Gupta-Bleuler condition and infrared-coherent states

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A recent claim by Zwanziger that the Gupta-Bleuler subsidiary condition and the infrared-coherence condition are inconsistent is examined. It is shown that the correct form of the Gupta-Bleuler condition agrees with the infrared-coherence condition. The effect of using a subsidiary condition proposed by Zwanziger as a substitute for the Gupta-Bleuler condition is discussed.

In a recent publication[1] Zwanziger proposed a new subsidiary condition to replace the one due to Gupta and Bleuler[2] for the Lorentz gauge formulation of quantum electrodynamics (QED). To support his contention that a change in the Gupta-Bleuler (GB) subsidiary condition is desirable, Zwanziger cites a number of defects from which he claims that the GB condition suffers. The most serious among the faults that he ascribes to the GB condition is incompatibility with the infrared-coherence condition.[3] Another difficulty, related to the first, is that the states that satisfy the GB condition form a Fock space, but the states that satisfy the infrared-coherence condition do not. Also related to the above is the claim that the GB condition has no localized solutions in the charged sector. Elsewhere[4] the argument is made that the solutions in the charged sector are not only nonlocalizable but also non-normalizable.

The purpose of this note is to point out some flaws in Zwanziger's criticism and to show that the GB subsidiary condition is a viable and satisfactory basis for the Lorentz gauge formulation of QED in charged as well as in neutral sectors, and that Zwanziger's subsidiary condition is neither an improvement nor really new, but is a special case of a set of subsidiary conditions that are related to the GB subsidiary condition by pseudounitary transformations.

Care is necessary in the comparison of the GB subsidiary condition and the infrared-coherence condition, because in Zwanziger's work both are written in terms of "in" operators, and, in the case of charged-particle states, these are difficult to treat correctly. In QED in the Lorentz gauge one may not assume that the "in" and "out" limits of field operators obey the equations of motion and commutation rules of noninteracting fields.[5] Such an assumption implies that when photons and electrons have not yet begun to interact (or are no longer close enough to interact), there are no couplings between properly normed electron states and any components of A_μ. In fact, in the physical sector of Hilbert space, charged particles are always coupled to an electric field, as is required by Gauss's law.

For example, in Ref. 1, the asymptotic form of $A_\mu(x)$ is given (in approximate adherence to Zwanziger's notation) by

$$A_\mu^{(\mathrm{in})}(x) = (2\pi)^{-3/2}\int d\vec{k}(2\omega)^{-1}[a_\mu^{(\mathrm{in})}(k)\,e^{ik\cdot x} + a_\mu^{(\mathrm{in})\dagger}(k)\,e^{-ik\cdot x}]\,. \quad (1)$$

If one were to simultaneously express the corresponding limiting form of the spinor field by

$$\psi^{(\mathrm{in})}(x) = (2\pi)^{-3/2}\int d\vec{p}(2E)^{-1}[b_s^{(\mathrm{in})}(p)\,e^{ip\cdot x}u_s(p) + d_s^{(\mathrm{in})\dagger}(p)\,e^{-ip\cdot x}v_s(p)]\,, \quad (2)$$

with canonical commutation rules for the "in" creation and annihilation operators, i.e.,

$$\{b_s^{(\mathrm{in})}(p), b_{s'}^{(\mathrm{in})\dagger}(p')\} = \{d_s^{(\mathrm{in})}(p), d_{s'}^{(\mathrm{in})\dagger}(p')\} = 2E\,\delta_{s,s'}\,\delta(\vec{p}-\vec{p}\,')$$

and

$$[a_\mu^{(\mathrm{in})}(k), a_{\mu'}^{(\mathrm{in})\dagger}(k')] = 2\omega\,\delta_{\mu,\mu'}\,\delta(\vec{k}-\vec{k}')\,,$$

then the use of these "in" operators to construct incident states would be clearly wrong. The procedure would generate charged-particle states which have vanishing expectation values of electric and magnetic fields, whereas in fact we know that the actual charged-particle states, even in the noninteracting "in" limit, are accompanied by electric and magnetic fields. The charged-particle states Zwanziger wants to select with the infrared-coherence condition are modified to take partially, but not completely, this fact into account. The infrared-coherence condition "dresses" the charged particles so that at large distances, which correspond to low-frequency components, they are accompanied by the electric field necessary to obey Gauss's law.

According to Zwanziger's account, the formulation of the infrared-coherence condition is properly covariant, so that the appropriate low-frequency components of the magnetic field also accompany "in" states for charged particles not at rest in the laboratory frame. In short, these "in" states are improvements over the "in" states that have no electromagnetic fields at all accompanying charged particles. However, even "in" states that obey the infrared-coherence condition have expectation values of electromagnetic fields that fail to obey Maxwell's equations, and thus they are not correct "in" states either. For that reason it seems advisable to us to avoid the "in" and "out" nomenclature for charged-particle theories unless the appropriate caveat about the electromagnetic field of charged-particle asymptotic states is explicitly stated.

In contrast to the infrared-coherence condition, the GB condition requires states to obey Gauss's law for all frequency components. We have previously shown[6] how to select a set of asymptotic states that satisfy the GB condition and how to formulate quantum electrodynamics in the space they define. These states also include coherent superpositions of photon components but can be transformed by a pseudounitary transformation, into elements of a Fock space. The charged-particle states selected in Ref. 6 are dressed with the coherent photon "ghosts" required to equip them with the Coulomb field appropriate for a charged particle in its rest frame; they lack all transverse field components. These charged-particle states therefore differ from the "in" states Zwanziger chose.[7] In spite of the discrepancy between these two sets of states, there is no conflict between the infrared-coherence condition and the GB condition. The infrared-coherence condition requires that Gauss's law hold in the low-frequency limit and that, in that same limit, charged particles are accompanied by the transverse fields necessary to preserve termwise manifest covariance of the S matrix. The GB condition requires that Gauss's law hold for all frequency components; it makes no requirements at all on transverse fields and can be satisfied by states having any or no transverse photons. There is no inconsistency between these two conditions and, in particular, the "correct" asymptotic "in" and "out" states would have to satisfy both.

The erroneous conclusion that the GB subsidiary condition contradicts the infrared-coherence condition [i.e., the inconsistency between Eqs. (1.14) and (1.15) in Ref. 1] is based upon a mistaken assumption about the $t\to\pm\infty$ limits of the Heisenberg fields A_μ and ψ. Reference 1 takes account of the fact that the infrared-coherence condition requires the $t\to\pm\infty$ limit of A_μ to differ from a free field by the potentials contributed by a moving charge. But the GB condition is treated in Ref. 1 as though the $t\to\pm\infty$ limit of A_μ were entirely free. It is this discrepancy that is responsible for the apparent inconsistency. A correct treatment of the GB condition neither requires nor even permits A_μ to behave like a free field in the $t\to\pm\infty$ limit when charges are present.

To see this in detail we represent the Heisenberg field A_μ as

$$A_\mu(x)=\frac{1}{(2\pi)^{3/2}}\int\frac{d\vec{k}}{2\omega}[a_\mu(k,x_0)e^{ik\cdot x}+a_\mu^\dagger(k,x_0)e^{-ik\cdot x}]\,. \qquad (3)$$

The assumption that is implicitly made in Ref. 1 is that, for the purpose of specifying the GB condition, in the limiting case $t\to-\infty$, $a_\mu(k,x_0)$ and $a_\mu^\dagger(k,x_0)$ become the time-independent $a_\mu^{(\text{in})}(k)$ and $a_\mu^{(\text{in})\dagger}(k)$, respectively; a similar assumption is made about the $t\to+\infty$ limit and the corresponding "out" operators. In the limit $t\to-\infty$, $A_\mu(x)$ then becomes $A_\mu^{(\text{in})}(x)$ given in Eq. (1). This is the assumption that is made when the Lehmann-Symanzik-Zimmermann (LSZ) formalism is applied to all components of A_μ in a manifestly covariant gauge, and this is also the assumption made in the earlier work of Yang and Feldman.[8] If we use the $t\to-\infty$ limit of $A_\mu(x)$, given in Eq. (1), to calculate the positive-frequency part of $\partial_\mu A_\mu$ we find that

$$(\partial_\mu A_\mu^{(\text{in})})^{(+)}=\frac{i}{(2\pi)^{3/2}}\int\frac{d\vec{k}}{2\omega}\,k\cdot a^{(\text{in})}(k)\,e^{ik\cdot x} \qquad (4)$$

and then the subsidiary condition becomes the one Zwanziger identifies as the GB condition, namely

$$k\cdot a^{(\text{in})}(k)\,|\phi\rangle=0\,. \qquad (5)$$

The use of Eq. (5) as the subsidiary condition implies that $A_\mu(x)$ has been assumed to behave like a free field in the $t\to-\infty$ limit, and leads to the following paradox. Since $\vec\nabla\cdot\vec{E}$ can be represented as

$$\vec\nabla\cdot\vec{E}=i\nabla^2A_4-\partial_0(\vec\nabla\cdot\vec{A})\,, \qquad (6)$$

it also has the "in" limit

$$\nabla\cdot E^{(\text{in})}=\frac{-1}{(2\pi)^{3/2}}\int\frac{d\vec{k}}{2}[k\cdot a^{(\text{in})}(k)\,e^{ik\cdot x}+k\cdot a^{(\text{in})\dagger}(k)\,e^{-ik\cdot x}]\,. \qquad (7)$$

Since the electron "in" fields $\psi^{(\text{in})}(x)$ and $\bar\psi^{(\text{in}}(x)$ commute with $A_\mu^{(\text{in})}(x)$, and if we choose an electron state $|e_p\rangle$ for Eq. (5), we get

$$k\cdot a^{(\mathrm{in})}(k)\,|e_p\rangle = 0 \tag{8}$$

as the subsidiary condition, and can infer that $\vec\nabla\cdot\vec{\mathrm{E}}$, taken between one-electron states, vanishes. But the equation $\langle e_p|\vec\nabla\cdot\vec{\mathrm{E}}|e_p\rangle = 0$ violates Gauss's law, and therefore Eq. (8) cannot be the correct form of the GB condition, since the latter not only cannot violate, but in fact implies the validity of Gauss's law in the physical subspace. The equivalence of Gauss's law and the GB condition has previously been discussed[9] and is summarized as follows: In the Feynman gauge (which we choose for convenience, though this argument could be repeated for any other one of the Lorentz gauges) $\vec\nabla\cdot\vec{\mathrm{E}} - \rho = -\partial_0(\partial_\mu A_\mu)$ is one of the Euler-Lagrange equations of motion.[10] Therefore the validity of $\partial_\mu A_\mu^{(+)}|\phi\rangle = 0$ and $\langle\phi|{*}\partial_\mu A_\mu^{(-)} = 0$ is equivalent to the validity of both, $\langle\phi|{*}\partial_\mu A_\mu|\phi\rangle = 0$ and $\langle\phi|{*}(\vec\nabla\cdot\vec{\mathrm{E}} - \rho)|\phi\rangle = 0$. It is not surprising that this paradox arises. When one assumes that A_μ is a free field in the $t\to\pm\infty$ limit, one does not allow for the obvious fact that part of A_μ must be available to accompany charges into the asymptotic region of space-time. Even when $t\to\pm\infty$, part of A_μ cannot be free, but must represent the longitudinal field required by Gauss's law. In different ways this same point is made in the papers listed in Ref. 5.

In addition to the foregoing, the following point can also be made. In general, the assumption that a field behaves like a free field in the $t\to\pm\infty$ limit neither follows from nor contradicts the equations of motion. But in QED, because of the coupling of A_μ to a conserved current, and the role that the indefinite-metric space plays in the theory, we can infer behavior in the $t\to\pm\infty$ limit that is incompatible with the assumption that A_μ is free in the $t\to\pm\infty$ limit. For example, from Eq. (3) and the equations of motion we find that

$$\partial_0[k\cdot a(k,x_0)] = \frac{i\omega}{(2\pi)^{3/2}}\int d\vec{\mathrm{x}}\,e^{-ik\cdot x}\left[-\rho(x) + i\int d\vec{\mathrm{y}}\,\mathfrak{D}(\vec{\mathrm{x}}-\vec{\mathrm{y}})\,\partial_0\rho(y)\Big|_{x_0=y_0}\right] \tag{9a}$$

and

$$\partial_0[\bar{k}\cdot a(k,x_0)] = \frac{i\omega}{(2\pi)^{3/2}}\int d\vec{\mathrm{x}}\,e^{-ik\cdot x}\left[\rho(x) + i\int d\vec{\mathrm{y}}\,\mathfrak{D}(\vec{\mathrm{x}}-\vec{\mathrm{y}})\,\partial_0\rho(y)\Big|_{x_0=y_0}\right], \tag{9b}$$

where $\bar{k}$ designates $(k_1, k_2, k_3, -k_4)$ and where

$$\mathfrak{D}(\vec{\mathrm{x}}-\vec{\mathrm{y}}) = (2\pi)^{-3}\int d\vec{\mathrm{k}}(\omega)^{-1}\exp[i\vec{\mathrm{k}}\cdot(\vec{\mathrm{x}}-\vec{\mathrm{y}})]\,.$$

In general the two time derivatives given in Eqs. (9) do not vanish as $t\to\pm\infty$ although they may vanish in special cases, as for example in the limit $\omega\to 0$, provided that the integrals on the right-hand side of Eqs. (9) remain finite. However, if we define

$$\rho(k,x_0) = e^{i(H+\omega)x_0}\rho(\vec{\mathrm{k}})\,e^{-iHx_0}, \tag{10}$$

then the combination

$$\Omega(k,x_0) = k\cdot a(k,x_0) + (2\pi)^{-3/2}\rho(k,x_0) \tag{11}$$

and its adjoint $\Omega^*(k,x_0)$ have vanishing time derivatives. We can make use of that fact to rewrite the A_0 component of A_μ in Eqs. (3) (the curl-free part of $\vec{\mathrm{A}}$ can be treated in the same way, but to save space only the results for A_0 will be given); we get that

$$A_0(x) = \frac{1}{(2\pi)^{3/2}}\int\frac{d\vec{\mathrm{k}}}{4\omega^2}\{[\bar{k}\cdot a(k,x_0)\,e^{ik\cdot x} + \bar{k}\cdot a^\dagger(k,x_0)\,e^{-ik\cdot x}] - \Omega(k,x_0)\,e^{ik\cdot x} - \Omega^*(k,x_0)\,e^{-ik\cdot x}\}$$
$$+ \frac{1}{(2\pi)^3}\int\frac{d\vec{\mathrm{k}}}{2\omega^2}\,\rho(k,x_0)\,e^{ik\cdot x}. \tag{12}$$

$\Omega(k,x_0)$ and $\Omega^*(k,x_0)$ in this equation both are time independent, and therefore are their own $t\to\pm\infty$ limits. $\Omega(k,x_0)$ and $\Omega^*(k,x_0)$ are c-number multiples of the positive- and negative-frequency parts of $\partial_\mu A_\mu$ which in turn obeys $\Box\,\partial_\mu A_\mu = 0$ and is an invariant local operator. These considerations dictate that the proper form for the GB condition is[11]

$$[k\cdot a(k)+(2\pi)^{-3/2}\rho(k)]\,|\phi\rangle=0\,. \qquad (13a)$$

Since we have shown that $[k\cdot a(k)+(2\pi)^{-3/2}\rho(k)]$ is its own $t\to\pm\infty$ limit, we are also entitled to write the GB condition as

$$[k\cdot a(k)+(2\pi)^{-3/2}\rho(k)]^{(\mathrm{in})}\,|\phi\rangle=0 \qquad (13b)$$

or, similarly, with an "out" designation. If we combine Eq. (12) with a similar expression for $\vec\nabla\cdot\vec{\mathrm{A}}$ and then use Eq. (6), we find that

$$\vec\nabla\cdot\vec{\mathrm{E}}-\rho=-(2\pi)^{-3/2}\int\frac{d\vec{\mathrm{k}}}{2}[\Omega(k)\,e^{ik\cdot x}+\Omega^*(k)\,e^{-ik\cdot x}] \qquad (14)$$

so that for physical states $\langle\phi\,|*(\vec\nabla\cdot\vec{\mathrm{E}}-\rho)\,|\phi\rangle=0$. The asymptotic behavior of A_0 dictated by Eq. (12) does not at all agree with the condition that, as $t\to\pm\infty$, A_0 is a free field. The components of A_0 that are proportional to Ω and Ω^* have vanishing expectation values in the physical subspace. Within the physical subspace A_0 is therefore constrained by Eq. (12) to "follow" the charge density ρ in a kinematic relation that implies Gauss's law. Of course A_0 is gauge dependent and has matrix elements that project beyond the physical subspace. Similar results for the curl-free part of $\vec{\mathrm{A}}$, and more details about the gauge structure of A_μ and ψ in QED, have previously been given elsewhere and will not be repeated here.[12] The transverse parts of $\vec{\mathrm{A}}$ may be assumed to behave like a free field as $t\to\pm\infty$ without affecting the GB condition in any way, although, as Zwanziger points out correctly, not without doing violence to the manifest covariance of the infrared-coherence condition.

How does Eq. (13b) compare with the infrared-coherence condition [Eq. (1.15) of Ref. 1]? In general, $k\cdot a(k,x_0)$ does not have a time-independent $t\to\pm\infty$ limit, so that $k\cdot a^{(\mathrm{in})}(k)$ cannot be defined. However, Eqs. (9) give us some confidence that, in the limit $\omega\to0$, and if the total amount of charge in the space is finite, $\partial_0[k\cdot a(k,x_0)]$ vanishes and the limit $\lim_{\omega\to0}[k\cdot a^{(\mathrm{in})}(k)]$ exists. In that case we may write that Eq. (13b) has, as a special case

$$\lim_{\omega\to0}[k\cdot a^{(\mathrm{in})}(k)+(2\pi)^{-3/2}Q]\,|\phi\rangle=0 \qquad (15)$$

since $Q=\lim_{\omega\to0}\rho(k)$. Thus the GB condition agrees with Zwanziger's form of the infrared-coherence condition [Eq. (1.15) in Ref. 1] and the inconsistency that he claims to have found is not there.

Suppose we were to identify $k\cdot a(k,x_0)+(2\pi)^{-3/2}\rho(k,x_0)$ as $k\cdot a'(k,x_0)$, and then construct the potentials $A_\mu(x)$ from the primed $a'_\mu(k,x_0)$ operators? If we identify $k\cdot a'(k)\,|\phi\rangle=0$ as the GB condition, and note that $a'_\mu(k,x_0)$ can be substituted for $a_\mu(k,x_0)$ in the expression for $\partial_\mu A_\mu$, we might believe that Zwanziger's paradox has some validity after all. But deeper reflection shows otherwise. The primed $a'_\mu(k,x_0)$ contain $\rho(x)$ implicitly and no longer have canonical equal-time commutation rules with $\psi(x)$. Moreover, in the expression for $\vec\nabla\cdot\vec{\mathrm{E}}$ [in Eq. (6)] the primed $a'_\mu(k,x_0)$ may *not* be substituted for the unprimed $a_\mu(k,x_0)$, and the $(2\pi)^{-3/2}\rho(k)$ reappear to restore consistency between the GB and the infrared-coherence conditions. This topic is considered in considerable detail in earlier publications.[13] The discussion will not be repeated here except to confirm that this essentially notational change does not alter anything of substance in the foregoing argument.

In Ref. 4 Maison and Zwanziger claim that states that satisfy the GB subsidiary condition [Eq. (13a)] cannot be normalized in charged sectors, and that no consistent theory can be based on the existence of charged states that satisfy the GB condition. However, that claim is not valid. The fact that QED is formulated in an indefinite-metric space completely eliminates this problem. To see this, we express the solutions of Eq. (13a) as

$$|\nu\rangle=e^{-D}\,|n\rangle\,, \qquad (16a)$$

where $|n\rangle$ are elements of the Fock space that constitutes the physical subspace of the indefinite-metric space for spinor QED in a Lorentz gauge. Equation (16a) leads to

$$|\nu\rangle=\exp\left[\frac{-1}{(2\pi)^{3/2}}\int\frac{d\vec{\mathrm{k}}}{4\omega^3}\,\bar k\cdot a^\dagger(k)\,\rho(k)\right]|n\rangle\,, \qquad (16b)$$

which agrees with Eq. (1.12) in Ref. 4. $|\nu\rangle$ includes a coherent superposition of photons and superficially may appear to have a divergent norm in the charged sector in which $\rho(0)\neq0$. However, $\bar k\cdot a(k)$ and $\bar k\cdot a^\dagger(k)$ commute, even though they are each others adjoints in the indefinite-metric space. For that reason the norm is not divergent, and, in fact,

$$\langle\nu_i^*|\nu_j\rangle=\langle n_i^*|n_j\rangle=\delta_{i,j} \qquad (17)$$

trivially. In QED the starred adjoint (P^* for operators P and $\langle i^*|$ for states $|i\rangle$) is the representation-independent adjoint that takes the place that the Hermitian adjoint has in a positively normed space. It is the only adjoint that needs to be introduced into the theory, and the norm $\langle i^*|i\rangle$ is the only one that can be given a meaningful interpretation. A self-contained algebra can be based on this adjoint and norm.[14] In a positive-semidefinite-metric Hilbert space $k\cdot a(k)$ and its adjoint could not commute, since such commuting adjoints imply the existence of non-null state vectors with vanishing norms. In that case (i.e., in a positive-semidefinite-metric space) a relationship such as that expressed in Eq. (16b) would lead to non-normalizability for the $|\nu\rangle$ when the $|n\rangle$ are normalized. But in QED Eqs. (16) are the ones that express the correct relationship between the elements of the Fock space, $|n\rangle$, and the states that satisfy the GB condition, $|\nu\rangle$, in charged as well as in the neutral sector. D is a nonlocal operator and its nonlocality is transmitted to the states it defines. We therefore have the option of representing the states $|\nu\rangle$ in a space defined by the $|n\rangle$, in which case they involve nonlocal superpositions of $|n\rangle$ states, but then the equations of motion are local. Or else we may use a Fock construction for the states $|\nu\rangle$ directly, but then the equations of motion are nonlocal. Both versions are discussed in Ref. 6. These nonlocalities are not faults or difficulties but are necessary for a nontrivial QED.[15]

In Ref. 1 Zwanziger recommends the use of a substitute subsidiary condition,

$$[k\cdot a^{(\text{in})}(k)+(2\pi)^{-3/2}f(k)]\,|\xi^{(\text{in})}\rangle=0 \tag{18}$$

(where f is some c-number function) in place of the GB condition. When $\omega\to 0$ then $f\to Q$, and Eq. (18) is designed to be consistent with the infrared-coherence condition. Earlier in this note we pointed out that such a device is not necessary. The question to be discussed here is what effect the substitution of Eq. (18) for Eq. (13) has on the theory. In Eq. (18) we are again faced with the question of how to interpret $k\cdot a^{(\text{in})}(k)$, since the "in" designation presupposes the existence of limits as $t\to-\infty$ that do not agree with Gauss's law. We will resolve this question in the following way. In previous work we defined operators[16]

$$D(0)=\frac{-1}{(2\pi)^{3/2}}\int\frac{d\vec{k}}{4\omega^3}\,[\bar{k}\cdot a(k,0)\,\rho(-k,0)-\bar{k}\cdot a^\dagger(k,0)\,\rho(k,0)] \tag{19}$$

and

$$D(x_0)=e^{iHx_0}D(0)\,e^{-iHx_0}\,. \tag{20}$$

$D(x_0)$ was used to generate the pseudounitary transformation

$$k\cdot a\,(k,x_0)=\exp[D(x_0)]\,\Omega(k)\exp[-D(x_0)]\,. \tag{21}$$

In the limit $x_0\to-\infty$, $\Omega(k)$ is its own limit, but $D(x_0)$ and $k\cdot a(k,x_0)$ approach no limit. We will, however, use Eq. (21) to write

$$k\cdot a^{(\text{in})}(k)=\exp(D^{(\text{in})})\,\Omega\exp(-D^{(\text{in})})\,, \tag{22}$$

even though the time-independent limits $k\cdot a^{(\text{in})}(k)$ and $D^{(\text{in})}$ do not exist, so that we can identify our operators with Zwanziger's. We will extend this same notation to similar transformations in which other form factors [such as the c number $f(k)$] are substituted for $\rho(k)$.

The set of states that satisfy Eq. (18) can be related to the set $|n^{(\text{in})}\rangle$ that satisfy $k\cdot a^{(\text{in})}(k)\,|n^{(\text{in})}\rangle=0$ by

$$|\xi^{(\text{in})}\rangle=\exp(-d^{(\text{in})})\,|n^{(\text{in})}\rangle\,, \tag{23a}$$

where

$$d^{(\text{in})}=-\frac{1}{(2\pi)^{3/2}}\int\frac{d\vec{k}}{4\omega^3}\,[\bar{k}\cdot a^{(\text{in})}(k)f(-k)-\bar{k}\cdot a^{(\text{in})\dagger}(k)f(k)]\,. \tag{23b}$$

And the set $|\xi^{(\text{in})}\rangle$ that satisfy Eq. (18) are related to the $|\nu^{(\text{in})}\rangle$ by

$$|\xi^{(\text{in})}\rangle=\exp(D'^{(\text{in})})\,|\nu^{(\text{in})}\rangle\,, \tag{24}$$

where $D'=D-d$.

The sets of states that can be related to each other by pseudounitary transformations give rise to transition amplitudes for which the following relation has been demonstrated[17]:

$$T_{f,i}=\overline{T}_{f,i}+(E_f-E_i)X+i\epsilon Y\,. \tag{25}$$

Here

$$T_{f,i}=\langle\xi_2|\,{}^*T\,|\xi_2\rangle\,,$$

$$\overline{T}_{f,i}=\langle\nu_f|\,{}^*T\,|\nu_i\rangle\,,$$

T is the transition operator

$$T=(H-H_0)+(H-H_0)(E_i-H+i\epsilon)^{-1}(H-H_0)\,,$$

and H_0 is the operator for which $|\xi_i\rangle$ (or $\nu_i\rangle$) is the eigenstate with eigenvalue E_i. X is nonsingular and Y may have $(i\epsilon)^{-1}$ singularities for wave-function renormalization diagrams, but is less singular otherwise. Except for renormalization constants Eq. (25) leads to $T_{f,i}=\overline{T}_{f,i}$ for on-shell transition amplitudes. Equation (25) therefore guarantees that the iterated S matrix will not suffer any damage from the substitution of any other subsidiary condition for the GB condition provided they are related by a pseudounitary transformation. Since Eq. (24) establishes that

$$[k\cdot a(k)+(2\pi)^{-3/2}\rho(k)]^{(\text{in})}=\exp(-D'^{(\text{in})})[k\cdot a^{(\text{in})}+(2\pi)^{-3/2}f(k)]\exp(D'^{(\text{in})})\,, \tag{26}$$

the use of Zwanziger's substitute subsidiary condition qualifies as one of the harmless substitutions. However, the situation changes when one considers the set of processes for which Zwanziger believes that his formulation is of particular importance, because in those cases more information is necessary than on-shell S-matrix elements can provide, and the conventional resolution of the infrared-divergence problem will not suffice. Such processes include, for example, collisions or decays in which the finite-time effects are important and the on-shell condition is modified in a nontrivial way by the time-energy uncertainty relation.[18] We have previously shown[19] that for these processes the use of the correct form of the GB condition is essential and errors even in lowest-order calculations can result from the use of a substitute subsidiary condition that would be harmless to on-shell S-matrix elements. In fact, in these cases the use of the correct "in" and "out" states is important.

Note added in proof. Zwanziger's remarks do not disprove the claim that I have made above. Zwanziger continues to represent the "in" and "out" limits of the Heisenberg field A_μ as though the latter had no long-range components. In so doing he ignores my argument that, when the Gupta-Bleuler subsidiary condition is imposed in Lorentz-gauge QED, the theory must be consistent with Gauss's law at all times, even in the asymptotic "in" and "out" limits. Zwanziger is of course correct that in his Eqs. (13) weak limits are indicated and that momentum-space operators must be defined with test-function packets. But these remarks have very little bearing on the issue here. The essential point is that when the Gupta-Bleuler condition is imposed, Gauss's law follows, so that $A_\mu^{(\mathrm{in})}$ and $A_\mu^{(\mathrm{out})}$ cannot be entirely detached from currents. Formal details, beyond those in this Comment, are given in my Refs. 6 and 12. Zwanziger partially obscures this issue by restricting his model to a c-number current which is largely contained within a finite region of space and cannot correspond to electrons participating in a scattering event. His $A_\mu^{(\mathrm{in})}$ and $A_\mu^{(\mathrm{out})}$ are therefore always evaluated in regions remote from the current. Zwanziger seems to have chosen the so-called "natural" infrared coherence condition because it does couple the $A_\mu^{(\mathrm{in})}$ and $A_\mu^{(\mathrm{out})}$ to currents. He then infers that this is inconsistent with the Gupta-Bleuler condition because he incorrectly maintains that the latter precludes that kind of coupling.

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