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Neutral-current–induced neutrino reactions in deuterium

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The total cross section for the reaction $\nu_\mu + d \to p + n + \nu_\mu$ is calculated for values of $40 \leq E_\nu \leq 1000$ MeV. The differential cross section $d\sigma/d|q^2|$ is also obtained for the same reaction at values of $|q^2|$ from threshold to 1.0 $(\mathrm{GeV}/c)^2$. The corresponding antineutrino reaction is also considered. The Weinberg-Salam model is the basis of these calculations, but the case of an entirely vector weak neutral current is also treated. The elementary-particle model is used to describe the matrix element of the hadronic part of the weak current. The described reaction is shown to be a suitable one for examing the axial-vector part of the weak neutral current.

I. INTRODUCTION

In the last few years gauge-theory models of the weak interactions, first proposed by Weinberg and Salam,[1] have been used successfully to explain many current experimental results and to provide a more basic understanding of the weak and electromagnetic interactions. It is therefore important to undertake calculations to carefully test these models.

In this paper we present a calculation of the total cross section for the neutral-current neutrino reaction[2] $\nu_\mu + d \to p + n + \nu_\mu$ for incident neutrino energies, $1000 \geq E_\nu \geq 40$ MeV, a range which includes LAMPF energies and the lower range of energies available at Argonne National Laboratory. We also calculate the differential cross section $d\sigma/d|q^2|$, $|q^2|$ from threshold to 1.0 $(\mathrm{GeV}/c)^2$ for the same reaction. This particular reaction is attractive for a number of reasons. First, at low neutrino energies, even up to several hundred MeV, the process is dominated by the axial-vector-current form factor.[3] It is therefore possible to isolate a particular part of the weak neutral current using this reaction and to test theories with pure vector neutral currents. Secondly, this reaction can be run below the threshold for muon production, thus avoiding a background due to charged-current processes.

This process has been calculated by the use of the impulse approximation at reactor and medium energies by a number of authors.[4] In this paper the elementary-particle-model approach is used as the method of calculation.[5] In this method the nuclei are treated as elementary particles of particular spin and parity. The form factors which are necessary for describing the matrix element of the weak charged vector current are generally obtained from the corresponding electromagnetic form factors via the CVC (conserved vector current) hypothesis. The form factors describing the axial-vector-current matrix element for a weak charged current are generally obtained from β decay results by making use of the PCAC (partially conserved axial-vector current) hypothesis and a result derived via the impulse approximation. Nuclear-structure effects are, of course, contained in the form factors.

In the case of a neutral-current process, the form factors can be obtained from form factors describing the corresponding charged-current processes via current commutation relations (discussed in Sec. II). This makes it possible to avoid using any impulse-approximation-derived results.

The advantage offered by the elementary-particle model, particularly in the calculation to be described over the conventional impulse-approximation treatment is that the elementary-particle-model approach avoids the use of nuclear wave functions. Cross sections calculated by the use of an impulse-approximation treatment sometimes depend sensitively on these wave functions, which in general are not well known.

In Sec. II of this paper we discuss the form of the matrix elements of the weak neutral current and give expressions for these matrix elements, assuming the Weinberg-Salam model of the weak and electromagnetic interactions. In Sec. III of this paper we obtain an expression for the differential cross section $d\sigma/d|q^2|$ for values of $|q^2|$ from threshold to 1.0 $(\mathrm{GeV}/c)^2$ and for the total cross section σ for E_ν from 40 to 1000 MeV. The antineutrino cross sections are also discussed. In Sec. IV we discuss the results obtained in Sec. III and conclude that the neutral-current neutrino disintegration reaction in deuterium would be a useful reaction for testing various models of the weak interactions. Comparison of these results with other calculations is also made.

II. GENERAL FORMULATION

The transition matrix element for the process $\nu_\mu + d \to p + n + \nu_\mu$ can be written, assuming the

Weinberg-Salam model and ignoring the q^2 dependence[6] of the W-particle propagator, as

$$M(\nu_\mu + d \to p + n + \nu_\mu) = \frac{1}{\sqrt{2}} G\langle pn|J_\lambda(0)|d\rangle \nu_\mu \gamma^\lambda (1-\gamma_5)\nu_\mu \quad (1)$$

where $G\,(=1.025\times 10^{-5}/m_p{}^2)$ is the weak coupling constant and $J_\lambda(0)$ is the non-strangeness-changing part of the hadronic weak neutral current. In the Weinberg-Salam model

$$J_\lambda(0) = J_\lambda^{(3)}(0) - \sin^2\theta_W J_\lambda^{(\mathrm{em})}(0)\,, \quad (2)$$

where

$$J_\lambda^{(3)}(0) = V_\lambda^{(3)}(0) - A_\lambda^{(3)}(0) \quad (3)$$

is the third component of an isotriplet of weak currents. The current $J^{(3)}$ has a vector and an axial-vector part as indicated. In Eq. (2) $J_\lambda^{(\mathrm{em})}$ is the electromagnetic current. The form of Eq. (1) follows immediately from the relevant term in the Weinberg-Salam Lagrangian[7]

$$(g^2+g'^2)^{1/2} Z^\mu \left[J_\mu^{(3)} - \sin^2\theta_W J_\mu^{(\mathrm{em})} + \tfrac{1}{2}\bar{\nu}\left(\frac{1-\gamma_5}{2}\right)\nu \right].$$

The weak currents $J_\mu^{(1)}$, $J_\mu^{(2)}$, and $J_\mu^{(3)}$ satisfy commutation relations of the form

$$[I^{(i)}, J_\mu^{(j)}] = i\epsilon_{ijk} J_\mu^{(k)}\,, \quad (4)$$

so that, for example,

$$\langle nn|J_\mu^\dagger|d\rangle = \langle nn|[I^-, J_\mu^{(3)}]|d\rangle = \sqrt{2}\,\langle np|J_\mu^{(3)}|d\rangle\,, \quad (5)$$

where $J_\mu^\dagger = J_\mu^{(1)} - iJ_\mu^{(2)}$. From Eq. (5) it is clear that the mathematical form of $\langle np|J_\mu^{(3)}|d\rangle$ and $\langle nn|J_\mu^\dagger|d\rangle$ (or $\langle pp|J_\mu|d\rangle$) must be the same. The form of the matrix elements $\langle nn|J_\mu^\dagger|d\rangle$ and $\langle pp|J_\mu|d\rangle$ have been obtained by the author in previous papers.[8,9,10] The results for $V_\lambda^{(3)}$ and $A_\lambda^{(3)}$ may be written as

$$\langle np|V_\lambda^{(3)}(0)|d\rangle = \eta\bar{u}(p_1)\left(\frac{F_1^{(3)}}{M_d{}^2}\epsilon_{\lambda\nu\rho\sigma}\xi^\nu Q^\rho d^\sigma + \frac{F_2^{(3)}}{M_d}\gamma^\nu \epsilon_{\nu\rho\sigma\lambda}\xi^\rho q^\sigma\right)\gamma_5 v(p_2)\,, \quad (6)$$

$$\langle np|A_\lambda^{(3)}(0)|d\rangle = \eta\bar{u}(p_1)(F_A^{(3)}\xi_\lambda + F_P^{(3)}\xi\cdot Q q_\lambda/M_d{}^2)\gamma_5 v(p_2)\,, \quad (7)$$

where the relationship among $J_\lambda^{(3)}$, $V_\lambda^{(3)}$ is given by Eq. (3). In Eqs. (6) and (7)

$$\eta = [M^2/(E_1E_2)]^{1/2}(2\pi)^{-1/2}(2d_0)^{-1/2}\,,$$

M being the nucleon mass, M_d the deuteron mass, E_1 and E_2 the nucleon energies, d_0 the deuteron energy, and ξ_λ the deuteron polarization vector. The form factors $F_1^{(3)}$, $F_2^{(3)}$, $F_A^{(3)}$, and $F_P^{(3)}$ are functions of three scalar variables Q^2, q^2, and $P\cdot d$ with

$$Q_\mu = (p_1+p_2)_\mu\,, \quad P_\mu = (p_1-p_2)_\mu\,, \quad q_\mu = (Q-d)_\mu\,. \quad (8)$$

The quantities $p_{1\mu}$, $p_{2\mu}$, and d_μ are the four-momenta of the nucleons and the deuteron respectively.

From Eq. (5), the relationship between the charged-current form factors and the neutral-current form factors is obvious[11]:

$$F_A^{(3)} = \pm F_A/\sqrt{2}\,, \quad F_P^{(3)} = \pm F_P/\sqrt{2}\,, \quad F_1^{(3)} = \pm F_1/\sqrt{2}\,, \quad F_2^{(3)} = \pm F_2/\sqrt{2}\,, \quad (9)$$

where[8,9,10] F_A and F_P are the form factors describing the matrix elements $\langle nn|A_\lambda|d\rangle$ and F_1 and F_2 are the matrix elements describing $\langle nn|V_\lambda|d\rangle$.

The form factor F_A is obtained from experimental data[12] on the reactions $\nu_\mu + d \to p + p + \mu^-$ and $\mu^- + d \to n + n + \nu_\mu$. Assuming a Nambu[13] form for the PCAC relation, F_P can be obtained from F_A:

$$F_P = -M_d{}^2 F_A/(q^2 - m_\pi^2)\,. \quad (10)$$

We find it convenient to parametrize the form factors F_i as

$$F_i = f_i(q^2)\mathfrak{F}_i(q^2, Q^2, P\cdot d)\,, \quad i = 1, 2, A, P \quad (11)$$

where[10]

$$|f_1(q^2) - f_2(q^2)| = (1 - q^2/M_V{}^2)^{-2}\,, \quad M_V = 0.84\ \mathrm{GeV}\,, \quad (12a)$$

and

$$f_A = (1 - q^2/M_A{}^2)^{-2}\,, \quad M_A = 0.912\ \mathrm{GeV}\,. \quad (12b)$$

The form factor $\mathfrak{F}_A(q^2, Q^2, P\cdot d)$ can be fitted by the expression[13] given below

$$|\mathfrak{F}_A(q^2, Q^2, P\cdot d)|^2 = \frac{(1.16 + 1.54\times 10^{-4} q\cdot D)}{[(5.33\times 10^{-4} q\cdot D - 2.26)^2 + 1.9]}\left\{1 - \frac{1}{M_d{}^{-4}}\left[0.75q^2 - q\cdot D - \frac{(q\cdot D)^2}{M_d{}^2}\right]\right\} R(q^2, \cos\theta)\,, \quad (13a)$$

where

$$R(q^2,\cos\theta)=[1.0+2.9\cos^2\theta+q^2(1.73\times10^{-5}+5.02\cos^2\theta)+q^4(3.27\times10^{-9}+9.48\times10^{-9}\cos^2\theta)]$$
$$\times\frac{1+0.12\exp[-9.8\times10^{-9}(q^2+0.02\times10^6)^2]}{1+\{9.90\times10^2+2.2\times10^3[1-\exp(-5.5\times10^{-12}q^4)]\}q^4}\,, \tag{13b}$$

and θ is the angle between $\vec{p}_1$ and $\vec{q}$ (as defined in Refs. 8 and 9).

Equation (13) is obtained by fitting the experimental data mentioned above. It is not unique but provides a very good fit to the existing data, which is all that is necessary in the model used here.

The matrix elements of $\langle np|V_\mu^{(3)}|d\rangle$ and $\langle np|J_\mu^{(\mathrm{em})}|d\rangle$ are obtained from electrodisintegration data, $e+d\to p+n+e$. At low energies $J_\mu^{(\mathrm{em})}\approx V_\mu^{(3)}$. For electrodisintegration data at a scattering angle of 180° only $V_\mu^{(3)}$ contributes to the differential cross section. As a result, from the existing data it is difficult to distinguish between $J_\mu^{(\mathrm{em})}$ and $V_\mu^{(3)}$. We therefore assume $J^{(\mathrm{em})}\approx J_\mu^{(3)}$. At energies up to these at LAMPF this will be approximately correct.

The existing experimental data[15] can be parametrized as indicated in Eqs. (11) and (12a) where

$$|\mathfrak{F}_1^{(3)}-\mathfrak{F}_2^{(3)}|^2\simeq|\mathfrak{F}_1-\mathfrak{F}_2|^2=\frac{(1.05+1.41\times10^{-4}q\cdot D)}{(5.21\times10^{-4}q\cdot D-2.26)^2+1.8}\left\{1-\frac{1}{M_d^4}\left[0.75q^2-q\cdot D-\left(\frac{q\cdot D}{M_d^2}\right)^2\right]^2\right\}R(q^2,\cos\theta). \tag{14}$$

We need the matrix element $\langle np|J_\lambda|d\rangle$ which by Eqs. (2), (3), and the preceding discussion is given by

$$\langle np|J_\lambda|d\rangle=\langle np|(J_\mu^{(3)}-\sin^2\theta_W J_\mu^{(3)})|d\rangle\equiv\langle np|[(1-\sin^2\theta_W)V_\mu^{(3)}-A_\mu^{(3)}]|d\rangle. \tag{15}$$

From Eqs. (6), (7), and (9), and Eqs. (10) through (14) it is clear that the form factors describing $\langle np|J_\lambda|d\rangle$; $F_1^{(3)}$, $F_2^{(3)}$, $F_A^{(3)}$, and $F_P^{(3)}$ are completely determined. For $\sin^2\theta_W$ we take the popular value 0.35, and so we have obtained the matrix element $\langle np|J_\lambda(0)|d\rangle$.

III. DIFFERENTIAL AND TOTAL CROSS SECTIONS FOR THE REACTION $\nu_\mu+d\to n+p+\nu_\mu$

Making use of Eqs. (1), (2), (3), (6), and (7) we calculate the transition matrix element squared for the process $\nu_\mu+d\to n+p+\nu_\mu$ to be

$$\begin{aligned}|\mathfrak{M}|^2=\frac{2}{3m_\nu{}^2M^2}\Bigg[&(\nu\cdot\nu'\vec{q}^2-\vec{\nu}\cdot\vec{q}\,\vec{\nu}'\cdot\vec{q})(1-\sin^2\theta_W)^2|F_1^{(3)}-F_2^{(3)}|^2\\&+|F_A^{(3)}|^2(p_1\cdot p_2+M^2)\Bigg((3\nu\nu'-\vec{\nu}\,\vec{\nu}')+2\frac{(-q\cdot\nu'\vec{q}\cdot\vec{\nu}-q\cdot\nu\vec{q}\cdot\vec{\nu}'+\nu\cdot\nu\vec{Q}^2}{q-m_\pi{}^2}\\&\qquad+\frac{\vec{Q}^2}{(q^2-m_\pi^2)^2}(2q\cdot\nu'q\cdot\nu-\nu\cdot\nu'q^2)\Bigg)\\&+(1-\sin^2\theta_W)|F_A^{(3)}|\,|F_1^{(3)}-F_2^{(3)}|(p_1\cdot p_2+M^2)(\nu\nu'-\vec{\nu}\cdot\vec{\nu}')\frac{(\nu+\nu')}{M_d}\Bigg].\end{aligned} \tag{16}$$

In the above, $\nu=(\nu,\vec{\nu})$ and $\nu'=(\nu',\vec{\nu}')$ are the incident and scattered neutrino four-momenta respectively.

The last term in Eq. (16) is the interference term for the axial-vector and vector parts of $J_\lambda(0)$. If antineutrinos rather than neutrinos are used, the sign of this term changes and the remainder of $|M|^2$ does not change. At low energies the processes $\nu_\mu+d\to n+p+\nu_\mu$ and $\bar{\nu}_\mu+d\to n+p+\bar{\nu}_\mu$ are indistinguishable.

As had been remarked earlier and as one can see from Eqs. (13), (14), and (16), at low energy transfers the form factors are large. However, for small q^2 values, because

$$q^2=q_0{}^2-\vec{Q}^2, \tag{17}$$

$\vec{Q}^2$ will be small, and hence terms proportional to $\vec{Q}^2$ (or to $|q^2|$) will be small. Hence the axial-vector-current contribution to the matrix element is quite large compared to that of the vector current.

In Fig. 1 we plot the differential cross section $d\sigma/d|q^2|(\nu_\mu+d\to p+n+\nu_\mu)$. We also plot the differential cross section for the charged-current process $d\sigma/d|q^2|(\nu_\mu+d\to p+p+\mu)$ for $|q^2|$ from threshold to 1.0 $(\mathrm{GeV}/c)^2$. In both cases we have folded in the Argonne neutrino spectrum.[12] For the neutral-current process we show two results: (1) using the Weinberg-Salam model of the weak interactions, and (2) using the Weinberg-Salam model but arbitrarily setting F_A, the axial-vector-current form factor, to be zero.

In Fig. 2, we plot the total cross section

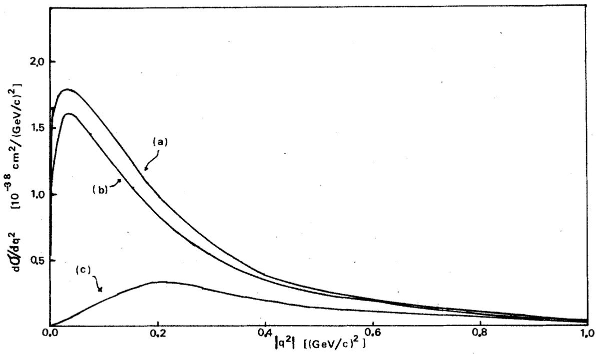

FIG. 1. Plot of the differential cross section $d\sigma/d|q^2|$ for a number of reactions. Curve (a) refers to the reaction $\nu_\mu + d \to n + p + \nu_\mu$ in the Weinberg-Salam model. Curve (b) refers to the reaction $\nu_\mu + d \to p + p + \mu^-$. Curve (c) refers to the reaction $\nu_\mu + d \to n + p + \nu_\mu$ in the Weinberg-Salam model with the axial-vector current arbitrarily set to zero.

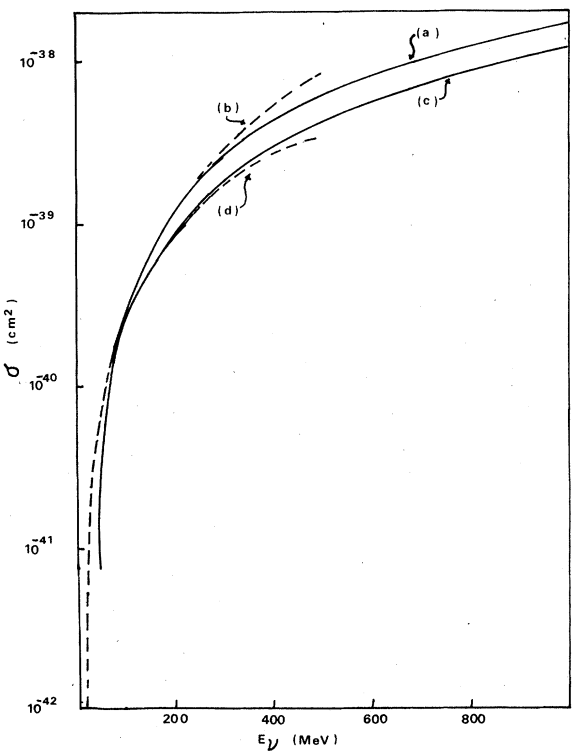

FIG. 2. Plot of the total cross section σ for the reactions ν_μ $(\bar{\nu}_\mu) + d \to n + p + \nu_\mu$ $(\bar{\nu}_\mu)$. Curve (a) refers to the calculation presented here for $\nu_\mu + d \to n + p + \nu_\mu$. Curve (b) refers to an impulse-approximation calculation for the same reaction. Curve (c) refers to the calculation presented here for the reaction $\bar{\nu}_\mu + d \to n + p + \bar{\nu}_\mu$ and curve (d) refers to an impulse-approximation calculation for the same reaction.

$\sigma(\nu_\mu + d \to p + n + \nu_\mu)$ (without the Argonne neutrino spectrum). We again use the Weinberg-Salam model; we also plot $\sigma(\bar{\nu} + d \to p + n + \bar{\nu})$. In Fig. 3 we plot the total cross section $\sigma(\nu_\mu + d \to p + n + \nu_\mu)$ using the Weinberg-Salam model but setting $F_A = 0$.

IV. CONCLUSION

From Fig. 1, curves (a) and (c), it can be clearly seen that in the Weinberg-Salam model the axial-vector-current form factor F_A dominates the differential cross section $d\sigma/d|q^2|$. Even at $|q^2| = 0.3$ $(\mathrm{GeV}/c)^2$, less than 40% of the differential cross section is attributable to the vector current. This same effect can also be noted by a comparison of curve (a) of Fig. 2 and curve (a) of Fig. 3. Thus it is apparent that the reaction $\nu_\mu + d \to \nu_\mu + p + n$ would be quite suitable for studying the axial-vector-current contribution to the neutral weak current.

In Fig. 1, curves (a) and (b) show the differential cross section $d\sigma/d|q^2|$ for the processes $\nu_\mu + d \to \nu_\mu + n + p$ (Weinberg-Salam model) and $\nu_\mu + d \to \mu^- + p + p$, respectively. To gain an approximate feeling for the relationship between these two cross sections we note that the weak coupling constant becomes (effectively) $G/\sqrt{2}$, see Eq. (1), for the neutral process, equal to $G/\sqrt{2}$ for the charged process. The form factor $F_A^{(3)} = F_A/\sqrt{2}$, and at low $|q^2|$

$$|\mathfrak{M}|^2 \propto |F_A|^2, \tag{18}$$

where $|\mathfrak{M}^2|$ is the matrix element squared [Eq. (16)]. Moreover, for the neutral-current process there are no identical particles in the final state, whereas for the charged-current process there are two protons in the final state and thus an extra factor of $\frac{1}{2}$ is present in the latter case. From these considerations one would expect that the differential cross section for the neutral-current process is similar in magnitude to that for the charged-current process, a conclusion which is seen to be approximately true at low $|q^2|$.

In Fig. 2, curves (a) and (b) show the results of this calculation and of an impulse-approximation calculation[16] of the total cross section $\sigma(\nu_\mu + d \to p + n + \nu_\mu)$ for the energy range in which they overlap. The agreement is very good, particularly in the $80 \leq E_\nu \leq 350$ MeV energy range. This region is the one in which the present calculation should be particularly reliable. Argonne[12] data used here do not cover the very-low-$|q^2|$ range (and hence the low-E_ν region) and the electron-scattering data[15] used here to obtain the vector-current matrix element are low to intermediate energy data. Therefore, we expect the best results in the intermediate energy range.

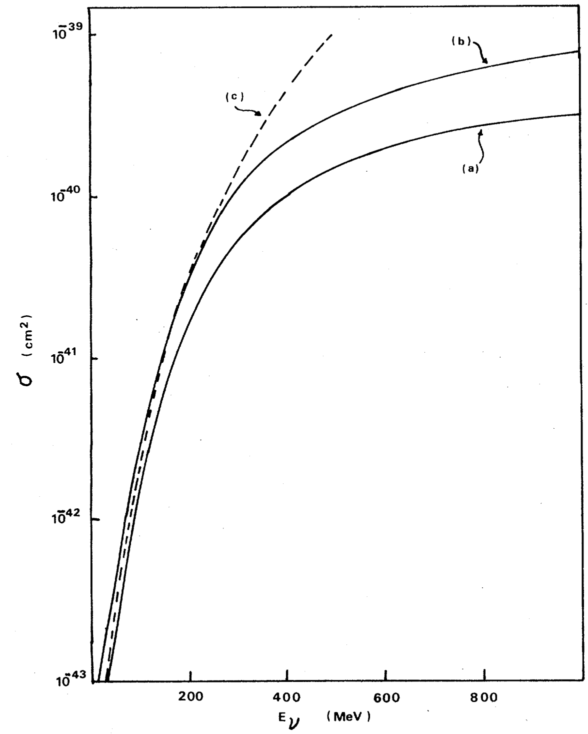

FIG. 3. Plot of the total cross section σ for the reaction $\nu_\mu + d \rightarrow n + p + \nu_\mu$. Curve (a) refers to the calculation presented here in which the Weinberg-Salam model is used but the axial-vector part of the current has been arbitrarily set to zero. Curve (b) refers to the Bég-Zee model calculation corresponding to curve (a). Curve (c) refers to an impulse-approximation calculation in the Bég-Zee model.

In Fig. 3 curve (a), the total cross section, $\sigma(\nu_\mu + d \rightarrow p + n + \nu_\mu)$ is given for the Weinberg-Salam model but for which F_A has been set equal to zero. This curve should have the same form as that which one would obtain using the Bég-Zee[17] model for the weak interactions. Under the assumptions used in this paper the net effect would be to multiply curve (a) [or curve (c) of Fig. 1] by approximately[3] 1.95 to obtain the Bég-Zee model results. If we examine Fig. 2 under this assumption, at $E_\nu = 100$ MeV the differential cross section which one would observe if the Bég-Zee model were correct is about 0.87% of that which one would observe if the Weinberg-Salam model were correct [Fig. 2(b)]. The difference between the two models is of course less striking at higher E_ν values.

Finally, we have plotted the antineutrino reaction $\bar{\nu}_\mu + d \rightarrow p + n + \bar{\nu}_\mu$ in Fig. 2 curve (c). At low E_ν values it is not easily distinguishable from the corresponding neutrino reaction.

Thus we conclude that the reaction $\nu_\mu + d \rightarrow p + n + \nu_\mu$ is, particularly at low E_ν (or low $|q^2|$ values), a promising vehicle for the study of the neutral weak current, as has been emphasized before. It may be further noted that LAMPF intends to run the charged-current reaction on deuterium using electron neutrinos. Data from this reaction can be used to obtain accurate predictions for the corresponding neutral-current process in the low-energy region.

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[2]The calculation for the reaction $\bar{\nu}_e(\nu_e) + d \rightarrow n + p + \bar{\nu}_e(\nu_e)$ is precisely the same as that for $\bar{\nu}_\mu(\nu_\mu) + d \rightarrow n + p + \bar{\nu}_\mu(\nu_\mu)$ yielding the same transition matrix element squared as that given by Eq. (16), and the same results as those given in Figs. 1, 2, and 3. We have not stressed this fact because the present calculation is not expected to be accurate at the reactor energies for which neutral-current electron-neutrino experiments are presently contemplated.

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[4]Impulse-approximation calculations have been performed by Yu V. Gapanov and I. V. Tyutin, Zh. Eksp. Teor. Fiz. 47, 1826 (1964) [Sov. Phys. JETP 20, 1231 (1965)]; S. K. Singh, Phys. Rev. D11, 2602 (1975) at reactor energies, and by A. Ali and C. A. Dominguez (see Ref. 16) at intermediate energies.

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[7]See, for example, S. Weinberg, Phys. Rev. D 5, 1412 (1972), or Ref. 5.

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[9]S. L. Mintz, Phys. Rev. D 10, 3017 (1974).

[10]S. L. Mintz, Phys. Rev. D 13, 639 (1976).

[11] The ± signs in Eq. (9) come from the fact that

$$\langle nn|J^+_\mu(0)|d\rangle = \sqrt{2}\ \langle np|J^{(3)}_\mu(0)|d\rangle$$

but

$$\langle pp|J^{(0)}_\mu|d\rangle = -\sqrt{2}\ \langle np|J^{(3)}_\mu(0)|d\rangle.$$

The ± signs play no role in this calculation since all terms in the transition matrix element squared contain a product of two form factors. We expect that the form factors $F(d \longleftrightarrow pp)$ and $F(d \longleftrightarrow nn)$ will have the same magnitude to the order in which we are calculating, and we shall not distinguish between them.

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