Charged- and neutral-current production of Δ(1236)

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Based on a quark-model approach previously developed by us which employs a q^2 continuation in terms of generalized meson-dominance form factors we study the weak production of the isobar Δ(1236). First we demonstrate that our model is in agreement with the Argonne data on charged-current production of the Δ. We then study neutral-current Δ production using four different gauge models, namely, the standard Weinberg-Salam model, a vectorlike model with six quarks, a five-quark model due to Achiman, Koller, and Walsh, and a variant of the Gürsey-Sikivie model. We fine that the results for the differential cross section in the forward region ($|q^2| \lesssim 0.1$ GeV^2) are very sensitive to the structure of the weak neutral current and suggest that measurements in this region constitute a stringent test of weak-interaction models. We also calculate the density-matrix elements measurable from decay correllations. The density-matrix elements are not so sensitive to the models containing some axial-vector contribution, whereas the vectorlike model shows a behavior quite distinct from the others.

I. INTRODUCTION

After the discovery of neutral currents in neutrino-induced reactions, there has been considerable interest in the experimental determination of the structure of the hadronic weak neutral current since this structure has a direct bearing on the construction of gauge models for weak and electromagnetic interactions. The most direct test of the hadronic weak neutral-current structure which requires only little additional theoretical input can be obtained through the elastic scattering processes $\nu(\bar{\nu})N \to \nu(\bar{\nu})N$. A detailed study of these elastic scattering processes has recently been undertaken in Refs. 1–3 using various representative gauge-model currents[1] and general spatial current structures,[2] the results of which have been compared with the recent Brookhaven data on $\nu(\bar{\nu})p \to \nu(\bar{\nu})p$.[4]

Next to the elastic scattering processes one expects also neutral-current data on the quasielastic production of the Δ(1236). In fact, some tentative evidence for neutral-current Δ production has recently been reported in a BNL experiment with neutrinos incident on complex nuclei[5] and in an Argonne experiment with neutrinos incident on a deuterium target.[6] Although Δ production is dynamically more complex than the elastic scattering processes, one can expect that an analysis of this production process will furnish additional information on the underlying neutral-current structure which will complement and strengthen the results of the neutral-current elastic scattering analysis. In addition, because the Δ has isospin $\frac{3}{2}$, the N-Δ transition has the advantage that one is obtaining separate information on the $\Delta I = 1$ piece of the neutral current. Furthermore, in the forward region of Δ production only the axial-vector current contributes so that this process affords a nice opportunity to separately study one isolated space-isospin component of the neutral weak current.

A natural framework for the formulation of dynamics of the N-Δ transition is to use the quark model since the hadronic sector of the gauge currents is given in terms of currents of quarks. Applications of the quark model up to date have been quite successful at $q^2 = 0$ (see e.g., Ref. 7). However, at $q^2 \neq 0$, the quark-model results tend to be unreliable (see e.g. the discussions in Refs. 8 and 9) and do not in general exhibit the correct structure of relativistic kinematics (see e.g., Ref. 10). We therefore use the quark model only at $q^2 = 0$ and continue to spacelike q^2 using constraint-free invariant form factors as has been done successfully in the corresponding electroproduction case.[11]

At $q^2 = 0$ our model is quite similar to the results of previous calculations of neutral-current Δ production including a static-model calculation,[2] a dispersion-theoretic treatment incorporating also the $I = \frac{1}{2}$ background contribution,[13] and a quark-model calculation.[14] At $q^2 \neq 0$ the results of the various calculations are in general different, reflecting the different q^2 behavior of the different dynamical schemes.

Regarding the neutral-current structure, we shall follow the procedure of Ref. 1 and study neutral-current Δ production for several specific gauge model currents. These include the Weinberg-Salam model,[15] vectorlike models using six quarks,[16] a five-quark model proposed by Achiman, Koller, and Walsh,[17] and one variant of the Gürsey-Sikivie model.[18] In view of the lack of data we feel this representative procedure to be adequate at the present time. A more general analysis that is independent of any specific gauge model should of course be attempted at a later stage when enough data become available.

In Sec. II we write down the neutral-current coupling strengths corresponding to the above four gauge models. In Sec. III we discuss the dynamics of the Δ production process and give the results of our quark-model approach for the $q^2=0$ values of invariant form factors and specify their $q^2\neq 0$ form-factor behavior. In Sec. IV we check our model against existing charged-current data of Δ production and compare our results with those of other model calculations. In Sec. V we treat the case of neutral-current production of the Δ. We present results for total cross sections, differential cross sections, and density matrix elements at a typical neutrino beam energy of $E=2$ GeV using the four different sets of neutral-current couplings discussed above. In Sec. VI we present our conclusions.

II. WEAK-CURRENT STRUCTURE

For the description of the hadronic coupling of the weak current we shall assume a basic SU(2) × U(1) gauge structure specified in terms of the weak current couplings to quarks. Whereas the charged-current couplings among ordinary quarks are determined *a priori*, the form of neutral-current couplings is model dependent. We shall, in the following, consider four definite models of neutral-current coupling, namely, the standard Weinberg-Salam (WS) model,[14] vectorlike (V) models with six quarks,[15] a five-quark model proposed by Achiman, Koller, and Walsh (AKW),[6] and one variant of the Gürsey-Sikivie (GS) model.[17] The charged- and neutral-current couplings of the quarks have the form

$$J^{\pm}_{\mu}=\sum_{L}\bar{\psi}_L\tau_{\pm}\gamma_{\mu}(1-\gamma_5)\psi_L+\sum_{R}\bar{\psi}_R\tau_{\pm}\gamma_{\mu}(1+\gamma_5)\psi_R, \tag{1}$$

$$J^{0}_{\mu}=\tfrac{1}{2}\sum_{L}\bar{\psi}_L\tau_{3}\gamma_{\mu}(1-\gamma_5)\psi_L+\tfrac{1}{2}\sum_{R}\bar{\psi}_R\tau_{3}\gamma_{\mu}(1+\gamma_5)\psi_R - 2\sin^2\theta_W J^{\rm em}_{\mu}\,, \tag{2}$$

TABLE I. $I=1$ neutral-current parameters in four different weak-interaction models.

	α	β
WS	$1-2x_W$	1
Vector	$2-2x_W$	0
GS	$\frac{3}{2}-2x_W$	$\frac{1}{2}$
AKW	$1-2x_W+\frac{1}{2}x_{\rm AKW}$	$1-\frac{1}{2}x_{\rm AKW}$

where the summation runs over the left-handed (L) and right-handed (R) doublet representations of the weak SU(2), and $\tau_{\pm}, \tau_3$ are the usual SU(2) isospin generators. The contribution of the admixed electromagnetic part in Eq. (2) is as usual

$$J^{\rm em}_{\mu}=\sum_{i}Q_i\bar{\psi}_i\gamma_{\mu}\psi_i, \tag{3}$$

where Q_i is the quark charge of the ith quark in units of e.

It is convenient and instructive to decompose the neutral weak current into pieces transforming as $I=1$ and $I=0$ of $SU(2)_{\rm strong}$. One has

$$J^{0}_{\mu}=\alpha V_{\mu}{}^{3}-\beta A_{\mu}{}^{3}+\tfrac{1}{3}\gamma V^{0}_{\mu}-\delta A^{0}_{\mu}\,. \tag{4}$$

In this paper we are only concerned with the first two contributions of Eq. (4) transforming as $I=1$ since the N-Δ transition is a pure $\Delta I=1$ transition. In Table I we list the model-dependent coupling values of α and β, where x_W is the usual Weinberg parameter and $x_{\rm AKW}=\cos^2\phi$ parametrizes the admixture of the up quark in the right-handed doublet in the AKW model.

III. MULTIPOLE STRUCTURE

Most of the recent calculations of weak Δ production[13,14,19] exhibit similar $q^2=0$ multipole amplitude structures, which are quite close to the simple multipole structure of the static model calculation and the nonrelativistic quark model. Latter models predict that the vector excitation is pure M_{1+} and that the axial-vector excitation is a definite mixture of $\mathcal{E}_{1+}$ and $\mathcal{L}_{1+}$. In terms of the invariant form factors of Ref. 20 defined by

$$\langle\Delta^{+}|J^{+}_{\mu}(0)|n\rangle=\frac{G\cos\theta_C}{\sqrt{2}}\bar{u}_{\beta}(p^{*})\Big[\frac{C^V_3}{m}(\not{q}g_{\beta\mu}-q_{\beta}\gamma_{\mu})\gamma_5+\frac{C^V_4}{m^2}(p^{*}\cdot q g_{\beta\mu}-q_{\beta}p^{*}_{\mu})\gamma_5+\frac{C^V_5}{m^2}(p\cdot q g_{\beta\mu}-q_{\beta}p_{\mu})\gamma_5+C^V_6 g_{\beta\mu}\gamma_5$$

$$+\frac{C^A_3}{m}(\not{q}g_{\beta\mu}-q_{\beta}\gamma_{\mu})+\frac{C^A_4}{m^2}(p^{*}\cdot q g_{\beta\mu}-q_{\beta}p^{*}_{\mu})+C^A_5 g_{\beta\mu}+\frac{C^A_6}{m^2}q_{\mu}p_{\beta}\Big]u(p), \tag{5}$$

these statements translate into $MC_4^V = mC_\beta^V$, $C_5^A \pm 0$ and all other $C_i^{V,A} = 0$. In Eq. (5) G denotes as usual the weak-interaction coupling constant and θ_C the Cabibbo angle. M and m are the masses of the Δ and the nucleon, and $q = p^* - p$.

In the relativistic quark model of Ref. 21 one has (for $\nu p \to l^- \Delta^{++}$)

$$\begin{aligned} &C_3^V(0) = 12\,\frac{Mm}{(M+m)^2}, \quad C_3^A(0) = 0, \\ &C_4^V(0) = -\frac{m}{M} C_3^V(0), \quad C_4^A(0) = 4\left(\frac{m}{M+m}\right)^2 Z, \\ &C_5^V(0) = 0, \quad C_5^A(0) = -4\,\frac{M}{M+m} Z, \\ &C_6^V(0) = 0, \quad C_6^A(0) = 0, \end{aligned} \tag{6}$$

where Z is the renormalization of the axial quark current which is set equal to $Z = 1.23 \times \frac{3}{5}$ in order to obtain agreement with the quark-model calculation of the axial-vector coupling of the nucleon.

Compared to the multipole structure of the static and nonrelativistic quark model the results in Eq. (6) still imply a pure vector M_{1+} transition, and for the axial-vector parts the simple $\mathcal{E}_{1+}, \mathcal{L}_{1+}$ structure.

The calculation of the $q^2 = 0$ values of the invariant form factors from the quark model of Ref. 21 has been described in detail in Ref. 21 for the corresponding charm-changing $\Delta C = 1$ case and need not be repeated here. In calculating the vector part of Eq. (6), we have as usual introduced a scaling factor $m/m_q \simeq 3$ (m_q = effective quark mass) which takes into account the different mass scales of the quark and particle magnetic moments. For the transition case one has a corresponding scaling factor $(m+M)/(m_{qi} + m_{qf})$ which gives the same factor 3 if the effective initial and final quarks masses are set equal to $m_{qi} = m/3$ and $m_{qf} = M/3$ (see also Ref. 22).

For the q^2 dependence of our invariant form factors we shall use a power-behaved generalized meson dominance q^2 dependence in the form

$$G^{V,A}(q^2) = \prod_{n=0}^{3} \left(1 - \frac{q^2}{m_{V,A}{}^2 + n\alpha'^{-1}}\right)^{-1}, \tag{7}$$

where $m_{V,A}$ are the masses of the lowest $I = 1$ vector and axial-vector mesons ρ and A_1, and α' is the Regge slope determining the positions of the recurrences ρ', ρ''... and A_1', A_2'.... We are using $m_\rho{}^2 = 0.593\ \text{GeV}^2$ and $m_{A_1}^2 = 1.21\ \text{GeV}^2$ and a universal slope $\alpha' = 1\ \text{GeV}^{-2}$.[23,24]

The motivation for using form factors in Eq. (7) has been discussed in detail in Refs. 11 and 21 and will not be repeated here. Let it suffice to remark that the vector-form-factor behavior of Eq. (7) leads to an excellent agreement with data on the electroproduction of the Δ.[11]

IV. CHARGED-CURRENT PRODUCTION OF Δ

In order to check the reliability of our model we apply our results to the charged-current production of the Δ for which there already exist some data.[30-32] We shall skip the details of how to obtain cross sections, density matrix elements, etc., from the invariant form factors $C_i^{V,A}$ since the necessary formulas have already been recorded in sufficient detail.[19,21,33]

In Fig. 1 we show the energy dependence of Δ^{++} production off protons.[34] The agreement with the Argonne data[31,32] is quite good, except perhaps in the energy region from threshold to $E \simeq 0.9$ GeV, where our cross section prediction is somewhat larger than the data, but is still lying within the statistical errors. The cross-section prediction is systematically lower than the data of the older CERN propane experiment.[30] Our cross section saturates rather quickly. Already at $E \simeq 1.1$ GeV the cross section is within 10% of its asymptotic value $\sigma = 0.68 \times 10^{-38}\ \text{cm}^2$. In Fig. 1 we have also plotted the results of the models of Adler[25] ($m_A = 0.84$ GeV), Ravndal,[19] Salin,[35] and Zucker.[36] The results of the quark-model calculation of the Orsay group[14] practically agree with the prediction of Ravndal's model and have therefore not been included in Fig. 1. From Fig. 1, one sees that our results are closest to the predictions of the models of Adler and Ravndal. A more detailed comparison of the various models will be given at the end of this section.

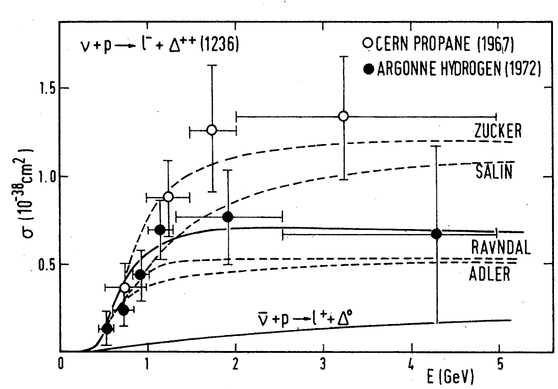

FIG. 1. Neutrino and antineutrino cross sections $\nu p \to l^- \Delta^{++}$ and $\bar{\nu} p \to l^+ \Delta^0$. CERN data are from Ref. 30. Argonne data are from Refs. 31 and 32. Our result: full line. Dashed lines are results of Adler (Ref. 25) ($m_A = 0.84$ GeV), Ravndal (Ref. 19), Salin (Ref. 35), and Zucker (Ref. 36).

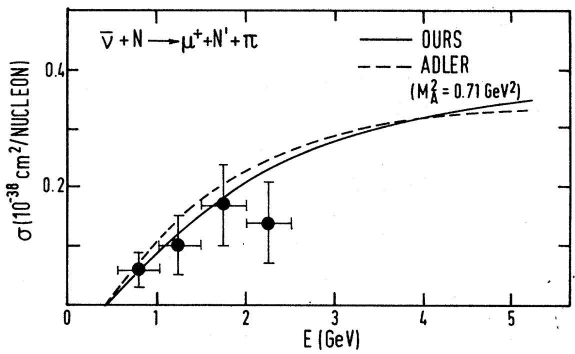

FIG. 2. Antineutrino cross section on a CF_3Br target. Data are from Refs. 25 and 37. Our result: full line. Dashed line is the result of Adler (Ref. 25).

In Fig. 2 we show our antineutrino cross section for antineutrinos incident on CF_3Br and compare it with the CERN data[37] and the results of the calculation of Adler ($m_A = 0.84$).[25] The agreement is satisfactory. We have not included nuclear physics corrections due to the Pauli exclusion principle as described in Ref. 25. These would amount to corrections of the order of $\lesssim 10\%$.[25] One notes that the antineutrino cross section saturates very slowly. Returning to Fig. 1, one sees that at $E = 5$ GeV $\sigma_{\bar{\nu}p \to l^+\Delta^0}$ is still 25% below its asymptotic value given by isospin symmetry as $\frac{1}{3}\sigma_{\nu p \to l^-\Delta^{++}}$. (The difference between the neutrino and antineutrino cross section is given by the VA interference term which is of the order E^{-1} relative to the VV and AA terms.)

In Fig. 3 we show the results for the differential cross section where we have folded our differential-cross-section predictions with the neutrino flux of the Argonne experiment as quoted in Ref. 33. Again the agreement is satisfactory. Since we are using the zero-lepton-mass approximation, our near-forward cross section does not show the pronounced forward dip of, e.g., the calculations in Ref. 33 where this approximation has not been made. This dip is due to pion exchange which produces a destructive effect in the very forward region for a small range of q^2 values $-q_{\min}{}^2 \leq -q^2 \lesssim m_\pi{}^2$. In this range the pion propagator provides an enhancement factor sufficient to counterbalance the kinematic $m_\mu{}^2$ factor (in the case of muon-neutrino scattering) in the cross section multiplying the spin-zero-exchange contribution. Since we are not concentrating on the very forward region we have not attempted to include this effect in our calculation. We are not presenting results on density matrix elements since these are not very different from the results of Ravndal[19] and the simplified Adler calculation given in Ref. 33.

As has been emphasized by Llewellyn Smith[20] a

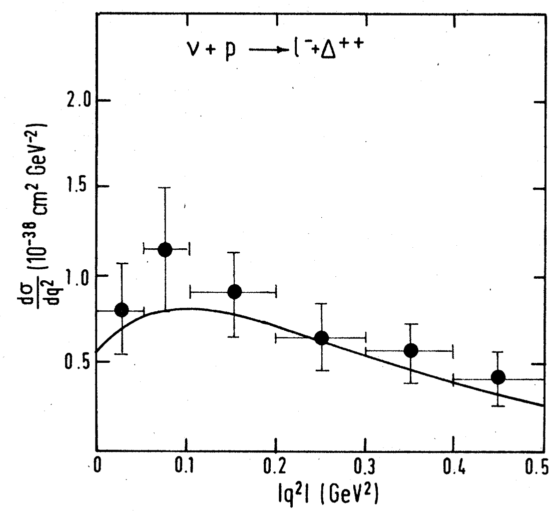

FIG. 3. Differential cross section $\nu p \to l^-\Delta^{++}$. Our differential-cross-section result has been folded with Argonne flux as given in Ref. 33. Data are taken from Ref. 33.

comparison of the various dynamical approaches to weak Δ production is facilitated by listing the results in terms of the invariant-form-factor predictions. In Table II we give the $q^2 = 0$ results of some recent model calculations together with our results and, where applicable, we also list the conserved vector current (CVC) and partially conserved axial-vector current (PCAC) predictions, where the CVC values have been extracted from Ref. 29. One should be reminded that pure M_{1+} dominance in the vector case corresponds to $MC_4^V(0)/mC_3^V(0) = -1$ and that the static-model result corresponds to $C_4^A(0) = 0$.

In Table III we compare the form-factor behavior of these models. For the model of Adler we have used the convenient parametrization given in Ref.

TABLE II. $q^2 = 0$ values of form factors in different models. In the case of Adler's model we use approximate equivalent values deduced by Bijtebier (Ref. 26). The last two columns list the results of the models of Ravndal (Ref. 19) and the Orsay group (Ref. 14). Results are for $\nu p \to l^-\Delta^{++}$.

	This work	CVC (PCAC)	Adler	Ravndal	Orsay
$C_3^V(0)$	2.95	3.56	3.20	2.95	2.66
$\frac{MC_4^V(0)}{mC_3^V(0)}$	-1	-0.74	-0.63	-1	-1
$C_5^A(0)$	-1.68	-2.08	-2.08	-1.68	-1.44
$\frac{C_4^A(0)}{C_5^A(0)}$	-0.33	...	0.29	-0.67	-0.24

TABLE III. q^2 dependence of (normalized) form factors $\tilde{C}_i^{V,A}$ in different models. For Adler's model we use the q^2 parametrization deduced by Bijtebier (Ref. 26). In this parametrization $\tilde{C}_i^V(q^2) = \tilde{C}_i^A(q^2)$ if the same dipole masses are used for the vector and axial-vector cases. Thus we list only $\tilde{C}_i^V(q^2)$ for Adler's model ($m_A = m_V = 0.84$ GeV). For the models of Ravndal (Ref. 19) and the Orsay group (Ref. 14) we give separately the behavior of $\tilde{C}_4^A$ and $\tilde{C}_5^A$. DT refers to the results of Dufner and Tsai (Ref. 38).

$\tilde{C}_i^V(q^2)$

$\lvert q^2 \rvert$ (GeV^2)	This work	DT	Adler	Ravndal	Orsay
0	1.00	1.00	1.00	1.00	1.00
0.1	0.75	0.72	0.72	0.76	0.75
0.2	0.58	0.55	0.54	0.59	0.57
0.5	0.30	0.29	0.26	0.32	0.28
1	0.13	0.14	0.10	0.15	0.11
5	3.7×10^{-3}	4.0×10^{-3}	2.0×10^{-3}	8.1×10^{-3}	2.3×10^{-3}
10	4.2×10^{-4}	4.7×10^{-4}	...	1.6×10^{-4}	2.5×10^{-4}

$\tilde{C}_i^A(q^2)$

$\lvert q^2 \rvert$ (GeV^2)	This work	Ravndal $\tilde{C}_5^A$	Ravndal $\tilde{C}_4^A$	Orsay $\tilde{C}_5^A$	Orsay $\tilde{C}_4^A$
0	1.00	1.00	1.00	1.00	1.00
0.1	0.84	0.90	−0.05	0.84	0.86
0.2	0.71	0.73	−0.23	0.72	0.74
0.5	0.45	0.41	−0.22	0.45	0.49
1	0.23	0.19	−0.12	0.23	0.26
5	1.1×10^{-2}	1.1×10^{-2}	-7.9×10^{-3}	9.2×10^{-3}	1.1×10^{-2}
10	1.4×10^{-3}	2.1×10^{-3}	-1.5×10^{-3}	1.2×10^{-3}	1.5×10^{-3}

26, which should, however, not be extended much beyond $\lvert q^2 \rvert \simeq 2$ GeV^2. In the vector case we have included the results of the phenomenological parametrization of the $N\Delta$ vector form factor of Dufner and Tsai[38] which has also been used in the analysis of Ref. 33. There is a remarkable agreement between this parametrization and our generalized vector-dominance model (GVDM) ansatz up to very high q^2 values. For smaller $\lvert q^2 \rvert$ our vector form factor lies between those of Refs. 19 and 38, whereas the form factor of Refs. 14 and 25 have a somewhat faster falloff behavior. In the axial-vector case, again the form factor of Ref. 25 falls faster than ours, although the form factors could of course be brought closer to one another by using a higher value for m_A in Adler's calculation. We practically agree with the results of the Orsay group,[14] which is quite a remarkable coincidence in view of the very elaborate form-factor expressions one extracts from the q^2 behavior of the quark-model helicity transitions given in Ref. 14, which also contain direct quark form factors. One should mention that the erratic behavior of the C_4^A form factor in Ravndal's model (see Table III) is a result of presenting Ravndal's results in terms of invariant form factors. In terms of the c.m. helicity amplitudes which are conventionally used for the formulation of the quark-model results the q^2 dependence of Ravndal's model is not strikingly different from ours.

From Table II it is apparent that the $q^2=0$ coupling values $C_3^V(0)$, $C_4^V(0)$, and $C_5^A(0)$ predicted in our quark-model approach are only in approximate agreement with the predictions of CVC and PCAC. From the application of PCAC in the axial N-N coupling case (Goldberger-Treiman relation) one knows that PCAC is only approximately fulfilled, whereas CVC is believed to be an exact statement. We have calculated the sensitivity of our results to these upward adjustments and have found that at $E \simeq 10$ GeV the neutrino cross section increases by $\simeq 30$ and $\simeq 35\%$ if the CVC values for $C_3^V(0)$ and $C_4^V(0)$ and the PCAC value for $C_5^A(0)$ are substituted for our quark-model values. Obviously the limited neutrino Δ production data do not allow one at present to accurately extract the relevant $q^2=0$ coupling values that would allow one to test the CVC and PCAC principles.

V. NEUTRAL-CURRENT PRODUCTION OF Δ

The calculation of the neutral-current excitation proceeds in complete analogy to the charged-current case discussed in Sec. IV. The spatial structure of the vector and axial-vector current matrix elements is the same as specified in Eqs. (6). The coupling strength of the neutral vector and axial-vector current can be read off from the $I=1$ piece in Eq. (4) with the parameters α and β determined in Table I according to the four different SU(2) × U(1) gauge models that we are considering here.

All four sets of neutral-current couplings involve the Weinberg angle as a free parameter. We shall take $x_W = \sin^2\theta_W \simeq 0.3-0.4$ which lies in a range favored by recent experiments.[1] This range also includes the unrenormalized value $x_W = \frac{3}{8}$ as calculated in grand unification schemes.[18,39] Our results will be given for the two values $x_W = 0.3$ and $x_W = 0.4$.

The AKW model involves a second free parameter $\cos^2\alpha_{AKW}$. At its extremes at $\cos^2\alpha = 0$ and $\cos^2\alpha = 1$, the AKW model can be seen to coincide with the WS model and the GS model, respectively. We shall not present any detailed results for various values of $\cos^2\alpha_{AKW}$ since an estimate of the dependence on this parameter can be obtained by interpolating between the extremal values represented by the WS and GS models.

In Fig. 4 we show the production cross section for $\nu p \to \nu\Delta^+$. The WS model gives the largest cross-section values, and the GS model the lowest. The results of the AKW model lie between these extremes depending on the value of the parameter $\cos^2\alpha_{AKW}$.

Antineutrino cross section $\bar{\nu}p \to \bar{\nu}\Delta^+$ are shown in Fig. 5. Again the results for the WS and GS models are comparatively large. One notes that the predictions of the vectorlike model are quite sensitive to the value chosen for x_W. As in the charged-current case the antineutrino cross section saturates more slowly compared to the neutrino cross section. In Figs. 6 and 7 we show differential cross sections for $\nu p \to \nu\Delta^+$ and $\bar{\nu}p \to \bar{\nu}\Delta^+$ at a typical beam energy of $E = 2$ GeV. There is a marked difference in the shape of the differential cross sections for the WS, GS, and V models in the small-$|q^2|$ region. Owing to the conserved nature of the vector current, only the axial-vector current contributes in the forward direction, and thus the differential cross section is very sensitive to the axial-vector current strength in this region.[40] If the differential cross sections could be determined with some degreee of accuracy in the region $|q^2| \lesssim 0.1$ GeV^2, one would have a very stringent test of the underlying weak-interaction model for the neutral-current coupling. One also notes from Figs. 6 and 7 that such a test would

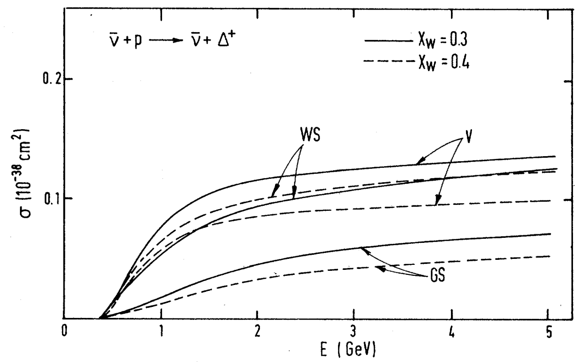

FIG. 5. Predictions for neutral-current antineutrino cross section $\bar{\nu}p \to \bar{\nu}\Delta^+$. Description is as in Fig. 4.

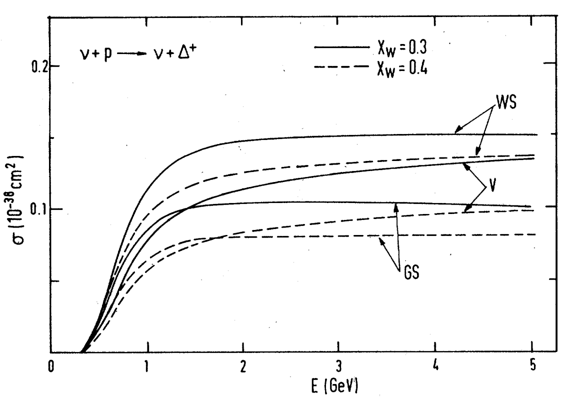

FIG. 4. Predictions for neutral-current neutrino cross section $\nu p \to \nu\Delta^+$ using neutral currents given by the models of Ref. 15 (WS), Ref. 16 (V), and Ref. 18 (GS) for two different values of the Weinberg parameter x_W.

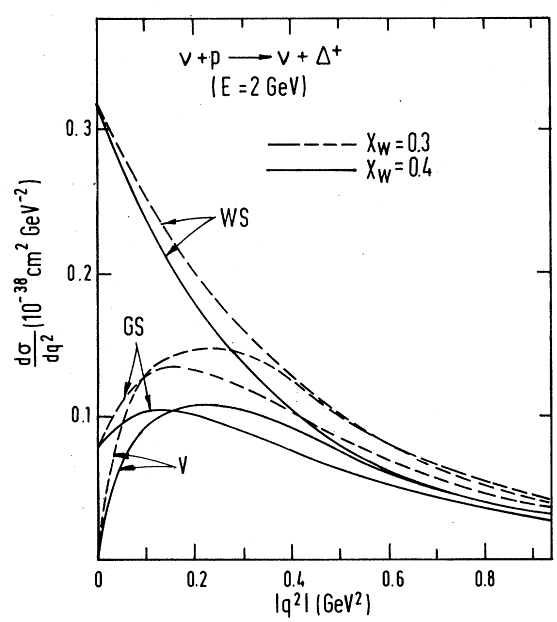

FIG. 6. Differential-cross-section prediction for neutral-current neutrino process $\nu p \to \nu\Delta^+$ at $E = 2$ GeV. Description is as in Fig. 4.

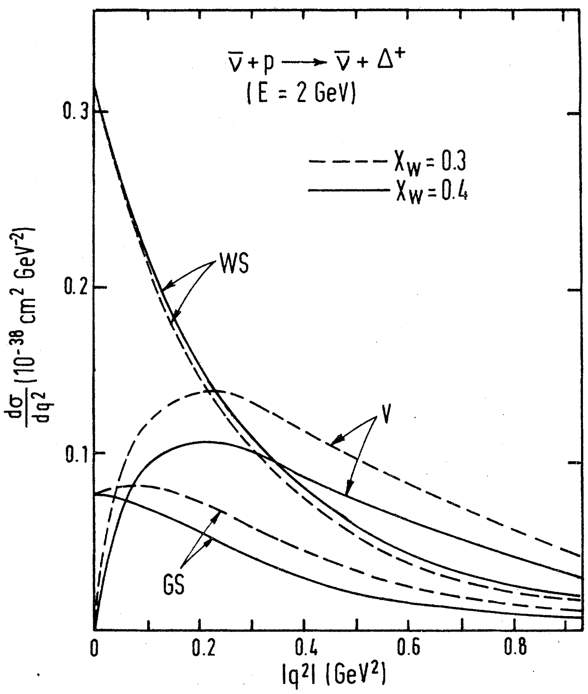

FIG. 7. Differential-cross-section prediction for neutral-current antineutrino process $\bar{\nu}p \to \bar{\nu}\Delta^+$ at $E = 2$ GeV. Description is as in Fig. 4.

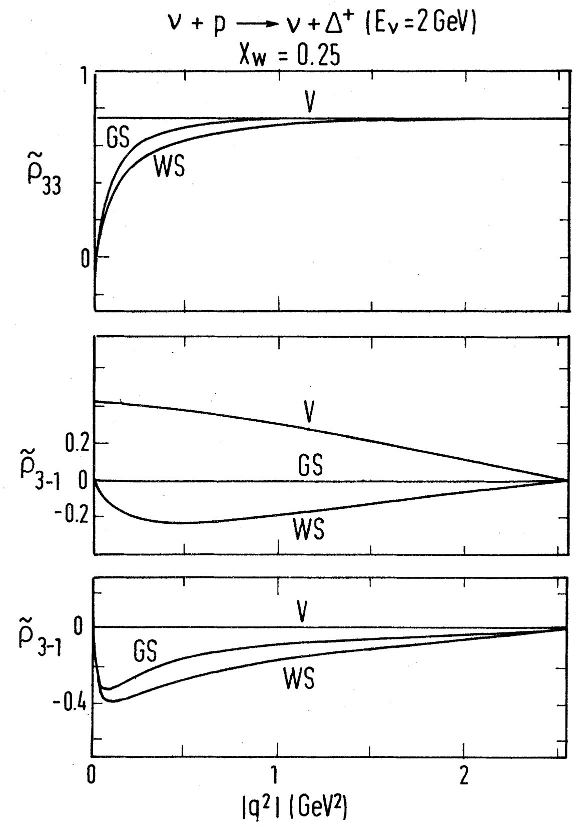

FIG. 8. Density matrix elements $\tilde{\rho}_{33}$, $\tilde{\rho}_{3-1}$, and $\tilde{\rho}_{31}$ for neutral-current reaction $\nu p \to \nu \Delta^+$ at $E = 2$ GeV and Weinberg parameter $x_W = 0.25$.

not depend very sensitively on the Weinberg parameter x_W.

In Fig. 8 we plot the predicted behavior of the three density matrix elements $\tilde{\rho}_{33}$, $\tilde{\rho}_{3-1}$, and $\tilde{\rho}_{31}$ (defined in Refs. 19 and 33) which can be determined from the angular correlations of the decay products of the Δ. The indicated q^2 range extends from the forward direction $q^2 = 0$ to the backward direction $q^2 = q_{max}{}^2$. For kinematical reasons $\tilde{\rho}_{33}$, $\tilde{\rho}_{3-1}$, and $\tilde{\rho}_{31}$ have to vanish in the forward direction $q^2 = 0$, and $\tilde{\rho}_{3-1}$ and $\tilde{\rho}_{31}$ have to vanish in the backward direction $q^2 = q^2_{max}$.[19] The reason that $\tilde{\rho}_{33}$ and $\tilde{\rho}_{3-1}$ do not in fact vanish in the forward direction in the vectorlike model is due to the dynamical vanishing of the forward cross section in this case. Since the density matrix elements are normalized to the differential cross section, the nonvanishing of $\tilde{\rho}_{33}$ and $\tilde{\rho}_{3-1}$ at $q^2 = 0$ is a spurious nonmeasurable result. The fact that $\tilde{\rho}_{31} \equiv 0$ over the whole q^2 range for the vectorlike model is due to the dynamics of the quark model, which predicts that the vector transition is purely transverse.

The difference in the predicted behavior of the density matrix elements between the vectorlike model on the one hand and the GS and WS models on the other hand is quite pronounced. Of course one should keep in mind that a test of the neutral-current structure through the density matrix ele-

TABLE IV. Predicted neutral-current ratios R^{ν}_{Δ} and $R^{\bar{\nu}}_{\Delta}$ for various gauge models at two energies $E = 2$ and 5 GeV, and for two values of the Weinberg parameter $x_W = 0.3$ and 0.4.

	$E = 2$ GeV				E = 5 GeV			
	R^{ν}_{Δ}		$R^{\bar{\nu}}_{\Delta}$		R^{ν}_{Δ}		$R^{\bar{\nu}}_{\Delta}$	
	$x_W = 0.3$	0.4	0.3	0.4	0.3	0.4	0.3	0.4
WS	0.21	0.18	0.91	0.99	0.22	0.20	0.74	0.72
Vector	0.16	0.12	1.14	0.84	0.20	0.14	0.79	0.58
GS	0.15	0.11	0.45	0.33	0.15	0.11	0.43	0.32

ments rather than the differential cross sections becomes meaningful only for $|q^2| \gtrsim 0.1\ \text{GeV}^2$ where there would be enough events in the case of the vectorlike model.

Finally we give our results for the ratio $R_\Delta^{\nu(\bar\nu)}$ of neutral- and charged-current production of the Δ defined by

$$R_\Delta^{\nu(\bar\nu)} = \frac{\sigma_{\nu(\bar\nu)p\to\nu(\bar\nu)\Delta^+}}{\sigma_{\nu(\bar\nu)p\to l^{-(+)}\Delta^{++(0)}}} \tag{8}$$

for two representative energy values $E=2$ and 5 GeV. It is apparent from Table IV that the dependence of R_Δ^ν on E and x_W is not very large, whereas for the antineutrino case $R_\Delta^{\bar\nu}$ the WS and vectorlike models show some energy dependence. At $E=2$ GeV the difference between the neutrino and antineutrino ratios R_Δ^ν and $R_\Delta^{\bar\nu}$ is quite large for the WS and V models, whereas there is not much difference in the GS model.

VI. SUMMARY AND CONCLUSION

We have presented a dynamical model of the weak excitation of the Δ(1236) which is simple and which possesses no adjustable parameters. The q^2 dependence of the isobar excitation form factors arises from the coupling of vector and axial-vector mesons and their recurrences. We have applied the model to charged-current Δ production and have found good agreement with the charged-current neutrino and antineutrino cross-section data.

We then applied the model to the neutral-current Δ production processes using various gauge model currents. We found that owing to the vanishing of the vector current contribution in the forward direction neutral-current Δ production in the near-forward region is quite sensitive to the underlying gauge structure and is an ideal process to separately gain information on the $I=1$ axial-vector piece of the neutral current. For the specific gauge model currents that have been considered here, the Weinberg-Salam and vectorlike models show the most extreme opposite behavior in the near-forward region ($|q^2| \lesssim 0.2\ \text{GeV}^2$). Whereas the WS model predicts a cross-section peak in the forward direction, the vectorlike model shows a strong dip in this region with a cross-section zero at $q^2=0$. The GS model shows a somewhat weaker dip which is, however, pronounced enough to set it aside from the prediction of the WS model. At larger q^2 values, the differences in the differential cross-section behavior of the various models are not so pronounced, so that experiments in this region would not be so useful for distinguishing among models. Owing to the forward peaking predicted for the WS model, this model also has the largest total cross section compared to the other models. The differences of the predicted cross sections of the various models are, however, not very large so that measurements of the total cross section with the expected large error bars of the first-generation experiments will not be so useful in discriminating between the various gauge models. The density matrix elements measureable from Δ decay are again quite sensitive to the underlying gauge structure. In particular, the vectorlike model shows a behavior which is quite distinct from the behavior of the other models.

It is clear from the above that the most useful test of the neutral-current structure in the case of Δ production would come from differential-cross-section measurements near the forward direction. Since Pauli effects and charge-exchange-scattering effects in heavy nuclei tend to wash out the near-forward structure,[41] it would be desirable to obtain such data using hydrogen or deuterium targets. Another experimental problem is the possible presence of non-negligible background contributions under the Δ-mass peak as has been observed in the corresponding charged-current case.[32] Background subtraction may pose a formidable experimental challenge, in particular since model calculations of the background contribution[13,42] are themselves dependent on the structure of the weak neutral current which one is attempting to investigate. One may hope that these problems can be overcome in the near future so that optimal use can be made of neutral-current Δ production in pinning down the hadronic structure of the weak neutral current.

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[23]The N-Δ form factors in Eq. (7) cannot *a priori* be compared to the elastic nucleon form factors since they are different dynamical entities. In the dispersion-theoretic model of Adler (Ref. 25) the N-Δ form-factor behavior arises from a Born-term contribution involving the elastic nucleon form factor. Within the context of this model one may define (q^2-dependent) equivalent dipole masses of N-Δ form factors. Using the explicit parametrization of Adler's results for $q^2 \simeq 0$ given in Ref. 26 our form factors have equivalent dipole masses of $\hat{m}_V = 0.92$ GeV and $\hat{m}_A = 1.28$ GeV at $q^2 \simeq 0$. The axial mass is close to the value $\hat{m}_A = (1.17 \pm 0.03)$ GeV obtained for the axial N-Δ form factor from a soft-pion analysis of threshold $\pi\Delta$ electroproduction at DESY (Ref. 27). The experimental value quoted for the true axial-vector form-factor mass m_A varies depending on the experimental data used and on the method of analysis [see H. H. Williams, in *Particles and Fields '76*, proceedings of the Annual Meeting of the Division of Particles and Fields of the APS, edited by H. Gordon and R. F. Peierls (BNL, Upton, New York, 1977), p. D 95] and lies in the range $m_A \simeq 0.9$–1.0 GeV.
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