Nonlinear realization and unified interaction

Y. M. Cho
Department of Physics, New York University, New York, New York 10003
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Spontaneously broken gauge theory viewed in terms of physical fields is identified as a nonlinear realization of the symmetry. In particular Nambu-Goldstone fields play the role of the auxiliary fields that are needed for a Callan-Coleman-Wess-Zumino nonlinear realization. Possible advantages of the nonlinear realization as an alternative approach to the Higgs mechanism in unifying color and flavor interactions are speculated.

I. INTRODUCTION

Several years ago Callan, Coleman, Wess, and Zumino[1,2] (CCWZ) showed how a symmetry can be realized nonlinearly under a group G yet linearly under a subgroup H. However, so far not much attention has been paid to the nonlinear realization mainly because of the nonrenormalizability aspect of the theory. In this paper we would like to point out that there exist renormalizable and yet nonlinearly realized theories, e.g., spontaneously broken gauge theories when viewed in terms of the physical fields, and then discuss some advantages of the nonlinear realization as an alternative approach to the Higgs mechanism in unifying color and flavor interactions.

Spontaneously broken gauge theories[3,4] have played an important role in recent theoretical physics. Clearly in spontaneously broken gauge theories both zero-mass Nambu-Goldstone fields and massive Higgs scalars transform linearly under the unbroken subgroup H. Then one can ask: How do they transform under the full group G? The answer, as one might have expected, is that they transform as nonlinear multiplets under the group G, which we confirm explicitly in the following. In particular, we show that Nambu-Goldstone fields play precisely the role of the auxiliary scalar fields that one needs for a nonlinear realization. Thus spontaneously broken gauge theory viewed in terms of physical fields is indeed an excellent example of nonlinear realization of the symmetry.

A nonlinear realization is often considered as the one which is linear *but constrained.*[2] However, a nonlinear realization does not always have to be of this kind, although a constrained linear representation is always a nonlinear realization. This point is particularly relevant since a gauge theory with linear but constrained multiplets is not, in general, renormalizable as it often does not allow the Higgs fields necessary for the renormalization. In the following by a nonlinear realization we will always mean a CCWZ nonlinear realization.

In constructing a unified theory of color and flavor interactions[5,6] as a spontaneously broken gauge theory, it is necessary to deal with a large number of scalar multiplets[6] (e.g., $912+1463$ or more fields in the case of E_7) to break the symmetry in a desirable way. Under these circumstances the nonlinear realization has some advantages as an alternative approach to the Higgs mechanism since in this case by utilizing nonlinear multiplets one can avoid dealing with a huge number of linearly realized scalar fields from the beginning, and can treat quantum flavor dynamics (QFD) practically independently of quantum chromodynamics (QCD) in breaking QFD further down to weak and electromagnetic interactions. We will discuss this in more detail in the following.

II. NONLINEAR REALIZATION: A BRIEF REVIEW

For the sake of notational convenience let us start by briefly reviewing the CCWZ nonlinear realization[1]. Any nonlinear realization of a group G that is realized linearly under a subgroup H can be constructed uniquely from its linear realization. To be specific let the subgroup H have m generators t_i $(i=1,2,\ldots,m)$ and the group G have $d=n-m$ additional ones u_a $(a=m+1,m+2,\ldots,m+d)$ and let ψ be a nonlinear realization of G which is linear under H. Now the nonlinear transformation law is obtained with the aid of a d-dimensional auxiliary scalar field $\vec{\xi}$ in the manner of CCWZ:

$$\begin{aligned}&\vec{\xi}\xrightarrow[g\in G]{}\vec{\xi}',\\&\psi\xrightarrow[g\in G]{}e^{-\vec{\alpha}'\cdot\vec{t}}\psi,\end{aligned}\tag{1}$$

where $\vec{\alpha}'$ and $\vec{\xi}'$ are determined uniquely by

$$\begin{aligned}e^{\vec{\xi}\cdot\vec{u}}g&=e^{\vec{\xi}\cdot\vec{u}}(e^{\vec{\alpha}\cdot\vec{t}}e^{\vec{\beta}\cdot\vec{u}})\\&=e^{\vec{\alpha}'\cdot\vec{t}}e^{\vec{\xi}'\cdot\vec{u}},\end{aligned}\tag{2}$$

$\vec{\alpha}$ and $\vec{\beta}$ being the unique "decomposed" parameters[1] of the group element $g\in G$. Notice that according to this rule one can always obtain a nonlinear realization from a linear realization ψ_L of G by defining

$$\psi_N = e^{\vec{\xi}\cdot\vec{u}}\psi_L\,. \tag{3}$$

Indeed under the gauge transformation one has

$$\begin{aligned}\psi_N &= e^{\vec{\xi}\cdot\vec{u}}\psi_L \xrightarrow[g\in G]{} e^{\vec{\xi}'\cdot\vec{u}}(e^{-\vec{\beta}\cdot\vec{u}}e^{-\vec{\alpha}\cdot\vec{t}})\psi_L\\ &= e^{-\vec{\alpha}'\cdot\vec{t}}e^{\vec{\xi}\cdot\vec{u}}\psi_L\\ &= e^{-\vec{\alpha}'\cdot\vec{t}}\psi_N\,.\end{aligned} \tag{4}$$

Now as for the gauge potentials one has A_μ^i and B_μ^a that transform canonically under $g\in G$,

$$\vec{A}_\mu'\cdot\vec{t}+\vec{B}_\mu'\cdot\vec{u} = g^{-1}(\vec{A}_\mu\cdot\vec{t}+\vec{B}_\mu\cdot\vec{u})g + \frac{1}{e}g^{-1}\partial_\mu g\,, \tag{5}$$

where e is the coupling constant. But one can introduce the "nonlinear" gauge potentials α_μ^i and β_μ^a suitable to a nonlinear realization by

$$\begin{aligned}\vec{\alpha}_\mu\cdot\vec{t}+\vec{\beta}_\mu\cdot\vec{u} &= e^{\vec{\xi}\cdot\vec{u}}(\vec{A}_\mu\cdot\vec{t}+\vec{B}_\mu\cdot\vec{u})e^{-\vec{\xi}\cdot\vec{u}} + \frac{1}{e}e^{\vec{\xi}\cdot\vec{u}}\partial_\mu e^{-\vec{\xi}\cdot\vec{u}}\,,\\ \vec{\alpha}_\mu'\cdot\vec{t}+\vec{\beta}_\mu'\cdot\vec{u} &= e^{\vec{\xi}'\cdot\vec{u}}(\vec{A}_\mu'\cdot\vec{t}+\vec{B}_\mu'\cdot\vec{u})e^{-\vec{\xi}'\cdot\vec{u}} + \frac{1}{e}e^{\vec{\xi}'\cdot\vec{u}}\partial_\mu e^{-\vec{\xi}'\cdot\vec{u}}\,.\end{aligned} \tag{6}$$

Notice that these nonlinear gauge potentials are defined as the ones that have "absorbed" the scalar fields $\vec{\xi}$. From their definition one can easily obtain the following transformation law for the nonlinear gauge potentials $\vec{\alpha}_\mu$ and $\vec{\beta}_\mu$:

$$\begin{aligned}\vec{\alpha}_\mu'\cdot\vec{t} &= e^{-\vec{\alpha}'\cdot\vec{t}}\vec{\alpha}_\mu\cdot\vec{t}\,e^{\vec{\alpha}'\cdot\vec{t}} + \frac{1}{e}e^{-\vec{\alpha}'\cdot\vec{t}}\partial_\mu e^{\vec{\alpha}'\cdot\vec{t}}\,,\\ \vec{\beta}_\mu'\cdot\vec{u} &= e^{-\vec{\alpha}'\cdot\vec{t}}\vec{\beta}_\mu\cdot\vec{u}\,e^{\vec{\alpha}'\cdot\vec{t}}\,.\end{aligned} \tag{7}$$

Observe that $\vec{\alpha}_\mu$ and $\vec{\beta}_\mu$ do not mix, and, moreover, $\vec{\beta}_\mu$ transforms covariantly under the nonlinear gauge transformation (7). This transformation law has been observed by CCWZ.[1]

III. AN EXAMPLE: SPONTANEOUSLY BROKEN GAUGE THEORY

Now we will have to construct a renormalizable nonlinear gauge theory. But since the requirement of the renormalizability will almost uniquely restrict the theory to be a spontaneously broken gauge theory[7] we would rather start with a spontaneously broken gauge theory to ensure the renormalizability, and then show that the theory is indeed a nonlinearly realized one in terms of the physical fields. So let us consider a spontaneously broken gauge theory of a group G with an unbroken subgroup H. Clearly in this case one has a d-dimensional Nambu-Goldstone field $\vec{\theta}$ and the Higgs multiplet $\vec{\chi}$ in the theory. Now we will show that under the group transformation of G Nambu-Goldstone fields $\vec{\theta}$ transform exactly like the auxiliary scalar fields $\vec{\xi}$ that one needs for a nonlinear realization. In doing so we will also confirm that the Higgs scalars transform as a nonlinear multiplet. For this remember that one can always decompose $\vec{\phi}$, the linearly realized scalar multiplet that is responsible for the symmetry breaking, in terms of the Nambu-Goldstone fields $\vec{\theta}$ and the "unsubtracted" Higgs multiplet $\vec{\rho}$:

$$\vec{\phi} = e^{-\vec{\theta}\cdot\vec{u}}\vec{\rho}\,. \tag{8}$$

Now since $\vec{\rho}$ must carry only the quantum numbers of the subgroup H one has

$$\begin{aligned}\vec{\rho} = e^{\vec{\theta}\cdot\vec{u}}\vec{\phi} \xrightarrow[g\in G]{} e^{-\vec{\omega}\cdot\vec{t}}\vec{\rho} &= e^{\vec{\theta}'\cdot\vec{u}}\vec{\phi}'\\ &= e^{\vec{\theta}'\cdot\vec{u}}e^{-\vec{\beta}\cdot\vec{u}}e^{-\vec{\alpha}\cdot\vec{t}}\vec{\phi}\\ &= e^{-\vec{\omega}\cdot\vec{t}}e^{\vec{\theta}\cdot\vec{u}}\vec{\phi}\end{aligned} \tag{9}$$

or

$$e^{\vec{\theta}\cdot\vec{u}}e^{\vec{\alpha}\cdot\vec{t}}e^{\vec{\beta}\cdot\vec{u}} = e^{\vec{\omega}\cdot\vec{t}}e^{\vec{\theta}'\cdot\vec{u}}\,. \tag{9$'$}$$

Then the uniqueness of the decomposition of the group element g, i.e., Eq.(2), guarantees that indeed $\vec{\theta}$ and $\vec{\rho}$ transform exactly like the auxiliary fields $\vec{\xi}$ and a nonlinear multiplet, respectively. Of course by assumption $\vec{\rho}$ must have a nonvanishing vacuum value $\vec{\rho}_0$ so that to obtain the physical Higgs scalars $\vec{\chi}$ one has to subtract the vacuum value from $\vec{\rho}$:

$$\vec{\chi} = \vec{\rho} - \vec{\rho}_0\,. \tag{10}$$

But since the vacuum $\vec{\rho}_0$ must remain invariant under the subgroup H, $\vec{\chi}$ itself must transform as a nonlinear multiplet:

$$\vec{\chi} \xrightarrow[g\in G]{} e^{-\vec{\omega}\cdot\vec{t}}\vec{\chi}\,. \tag{11}$$

Thus the Nambu-Goldstone fields in a spontaneously broken gauge theory play the role of the auxiliary scalar fields that one needs in a nonlinear realization, and the corresponding Higgs scalars transform as a nonlinear multiplet under the group G.

Now one can rewrite the whole Lagrangian in terms of the nonlinear multiplets using Eqs. (3), (6), (8), and (10), excluding the Nambu-Goldstone fields, and obtain an effective Lagrangian. In the effective Lagrangian, of course, the covariant vector field $\vec{\beta}_\mu$ will acquire mass, and the scalar potential now written purely in terms of Higgs scalars will remain quartic and by assumption will exhibit no more spontaneous symmetry breaking. Also, since the nonlinear multiplets are in general reducible under H, their mass can be different for different irreducible multiplets of H in the effective Lagrangian.

At this point it is easy to see why a gauge theory with linear but constrained multiplets does not yield a renormalizable theory. The system in general does not contain the Higgs scalars[4] necessary for the renormalization.

As an application of the nonlinear scheme we will now consider a two-step symmetry breaking utilizing nonlinear multiplets which has some advantages over the conventional one-step spontaneous symmetry breaking in unifying color and flavor interactions. To be explicit let us consider E_7 as the unified group G of color and flavor interactions[6] and its maximal subgroup $SU(3)_C \otimes SU(6)_F$ as H. Clearly all the fields are realized linearly under the subgroup $SU(3)_C \otimes SU(6)_F$. Let us denote the multiplets of $SU(3)_C \otimes SU(6)_F$ by (p, q) and choose the quarks to be (3, 6) and the leptons to be (1, 20) with the standard charge assignment.[6] Then out of 133 gauge bosons one has one (8, 1) that is responsible for QCD, and one (1, 35) for QFD. Now by breaking E_7 first to $SU(3)_C \otimes SU(6)_F$ with a linear 133-dimensional scalar multiplet one can make the remaining $(3, \overline{15}) + (\overline{3}, 15)$ "leptoquark" gauge bosons [and (8, 1) + (1, 35) Higgs scalars] superheavy (for the sake of the proton stability), while keeping the fermions massless at this first stage of the symmetry breaking. Then by introducing another set of *nonlinear* scalar multiplet which carries *only* the flavor quantum numbers one can break the flavor subgroup further down to weak and electromagnetic, without destroying the renormalizability of the theory. At this second stage of the symmetry breaking fermions can pick up masses. Clearly this two-step symmetry-breaking mechanism utilizing the nonlinear realization is different from the conventional one-step spontaneous symmetry breaking, and has some advantages since in this case one does not have to deal with a huge number of linearly realized scalars from the beginning, and thus can bypass the rather formidable yet crucially important (to show the validity of the unified scheme) task of constructing the most general Higgs potential and determining its vacuum. More importantly one can treat QFD practically independently of QCD in breaking QFD further down to weak and electromagnetic interactions without losing the unified nature of the theory. Also, this two-step symmetry breaking can naturally accommodate itself with two enormously different mass scales involved, one of the order of Planck mass ($\sim 10^{20}$ GeV) and the other of the order of a weak gauge boson mass ($\sim$100 GeV). A detailed example of the two-step symmetry breaking utilizing nonlinear multiplets will be presented elsewhere.[8]

IV. CONCLUSION

We have shown explicitly that spontaneously broken gauge theory is indeed a nonlinear realization of the symmetry when viewed in terms of the physical fields. In particular, Nambu-Goldstone fields of the theory play precisely the role of the auxiliary scalars that is needed in a nonlinear realization. Thus although nonlinearly realized gauge theories are not in general renormalizable there exist renormalizable ones. In unifying color and flavor interactions some advantages of the nonlinearly realized two-step symmetry breaking over the conventional one-step spontaneous symmetry breaking is discussed.

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[1]S. Coleman, J. Wess, and B. Zumino, Phys. Rev. 177, 2239 (1969); C. G. Callan, S. Coleman, J. Wess, and B. Zumino, *ibid.* 177, 2247 (1969).

[2]A. Salam and J. Strathdee, Phys. Rev. 184, 1750 (1969); 184, 1760 (1969).

[3]P. W. Higgs, Phys. Lett. 12, 132 (1964); G. S. Guralnik, C. R. Hagen, and T. W. B. Kibble, Phys. Rev. Lett. 13, 585 (1964).

[4]G. 't Hooft, Nucl. Phys. B35, 167 (1971); B. W. Lee and J. Zinn-Justin, Phys. Rev. D 7, 1049 (1973).

[5]J. Pati and A. Salam, Phys. Rev. D 8, 1240 (1973); H. Georgi and S. Glashow, Phys. Rev. Lett. 32, 438 (1974); M. Fritzsch, M. Gell-Mann, and P. Minkowski, Phys. Lett. 59B, 156 (1976); F. Gürsey and P. Sikivie, Phys. Rev. Lett. 36, 775 (1976).

[6]P. Ramond, Nucl. Phys. B126, 509 (1977); P. Sikivie and F. Gursey, Phys. Rev. D 16 816 (1977).

[7]J. M. Cornwall, D. N. Levin, and G. Tiktopoulos, Phys. Rev. Lett. 32, 498 (1974); Phys. Rev. D 10, 1145 (1974).

[8]Y. M. Cho (unpublished).