ MeV. We now suggest two variations on a string model in which the regularities we have noted may be accounted for. It is our belief, however, that independent of the validity of the models discussed here, the empirical regularity summarized by Table II is of some significance in itself, and that it is one which any theory of the strong interaction should attempt to explain.

THE STRING MODEL

As in Tryon's string model, we assume that mesons consist of $q\bar{q}$ pairs bound by a one-dimensional massless flexible string, which presumably consists of quantized field lines collapsed into a flux bundle [11] We assume further that

(1) the quarks are *zero*-rest-mass objects, and hence travel at the speed of light;

(2) the string is in rotation with the centrifugal force on the relativistically massive $q\bar{q}$ balancing the tension for a meson in its unexcited state;

(3) the tension in the string remains constant as the string is stretched (a reasonable assumption for a *linear* collection of field lines);

(4) the π and K mesons are assumed to have strings of length $l = 2l_0$, consistent with their measured charge radii, $r_\pi = r_K = l_0$.

We may now show that the string for a resonant state which has $L = n\hbar$, and which results when a π or K meson becomes excited, has an equilibrium string length $l = 2nl_0$.

Suppose that a π- or K-meson string initially of length $2l_0$ acquires angular momentum $n\hbar$ as a result of an interaction, causing it to become excited and stretch. Before stretching, a massless string having oribtal angular momentum $n\hbar$ terminates on quarks having a relativistic mass

$$m^* = \frac{n\hbar}{2l_0 c} = n\frac{\mu}{2},$$

and after stretching the relativistic quark masses are

$$m = \frac{n\hbar}{lc}, \tag{1}$$

where $2l_0$ and l are, respectively, the string lengths before and after the string stretches. If all the work done in stretching the string from length $2l_0$ to length l equals the change in the relativistic quark energies, we have

$$T(l - 2l_0) = 2m^*c^2 - 2mc^2 \ . \quad (2)$$

For the constant string tension we use $T = 2\mu c^2/2l_0$, assuming the pair of energy $2\mu c^2$ is created when the string is stretched from length zero to $2l_0$. With the aid of Eqs. (1) and (2), we then find that

$$l = 2nl_0 \ ,$$

where

$$n = L/\hbar \ .$$

Thus showing that an excited state of a π or K meson has an equilibrium string length which is an integral multiple of $2l_0$. When the string length equals $2nl_0$, it becomes energetically possible to create a zero-rest-mass $q\bar{q}$ pair, and the string snaps.

This model can account for the quantized-lifetime hypothesis. For a string of initial length $2l_0$ to reach the point where it can snap, its ends must travel distances which in the meson rest frame are integral multiples of l_0. Since the quarks at the ends of the string travel at the speed of light, the time for a snap to occur in the rest frame of the excited state is therefore $\tau = n\tau_0$, $n = 1, 2, 3, \ldots$.

While the string model may possibly account for the regularities we have noted, we shall now extend the model to one with fewer arbitrary assumptions, e.g., that the tension in the string remains constant as it is stretched.

THE STRING-LATTICE MODEL

We keep postualtes (1), (2), and (4) and now add

(5) In the rest frame of a string, and *rotating with it*, there exists a space-time lattice, of lattice spacing $l_0 = 0.66$ fm, and $\tau_0 = 0.66$ fm$/c$.

(6) The $q\bar{q}$ at the ends of the string may only lie on lattice sites at any time, τ.

(7) At the time $\tau + \tau_0$, the $q\bar{q}$ may either be found at the same positions as at time τ (a rotating string of fixed length), or at adjacent lattice sites (a rotating string of varying length). In either case, the ends of the string move at the speed c in the laboratory frame.

It is clear that in this model, the lifetimes of excited states must be quantized in units of τ_0, since for an excited state to decay, the quarks at the ends of the string must travel a distance $l = nl_0$ in a time $\tau = n\tau_0, n = 1, 2, 3 \ldots$. Moreover, the model would also require stable mesons to have charge radii that are integer multiples of l_0.

For the charge radii of baryons, the string-lattice model yields an interesting result that is perhaps not something that would have been expected for the string model without a space-time lattice. Suppose we arrange the *uud* quarks in the proton on a spatial lattice, making the arrangement as close to being equilateral as possible (Fig. 1). We have placed the *ud* quarks a distance $2l_0$ apart (rather than l_0), since the quarks in the pion appear to have this spacing, based on its measured charge radius. Also, the two unlike quarks are placed at the ends of the short side of the *uud* triangle owing to the Coulomb attraction of their unlike charges. We find, using $l_0 = 0.66 \pm 0.01$ fm, for the rms charge radius of this *uud* arrangement,

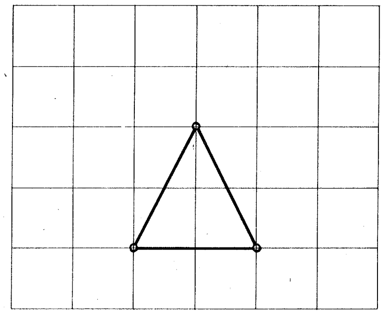

FIG. 1. A three-quark arrangement on a rectangular spatial lattice that is as close to being equilateral as possible. As noted in the text, the computed charge radius for this *uud* arrangement is very close to the experimental value of $\langle r^2 \rangle_p{}^{1/2}$.

$$\langle r^2 \rangle_{uud}{}^{1/2} = 0.85 \pm 0.01 \text{ fm} \ ,$$

which may be compared with the experimental value for the proton,[12]

$$\langle r^2 \rangle_p{}^{1/2} = 0.84 \pm 0.01 \text{ fm} \ .$$

If we compute the rms charge radius for exactly the same *ddu* configuration, we find

$$\langle r^2 \rangle_{ddu} = -0.07 \langle r^2 \rangle_{uud} \ .$$

While the computed *ddu/uud* ratio is small and of the right sign, the agreement with the experimental value[13]

$$\langle r^2 \rangle_n = (-0.225 \pm 0.035) \langle r^2 \rangle_p$$

is now considerably worse. However, it should be expected that the computed neutron charge radius would be much more sensitively dependent than the proton radius on an effect we have ignored: possible superpositions of other spatial arrangements of the *ddu* quarks and of other quark flavors.

If the quark configurations for neutron and proton are actually as regular as we have assumed, there are several reasons why this might not clearly show up in n,p form-factor measurements:

(1) These are only the *average* configurations,

with a superposition of quark states and the uncertainty principle leading to a smeared-out charge distribution.

(2) A high-energy (hadronic) photon "sees" only one quark and not the whole structure.

We note that the existence of an elementary length and time, and their possible manifestation in a space-time lattice, may offer explanations for some fundamental questions. Kadyshevsky *et al.*[14] have, for example, shown that the quantization of electric charge is a direct consequence of the existence of a fundamental length. Moreover, we may combine the lattice of space-time with lattices in hyperchange space as well as all other quantized spaces. We could then reformulate the exclusion principle to say that no two quarks (or other pointlike particles) may fall at the same point in the combined space. This could then explain why, for example, a pair of pointlike leptons obey the exclusion principle, but extended $q\bar{q}$ mesons do not, since q and $\bar{q}$ would not lie at the same point in the combined space. However, baryons consisting of three quarks would again have to satisfy the exclusion principle.

The empirical indications we have discussed here for the existence of elementary units of length and time may be relevant to the lattice gauge theories Wilson[15] and others.[16,17] Usually the lattice in lattice gauge theory is not treated as anything more than a calculational device, because of its apparent inconsistency with most of the space-time symmetries of relativistic field theories.[16,18,19,20] However, Chodos and Healy show that this view is too simplistic.[21] They demonstrate, as did Snyder,[22] in 1947, that a lattice version of a Lorentz-invariant theory can be constructed in which spatial coordinates are discrete and time is continuous. The existence of a Lorentz-invariant theory has also been demonstrated for the converse case of quantized time and continuous space.[23] Yang has also shown how to modify Snyder's result to make a lattice theory invariant under translations as well as proper Lorentz transformations assuming a constant-curvature de Sitter space.[24]

An alternative simple demonstration that a lattice theory is compatible with continuous translations in space or time can be achieved if we regard the lattice as defined by the discrete eigenvalues for *observer*-dependent position, and time operators.[25] The origin and orientation of each observer's lattice might be defined by his observation of a spinning particle at some point in space-time. The two lattices may then be connected by *continuous* space-time transformations even if *both* position and time operators have discrete eigenvalues. The observer dependence of the lattice also means that there are no preferred spatial directions, and the existence of the lattice is no more incompatible with rotational invariance than is the fact that spins are found to be qunatized along any particular observer-defined direction. Cole[25] has adopted such an approach using an observer-dependent space-time lattice, and he shows that space-time lattices are compatible with covariance, given specific restrictions on the transformation of operators between two observer's lattices.

Contrary to the reservations of lattice gauge theorists in treating the lattice as simply a calculational device, it may well be that the reason their theory works so well is that a space-time lattice in fact exists. There are several possible experimental tests of the model discussed here. For example, resonant-state form factors should reveal a decreasingly dense charge distribution extending from $r = l_0$ (the pion radius) to $r = (n+1)l_0$, where $n = \tau/\tau_0$ (the point where the meson string snaps). In addition, resonant-state lifetimes can be more accurately measured, although the model dependence of fits to particle decay widths may make a clear resolution impossible, particularly for baryon states. There are, however, a number of experimentally verifiable tests which could serve to distinguish between a $q\bar{q}$ string rotating in a plane and a spherical charge distribution. For example, consider a process such as $e^-p \to K^*\Lambda^0$, which could be used to study the charge structure of the K^*. According to the Feynman diagram

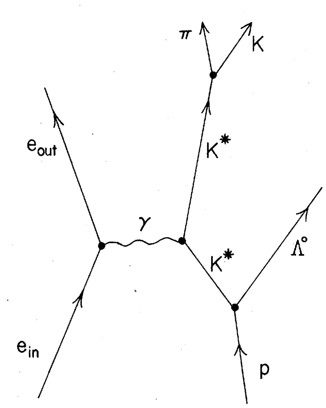

FIG. 2. A Feynman diagram for the process $e^-p \to e^-K^*$. If the e_{in}, e_{out}, p, and Λ^0 momenta are in the plane of the paper, then the model of a rotating string for mesons requires the K^* decay products to always be either in the plane of the paper or perpendicular to it. A lattice model of rotating strings would also yield the suggested result. However, in a more conventional model of $q\bar{q}$ in a p-wave orbital, all spatial directions should be possible, since the meson is considered to be effectively a "blob" not a "disk" of charge.

shown in Fig. 2, if we select a subsample of data in which the e_{in}, e_{out}, p, and Λ^0 momenta lie in the plane of the paper, then the virtual photon and virtual K^* spins are normal to this plane. The scattered K^* spin must then either be perpendicular or parallel to this plane. If the K^* consists of a string rotating in a plane, rather than a spherically symmetric charge distribution, the K^* decay products will either lie in the plane of the paper or perpendicular to this plane. Other diagrams give the same result.

ACKNOWLEDGMENTS

I am grateful to Dr. Darryl Leiter and Dr. Robert Pease for some helpful discussions. I would also like to thank Dr. George Snow for some helpful criticisms of a preliminary version of this paper.

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