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Test of equivalence principle for particles with spin

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We consider a simple modification of the Dirac equation, such that spin-1/2 particles violate the equivalence principle, but the latter is restored by averaging over spins. An experiment is suggested to test the existence of such an effect.

It is well known[1,2] that, although the usual Eötvös experiment tests only the weak equivalence principle,[3] it could become a test for the strong equivalence principle if performed with polarized bodies (such as test bodies with aligned spins). The purpose of this paper is twofold: First we devise a theoretical model whereby spinning particles violate the equivalence principle, but the latter is restored by averaging over spins. This model involves a dimensionless coupling constant. Then we suggest an experiment which could set an upper limit on its value.

Let $\vec{g}$ denote the local acceleration of gravity ($g = 980\,\mathrm{cm/sec}^2$). The simplest modification of the Dirac Lagrangian involving $\vec{g}$ is to add a term proportional to $\bar{\psi}\gamma^\mu g_\mu \psi$. However, in a static gravitational field, $g_0 = 0$ and $\vec{g}$ is a gradient, therefore, such a term can be transformed away (it is similar to adding a gradient to A_μ).[4]

The simplest nontrivial modification of the Dirac Lagrangian is a term proportional to $i\bar{\psi}\gamma_5\gamma^\mu g_\mu \psi$. The Dirac equation then becomes

$$i\hbar\frac{\partial\psi}{\partial t} = (c\vec{\alpha}\cdot\vec{p} + \beta mc^2 + ik\hbar c^{-1}\gamma_5\vec{\alpha}\cdot\vec{g})\psi\,.$$

In the last term, which conserves CP but not C and P separately, the factor $\hbar c^{-1}$ has been introduced so that the "coupling constant" k is dimensionless. In the nonrelativistic limit, the additional term in the Hamiltonian is simply $\pm k\hbar c^{-1}\vec{\sigma}\cdot\vec{g}$, with opposite signs for particles and antiparticles.

Such a term would mean that a spin-$\frac{1}{2}$ particle carries a gravitational dipole moment $k\hbar c^{-1}\vec{\sigma}$. In classical language, its center of mass and center of gravity are separated by a distance $k\hbar/mc$. It is therefore unlikely that k is a large number.[5]

Let us examine the consequences of our hypothesis. First, we note that a degenerate energy level would be resolved into two close ones separated by $2k\hbar c^{-1}g$. As $2\hbar c^{-1}g = 4.30\times10^{-23}$ eV, such a splitting would be considerably smaller than the present limit on a possible violation of the equivalence principle by weak interactions.[6] Moreover, the spin of a particle would precess around the vertical axis with a frequency $2kc^{-1}g$. Note that $c^{-1}g = 1.03$ rad/yr. For $k = 1$, this precession is much too slow to be observable in neutron interference experiments.[7] Yet it is more than a million times faster than the one predicted by general relativity due to the dragging of inertial frames by the rotation of the earth.[8] However, it affects only spin, not angular momentum in general. Indeed, averaging over spins cancels the $\vec{\sigma}\cdot\vec{g}$ term and the equivalence principle is restored on a macroscopic scale.

Consider now a *polarized* macroscopic body such as a permanent magnet. It would have an additional energy $\pm 2kc^{-1}\vec{S}\cdot\vec{g}$ where $\vec{S} = (\hbar/2)\sum\vec{\sigma}$ is the total spin. This induces a *torque*[9] $\pm 2kc^{-1}\vec{S}\times\vec{g}$, which could be observed in the following way:

Let the permanent magnet, thoroughly shielded from external magnetic fields, hang freely in such a way that in its equilibrium position $\vec{S}$ is approximately horizontal. Then, if the magnetization is destroyed by heating it above the Curie point, the equilibrium position will be shifted by an angle θ such that $MgH\theta = 2kc^{-1}Sg$, where M is the mass of the magnet and H the height of the point of suspension above its center of gravity. Thus,

$$\theta = 2kc^{-1}S/MH = k\hbar c^{-1}/mH\,,$$

where m is the mass of an atom (more generally, the mass associated with spin $\hbar/2$).[10] For iron, we get $\theta = k(3.8\times10^{-16}\,\mathrm{cm}/H)$.

Unless k is very large, the main difficulties in such an experiment, apart from observing such a small angle, would be the following:

(a) External magnetic fields must be *completely* shielded away. Even a single quantum of magnetic flux $\pi\hbar/e$, spread over an area A so that $B = \pi\hbar/eA$, would introduce in the Hamiltonian a term similar to the one we are considering, but with a coefficient $e\hbar B/2m_e = \pi\hbar^2/2m_eA$ instead of $k\hbar c^{-1}g$. We would thus need

$$A \gg \pi c/2k\hbar g m_e \simeq (7500\ \mathrm{cm})^2/k$$

to be able to neglect such a term.

(b) The demagnetization process may upset the mechanical equilibrium of the test body because of the Einstein-de Haas effect[11] and because of magnetostriction.[11]

It seems that the proposed experiment, although very difficult, could be feasible in the near future.

Note added in proof. For an alternative approach to this problem, see N. D. Hari Dass, Ann. Phys. (N.Y.) 107, 337 (1977); Gen. Relativ. Gravit. 8, 89 (1977).

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[1] T. A. Morgan and A. Peres, Phys. Rev. Lett. 9, 79 (1962).

[2] W. T. Ni, Phys. Rev. Lett. 38, 301 (1977).

[3] The strong equivalence principle asserts that in a freely falling, nonrotating laboratory, not only do all free particles move with constant velocities—this is the weak equivalence principle—but *all* the laws of physics are the same in that laboratory, independent of its position in space and time.

[4] In this paper, we made the simple assumption that the gravitational field is the gradient of a scalar field. It is well known that no scalar theory of gravitation can account for the experimental facts. However, mixed scalar-tensor theories [C. H. Brans and R. H. Dicke, Phys. Rev. 124, 925 (1961)] or bimetric theories [N. Rosen, Ann. Phys. (N.Y.) 84, 455 (1974)] cannot be experimentally ruled out. In such theories, in the quasistatic case, it is not difficult to construct scalars analogous to Newton's potential.

[5] The experimental limit on the electric dipole moment of neutrons [W. B. Dress *et al.*, Phys. Rev. D 15, 9 (1977)] implies that for neutrons, $k < 14\,000$. For electrons, the experimental limit on k is much higher.

[6] M. P. Haugan and C. M. Will, Phys. Rev. Lett. 37, 1 (1976).

[7] R. Colella, A. W. Overhauser, and S. A. Werner, Phys. Rev. Lett. 34, 1472 (1975).

[8] L. I. Schiff, Phys. Rev. Lett. 4, 215 (1960). The frequency of the Schiff precession is about $(g/c)(R\omega/c)$ where R is the earth radius and $\omega = 2\pi/\mathrm{day}$.

[9] Besides this torque, there is also a net *force* due to the gradient of $(\vec{S}\cdot\vec{g})$. This is, however, a much smaller effect.

[10] This exactly corresponds to a horizontal shift of the center of gravity by $k\hbar/mc$. We see how the principle of equivalence is violated in the present theory: A magnet suspended in an accelerated laboratory, instead of a gravitational field, would *not* tilt.

[11] L. D. Landau and E. M. Lifshitz, *Electrodynamics of Continuous Media* (Pergamon, Oxford, 1960).