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Radiative $M1$ transitions of the narrow resonances

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We study the radiative transitions among the ψ', $\hat{\chi} = \chi(3.45)$, J/ψ, and $X(2.83)$ states, within the framework of the charmonium model. Numerical results are given for a superposition of a linear vector potential, a linear scalar potential, and an r^{-1} term. We show that the systematic inclusion of relativistic effects leads to discrepancies between experiment and theory which are considerably smaller than those obtained in calculations in which these effects are ignored. In particular, the factor of 2×10^3 between the experimental and theoretical value for $B(\psi'\to\hat{\chi}+\gamma)B(\hat{\chi}\to\psi+\gamma)$ can be reduced to a factor of 20 or so, without the introduction of new parameters. An analysis in which the spin-spin interaction is treated nonperturbatively is also developed.

I. INTRODUCTION

The discovery of the narrow resonances in the 3-to-4-GeV range in the last few years[1] has focused a great deal of theoretical attention on an understanding of the interactions among "heavy" quarks. The viewpoint which has been most vigorously pursued is that these particles are a charmed-quark-antiquark ($c\bar{c}$) system bound nonrelativistically—the charmonium model.[2] The choice of the interaction potential for the $c\bar{c}$ system has been largely guided by the ideas of quantum chromodynamics (QCD). This leads to a long-range linear confining potential, as suggested by lattice gauge theory,[3] to which there is added a short-range Coulomb potential, arising from the exchange of a single color gluon between the quarks,

$$V(r) = ar - \frac{4}{3}\frac{\alpha_s}{r} + b\ . \tag{1}$$

An immediate consequence of the potential (1) is the existence of both orbital excitations, e.g., ^3P_J states, and of $^1S_0$ states—hyperfine partners of the ψ and ψ', interpreted as $^3S_1$ states. Although candidates for ^3P_J and $^1S_0$ states were subsequently discovered,[4,5,6] the level spacings between these states cannot be explained in the "naive" charmonium model, if spin-dependent interactions are of relative order v^2/c^2 as in QED. In a nonrelativistic description one may write $H = H_0 + H_{\rm spin}$ with

$$H_0 = \frac{\vec{p}^2}{m_c} + 2m_c + V(r)\ ,$$
$$H_{\rm spin} = V_{\rm so}(\vec{\sigma}_1+\vec{\sigma}_2)\cdot\vec{l} + V_{\rm ss}\vec{\sigma}_1\cdot\vec{\sigma}_2 + V_t(3\vec{\sigma}_1\cdot\hat{r}\vec{\sigma}_2\cdot\hat{r} - \vec{\sigma}_1\cdot\vec{\sigma}_2)\ , \tag{2}$$

where $V_{\rm so}(r)$, $V_{\rm ss}(r)$, and $V_t(r)$ are spin-orbit, spin-spin, and tensor potentials. If $V(r)$ arises from exchange of color gluons, then at short distances, where the exchange of a single gluon is supposed to yield a good approximation to the potential V, one finds[7]

$$V_{\rm so} = \frac{1}{4m_c^2}\left(\frac{3}{r}\frac{dX_V}{dr} - \frac{1}{r}\frac{dX_S}{dr}\right)\ , \tag{3a}$$

$$V_{\rm ss} = \frac{1}{6m_c^2}\nabla^2 X_V\ , \tag{3b}$$

$$V_t = \frac{1}{12m_c^2}\left(\frac{1}{r}\frac{dX_V}{dr} - \frac{d^2X_V}{dr^2}\right)\ , \tag{3c}$$

where $X_S(r)$ and $X_V(r)$ denote scalar and vector parts of $V(r)$. It has become popular to assume that (3) holds for large r also. If this is done, then with $V(r)$ given by (1), the ratio R_1 of the ^3P_J state splittings has a lower bound[8] of 0.8 while the favored assignment of states[9] gives $R_1 \approx 0.42$. Furthermore, the splitting of the $1S$ state $M(1\,^3S_1) - M(1\,^1S_0)$ is then found to be less than 100 MeV,[10] a factor of 3 or so smaller than the experimental value of 270 MeV, if the $X(2.83)$ (Ref. 6) is to be identified as the $1\,^1S_0$ state.

A variety of suggestions have been made to deal with these discrepancies, for which we refer the reader to recent reviews.[2] Here we shall only note that a satisfactory S-state and P-state level structure can be obtained[11,12] by (i) writing the linear part of V in the form $ar = (1-\lambda)ar + \lambda ar$ and apportioning the $(1-\lambda)ar$ part to a scalar interaction and the λar part to a vector interaction and (ii) ascribing a large "color-magnetic" moment $1+\kappa$ to the quarks.

The level structure clearly gives important information concerning the nature of the binding potential. However, further restrictions can be obtained from a study of the radiative decays.

As stressed in earlier work[13] the radiative $M1$ transitions can provide severe constraints on the binding potential for a $c\bar{c}$ system, especially when the bound states admit a nonrelativistic description. For then there can be "hindered" or "relativistic" transitions, i.e., $M1$ transitions between S states with approximately orthogonal radial wave functions, such as $\chi(3.45)\to\psi+\gamma$ in the naive charmonium model with $\chi(3.45)$ assigned to $2\,{}^1S_0$. As shown in Ref. 13 the amplitude for such a decay is very sensitive to the nature of the Dirac covariants entering a relativistic two-body $c\bar{c}$ interaction. This is in contrast to the $E1$ transition amplitudes which, although much larger than the $M1$ amplitudes, are relatively insensitive to the nature of the potential.[14,15] The purpose of this paper is to study the question of the $M1$ decays in greater depth, in the light of the new and more precise experimental information which has accumulated in the past several years.[4,5,6]

In Sec. II we compute, using the formalism of Ref. 13, the $M1$ decay amplitude for a superposition of scalar and vector linear potentials, with the Coulomb-type interaction included. Motivated by the large ${}^1S_0$-${}^3S_1$ mass difference we study, in Sec. III, the effect of treating the spin-spin interaction $\vec{\sigma}_1\cdot\vec{\sigma}_2 V_{ss}$ nonperturbatively. The results are discussed in Sec. IV.

II. CALCULATION OF $M1$ DECAY RATES

The theory of radiative decays of a bound system of two spin-$\frac{1}{2}$ particles has been reviewed recently.[16] For an $M1$ transition between ${}^3S_1$ and ${}^1S_0$ $c\bar{c}$ bound states, the decay rate is given by

$$\Gamma=\frac{16}{9}\frac{\alpha}{m_c{}^2}\frac{1}{2S_i+1}k^3|I|^2\frac{M_i{}^2+M_f{}^2}{2M_i{}^2}\,, \tag{4}$$

where S_i is the spin of the initial state, k is the momentum of the emitted photon in the c.m. frame, and M_i and M_f are the masses of the initial and the final states, respectively. The quantity I is a radial matrix element which may be written in the form

$$I\approx I_1+I_2+I_3+I_4\,, \tag{5}$$

where the I's are defined as follows:

$$I_1=\left\langle f\left|-j_0\left(\frac{kr}{2}\right)\right| i\right\rangle\approx\left\langle f\left|-1+\frac{k^2r^2}{24}\right|i\right\rangle, \tag{6}$$

$$I_2=\left\langle f\left|\frac{2\vec{p}^2}{3m_c{}^2}\right|i\right\rangle, \tag{7}$$

$$I_3=\pm4\frac{\langle f|V_{ss}|i\rangle}{E_i-E_f}\,, \tag{8}$$

$$I_4=\left\langle f\left|\frac{X_s(r)}{m_c}j_0\left(\frac{kr}{2}\right)\right|i\right\rangle. \tag{9}$$

[In Eq. (8), the upper (lower) sign refers to the case $S_i=0$ ($S_i=1$)]. The term I_1 is the familiar nonrelativistic decay amplitude, with retardation effects included. I_2 is a kinetic term obtained in a relativistic treatment. The amplitude I_3 arises from the effect of the spin-spin interaction on the radial wave functions and is present only when the principal quantum numbers are not the same. Finally, I_4 arises from virtual-pair effects; the vector potential turns out not to contribute to these in the first approximation. The integrations in (6)–(9) are over the radial S-state eigenfunctions of H_0.

We have used two sets of parameters for the potential (1). Eichten *et al.*[14] fixed parameters with $M(2\,{}^3S_1)-M(1\,{}^3S_1)=590$ MeV and $\Gamma(J/\psi\to e^+e^-)=5.3$ keV (1 standard deviation above the measured value) as input; α_s is a semifree parameter, with the constraint that v^2/c^2 turn out to be small. Their preferred choice, which we denote by (I) is

$$\text{(I)}\quad a=0.233\text{ GeV}^2,\quad m_c=1.65\text{ GeV}\qquad \alpha_s=0.10,\quad b=-0.869\text{ GeV}\,. \tag{10}$$

We fixed the four parameters with $M(2\,{}^3S_1)-M(1\,{}^3S_1)=590$ MeV, $\Gamma(J/\psi\to e^+e^-)=4.8$ KeV, and $\Gamma(J/\psi\to\text{hadrons})=57$ keV as input, using the usual QCD formula for $\psi\to$ three gluons. Thus we get a second set (II),

$$\text{(II)}\quad a=0.198\text{ GeV}^2,\quad m_c=1.37\text{ GeV}\qquad \alpha_s=0.202\,,\quad b=-0.195\text{ GeV}\,. \tag{11}$$

We have calculated the decay amplitudes for $M1$ transitions for the following family of potentials. We write V as a sum of a vector part X_V and a scalar part X_S,

$$V=X_V+X_S\,, \tag{12}$$

with

$$X_V=\lambda ar-4\alpha_S/3r+b\,,\quad X_S=(1-\lambda)ar\,. \tag{13}$$

The constant "b" is regarded as contributing only to X_V so that it does not enter the calculation of the decay rates. The total amplitude $I(\lambda)$ is then

$$I(\lambda)=\sum_{i=1}^{4}I_j(\lambda)\,,$$

where the $I_j(\lambda)$ are defined by Eqs. (6)–(9) and the decomposition (13). With the definitions

$$I_j^S=I_j(0)\,,\quad I_j^V=I_j(1)$$

we then have, on noting that both $I_1(\lambda)$ and $I_2(\lambda)$ are independent of λ, the result

$$I(\lambda)=I^S+\lambda(I_3^V-I_3^S-I_4^S)\,. \tag{14}$$

The values of I_j^V and I_j^S are given in Tables I and II, respectively. From these, $I(\lambda)$ may be computed from Eq. (14).

III. NONPERTURBATIVE SPIN-SPIN INTERACTION EFFECTS

We have also pursued the possibility of including the spin-spin interaction as part of the zero-order

H_0'. This is motivated by the mass difference between the $1\,^3S_1$ and $1\,^1S_0$ states, about 270 MeV, which is almost one-half of the 2^3S_1-1^3S_1 splitting, about 600 MeV. Thus we replace H_0 by H_0':

$$H_0' = H_0 + V_{ss}\vec{\sigma}_1 \cdot \vec{\sigma}_2 . \quad (15)$$

If we take Eqs. (3b) and (15) literally, we get

$$V_{ss}(r) = \frac{1}{6m_c^2}\left[\frac{2\lambda a}{r} + \frac{16\pi\alpha_s}{3}\,\delta(\vec{r})\right]. \quad (16)$$

Unfortunately, for the case of an *attractive* three-dimensional δ-function potential, the Schrödinger equation for H_0' is in general not well defined. In particular for the case of a $^1S_0$ state, where $\vec{\sigma}_1 \cdot \vec{\sigma}_2 \to -3$ the spectrum of the Hamiltonian H_0' is not bounded below. Thus (16) is not a useful form for V_{ss} if one wishes to go beyond perturbation theory. To overcome this difficulty, we note that in fact the formula (3) for V_{ss} *cannot* be taken literally for $r \to 0$ even if, on the whole, a nonrelativistic description is approximately valid. To see this we observe that from the viewpoint of a relativsitic description, the spin-spin contact interaction arises from a reduction to Pauli form of the single-gluon-exchange potential

$$V_{\text{gluon}} = -\frac{4\alpha_s}{3}(1 - \vec{\alpha}_1 \cdot \vec{\alpha}_2)/r . \quad (17)$$

The reduced form of the $\vec{\alpha}_1 \cdot \vec{\alpha}_2/r$ part of V_{gluon} is given by

$$(\vec{\alpha}_1 \cdot \vec{\alpha}_2/r)_{\text{red}} = \zeta_1\zeta_2(\vec{\alpha}_1 \cdot \vec{\alpha}_2\, r^{-1}) + \zeta_1(\vec{\alpha}_1 \cdot \vec{\alpha}_2 r^{-1})\zeta_2 + \text{H.c.}, \quad (18)$$

where $\zeta_1 = \vec{\alpha}_1 \cdot \vec{p}/[m_c + E(\vec{p})]$, $\zeta_2 = -\vec{\alpha}_2 \cdot \vec{p}/[m_c + E(\vec{p})]$. *If* one replaces $E(\vec{p}) = (m_c^2 + \vec{p}^2)^{1/2}$ by m_c, the spin-spin contact term emerges when one commutes $\zeta_1\zeta_2$ past r^{-1} in the first term and uses

$$\delta_{ij}\nabla_i\nabla_j r^{-1} = -4\pi\delta(\vec{r}). \quad (19)$$

However, at *short* distances (large momenta) the approximation $E(\vec{p}) \sim m$ becomes invalid. If this fact is taken into account, the δ function is seen to

TABLE I. $M1$ decay amplitudes I and transition rates Γ for a vector linear potential. The quantities I_j are defined by Eqs. (6)–(9) of the text. The symbols X and $\hat{\chi}$ refer to the states at 2.83 and 3.45 GeV, respectively. The columns labelled (I) and (II) refer to the parameter choices (10) and (11) of the text and column (III) shows values obtained when the spin-spin interaction is treated nonperturbatively.

		(I)	(II)	(III)
$\Gamma(\psi \to X+\gamma)$	I_1	−0.987	−0.984	−0.920
	I_2	0.113	0.145	0.137
	I_3	0.000	0.000	0.000
	I_4	0.000	0.000	0.000
	I	−0.874	−0.840	−0.783
	Γ	19 keV	25 keV	15 keV
$\Gamma(\psi' \to \hat{\chi}+\gamma)$	I_1	−0.967	−0.960	−0.865
	I_2	0.184	0.221	0.163
	I_3	0.000	0.000	0.000
	I_4	0.000	0.000	0.000
	I	−0.783	−0.739	−0.701
	Γ	0.10 keV	13 keV	8.2 keV
$\Gamma(\psi' \to X+\gamma)$	I_1	−0.092	−0.106	−0.269
	I_2	0.094	0.121	0.156
	I_3	−0.052	−0.088	0.000
	I_4	0.000	0.000	0.000
	I	−0.050	−0.074	−0.113
	Γ	1.4 keV	4.3 keV	7.0 keV
$\Gamma(\hat{\chi} \to \psi+\gamma)$	I_1	−0.021	−0.024	0.290
	I_2	0.095	0.121	0.063
	I_3	0.124	0.212	0.000
	I_4	0.000	0.000	0.000
	I	0.198	0.309	0.353
	Γ	6.5 keV	23 keV	21 keV

TABLE II. $M1$ decay amplitudes and transition rates for a scalar linear potential. See caption of Table I for definitions of the symbols.

		(I)	(II)	(III)
$\Gamma(\psi \to X+\gamma)$	I_1	−0.987	−0.984	−0.882
	I_2	0.113	0.145	0.135
	I_3	0.000	0.000	0.000
	I_4	0.281	0.306	0.186
	I	−0.593	−0.533	−0.560
	Γ	8.6 keV	10 keV	7.7 keV
$\Gamma(\psi' \to \hat{\chi}+\gamma)$	I_1	−0.967	−0.960	−0.817
	I_2	0.184	0.221	0.147
	I_3	0.000	0.000	0.000
	I_4	0.496	0.555	0.383
	I	−0.287	−0.185	−0.287
	Γ	1.4 keV	0.8 keV	1.4 keV
$\Gamma(\psi' \to X+\gamma)$	I_1	−0.092	−0.106	−0.287
	I_2	0.094	0.121	0.159
	I_3	−0.019	−0.050	0.000
	I_4	−0.064	−0.061	0.004
	I	−0.081	−0.097	−0.124
	Γ	3.6 keV	7.4 keV	8.4 keV
$\Gamma(\hat{\chi} \to \psi+\gamma)$	I_1	−0.021	−0.024	0.365
	I_2	0.095	0.121	0.050
	I_3	0.046	0.121	0.000
	I_4	−0.109	−0.119	−0.197
	I	0.011	0.099	0.217
	Γ	0.02 keV	2.4 keV	7.8 keV

be simply an approximation which can and must be abandoned when the chips are down. The exact coefficient of the $\vec{\sigma}_1 \cdot \vec{\sigma}_2$ term contained in (18) is a nonlocal operator. A rough equivalent to it can be obtained by noting that in momentum space, the relevant integral to be studied for an S state is

$$\frac{p^2}{[E(\vec{p})+m]^2} \int \frac{d\vec{p}'}{(\vec{p}-\vec{p}')^2} \tilde{\psi}(\vec{p}') . \tag{20}$$

If we neglect $\vec{p}'$ relative to $\vec{p}$ this is proportional to $[E(\vec{p})+m]^{-2}\psi(0) \sim E^{-2}(\vec{p})\psi(0)$ for *large p*. Thus we are led to consider instead of

$$\delta(\vec{r}) = \int \frac{d\vec{p}}{(2\pi)^3} e^{i\vec{p}\cdot\vec{r}} , \tag{21}$$

the quantity

$$D(\vec{r}) = \int \frac{d\vec{p}}{(2\pi)^3} \frac{m^2}{E^2(\vec{p})} e^{i\vec{p}\cdot\vec{r}} = \frac{m^2}{4\pi} \frac{e^{-mr}}{r} . \tag{22}$$

This suggests that in order to study higher-order effects of spin-spin interactions we replace $\delta(\vec{r})$ in (16) by

$$D_0(r) = \frac{1}{4\pi r_0^2} \frac{e^{-r/r_0}}{r} , \tag{23}$$

where r_0 has the dimensions of a length. With this replacement for $\delta(\vec{r})$, the Hamiltonian becomes

$$H_0'' = 2m_c + \frac{p^2}{m_c} + V(r) + \frac{1}{6m_c^2} \left[\frac{2\lambda a}{r} + \frac{16\pi\alpha_s}{3} D_0(r) \right] \vec{\sigma}_1 \cdot \vec{\sigma}_2 . \tag{24}$$

We have solved the Schrödinger equation numerically for the Hamiltonian H_0'' using a variational method. Since our main purpose is to see what the qualitative effects of treating the spin-spin interaction nonperturbatively are, we have confined our attention to the cases $\lambda = 1$ and $\lambda = 0$. For the radial wave function we take a polynomial times an exponential with the correct asymptotic behavior. It turns out that the spectrum of the triplet states is extremely insensitive to the value of r_0 and the variational procedure converges very quickly. On the other hand, the singlet states are sensitive to r_0 and the convergence is slow. We needed a trial function with a polynomial of degree 24 to get a hyperfine splitting as large as 270 MeV. But other physical quantities such as the mass of $\hat{\chi} \equiv \chi(3.45)$ and decay amplitudes (5)–(9) are not sensitive to r_0. The parameters a, b, α_s and m_c were determined again from the triplet-state data $M(\psi)$, $M(\psi')$, $\Gamma(\psi \to l\bar{l})$, and $\Gamma(\psi \to \text{hadrons})$. The value of r_0 was fixed by using the mass of the $X(2.83)$ as input.

The quantity λ enters explicitly in the Hamiltonian H_0'' and the same input gives slightly different parameters depending upon the precise value of λ. With a trial wave function which has a polynomial of degree 36 we find, on denoting the case $\lambda = 1$ by $(\text{III})_V$ and $\lambda = 0$ by $(\text{III})_S$, the results

$$\begin{aligned}
(\text{III})_V\text{:}\quad & a = 0.219\ \text{GeV}^2, \quad m_c = 1.64\ \text{GeV} \\
& \alpha_s = 0.200, \quad b = -0.763\ \text{GeV}, \quad r_0 = 0.036\ \text{GeV}^{-1} \\
(\text{III})_S\text{:}\quad & a = 0.215\ \text{GeV}^2, \quad m_c = 1.65\ \text{GeV} \\
& \alpha_s = 0.200, \quad b = -0.753\ \text{GeV}, \quad r_0 = 0.018\ \text{GeV}^{-1} .
\end{aligned} \tag{25}$$

The $M1$ decay amplitudes and the transition rates in these cases are shown in the last column of Tables I and II.

Note that the value of r_0^{-1} needed to get a fit to the hyperfine splitting exceeds m_c by more than an order of magnitude; in view of the simplicity of our ansatz, not too much physical significance should be attached to the precise value of the cutoff r_0^{-1}. We now turn to a comparison of the results of this and the preceding section with experimental data.

IV. DISCUSSION

A. Summary of numerical results

An inspection of Tables I and II reveals the following features:

(i) *Relativistic corrections:* These are always appreciable. For the $M1$ transitions between the states with the same radial quantum number ("favored" transitions) the correction terms are always of opposite sign to the nonrelativistic terms. As a result we find smaller $M1$ decay rates than those obtained by other authors[14,17] who considered only the I_1 terms. For transitions between states with different radial quantum numbers ("hindered" transitions) the smallness of the overlap integral I_1 makes the correction terms very important and often the latter dominate over the former.

(ii) *Nonperturbative spin-spin interactions:* There is substantial overlap between the states with different radial quantum numbers and less overlap when the quantum numbers are the same. This tends to produce smaller decay rates for favored transitions and larger values for the hindered transitions.

(iii) *Parameter dependence:* Only the rate for the hindered transition $\hat{\chi} \to \psi + \gamma$ is sensitive to the choice (I) or (II) of the parameters a, m_c, and α_s. In the pure vector case the rates differ by a factor of 4, whereas in the pure scalar case they differ by a factor of 100, as a consequence of a small value for I_3 and cancellations among other terms.

(iv) *Nature of confining potential:* A comparison of Tables I and II shows that the rate for the favored transition $\psi' \to \hat{\chi} + \gamma$ and the hindered transition $\hat{\chi} \to \psi + \gamma$ are both sensitive to the relative

TABLE III. Comparison of experimental upper bounds for certain radiative decay modes of ψ and ψ' with theoretical values. See caption of Table I for explanation of the choices (I), (II), and (III).

	Experimental data	Vector linear potential (I)	(II)	(III)	Scalar linear potential (I)	(II)	(III)
$B(\psi \to X\gamma)$	$< 1.7 \times 10^{-2}$ [a]	28×10^{-2}	37×10^{-2}	22×10^{-2}	13×10^{-2}	15×10^{-2}	12×10^{-2}
$B(\psi' \to \hat{\chi}\gamma)$	$< 2.5 \times 10^{-2}$ [a]	4.4×10^{-2}	5.7×10^{-2}	3.6×10^{-2}	0.61×10^{-2}	0.35×10^{-2}	0.61×10^{-2}
$B(\psi' \to X\gamma)$	$< 1 \times 10^{-2}$ [a]	0.6×10^{-2}	1.9×10^{-2}	3.1×10^{-2}	1.6×10^{-2}	3.2×10^{-2}	3.7×10^{-2}

[a]See Ref. 20.

amounts of linear scalar versus linear vector parts of the potential V, varying by a factor of 10 in going from $\lambda = 0$ (pure scalar) to $\lambda = 1$ (pure vector). The dependence of the rates on λ is monotonic in the interval $0 \leq \lambda \leq 1$, as can be ascertained from use of Eq. (14). [The difference between $\Gamma(\hat{\chi} \to \psi + \gamma)$ for $\lambda = 0$ and $\lambda = 1$ is a factor of 300, for the parameter choice (I); however, this is an artifact of the cancellation mentioned above.]

B. Comparison with experiment

The evidence for $X = X(2.83)$ and $\hat{\chi} = \chi(3.45)$ comes mainly from the three-photon decay of the ψ,[6,18] and the two-photon cascade process $\psi' \to \psi\gamma\gamma$, respectively.[19] Further relevant information comes from upper bounds on certain branching ratios and products of branching ratios.[18,20]

In Table III we compare the experimental upper bounds for the branching ratios $B(\psi \to X\gamma)$, $B(\psi' \to \hat{\chi}\gamma)$ and $B(\psi' \to X\gamma)$ with calculated widths, by giving the values of the ratios $\Gamma_{th}(\psi \to X\gamma)/\Gamma_{exp}(\psi \to \text{all})$, etc. As can be seen, the calculated values for the last two branching ratios are all roughly compatible with the present upper bounds. In the case $\psi \to X\gamma$, the calculated values are still an order of magnitude or so larger than the experimental upper bound; however, the inclusion of relativistic effects has decreased the discrepancy.

In Table IV we list, in the second column, experimental upper bounds for $B(\psi' \to X\gamma)B(X \to \gamma\gamma)$ and $B(\psi' \to \hat{\chi}\gamma)B(\hat{\chi} \to \gamma\gamma)$ together with the reported values for $B(\psi \to X\gamma)B(X \to \gamma\gamma)$ and $B(\psi \to \hat{\chi}\gamma)B(\hat{\chi} \to \psi\gamma)$.

To compare these with theoretical values we need not only the values for radiative widths but also theoretical estimates for the two-photon width and the hadronic width of both X and $\hat{\chi}$. As usual, we use the QCD formula

$$\Gamma(^1S_0 \to \text{hadrons}) = \frac{8\alpha_s^2}{3M^2}|R(0)|^2 \tag{26}$$

for the hadronic widths and the analogous QED formula for the two-photon widths. The resulting values are shown in the remaining columns of Table IV. As can be seen, the theoretical values for the three-photon decays of ψ' are compatible with the present upper bounds. For the product $B(\psi \to X\gamma)B(X \to \gamma\gamma)$ the cases (II) and (III) lead to values more or less comparable with the experimental value; the larger value for case (I) is a consequence of the small associated value of $\alpha_s \sim 0.1$.

Perhaps the most interesting consequence of our calculations emerges when we consider the last line in Table IV, for the product $B(\psi' \to \hat{\chi}\gamma)B(\hat{\chi} \to \psi\gamma)$. Previous analyses[2] of this had led to the conclusion that the use of a QCD formula for the denominator in the branching ratio $B(\hat{\chi} \to \psi\gamma) = \Gamma(\hat{\chi} \to \psi\gamma)/\Gamma(\hat{\chi} \to \text{hadrons})$ leads to a value for the product which is smaller than the experimental one by a factor of order 2×10^3. From Table IV, we see that for the case of a vector linear potential and for either the Cornell choice (I) or our choice (II) of the parameters a, m_c, and α_s. the discrepancy factor is "only" a factor of 20 or so. The reduction of the discrepancy by almost two orders of magnitude is primarily a consequence of taking into account simultaneously the relativistic corrections for the hindered decay $\chi(3.45) \to \psi + \gamma$ and the

TABLE IV. Comparison of experimental bounds and values for branching-ratio products involving ψ, ψ', X, and $\hat{\chi}$ with theoretical values. See caption of Table I for explanation of the choices (I), (II), and (III).

	Experimental data	Vector linear potential (I)	(II)	(III)	Scalar linear potential (I)	(II)	(III)
$B(\psi' \to X\gamma)\ B(X \to \gamma\gamma)$	$< 3.4 \times 10^{-4}$ [a]	2.9×10^{-5}	2.2×10^{-5}	3.7×10^{-5}	7.7×10^{-5}	3.7×10^{-5}	4.4×10^{-5}
$B(\psi' \to \hat{\chi}\gamma)\ B(\hat{\chi} \to \gamma\gamma)$	$< 3.1 \times 10^{-4}$ [a]	2.1×10^{-4}	6.6×10^{-5}	4.2×10^{-5}	2.9×10^{-5}	0.4×10^{-5}	0.7×10^{-5}
$B(\psi \to X\gamma)\ B(X \to \gamma\gamma)$	$(1.2 \pm 0.5) \times 10^{-4}$ [b]	14×10^{-4}	4.3×10^{-4}	2.6×10^{-4}	6.3×10^{-4}	1.7×10^{-4}	1.4×10^{-4}
$B(\psi' \to \hat{\chi}\gamma)\ B(\hat{\chi} \to \psi\gamma)$	$(8 \pm 4) \times 10^{-3}$ [c]	2.7×10^{-4}	3.5×10^{-4}	0.1×10^{-4}	1.1×10^{-7}	22×10^{-7}	3.5×10^{-7}

[a]See Ref. 18. [b]See Ref. 6. [c]See Ref. 19.

r^{-1} term in the potential V_{ss}.

Although the picture that emerges from our study is not as gray (black?) as that which has been painted previously, it is still far from rosy. It should be emphasized that the potential [(12) and (3)] used in this exploratory analysis does not reproduce the $^1S_0$ or ^3P_J spectrum; we have simply used the experimental masses in the computation of the rates. Further study of $M1$ rates, using a potential which fits the spectrum and which is used consistently in the calculation of the effects involving virtual pairs would seem to be very worthwhile.

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