PHYSICAL REVIEW D VOLUME 18, NUMBER 1 1 JULY 1978

# Quark van der Waals forces

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(Received 20 December 1977; revised manuscript received 14 March 1978)

The possibility of long-range forces between quarks which are constituents of different hadrons is considered, in analogy to the van der Waals forces between electrically neutral atoms. The two-virtual-gluon-exchange force, implied by quantum chromodynamics, can be estimated and is uninterestingly small even if there exist rather low-lying color-octet hadrons. If there are van der Waals–type forces associated with a quark-confining potential, then there may arise a conflict with the experimentally well determined one-pion-exchange tail of the nuclear force unless any color-octet baryons lie very high in energy. This condition is not met in the model in which the confining potential arises from the small-$k$ singularity of "dressed" one-gluon exchange.

## I. INTRODUCTION

Experimental and theoretical developments in the last few years have led to the view that the observed hadrons are color-singlet bound states of colored quarks and antiquarks. The quarks and antiquarks are permanently confined. It is further speculated that a non-Abelian gauge theory built on the SU(3) of color is the fundamental dynamical theory (called quantum chromodynamics—QCD); the forces between quarks are mediated by massless-colored-gluon exchange. In the nonrelativistic limit simple one-gluon exchange (OGE) leads to a static interaction potential between two quarks, or between a quark and an antiquark, which falls off as $1/r$, just as in QED. Thus simple one-gluon exchange does not lead to confinement. No one has yet been able to calculate the long-range interaction and demonstrate confinement, starting from QCD. Rather one may proceed phenomenologically and assume that somehow there exists a confining potential, i.e., a potential energy between a color-symmetric $q\overline{q}$ pair, or between the $qq$ pairs in the color-antisymmetric three-quark system, which goes to $+\infty$ as $r\rightarrow\infty$. Because one has not provided a derivation from QCD, one does not know the theoretical domain of validity of the potential picture of long-range quark interactions; for example, it is believed that the confining-potential description may break down at large distance through polarization of the vacuum. Nevertheless, the considerable phenomenological success of the nonrelativistic quark model argues for the validity of a potential picture over distances comparable to the sizes of observed hadrons (say 0.1 to a few F).

In any event, whether one describes the interaction between quarks bound in a hadron by virtual-gluon exchange or by a sum of two-body potentials, the question of the interaction between quarks in different hadrons presents itself. This interaction would be the color analog of the electric dipole-dipole van der Waals interaction between neutral composite systems in atomic and molecular physics. In QED, the interaction between the elementary constituents of atoms (electrons and nuclei) is the Coulomb interaction arising from single-photon exchange. In the static limit, the van der Waals interaction between two neutral composites can be computed in second-order nonrelativistic perturbation theory[1] starting from the Coulomb interaction between pairs of constituents, one from each composite. The result is an interaction energy $\sim\text{const}\times R^{-6}$. For atoms, for distances greater than $137a_0$, retardation effects become important and the leading interaction term is $\sim\text{const}'\times R^{-7}$. This result can also be obtained by a relativistic dispersion relation treatment of the sum of two-photon-exchange Feynman diagrams.[2] In this paper we investigate the conditions, if any, under which the quark van der Waals forces between different color-singlet hadrons are absent, or at least negligible compared to the known one-pion-exchange tail of the observed nucleon-nucleon interaction.

## II. THE TWO-GLUON-EXCHANGE INTERACTION

We first consider the two-virtual-gluon-exchange interaction because it surely exists if QCD is the underlying theory, and because we can fairly reliably estimate it by making simple modifications of the analogous atomic calculations. We prefer to make the required modifications in the static calculation, even though we will see that the retardation effects probably enter for any interesting distance in this problem. Our first reason is that to the required order of perturbation theory QCD differs from QED just by the replacement of a numerical coupling (charge) by a matrix coupling;

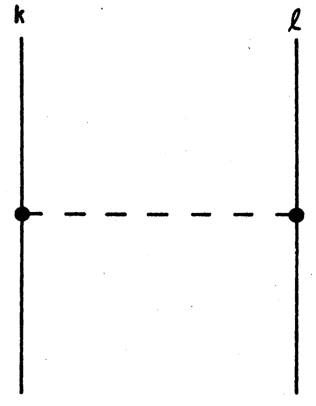


FIG. 1. One-gluon exchange between two quarks.

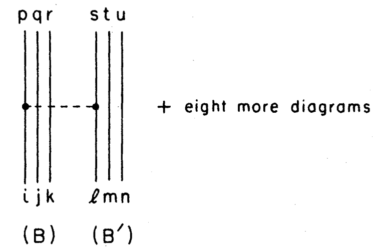


FIG. 2. One-gluon exchange between two baryons.

so the cancellation of leading multipole terms between different pairs is more complicated. We want to demonstrate this cancellation explicitly, which requires a consideration of the quark constituents of the hadrons, which is not included in the dispersion approach to two-photon (two-gluon) exchange. Second, the dispersion approach depends on the threshold behavior of the spectral functions, which is not the same in higher orders of perturbation theory for QCD as for QED. Indeed, one speculation is that confinement in QCD arises from a more singular infrared behavior. In the next section we will consider this speculation; its implementation is simple in the nonrelativistic perturbation approach. Third, since one knows the effect of retardation in two-photon exchange,[1,2] one can simply take it into account at the end by multiplying the static result by $(\Delta E)^{-1}R^{-1}$.

The starting point is the static interaction potential between two quarks, arising from one-gluon exchange (Fig. 1).

$$V_{qq}(r)=\frac{\alpha_s}{r}\left(\frac{\lambda_a}{2}\right)_{ki}\left(\frac{\lambda_a}{2}\right)_{lj}, \tag{2.1}$$

where $i,j,k,l$ are quark color indices and $\alpha_s$ is a coupling constant $(g^2/4\pi)$ estimated to be $0.2<\alpha_s<0.5$ when renormalization is carried out at a mass on the scale of hadron masses. Then one can write the one-gluon-exchange interaction between two baryons (Fig. 2):

$$\begin{aligned}H'=\frac{\alpha_s}{4}\{&(\lambda_a)_{pi}\delta_{qj}\delta_{rk}[(\lambda_a)_{sl}\delta_{tm}\delta_{un}|\vec{R}+\vec{r}_1'-\vec{r}_1|^n+(\lambda_a)_{tm}\delta_{sl}\delta_{un}|\vec{R}+\vec{r}_2'-\vec{r}_1|^n+(\lambda_a)_{un}\delta_{sl}\delta_{tm}|\vec{R}+\vec{r}_3'-\vec{r}_1|^n]\\&+(\lambda_a)_{qj}\delta_{pi}\delta_{rk}[(\lambda_a)_{sl}\delta_{tm}\delta_{un}|\vec{R}+\vec{r}_1'-\vec{r}_2|^n+(\lambda_a)_{tm}\delta_{sl}\delta_{un}|\vec{R}+\vec{r}_2'-\vec{r}_2|^n+(\lambda_a)_{un}\delta_{sl}\delta_{tm}|\vec{R}+\vec{r}_3'-\vec{r}_2|^n]\\&+(\lambda_a)_{rk}\delta_{pi}\delta_{qj}[(\lambda_a)_{sl}\delta_{tm}\delta_{un}|\vec{R}+\vec{r}_1'-\vec{r}_3|^n+(\lambda_a)_{tm}\delta_{sl}\delta_{un}|\vec{R}+\vec{r}_2'-\vec{r}_3|^n+(\lambda_a)_{un}\delta_{sl}\delta_{tm}|\vec{R}+\vec{r}_3'-\vec{r}_3|^n]\}.\end{aligned} \tag{2.2}$$

$R$ is the separation of the centers of mass of the two baryons and $\vec{r}_i, \vec{r}_j'$ are the (c.m.) coordinates of the respective constituent quarks. For simple one-gluon exchange, $n=-1$. We have written the general $n$ here for future reference. For color-singlet baryons, the color wave function is

$$B_{ijk}=(1/\sqrt{6})\epsilon_{ijk}. \tag{2.3}$$

Then

$$\langle BB'|H'|BB'\rangle=\frac{1}{36}\int d\tau_1 d\tau_2 u^*(\underline{r})u'^*(\underline{r}')\epsilon_{pqr}\epsilon_{stu}H'\epsilon_{ijk}\epsilon_{lmn}u(\underline{r})u'(\underline{r}')=0, \tag{2.4}$$

where

$$\underline{r}=\vec{r}_1,\ \vec{r}_2,\ \vec{r}_3 \text{ and } d\tau=d^3r_1d^3r_2d^3r_3\delta(\textstyle\sum\vec{r}_i).$$

The matrix element is zero because all terms are proportional to $[\mathrm{Tr}(\lambda_a)]^2=0$. This is the statement that OGE between two color-singlet baryons vanishes, the analog of no Coulomb force between electrically neutral composites.

To get an interaction energy between two color-singlet baryons, one has to go to second-order perturbation theory (the nonrelativistic version of two-gluon exchange),

$$W(R)=\sum_{\overline{B},\overline{B}'}\frac{|\langle\overline{B}\overline{B}'|H'|BB'\rangle|^2}{E_{BB'}-E_{\overline{B}\overline{B}'}}. \tag{2.5}$$

Because the gluon is a color octet, the intermediate states which contribute are color-octet states[3]:

$$\overline{B}^a_{pqr} = \tfrac{1}{2}\epsilon_{pqk}(\lambda_a)_{rk}. \tag{2.6}$$

Then, in terms of the group-theory constants,

$$\overline{B}^{b*}_{pjk}\lambda^a_{pi}\epsilon_{ijk} = b_1\delta_{ab} = -\delta_{ab}, \tag{2.7a}$$

$$\overline{B}^{b*}_{iqk}\lambda^a_{qj}\epsilon_{ijk} = b_2\delta_{ab} = -\delta_{ab}, \tag{2.7b}$$

$$\overline{B}^{b*}_{ijr}\lambda^a_{rk}\epsilon_{ijk} = b_3\delta_{ab} = 2\delta_{ab}. \tag{2.7c}$$

The matrix element in (2.5) is

$$\begin{aligned}\langle B^b B'^c|H'|BB'\rangle = \delta_{bc}\frac{\alpha_s}{96}\int d\tau\, d\tau'\, \overline{u}^*(\underline{r})\overline{u}'^*(\underline{r}') \\ \times\{b_1[b_1|\vec{R}+\vec{r}_1'-\vec{r}_1|^n + b_2|\vec{R}+\vec{r}_2'-\vec{r}_1|^n + b_3|\vec{R}+\vec{r}_3'-\vec{r}_1|^n] \\ + b_2[b_1|\vec{R}+\vec{r}_1'-\vec{r}_2|^n + b_2|\vec{R}+\vec{r}_2'-\vec{r}_2|^n + b_3|\vec{R}+\vec{r}_3'-\vec{r}_2|^n] \\ + b_3[b_1|\vec{R}+\vec{r}_1'-\vec{r}_3|^n + b_2|\vec{R}+\vec{r}_2'-\vec{r}_3|^n + b_3|\vec{R}+\vec{r}_3'-\vec{r}_3|^n]\}u(\underline{r})u'(\underline{r}').\end{aligned} \tag{2.8}$$

Next make the "multipole" expansion

$$|\vec{R}+\vec{\rho}|^n = R^n + nR^{n-1}\left(\frac{\vec{R}}{R}\right)\cdot\vec{\rho} + \frac{n}{2}R^{n-2}\left[\rho^2 + (n-2)\left(\frac{\vec{R}}{R}\cdot\vec{\rho}\right)^2\right] + \cdots, \tag{2.9}$$

where $\vec{\rho}$ is any of the $\vec{r}_i' - \vec{r}_i$. When this expansion is substituted into (2.8) and the condition $b_1+b_2+b_3=0$ is used, we see that the $R^n$ and $R^{n-1}$ terms both add to zero, so our order-of-magnitude estimate for the matrix element is

$$\langle \overline{B}^b\overline{B}'^c|H'|BB'\rangle \sim \tfrac{1}{10}\,\delta_{bc}\alpha_s a^2 R^{n-2}, \tag{2.10}$$

where $a \sim (\langle r^2\rangle)^{1/2}$ is a length of the order of magnitude of a hadron size. Then our order-of-magnitude estimate[4] for (2.5) is

$$W(R) \sim -\alpha_s^{\,2}\frac{a^4R^{2n-4}}{\Delta E_c}, \tag{2.11}$$

where $\Delta E_c$ is the energy difference between the lightest color-octet hadron state and the ordinary baryons. Again, for simple two-gluon exchange $n=-1$.

For atoms the result const $\times R^{-6}$ is valid for distances $\lesssim 137a_0$. For distances larger than this, retardation effects lead to a const$'\times R^{-7}$ interaction potential. The distance at which retardation effects become important is $R \gtrsim \lambda \sim \Delta E^{-1}$. Since $\Delta E_c$ may be large (infinite?), we will multiply (2.11) by $(\Delta E)^{-1}R^{-1}$ to account for retardation:

$$W(R) \sim -\alpha_s^{\,2}\frac{a^4R^{2n-5}}{(\Delta E_c)^2}. \tag{2.11$'$}$$

Then the order of magnitude of the two-gluon-exchange force between two nucleons is estimated to be

$$F(R) \sim \alpha_s^{\,2}\frac{a^4}{(\Delta E_c)^2}\frac{1}{R^8} \sim \frac{1.5\times10^{-6}}{[\Delta E_c\ (\mathrm{GeV})]^2}\ \mathrm{GeV}^2 \quad (R = 2\ F \simeq 10\ \mathrm{GeV}^{-1}). \tag{2.12}$$

The tail of the one-pion-exchange (OPE) force between nucleons is well known from *NN* phase-shift analyses. Neglecting spin and *i*-spin factors, it is

$$F(R)_{\mathrm{OPE}} \sim f^2\frac{\mu}{R}e^{-\mu R}, \tag{2.13}$$

where

$$\begin{aligned} &f^2 \simeq 0.08, \quad \mu \simeq 0.14\ \mathrm{GeV}, \\ &F(R)_{\mathrm{OPE}} \sim 300\times10^{-6}\ \mathrm{GeV}^2 \quad (R = 2F). \end{aligned} \tag{2.13$'$}$$

Because of the light mass of the pion, the $R^{-8}$ force actually falls faster than the OPE force for distances out to $R \gtrsim 7\mu^{-1} \simeq 10$ F where both forces are negligible (orders of magnitude less than Coulomb). Thus the two-gluon-exchange force between nucleons, implied by QCD, is comfortably smaller than the OPE tail, even without invoking a large $\Delta E_c$.

## III. VAN DER WAALS FORCE ASSOCIATED WITH THE CONFINING POTENTIAL

The situation may be more interesting if there is a van der Waals-type multipole force associated with the long-range quark-confinement potential. Since no one has yet derived the confining potential from QCD, one also can not derive the associated van der Waals forces, if any. But if quarks and gluons are confined, the perturbative two-gluon exchange considered in the previous section cannot be the dominant feature at distances greater than one fermi. We will consider one possible model of the origin of the confining potential, for which we can make a more or less plausible guess for the associated van der Waals force.

The model[5] is based on the speculation that the running coupling constant $\alpha_s(k^2)$ may behave like $k^{-2}$ as $k \to 0$ so that for small $k$ one-gluon exchange is effectively $\alpha_s(k^2)/k^2 \sim \text{const} \times k^{-4}$, whose static Fourier transform leads to the linear potential, $Kr$. The guess for the associated van der Waals force arising from effective two-gluon exchange is just to replace $n=-1$ by $n=+1$ in the formulas of the previous section. Then we have [including the factor $(\Delta E)^{-1}R^{-1}$ for retardation]

$$F(R) \sim K^2\left(\frac{a}{R}\right)^4 \frac{1}{(\Delta E_c)^2} \; . \tag{3.1}$$

$K$ is known from phenomenological fits. It is $K \simeq 0.2$ GeV$^2$. Then

$$F \underset{R=2F}{\simeq} \frac{0.0025}{[\Delta E_c\ (\text{GeV})]^2}\ \text{GeV}^2 , \tag{3.2}$$

which requires $\Delta E_c > 10$ GeV in order not to be in conflict with OPE in $NN$ scattering. But this condition is not met in the model in which the confining potential arises from the small-$k$ singularity of OGE because, in this model, the two-body potential has the same group-theory factors as (2.1), and a simple calculation with the wave functions (2.3) and (2.6) shows that the total potential energy of a color-octet baryon is the same sign and order of magnitude as that of a color-singlet baryon. Then (3.1) and (3.2) are in conflict with our knowledge of the nucleon-nucleon force and provide phenomenological evidence that effective (or "dressed") OGE is not the dominant mechanism for generating the confining potential. (Note that this problem does not arise for mesons. The OGE potential is repulsive for color-octet mesons, i.e., we may take $\Delta E_c = \infty$ for mesons.)

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[1]F. London, Z. Phys. 63, 245 (1930). For an accessible text-book treatment see, e.g., L. I. Schiff, *Quantum Mechanics* (McGraw-Hill, New York, 1968), 3rd ed., p. 259.

[2]G. Feinberg and J. Sucher, Phys. Rev. A 2, 2395 (1970).

[3]There are two possible octet states corresponding to the mixed symmetry Young tableaux (1, 1): symmetric in one pair, or antisymmetric in one pair. Only the latter will contribute to the matrix element in (2.5) because of the total antisymmetry of the color-singlet ground state.

[4]The closure approximation guarantees that the summation in (2.5) converges, L. I. Schiff, Ref. 1.

[5]This basis for a linear confinement potential has been proposed by a number of people. The most detailed elaboration is probably that of J. M. Cornwall, Nucl. Phys. B 128, 75 (1977). Professor Cornwall has informed me that he has also noted the difficulties involved with the van der Waals force associated with a linear confinement potential.