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Derivation of interaction potentials from field theory

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A general formulation is given for the derivation of relativistic equations and associated potentials describing the field-theoretical interaction of two or more particles using a recently developed functional, field-theoretical formalism. The latter allows an exact single-time treatment which makes possible a global (i.e., off-shell) definition of the potential as a certain set of "connected" contributions. The potential thus obtained is a nonlocal operator with a precisely defined energy dependence. Simple, illustrative applications to N-electron and N-nucleon systems are given, and the one-boson exchange and annihilation potentials for the nucleon-antinucleon system are calculated. The one-boson annihilation contribution is shown to be quite important. Remarks concerning the applicability of the formalism as well as extension to non-Abelian gauge theories are presented.

I. INTRODUCTION

This paper is devoted to the presentation of a general formalism for developing two- as well as many-body equations describing the effective field-theoretical interaction of systems of fermions and antifermions which are pure enough in their particle composition to allow a useful description by means of such equations. This procedure is based upon a recently developed, functionally based, field-theoretical formalism,[1] and differs from the existing ones in several important respects. In addition to illustrative applications, a calculation of (relativistic, off shell) single-boson exchange and annihilation nuclear potentials is given which shows the essential importance of the latter for nucleon-antinucleon systems.

A survey of the work on the problem of representing the field-theoretical interaction of a pair of particles by means of wave equations and potentials shows that it originated with the consideration of quantum-electrodynamical (QED) corrections to to the Coulomb potential as early as the birth of quantum field theory itself.[2] Subsequently, in the efforts to deduce the interaction of nucleons from meson field theories, this problem acquired a new significance which led to an extensive investigation involving many workers in the field.[3] While the latter has continued to the present at a more or less undiminished pace, the recent discoveries of heavy mesons and the emergence of quantum chromodynamics (QCD) as a candidate for a theory of strong interactions have posed the similar problem of deducing the quark-antiquark interaction suitable for the description of these heavy mesons as loosely bound, particle-antiparticle states of the theory.[4] While the problem has thus widened in scope and areas of application, the methods of solution have not achieved a parallel degree of generality and perfection, as will be evident shortly.

A survey of the literature in Ref. 3 (which represents the bulk of the efforts on the problem in question) shows that the various methods of constructing interaction potentials from field theory have in the past decade converged upon the reduction of the Bethe-Salpeter[5] (BS) equation to a three-dimensional form.[6] This convergence, however, has at the same time given rise to a large number of alternatives, each with a claim to supremacy, and some with important differences.[7] The cause is well known: The elimination of the unwanted relative-time variable in the BS equation is a nonunique process. Even within the context of QED, where owing to the weakness of the interaction one is facing an essentially unambiguous situation, and where the principal concern is a direct calculation of small corrections to energy levels rather than effective interaction potentials, the process of reducing the BS equation to a three-dimensional form is a subtle and nonunique task.[8] Another main disadvantage of the BS formalism stems from its essentially perturbative structure which prevents a closed expression of its contents and renders any systematic treatment other than a perturbative one a tedious task. Finally, we note that the BS formalism is a two-body formulation whose many-body extension is a rather useless scheme with an N-fermion equation that is a partial-differential system of order N.

In contrast to the existing schemes, the present formulation is based on a field-theoretical equation (FTE) which is free of the relative-time variable in the center-of-mass frame.[1,9] Moreover, because the entire scheme is based on a functional formulation, the resulting formalism is logically independent of perturbation theory and expressible

in closed form (involving functional derivatives). More specifically, one deals with an exact two-body (or many body) equation of the form[10]

$$i\frac{\partial}{\partial t}\chi=(h+\Omega)\chi\,,$$

where χ is a bispinor which depends upon (the common time) t and a pair of spatial variables corresponding to the two interacting particles, h is the sum of two corresponding Dirac Hamiltonians, and Ω represents the interaction by virtue of the presence of functional variables and derivatives therein. The sought-after equation and the associated interaction potential are simply obtained by the conversion of the various (functionally expressed) contributions contained in Ω into numerical interaction terms in accordance with some approximation scheme. In this way, one is naturally led to a relativistic equation (RE) of the form first considered by Breit[2] (in its time-independent version),

$$i\frac{\partial}{\partial t}\chi=(h+V)\chi\,.$$

In short, the passage from the (exact) FTE to the RE is effected without the modification of the kinematic structure of the equation as represented by the Green's function $(i\partial/\partial t-h)^{-1}$. This is the basis of the advantageous features enjoyed by the present method. It is important to note that a further step of turning the RE into a Schrödinger equation is a matter of a nonrelativistic approximation which may or may not be desirable or accurate,[11] and one which is not necessary.[12]

The single-time property of the FTE and the resulting identity of its kinematic structure to that of the RE also form the basis of a global (i.e., off-shell) definition of the interaction potential. This is a very significant improvement not only in the accuracy of the potential, but also in a precise definition of it for many-body problems where the customary on-shell definitions are inherently ambiguous.

Finally, we emphasize that annihilation effects attending the presence of fermion-antifermion pairs are fully incorporated into the FTE. In fact, an interesting result of our calculation of the single-boson interaction for the $N\bar{N}$ system is that the vector-meson (virtual) annihilation contribution to the interaction is a short-range effect of a magnitude comparable to the corresponding exchange contribution. In any case, in view of the fundamental importance of fermion-antifermion systems for all interactions, the inclusion of annihilation effects in the formalism is clearly an essential feature.

Some of the properties discussed above have already been demonstrated in more or less direct applications of the FTE to QED bound-state problems.[1,9] Our main concern here is of course equivalent potentials, and for the most part, this will be discussed in the context of model nucleon interactions. As will be evident from the derivations, however, the formulation is equally applicable to the case of coupling to non-Abelian gauge fields such as QCD.

This paper is organized as follows: In Sec. II the derivation of the two-body FTE for the $N\bar{N}$ system with pseudoscalar coupling (as a model interaction) is presented. An integral variant of this equation, which is more suitable to the considerations of this paper, is also derived. In Sec. III, the general form of the FTE is considered, and an equivalent interaction kernel is defined as a certain set of "connected" contributions. The global nature of this interaction potential and its energy dependence are illustrated in Sec. IV in the context of a many-electron system and a simple nuclear model. The many-body FTE is also derived in this section. Section V presents the derivation of one-boson exchange (OBE) and annihilation contributions and a brief comparison with customary OBE potentials as well as an estimate of the importance of the annihilation part. Various comments regarding the validity of the procedures of this paper and its practical use are contained in Sec. VI.

II. DERIVATION OF THE TWO-BODY EQUATION

In this section we present the derivation of the FTE for the nucleon-antinucleon system within the pseudoscalar coupling model of meson-nucleon coupling. This will serve as a prototype for fermion-antifermion (as well as the simpler fermion-fermion) systems in field theories with linear boson-fermion coupling. First, we sketch the derivation of the field-theoretical formalism. This derivation is strictly analogous to the original derivation developed within QED.[1,9] Using the notation of Ref. 9, we write the Lagrangian for the meson-nucleon system as

$$\mathfrak{L}=\mathfrak{L}_0-\vec{\pi}\cdot\vec{\mathrm{j}},\quad \vec{\mathrm{j}}=\frac{i}{2}g_P[\bar{N},\gamma^5\vec{\tau}N]\,, \tag{1}$$

where π and N are, respectively, the meson and the nucleon fields, and $\vec{\mathrm{j}}$ is the usual pseudoscalar, isovector current of the nucleon. Then we consider a (fictitious) perturbation of the above (fully interacting) system by means of coupling to a c-number current $\vec{\mathrm{J}}$,

$$\mathfrak{L}^{\vec{\mathrm{J}}}=\mathfrak{L}-\vec{\pi}\cdot\vec{\mathrm{J}}\,, \tag{2}$$

with $\vec{\mathrm{J}}$ constrained to vanish at distant times. Thus

the physical system is regained at distant times as well as in the limit of $\vec{J}=0$.

The object of interest, the two-body amplitude, is defined by

$$\chi^G_{\alpha\beta}(x,y)=Z^{\vec{J}}\langle 0,\text{out}\,|\,T[N^{\vec{J}}_\alpha(x)N^{G\vec{J}}_\beta(y)]Q^{\vec{J}}\,|N\bar{N},\text{in}\rangle^{\vec{J}},$$
$$Z^{-1}={}^{\vec{J}}\langle 0,\text{out}\,|\,0,\text{in}\rangle^{\vec{J}}, \tag{3}$$

where N^G is the G-conjugate field to N, "in" ("out") stands for an incoming (outgoing) state, isospin indices have been suppressed, and $\vec{J}$ everywhere signifies the presence of the external source. The projection operator $Q^{\vec{J}}$, appropriate for the description of annihilation effects, is defined by

$$Q^{\vec{J}}=1-Z\,|0,\text{in}\rangle^{\vec{J}\,\vec{J}}\langle 0,\text{out}\,|\,. \tag{4}$$

The use of N^G rather than the usual (charge) conjugate field N^c in the above is on account of the simplicity of the corresponding field equation,

$$(\gamma^\mu p_\mu - m - ig_P\gamma^5\vec{\tau}\cdot\vec{\pi})N=0,$$
$$(\gamma^\mu p_\mu - m - ig_P\gamma^5\vec{\tau}\cdot\vec{\pi}^G)N^G=0, \tag{5}$$

where, as usual, $\pi^G=(-1)^G\pi$, where the exponent represents the G parity of the meson. Note that according to the usual convention,

$$N=\begin{pmatrix}p\\n\end{pmatrix},\quad N^c=\begin{pmatrix}p^c\\n^c\end{pmatrix},\quad N^G=\begin{pmatrix}n^c\\-p^c\end{pmatrix}, \tag{6}$$

and that the usual $N\bar{N}$ amplitude is simply given by $i\chi^G\tau_2$.

Using a procedure strictly analogous to that given in Ref. 9, we arrive at the following two-body FTE:

$$n^\mu(\gamma^{(1)}_\mu\mathcal{L}^{(1)}_x+\gamma^{(2)}_\mu\mathcal{L}^{(2)}_y)\chi^G(x,y)$$
$$=n^\mu(\gamma^{(1)}_\mu\mathcal{F}^{(1)}_x+\gamma^{(2)}_\mu\mathcal{F}^{(2)}_y)S(x,y)C(-i\tau_2), \tag{7}$$

where

$$\mathcal{L}^{(i)}_x=\gamma^{(i)}_\mu p^\mu_x-ig^{(i)}_P\gamma^{5(i)}\vec{\tau}^{(i)}\cdot\vec{\Phi}(x)-m, \tag{8}$$
$$\mathcal{F}^{(i)}_x=ig^{(i)}_P\gamma^{5(i)}\vec{\tau}^{(i)}\cdot\vec{F}(x), \tag{9}$$
$$\vec{F}(x)=-g_P\int d^4x'\Delta^0(x-x')\times\text{tr}[\gamma^5\vec{\tau}\chi^G(x',x')Ci\tau_2], \tag{10}$$
$$g^{(1)}_P=g_P,\quad g^{(2)}_P=(-1)^Gg_P,$$
$$\vec{\Phi}(x)=\vec{\varphi}(x)+i\int d^4z\,\Delta(z,x)\frac{\delta}{\delta\vec{\varphi}(z)}, \tag{11}$$
$$C=i\gamma^2\gamma^0.$$

Note that the original functional variable $\vec{J}$ has been eliminated in favor of $\vec{\varphi}$ which has the geometrical structure of the meson field. The unit vector n^μ is the four-velocity of the two-body state. The superscript (1) [(2)] corresponds to the nucleon (antinucleon) variables and specifies that the γ- and τ-matrices act on the corresponding first (second) index of their operand. The absence of such superscripts, as in the trace operation of (10), signifies the usual matrix multiplication. The two-body equation (7) is supplemented by a pair of Schwinger equations[5]

$$S(x,y)=S^0(x-y)+ig_P\int d^4zS^0(x-z)\gamma^5\vec{\tau}\cdot\vec{\Phi}(z)S(z,y), \tag{12}$$
$$\Delta_{ij}(x,y)=\delta_{ij}\Delta^0(x-y)+g_P\int d^4zd^4z'\Delta^0(x-z)\times\text{tr}[\gamma^5\tau_i\delta S(z,z)/\delta\varphi_i(z')]\times\Delta_{ij}(z',y), \tag{13}$$

where S^0 and Δ^0 are free nucleon and meson propagators. Equation (7) together with the pair of Schwinger equations constitute an exact, closed set of functional differential equations which serve to determine χ^G. The latter, in turn, serves to determine bound-state energies or scattering amplitudes for the corresponding states. We also note in passing that the right-hand side of (7) is specifically a result of the particle-antiparticle nature of the system and would be absent otherwise.

Equation (7) is covariant and thus valid in any coordinate system. In the center-of-mass system, where n^μ has only a temporal component, it assumes the form

$$\left[i\left(\frac{\partial}{\partial x_0}+\frac{\partial}{\partial y_0}\right)-h-U\right]\chi^G(x,y)$$
$$=[\gamma^{0(1)}\mathcal{F}^{(1)}_x+\gamma^{0(2)}\mathcal{F}^{(2)}_y]S(x,y)C(-i\tau_2), \tag{14}$$

with

$$U=U^{(1)}_{(x)}+U^{(2)}_{(y)},$$
$$U^{(i)}_{(x)}=ig^{(i)}_P\gamma^{0(i)}\gamma^{5(i)}\vec{\tau}^{(i)}\cdot\vec{\Phi}(x), \tag{15}$$
$$h=h^{(1)}_x+h^{(2)}_y,\quad h^{(i)}_x=\vec{\alpha}^{(i)}\cdot\vec{p}_x+\beta^{(i)}m.$$

Note that, by definition, the mode of operation of h and U upon χ^G is bispinorial. The occurrence of the temporal derivatives in an additive combination immediately renders the relative time x^0-y^0 a free parameter in the equation. Thus we adopt the common time $t=\frac{1}{2}(x^0+y^0)$, setting the relative time equal to zero, and obtain the single-time equation

$$\left(i\frac{\partial}{\partial t}-h\right)\chi^G=\Omega\chi^G\quad(x^0=y^0=t), \tag{16}$$

where, noting that the right-hand side of (14) is a linear operation on χ^G [see Eqs. (9) and (10)], we have lumped all interaction-bearing terms of (14) into a grand sum Ω. We have thus arrived at the FTE discussed in the Introduction. It is worth emphasizing that (16) is an *exact* equation which (in conjunction with the pair of supplementary equations) serves to determine the equal-time, field-theoretical amplitude $\chi^G(t,\vec{x},\vec{y})$.

A variant of the FTE just discussed will be more suitable for subsequent applications. To derive this version, we note that Eq. (3) may, in view of (4), be written

$$\chi^G_{\alpha\beta}(x,y)=\Upsilon^G_{\alpha\beta}(x,y)-Z^{\vec{J}}\langle 0,\text{out}\,|\,T[N^{\vec{J}}_\alpha(x)N^{G\vec{J}}_\beta(y)]\times|\,0,\text{in}\rangle^{\vec{J}}K,$$

$$K=Z^{\vec{J}}\langle 0,\text{out}\,|\,N\bar{N},\text{in}\rangle^{\vec{J}}, \tag{17}$$

which simply states that Υ^G is given by the right-hand side of (3) with the projection operator $Q^{\vec{J}}$ removed. But in the physical limit of $\vec{J}=0$, K vanishes, implying that in the limit χ^G and Υ^G are identical. Furthermore, an examination of the derivation of (7) in Ref. 9 reveals that Υ^G satisfies the simpler equation

$$n^\mu(\gamma^{(1)}_\mu\mathfrak{L}^{(1)}_x+\gamma^{(2)}_\mu\mathfrak{L}^{(2)}_y)\Upsilon^G(x,y)=0, \tag{18}$$

which is, as already pointed out, the equation χ^G would satisfy if the state were not a particle-antiparticle one. This is also evident from the fact that K can be nonzero only for particle-antiparticle systems, regardless of the presence of $\vec{J}$.

Equations (17) may be written more compactly as

$$\chi^G(x,y)-\Upsilon^G(x,y)=S(x,y)C\tau_2K, \tag{19}$$

which serves to relate the right-hand sides of Eqs. (7) and (19). Equation (18) may now be cast in an integral form, with the information contained in (19) incorporated therein. To do this, we rewrite (18) in the center-of-mass frame [cf. Eq. (14)],

$$\left(i\frac{\partial}{\partial t}-h\right)\Upsilon^G=U\Upsilon^G, \tag{20}$$

and define the corresponding Green's function by

$$\left(i\frac{\partial}{\partial t}-h\right)G(t,\vec{x},\vec{y}\,|\,t',\vec{x}',\vec{y}')=\delta(t-t')\delta(\vec{x}-\vec{x}')\times\delta(\vec{y}-\vec{y}'), \tag{21}$$

which, writen more compactly, reads

$$\left(i\frac{\partial}{\partial t}-h\right)G(t\,|\,t')=\delta(t-t').$$

The boundary conditions to be imposed upon G are the analogs of those of Feynman for the present formulation; they are given in Eqs. (56) and (57).

The integral form of (20) can now be written down,

$$\Upsilon^G=\Upsilon^a+GU\Upsilon^G. \tag{22}$$

It is at this juncture that the considerations culminating in (19) are utilized to determine the homogeneous solution Υ^a; to wit, Eq. (19) yields

$$\Upsilon^a(x,y)=\chi^a(x,y)-S^0(x,y)C\tau_2K. \tag{23}$$

Here χ^a, which of course satisfies

$$\left(i\frac{\partial}{\partial t}-h\right)\chi^a=0, \tag{24}$$

is an appropriate superposition of energy eigensolutions of h [see Eq. (37) in Sec. III]. This leaves us with a determination of K, the matrix element for the process of $N\bar{N}\to$ vacuum in the presence of $\vec{J}$, in terms of χ^a. We mention in passing that while K vanishes in the absence of $\vec{J}$, its functional derivatives, which represent annihilation effects, do not. Indeed the quantity $\vec{F}$, which appeared in the original FTE, is a functional derivative of K; i.e.,

$$\vec{F}(x)=[\delta/\delta\vec{J}(x)]K, \tag{25}$$

as may be seen in the original derivation.[9]

Recalling the definition of K in Eq. (17), we employ the usual procedure of relating matrix elements to vacuum expectation values, modified to suit the present one-time formulation, to obtain

$$K=\int d\vec{x}d\vec{y}^{\vec{J}}\langle 0,\text{out}\,|N^{\dagger\vec{J}}_\alpha(x)N^{G\dagger\vec{J}}_\beta(y)\,|\,0,\text{in}\rangle^{\vec{J}}\times\chi^a_{\alpha\beta}(x,y)\quad(x^0=y^0=t\to-\infty), \tag{26}$$

and the subsequent expression

$$K=-\int dtd\vec{x}d\vec{y}\,\frac{\partial}{\partial t}\times\{^{\vec{J}}\langle 0,\text{out}\,|\,T[N^{\dagger\vec{J}}_\alpha(x)N^{G\dagger\vec{J}}_\beta(y)]\,|\,0,\text{in}\rangle^{\vec{J}}\chi^a_{\alpha\beta}(x,y)\}\quad(x^0=y^0=t). \tag{27}$$

The matrix element occurring in (27) may be expressed in terms of the (equal time) fermion Green's function S, leading to

$$K=i\int dtd\vec{x}d\vec{y}\,\frac{\partial}{\partial t}\,\text{tr}\,[i\tau_2C\gamma^0S(y,x)\chi^a(x,y)]\quad(x^0=y^0=t). \tag{28}$$

At this juncture we use Eq. (24), integrate by parts, and use the equation of motion (20) (which is obeyed by G for equal times) in the familiar

way to arrive at the result

$$K = i\int dtd\vec{x}d\vec{y}\,\mathrm{tr}\{\tau_2\gamma_2[U(y,x)S(y,x)C\tau_2] \times\gamma_2\tau_2\chi^a(x,y)\}\quad (x^0=y^0=t). \quad (29)$$

Recall that the matrix mode of operation of U upon SC is bispinorial [cf. Eqs. (14) and (15)]

With K thus determined, Eq. (23) gives Υ^a in terms of χ^a and S, which in turn completes Eq. (22). To record the sought after equation conveniently, we revert to a compact, symbolic notation. Using the formal solution of (22),

$$\Upsilon^G = (1-GU)^{-1}\Upsilon^a, \quad (30)$$

we can write the related solution for χ^G as follows:

$$\chi^G = SC\tau_2 K + (1-GU)^{-1}(\chi^a - S^0C\tau_2 K). \quad (31)$$

Similarly, Eq. (29) may be written compactly as the linear operation

$$K = \int dtd\vec{x}d\vec{y}\,\mathrm{tr}[\zeta^\dagger(x,y)\chi^a(x,y)] \equiv \zeta\chi^a \quad (x^0=y^0=t), \quad (32)$$

where $\zeta^\dagger$ signifies the adjoint of ζ with respect to its indices. Equations (29) and (32) serve to define ζ.

The result may now be recorded,

$$\chi^G = SC\tau_2 K + (1-GU)^{-1}(1-S^0C\tau_2\zeta)\chi^a, \quad (33)$$

which is a variant of the FTE (7) in an integral form.

III. DEFINITION OF THE INTERACTION POTENTIAL

Having developed a suitable field-theoretical formalism, we are now prepared to define an equivalent relativistic equation. Before we do so, however, we wish to underline some relevant points. Let us first emphasize the obvious fact that unless some suitable approximation allows a simplification of the grand kernel Ω, any attempt at a potential representation is futile; in fact Eq. (16) is as "reduced" a form as one would hope for.[13] Second, even when some simplifications are possible, one must in general be prepared for a rather complicated (e.g., energy-dependent, nonlocal, complex) potential. Third, it is important that the basic definition of an equivalent interaction potential respect the symmetries of the original theory, particularly its relativistic structure.[11] It is thus considered a very significant aspect of the present procedure that the equivalent system possesses the same (relativistic) kinematic structure as the original one, and that one is naturally led to a physically very pleasant equation of the Breit type. As noted in the Introduction, the nonrelativistic approximation of reducing the latter to a Schrödinger equation (which may or may not be valid) is to be considered a step beyond our basic procedure and logically independent of it.

Finally, we emphasize the important feature, mentioned before, that in the present definition any reference to the "energy shell" will be avoided, and a globally valid kernel of interaction will be derived. This general scheme necessitates a corresponding general form for the potential in the form of an integral operator in space as well as in time variables. The temporal nonlocality, being rather trivial as a consequence of time-translation invariance, is in a way a generalization of the energy dependence of the "on-shell" methods, but with the advantage of being well defined for any number of particles.

To state the definition of the potential, we begin with the prototype FTE written in the center-of-mass frame,

$$\left(i\frac{\partial}{\partial t} - h\right)\chi(t\,|\,\vec{J}) = \Omega(t\,|\,\vec{J})\chi(t\,|\,\vec{J}), \quad (16)$$

where, for convenience, we have suppressed indices and spatial variables. The objective is to construct a kernel V such that the solution of the equation

$$\left(i\frac{\partial}{\partial t} - h\right)\chi(t) = \int dt' V(t-t')\chi(t') \quad (34)$$

is the physical (i.e., $\vec{J}\to 0$) limit of that given by (16) for *all times*, i.e.,

$$\lim_{\vec{J}\to 0}\chi(t\,|\,\vec{J}) = \chi(t), \quad (35)$$

or equivalently

$$\lim_{\vec{J}\to 0}\Omega(t\,|\,\vec{J})\chi(t\,|\,\vec{J}) = \int dt' V(t-t)\chi(t'). \quad (36)$$

To implement this condition, let us consider a complete set of bispinors $\{\lambda_\alpha\}$, conveniently taken to be eigenstates of h, and the corresponding set $\{\Lambda_\alpha(t)\}$, where

$$\Lambda_\alpha(t) = e^{-iht}\lambda_\alpha. \quad (37)$$

Next, we define a resolvent R for Eq. (16), symbolically given by

$$R(\vec{J}) = [1 - G\Omega(\vec{J})]^{-1}, \quad (38)$$

where G is as defined in Eq. (21). Then, a general solution of (16) is given by

$$\chi(t\,|\,\vec{J}) = \int dt' R(t,t'\,|\,\vec{J})\chi^a(t'), \qquad \chi^a(t) = \sum_\alpha c_\alpha\Lambda_\alpha(t), \quad (39)$$

where c_α are arbitrary coefficients. A similar so-

lution constructed from (34) in the usual manner is

$$\chi(t) = \int dt' R(t-t') \sum_\alpha c_\alpha \Lambda_\alpha(t') ,$$
$$R = (1 - GV)^{-1} . \tag{40}$$

The equality implied by (36) is now called upon to assert that

$$\int dt' \left(V \frac{1}{1-GV} \right) (t-t') e^{-i\hbar t} \sum_\alpha c_\alpha \lambda_\alpha = \lim_{\vec{J}\to 0} \int dt' \Omega(t|\vec{J}) R(t,t'|\vec{J}) e^{-i\hbar t'} \sum_\alpha c_\alpha \lambda_\alpha . \tag{41}$$

In the physical limit, one recovers time-translation invariance and may therefore define

$$\mathfrak{M}(t-t') = \lim_{\vec{J}\to 0} \Omega(t|\vec{J}) R(t,t'|\vec{J}) . \tag{42}$$

Note that the dependence of $\mathfrak{M}$ upon the spatial center-of-mass coordinates [suppressed in Eq. (42)] is similarly translation invariant.

Using the completeness of λ_α, one obtains from (41),

$$\int dt' \left(V \frac{1}{1-GV} \right) (t-t') e^{-i\hbar t'} = \int dt' \mathfrak{M}(t-t') e^{-i\hbar t'} . \tag{43}$$

We now assert that a solution to (43) is given by the formula

$$V(t) = \mathfrak{M}_c(t) , \tag{44}$$

where the subscript denotes connectedness as defined below. Consider the usual perturbation expansion of $\Omega(\vec{J})R(\vec{J})$,

$$\Omega(\vec{J})R(\vec{J}) = \Omega(\vec{J}) + \Omega(\vec{J})G\Omega(\vec{J}) + \cdots . \tag{45}$$

In the physical limit of $\vec{J}\to 0$, a general term of (45) will have the typical form

$$\{\lim_{\vec{J}\to 0}[\Omega(\vec{J})G\cdots\Omega(\vec{J})]\}G\{\lim_{\vec{J}'\to 0}[\Omega(\vec{J}')G\cdots\Omega(\vec{J}')]\}\cdots , \tag{46}$$

where we have explicitly indicated the independence of the limiting processes occurring in each "connected" piece of the above chain. In analogy to the usual definition, if a term consists of only one such piece (i.e., if it is not possible to break it up into more than one limiting process), it is said to be connected. Otherwise, it is disconnected, consisting of a chain of connected pieces linked by means of the Green's function G.

To demonstrate the validity of Eq. (44), we consider an f-piece disconnected contribution to the right-hand side of (43):

$$\int dt_1 \cdots dt_{2f-2} dt' \mathfrak{M}_c^{(n_1)}(t-t_1) G(t_1-t_2) \times \mathfrak{M}_c^{(n_2)}(t_2-t_3)\cdots G(t_{2f-3}-t_{2f-2}) \times \mathfrak{M}_c^{(n_f)}(t_{2f-2}-t') e^{-i\hbar t'} , \tag{47}$$

where the superscripts indicate the number of times Ω has occurred in a connected piece, i.e., its order in the expansion. Using (44) we can rewrite the above expression as

$$\int dt_1 \cdots dt_{2f-2} dt' V^{(n_1)}(t-t_1) G(t_1-t_2) \times V^{(n_2)}(t_2-t_3)\cdots G(t_{2f-3}-t_{2f-2}) \times V^{(n_f)}(t_{2f-2}-t') e^{-i\hbar t'} , \tag{48}$$

where $V^{(n)}$ is the nth-order contribution to V. Now, summing (47) over all integer values of n_i and f gives $\mathfrak{M}$. On the other hand, summing (48) over n_i first simply replaces each $V^{(n_i)}$ by V, leaving

$$\sum_{f=1}^{\infty} \int dt_1 \cdots dt_{2f-2} dt' V(t-t_1) G(t_1-t_2) \cdots \times V(t_{2f-2}-t') e^{-i\hbar t'} , \tag{49}$$

which is easily recognized as the left-hand side of (43).

The temporal nonlocality of the potential, anticipated in (34) by the expression

$$\int dt' V(t-t')\chi(t') ,$$

is for isolated systems of total energy (i.e., invariant mass) W equivalent to an energy dependence. To see this in a general context, we make the spatial center-of-mass coordinates also explicit in the above, that is, we write

$$\int dt' d\vec{R}' V(t-t', \vec{R}-\vec{R}')\chi(t', \vec{R}') ,$$

where, it will be noted, the relative spatial variables as well as indices are still suppressed in this expression. Translational invariance is evident in V, and for χ it implies that

$$\chi(t, \vec{R}) = e^{-i\vec{W}t}\chi ,$$

since we are working in the center-of-mass system. Thus the original expression is equivalent to

$$\left[\int dt' d\vec{R}' e^{i\vec{W}t'} V(t', \vec{R}') \right] \chi(t) ,$$

which defines an energy-dependent potential inside the brackets. More importantly, however, whatever the consequences of temporal nonlocality, they are unambiguously defined by the original time-dependent version regardless of the particu-

lar situation at hand. An example of this for a many-body system will be considered in the next section.

IV. TWO EXAMPLES

To elucidate the features of the definition given in the last section, we shall consider here two examples, both in the framework of a many-body system, which will serve to illustrate the main points. These examples will also serve to present the generalization to a many-body system of the formalism given in Sec. II. Of course, the two-body interaction will be apparent as a special case.

The first example will be the familiar Coulomb interaction in an N-electron system, where we shall see that the instantaneous character of the interaction reduces the potential to a sum of two-body potentials independent of the many-body environment. In contrast to this, retardation effects will be evident in the second example of an N-nucleon system interacting via scalar mesons. This example will also illustrate the manner in which temporal nonlocality gives rise to N-body effects in two-body interactions.

The FTE for the N-electron system is obtained by means of a straightforward generalization of the original derivation. The amplitude is defined by

$$\chi_{\alpha_1\cdots\alpha_f}(x_1,\ldots,x_f) = {}^J\langle 0,\text{out}\,|T[\psi^J_{\alpha_1}(x_1)\cdots\psi^J_{\alpha_f}(x_f)] \times |P,\text{in}\rangle^J , \tag{50}$$

where ψ is the electron field operator, P is the total momentum of the system, and J represents the external current source. The equation obeyed by χ is found to be

$$\left(n_\mu \sum_{i=1}^{N} \gamma_\mu^{(i)} \mathcal{L}_{x_i}^{(i)}\right)\chi = 0 , \tag{51}$$

where

$$\mathcal{L}_x^{(i)} = \gamma_\mu^{(i)}\left[i\frac{\partial}{\partial x_{i\mu}} - e\Phi^\mu(x_i)\right] - m ,$$
$$\Phi^\mu(x) = A^\mu(x) + i\int d^4z D^{\nu\mu}(z,x)\delta/\delta A^\nu(z) . \tag{52}$$

These equations are as usual supplemented by a pair of Schwinger equations which need not be recorded here.

Just as in the two-body case, the N-time equation (51) reduces to a single-time FTE of the familiar form

$$\left(i\frac{\partial}{\partial t} - h\right)\chi = U\chi, \quad h = \sum_{i=1}^{N} h^{(i)}$$
$$h^{(i)} = \vec{\alpha}^{(i)}\cdot\vec{p}_{x_i} + \beta^{(i)}m , \quad U = \sum_{i=1}^{N} U^{(i)} \tag{53}$$
$$U^{(i)} = e\beta^{(i)}\gamma_\mu^{(i)}\Phi^\mu(x_i) \quad (x_i^0 = t ,\ i = 1,\ldots,N) .$$

The Coulomb interaction is conveniently obtained by replacing the full propagator $D^{\nu\mu}$ in Φ by the free propagator in the Coulomb gauge and retaining the temporal components

$$D_{\nu\mu}(z,x) \to \delta_{\nu 0}\delta_{\mu 0}D_c^{00}(z-x) . \tag{54}$$

We shall denote the corresponding interaction term by V_c.

Let us now consider the one-Coulomb photon-exchange potential V_c as given by (44),

$$V_c = \lim_{\vec{J}\to 0} U_c G U_c$$
$$= \lim_{A_0\to 0}\sum_{\substack{i,j=1\\ i\neq j}}^{N} e\Phi^0(t,\vec{x}_i)G(t,\vec{x}_1,\ldots,\vec{x}_N|t',\vec{x}_1',\ldots,\vec{x}_N')\times e\Phi^0(t',\vec{x}_j') . \tag{55}$$

Note that the restriction in the above sum serves to eliminate self-energy terms.

At this point, the boundary conditions obeyed by G must be precisely stated. First, note that G always occurs in a combination of the type $U^{(i)}GU^{(j)}$, where, as before, the superscript indicates the particle whose coordinates occur in a given term. We now define[9]

$$G^{(i)}(t-t') = -ie^{-ih^{(i)}(t-t')}[\theta(t-t')\Lambda^{+(i)} - \theta(t'-t)\Lambda^{-(i)}] , \tag{56}$$

where $\Lambda^\pm$ are the usual energy projection operators. Using this definition, we can state the boundary conditions:

$$U^{(i)}GU^{(j)} = \tfrac{1}{2}U^{(i)}(G^{(i)}+G^{(j)})U^{(j)} . \tag{57}$$

In short, G obeys Feynman boundary conditions with respect to the coordinates that occur immediately next to it, treating the two sides symmetrically.

Using the definition just given and Eq. (54), we can convert (55) into

$$V_c = \frac{e^2}{4\pi}\sum_{\substack{i=j=1\\ i<j}}^{N}\tfrac{1}{4}\{[(\Lambda^{+(i)}-\Lambda^{-(i)})+(\Lambda^{+(j)}-\Lambda^{-(j)})]\times|\vec{x}_i-\vec{x}_j|^{-1}+|\vec{x}_i-\vec{x}_j|^{-1}\times[(\Lambda^{+(i)}-\Lambda^{-(i)})+(\Lambda^{+(j)}-\Lambda^{-(j)})]\} . \tag{58}$$

Thus the instantaneous character of the Coulomb propagator has reduced the interaction to a sum of instantaneous two-body potentials: Retardation effects are absent. We mention in passing that to the lowest order, the projection operators $\Lambda^\pm$ in (58) may be replaced by unity and zero, respectively, and the familiar form regained. The remaining terms, known as Coulomb corrections,

are known in atomic physics.[15]

The second example is obtained by analogy to the first and with the interaction term as given by

$$U^{(i)}=g_S\beta^{(i)}\Phi(x_i),$$
$$\Phi(x_i)=\varphi(x)+i\int d^4z\,\Delta(z,x)\delta/\delta\varphi(z), \tag{59}$$

where Δ is the (scalar) boson propagator. Again we shall consider the one-boson-exchange interaction V_S which is obtained from the above by taking Δ to be the free propagator and treating $\mathfrak{M}$ up to one exchange. In analogy with the first example, we get

$$V_S(t-t')=ig_S{}^2\sum_{\substack{i,j=1\\ i\neq j}}^{N}\tfrac{1}{2}\beta^{(i)}\Delta(t-t',\vec{x}_i-\vec{x}_j')(G^{(i)}+G^{(j)})(t,\vec{x}_1,\ldots,\vec{x}_N|t',\vec{x}_1',\ldots,x_N')\beta^{(j)}, \tag{60}$$

with the resulting energy-dependent potential

$$V_S^W=\int dt'e^{iWt'}V_S(t'). \tag{61}$$

It is clear from Eq. (60) that V_S^W is a sum of "two-body" potentials each of which involves the coordinates of the remaining N-2 particles as well (these enter via G). To make this feature more transparent, let us adapt this calculation to a model nucleus whose nucleons, while bound by a common potential, exchange scalar mesons. The exchange potential for this model is obtained from (60) by taking $h^{(i)}$ to include the common potential. To make the result still more transparent, we will consider the nonrelativistic limit of (60). Proceeding with these steps, then, we obtain

$$V_S^W(\vec{x}_1,\ldots,\vec{x}_N|\vec{x}_1',\ldots,\vec{x}_N')=-(2\pi)^{-3}g_S{}^2\sum_{\substack{i,j=1\\ i\neq j}}\int d\vec{q}e^{i\vec{q}\cdot\vec{x}_i}(\vec{q}^{\,2}+\mu^2)^{-1}[(\vec{q}^{\,2}+\mu^2)+h-W-i\epsilon]^{-1}e^{-i\vec{q}\cdot\vec{x}_j'}, \tag{62}$$

where the dependence of h upon the spatial variables is the same as that of V_S^W. It is clear now that V_S^W is a sum of two-body potentials only if a product-basis formed of the eigenfunctions of h is employed (in which case the potential has the usual Yukawa form). In general, however, V_S^W is a sum of what may be called quasi-two-body potentials with an energy-dependence specified by (62). Needless to say, the complications of nonlocality in time and its consequences are manifestations of the global definition of the potential.

V. ONE-BOSON-EXCHANGE ANNIHILATION POTENTIALS

In this section, we return to the two-body case and derive the nucleon-antinucleon potential resulting from the exchange of one boson in the forward (usual exchange) and in the crossed (virtual annihilation) channels according to the definition given in Eq. (44).[16] Our aim here is not the development of a comprehensive potential for the $N\bar{N}$ interaction.[17] Rather, limiting ourselves to single-exchange contributions (which, among other things, suffer from the lack of important effects of real annihilation into two or more mesons), we wish to apply the procedures of this paper to deriving relativistic, off-shell versions of well-known OBE potentials as well as the (usually ignored) one-boson-annihilation interaction (which will be shown to be quite significant).

To obtain the desired terms, we turn to the integral version of the $N\bar{N}$ FTE, Eq. (33), and use definition (44) to obtain the expression for the full potential,

$$[\lim_{\vec{J}\to 0}U(1-GU)^{-1}(1-S^0c\tau_2\zeta)]_c. \tag{63}$$

Note that in writing (63) we have dropped a term present in Eq. (33) because it vanishes in the physical limit anyway. We are only interested in one-quantum exchanges:

$$V=\lim_{\vec{J}\to 0}U(GU-S^0c\tau_2\zeta), \tag{64}$$

where it is understood that ζ shall be approximated up to one power of U.

The first term in (64) gives the usual exchange contribution (already encountered in Sec. IV), and the second yields the annihilation part.

Proceeding with the former for an isoscalar, vector exchange (with direct coupling), we obtain

$$V_{VE}(t-t')=(-1)^G g_V{}^2\sum_{\substack{i,j=1\\ i\neq j}}^{2}\tfrac{1}{2}\alpha^{(i)\nu}\Delta^0(t-t',\vec{x}_i-\vec{x}_j')(G^{(i)}+G^{(j)})(t,\vec{x}_1,\vec{x}_2|t',\vec{x}_1',\vec{x}_2')\alpha_\nu^{(j)}, \tag{65}$$

where the exponent G represents the G parity of the exchanged meson, and

$$\alpha_\nu = \gamma_0\gamma_\nu ,$$
$$\Delta^0(x) = (2\pi)^{-4}\int d^4k e^{-ik\cdot x}(k^2 - \mu^2 + i\epsilon)^{-1} .$$

Performing a temporal integration and reverting to momentum representation, we obtain

$$V^W_{VE}(\vec{p}\,|\vec{p}\,') = (-1)^G(2\pi)^{-3}g_V{}^2(\tfrac{1}{2}) \times [\alpha_\nu^{(2)}Q(\vec{p}\,;-\vec{p})\alpha^{(1)\nu} + \alpha_\nu^{(1)}Q(\vec{p},-\vec{p}\,')\alpha^{(2)\nu}] , \tag{66}$$

$$Q(\vec{p}_1,\vec{p}_2) = (\omega + h - W)^{-1} \times \{\omega^{-1} - (\omega + W - h)^{-1}[\Lambda^{-(1)}(\vec{p}_1) + \Lambda^{-(2)}(\vec{p}_2)]\} ,$$
$$\omega = (\vec{p}_1 - \vec{p}_2)^2 + \mu^2 , \quad h = h^{(1)}(\vec{p}_1) + h^{(2)}(\vec{p}_2) . \tag{67}$$

For scalar (pseudoscalar) exchange, $g_V{}^2$ is replaced by $-g_S{}^2$ ($g_P{}^2$) and γ^0 ($\gamma^0\gamma^5$) substituted for α^ν. As usual, the isospin factor $\vec{\tau}^{(1)}\cdot\vec{\tau}^{(2)}$ is to be appended for isovector exchange.

We now proceed with the annihilation contribution in (64). Recalling the definition of ξ in (32), we obtain from (64)

$$V_{VA}(x_1,x_2\,|x_1',x_2') = -i\lim_{\tau\to 0}[U(x_1,x_2)S^0(x_1-x_2)c\tau_2] \times \{\tau_2\gamma_2[U(x_2',x_1')S^0(x_2'-x_1')c\tau_2]\gamma_2\tau_2\} \quad (x_1^0 = x_2^0 = t\,,\ x_1'^0 = x_2'^0 = t') , \tag{68}$$

where U is the quantity

$$U(x_1,x_2) = g_V[\alpha_\nu^{(1)}\vec{\Phi}^\nu(x_1)\cdot\vec{\tau}^{(1)} + (-1)^G\alpha_\nu^{(2)} \times \vec{\Phi}^\nu(x_2)\cdot\vec{\tau}^{(2)}] , \tag{69}$$
$$\vec{\Phi}_\mu(x) = \vec{\varphi}_\mu(x) + i\int d^4z\,\Delta^0_{\mu\nu}(x-z)\,\delta/\delta\vec{\varphi}_\nu(z) .$$

Upon performing the functional differentiation and after removing the center-of-mass coordinates, we obtain

$$V^W_{VA}(\vec{x}\,|\vec{x}\,') = (-1)^G g_V{}^2(W^2-\mu^2)^{-1}\vec{f}^{\,\nu}(\vec{x})\cdot\vec{f}^{\,\dagger}_\nu(\vec{x}') , \tag{70}$$

where

$$\vec{f}^{\,\nu}(\vec{x}) = \vec{\tau}[\gamma^0\gamma^\nu S^0(0,\vec{x}) + (-1)^G S^0(0,\vec{x})\gamma^\nu\gamma^0]c\tau_2 .$$

The isospin factors can be explicitly calculated and reduced to the conservation condition requiring the equality of the isospin of the $N\bar{N}$ state (T) and of the annihilation boson (T_B). Thus the potential, now written in momentum representation, appears as

$$V^W_{VA}(\vec{p}\,|\vec{p}\,') = -2\delta_{T,T_B}(-1)^G g_V{}^2(2\pi)^{-3} \times (W^2-\mu^2)^{-1}\hat{f}^\nu(\vec{p})\hat{f}_\nu(\vec{p}\,') , \tag{71}$$

where

$$\hat{f}^\nu(\vec{p}) = \left[\alpha^\nu\frac{h(\vec{p})}{2E(\vec{p})} + (-1)^G\frac{h(\vec{p})}{2E(\vec{p})}\alpha^\nu\right]\gamma^0 C , \tag{72}$$
$$E(\vec{p}) = (\vec{p}^2 + m^2)^{-1} .$$

For scalar (pseudoscalar) annihilation, $g_V{}^2$ is replaced by $g_S{}^2(g_P{}^2)$ and γ^0 ($\gamma^0\gamma^5$) substituted for α^ν. The explicit representation of the matrix elements of (71), namely, the equation

$$\langle\chi_2|\,V^W_{VA}\,|\chi_1\rangle = -2\delta_{T,T_B}(-1)^G g_V{}^2(2\pi)^{-3}(W^2-\mu^2)^{-1} \times \int d\vec{p}\,\mathrm{tr}[\chi_2^\dagger(\vec{p})\hat{f}^\nu(\vec{p})] \times \int d\vec{p}\,'\,\mathrm{tr}[\hat{f}^\dagger_\nu(\vec{p}\,')\chi_1(\vec{p}\,')] , \tag{73}$$

underlines the separable nature of the annihilation potentials.

Equations (66), (67), (71), and (72), supplemented by the rules given subsequent thereto for various mesons, define our one-boson exchange and annihilation potentials, V_E and V_A, respectively [see also Eq. (74) below]. As remarked before, these are not considered to constitute a complete interaction potential for the $N\bar{N}$ system. Furthermore, a proper comparison of these with the existing versions of OBE potentials requires a numerical solution of the resulting RE,

$$[W - h^{(1)}(\vec{p}) - h^{(2)}(-\vec{p})]\chi(\vec{p}) = \int d\vec{p}\,'V(\vec{p}\,|\vec{p}\,')\chi(\vec{p}\,') . \tag{74}$$

Nevertheless, it is not hard to see that the exchange potentials given here differ significantly from the static, static-plus-$\vec{p}^2$ corrections, or the so-called relativistic versions of OBE potentials.[3] In all of these, the second term contributing to Q in Eq. (67) is absent, and the first term is replaced with ω^{-2} except for the off-shell potentials that are essentially arbitrary continuations of the on-shell ones. Obviously, the quantity W-h in Q, which is responsible for part of the off-shell effects, is far from negligible for $|\vec{p}_1|$ and $|\vec{p}_2|$ of the order of vector-meson masses, or, loosely speaking, at distances corresponding to the range of vector-meson potentials. For essentially the same reason, and at comparable values of momenta, the nonrelativistic approximation (i.e., the reduction to a Schrödinger equation) suffers from serious errors.

Finally, it is clear from (71) that the annihilation potential, which is ignored in the current OBE models of $N\bar{N}$ interaction, is a sizeable effect. Its range (the nucleon Compton wave length) is comparable to that of vector-meson potentials, and so is its strength. Taking the ω potential as an example, and considering the $T=0$, $J=1$, $N\bar{N}$ state, we have for the ratio of the potentials at zero momenta,

$$V_{\mathrm{VA}}(0|0)/V_{\mathrm{VE}}(0|0) = -(\mu_\omega/m)^2 . \tag{75}$$

Since the mass of ω is comparable to that of the nucleon, one can see that the ω-annihilation potential in this example substantially cancels the ω-exchange potential, which in turn is a dominant short-range contribution in OBE models. In short, it is inconsistent to use a short-range (i.e., range comparable to nucleon Compton wavelength) exchange potential without at the same time including the corresponding (virtual) annihilation contribution.

VI. DISCUSSION

In this paper we have developed a procedure for deriving N-body equations and associated interaction potentials from field-theoretical models with linear fermion-boson coupling. As usual, a description of systems of two or more particles in terms of such equations is useful when the particle composition enjoys a fair degree of purity as in ordinary atomic and nuclear systems and perhaps also in charmonium and possibly in other meson and baryon states as well. Of course, the question of the validity of a given field-theoretical model is additional, and one which is logically separate from our considerations. Be that as it may, one must be aware that in field theories where there is a strong coupling which does not weaken with increasing momentum transfer, the above description becomes doubtful for high-momentum regions. The pseudoscalar model of nuclear coupling is presumably an example of such unwanted behavior. Fortunately, the possibility that QCD (or a similarly asymptotically free gauge theory) is the underlying theory of strong interactions is encouraging in this respect. We should also add that the above-mentioned breakdown is additional to the similar malady that would occur as a result of a nonrelativistic description such as the use of the Schrödinger equation. Thus a description in terms of relativistic equations is seen to be a basic advantage of the present scheme. As for the feasibility of the methods, suffice it to say that the customary momentum-space techniques of solution would be adequate. In this respect, we mention a recent angular-momentum resolution and numerical integration of the RE (the Breit equation in this case) with the static Coulomb potential for the positronium atom.[12]

The main application given in this paper is to nuclear forces within the one-boson interaction model. While this afforded the possibility of a brief comparison with existing models, it should be emphasized that we are here invoking the use of RE, rather than the customary Schrödinger equation, for the description of NN (or $N\bar{N}$) systems. The case for a relativistic description is well known.[18] It should be emphasized, however, that any description of the OBE kind must be amended by a phenomenological treatment for short distances. Note that such treatments as the use of cores or boundary conditions are easily adapted to the RE. Note also that, as a consequence of the relativistic structure of the present formalism, the matter of subtracting iterations of a lower-order potential from a higher-order one (which comes about as a result of trading one Green's function for another) is trivially eliminated here by virtue of the connectedness property defined in Sec. V. The latter property is essentially one of two-particle irreducibility within the present formalism (which is slightly different from its counterpart in Feynman theory).

Despite their exclusion in this paper, applications to the quark interactions are perhaps the most interesting ones within the present formalism. As already mentioned, the FTE corresponding to QCD (or other non-Abelian gauge theories) are trivial generalizations to those given in this paper, except, of course, for the supplementary equation corresponding to (13). The latter is qualitatively more complicated on account of the self-coupling of the gauge field. This complication and the related infrared problems notwithstanding, there are useful areas of application which have lead to results bearing on heavy-meson spectroscopy and on a possible quark phase of nuclear matter. Such areas therefore seem the most suitable for future applications of the present formalism.

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[4]For a recent example containing references to similar work, see F. Feinberg, Phys. Rev. D 17, 2659 (1978).

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[6]We are not here concerned with the validity of the field-theoretical models employed or their relative merits *vis-a-vis* other methods such as dispersion-theory techniques.

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[8]See, e.g., G. Feldman, T. Fulton, and J. Townsend, Phys. Rev. A 8, 1149 (1973); and G. P. Lepage, *ibid.* 16, 863 (1977), and references therein.

[9]M. H. Partovi, Phys. Rev. D 14, 3525 (1976).

[10]An equation of the Breit type and a slight generalization thereof [both special cases of Eq. (7) of Refs. 1 and 9] were considered by H. Suura [Phys. Rev. Lett. 38, 636 (1977)] for the study of the meson spectrum within the quark model. In this work, the equation in question was proposed as an essentially *ad hoc*, suitable generalization of the Dirac equation without mention of its rigorous derivation from field theory in Refs. 1 and 9.

[11]In the case of nucleon-nucleon interaction, for example, relativistic effects are known to be non-negligible; see M. H. Partovi and E. L. Lomon, Phys. Rev. D 2, 1999 (1970); see also the second paper of Ref. 3.

[12]The Breit equation may be resolved into partial waves and solved numerically in much the same way as the Schrödinger equation; see M. H. Partovi and A. Jabbarian-Lotfahadi, Phys. Rev. A 17, 1575 (1978).

[13]Noting that the mathematical contents of a field theory comprise field operators obeying certain commutation relations as well as certain partial differential equations, and that functional differentiation furnishes a convenient representation of the former, we see that functional partial differential equations are a minimal way of representing the contents of a field theory.

[14]It is always possible to add a piece to h and proceed as before with a kernel Ω that is similarly modified. This could be advantageous when it is desired to incorporate a known part of the potential into the propagator, as in the second example of Sec. IV.

[15]See the first paper of Ref. 8.

[16]Of course the exchange parts are appropriate to NN interaction as well.

[17]For a recent reference see I. S. Shapiro, Phys. Rep. 35C, 129 (1978).

[18]See Ref. 11 and the second paper of Ref. 3.